An argument based on the content fit among a college course, the American College Testing Program (ACT) Assessment tests, and students' high school course work is described to justify use of ACT scores and self-reported high school grades for placement of college freshmen in undergraduate remedial education. A utility-based approach to quantifying the effectiveness of placement rules is described; and its relationship to traditional predictive validity statistics, such as the multiple correlation and standard error of estimate, is explained. An example that is based on the ACT scores, self-reported high school grades, and freshmen English course grades of 5,609 students is presented. Results indicate that a sound argument for the validity of a placement rule defined in terms of ACT Assessment scores can be based on the fit between the skills measured by the test battery and the skills required for success in a course. Demonstrating statistical relationships between test scores and performance in the course does not by itself provide a logical justification for the placement system, although it lends credibility to an argument based on content fit. Statistical decision theory provides a means by which an institution can evaluate the benefits and costs of a placement system. Utility-based statistics, developed in the context of a decision model, provide more appropriate information on the practical effectiveness of a placement test than do traditional validity statistics. Seven data tables and three graphs are included. (TJH)
Validating the Use of ACT Assessment Scores and High School Grades for Remedial Course Placement in College

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August 1989
VALIDATING THE USE OF ACT ASSESSMENT SCORES AND HIGH SCHOOL GRADES
FOR REMEDIAL COURSE PLACEMENT IN COLLEGE

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The author thanks Michael Kane for his advice on the issue discussed in this paper, and Mark Houston, Jim Maxey, and Julie Noble for their comments on earlier drafts.
ABSTRACT

An important use of ACT Assessment test scores and self-reported high school grades is college freshman course placement. An argument, based on the content fit between a college course, the ACT Assessment tests, and students' high school course work, is described for justifying this use at particular institutions. A utility-based approach to quantifying the effectiveness of placement rules is then described, and its relationship to traditional predictive validity statistics, such as the multiple correlation and the standard error of estimate, is explained.
A typical and important use of ACT Assessment data is college freshman course placement, i.e., the matching of students with appropriate instruction. For example, students with low probability of success in a standard freshman English course might, on the basis of their ACT test scores and self-reported high school grades, be advised or required to enroll in a remedial English course. On the other hand, students with high probability of success in an accelerated English course might be encouraged to enroll in it. This paper considers the problem of validating placement procedures, that is, of determining their educational appropriateness.

Cronbach (1988) argued that validating a test use is really evaluation, and must not only address the particular educational functions for which the test was designed, but must also consider the broader educational, political, and social consequences of using the test from the perspectives of different value systems. By this standard, validation is an enormous and never-ending task. This paper is concerned only with the traditional issue in Cronbach's agenda, namely determining whether a test score serves the particular educational function it was designed to serve (e.g., course placement), and whether it does so in an economical way. For a discussion of procedures by which other aspects of placement systems can be evaluated, see Frisbie (1982).

An impressive theoretical methodology, based on statistical decision theory, has been developed during the past twenty-five years for determining the effectiveness of selection systems. A principal goal of this paper is to interpret the decision theory methods developed by Cronbach and Gleser (1965) and by Petersen and Novick (1976) in the context of recent ideas about validation (Kane, 1987). Further, their decision models are extended to one
that more easily addresses the concerns of an institution in measuring the effectiveness of its placement system. Methodological issues in quantifying system effectiveness are then examined, and are illustrated by an example.

Remediation

Many different techniques exist for matching college students with appropriate instruction. Willingham (1974) described in detail and classified various placement techniques that are based on test scores and other measures of academic ability. This paper focuses on only one particular placement function, remediation.

At many postsecondary institutions, there are two levels of freshman courses: a standard level course in which most freshmen enroll, and a lower level course for students who are not academically prepared for the standard course. (At some institutions, there are "developmental" or "review" courses in addition to the remedial course; in this paper, we consider only the single remedial lower-level course). In Willingham's classification of placement procedures, this instructional treatment is called "remediation" (Model 4).

A placement device, such as a test, is but one component of a placement system. To be educationally effective, the placement system must have at least all the following characteristics:

a. There must be some way to identify accurately those students who have a small chance of succeeding in the standard course.

b. Appropriate remediation must be provided to the high risk students.

c. Both the students who originally enrolled in the standard course, and the students who were provided remediation, must do satisfactory work in the standard course.
Note that merely accurately identifying high risk students is insufficient for the placement system as a whole to be effective. If these high risk students eventually drop out or fail in the standard course, no useful purpose will have been served by the placement system; on the contrary, both the institution's and the students' resources will have been wasted. The validation strategies described here address issues a. and c.

**Types of Placement Selection Rules**

An institution's placement rule is assumed to have the following general form: If a student is predicted to have a small chance of succeeding in the standard freshman course, then the student is selected for the remedial course. Thus, the strategy is to reduce the chances of failure by identifying students who are at high risk, then to offer them instruction at an appropriate lower level.

Placement is often based on a single test score related to a course. For example, placement in freshman English may be determined only from students' scores on the ACT English Usage test. A critical score level for selection is determined either by reviewing the contents of the test and college course, or by analyzing the statistical relationship between test score and course grade.

Single score cutoffs are also frequently determined on the basis of local normative information, the goal being that a predetermined number or percentage of students automatically be selected for each level. This has obvious administrative advantages in sectioning courses, and may be effective in correctly placing students, if the academic skills of students selected for the different levels are appropriate. To ensure a match between students' skills and course requirements, however, the contents of the courses into which students are placed might need to be adjusted.
ACT recommends that placement rules based on ACT Assessment test scores and high school grades be expressed in terms of grade predictions:

\[ \hat{Y} = a_0 + a_1 \times \text{ACT English Usage score} + a_2 \times \text{ACT Mathematics Usage score} + a_3 \times \text{ACT Social Studies Reading score} + a_4 \times \text{ACT Natural Sciences Reading score} + a_5 \times \text{self-reported high school English grade} + a_6 \times \text{self-reported high school mathematics grade} + a_7 \times \text{self-reported high school social studies grade} + a_8 \times \text{self-reported high school natural science grade} \]

In this equation, \( \hat{Y} \) is the predicted grade in a specific freshman course, and \( a_0, a_1, \ldots, a_8 \) are regression coefficients estimated from students' ACT Assessment data and from course grades provided by the institution. The advantage of using predictions based on several variables for placement is that the predictions are potentially more accurate, and therefore more likely to result in correct placement decisions. The opportunity to develop prediction equations is routinely provided through ACT's Standard Research Service (ACT, 1988). ACT will also, at the request of institutions, calculate the predicted grades of future applicants in terms of their grade expectancies (chances of earning a given grade or higher) and print the expectancies on the applicants' score reports. For a discussion of how ACT calculates grade expectancies, see *Your College Freshmen* (ACT, 1981). For information on the technical characteristics of ACT scores and self-reported high school grades, see the *ACT Assessment Program Technical Manual* (ACT, 1987).

Some institutions select for an accelerated course or for advanced placement those students who have a very high chance of success in the standard level course. This procedure may work satisfactorily in practice,
but it is not as directly linked to the placement goal as is the procedure described previously. The reason is that students who are predicted to do well in the standard level course could, nonetheless, be ill-prepared for the advanced course; this could occur if the skills required for success in the accelerated or advanced course were not measured by the ACT Assessment, and if they differed significantly from those required in the standard course. The practical advantage of this alternative placement method is that it requires developing only one prediction equation for all three course levels (remedial, standard, accelerated/advanced), rather than two prediction equations.

Validating Placement Rules Based on ACT Data

In validating tests for use in college admissions and placement, researchers have traditionally emphasized documenting time-ordered statistical relationships between test scores and relevant criteria. Typically, this documentation has consisted of correlation coefficients and associated tests of statistical significance. To the extent that the usefulness of a test depends on the existence of statistical relationships, such evidence is relevant to validation. There are advantages however, to moving beyond simple documentation of validity statistics to a more theoretically oriented validation strategy. In the more theoretical approach to validity, the use of a test for a particular purpose is seen as the logical conclusion of a set of assumptions which need to be justified; the statistical analyses usually thought of as constituting the validation become the means by which some of the assumptions can be justified, either directly or indirectly. Such a procedure can be thought of as applying the scientific method to test validation. Angoff (1988, p. 30), in summarizing the trend during the past three decades to think of validation this way, stated, "...as it became clear
that it was not the test, but the inferences drawn from the test scores that were on trial, it also followed that the theory that dictated the inferences was also on trial."

Purely empirical validation through extended replication and variation can, within a limited sphere, make certain claims credible and certain predictions safe to use---much of early human knowledge, for example, probably developed atheoretically from millenia of trial-and-error validation. A theoretical approach to validity, however, offers the possibility of understanding why empirical relationships exist, of making educated guesses about what limitations there may be in extrapolating them to different situations, and even of figuring out how new tools might be devised that are more effective in accomplishing our goals.

Kane (1987) proposed a paradigm for validating uses of test scores that is theoretically oriented. In Kane's approach to validation, one first states as clearly as possible the particular use being made of test scores, and the logical chain of assumptions by which the use can be justified. Next, one examines the plausibility of each of the assumptions. One then investigates more thoroughly those assumptions which are least plausible, based on available evidence; usually, this will involve collecting and analyzing relevant data. The final step is to review the overall plausibility of the logical chain of inferences, and to determine how the plausibility can be enhanced, either by modifying the test or the use made of the test. Although Kane proposed this method in the context of professional licensure and certification, it is easily transferred to other contexts, including course placement.

Clearly, Kane's approach to validation would require a different validity argument for different tests or different uses of the same test; presumably,
different arguments could also be made for the same test and the same use. Following is a simple argument for using ACT test scores and self-reported high school grades to identify high risk students. It is based on two assumptions:

1. The academic skills a student acquires from taking a college course are directly related to, among other things, the academic skills the student has previously acquired. Moreover, there are minimum skills the student must bring to the course before he or she can be expected to derive much benefit. The skills required are particular to each college course: they may overlap to some extent (e.g., reading skills are necessary for most courses), but also have unique elements (e.g., knowledge of analytic geometry is a prerequisite for calculus, but not for any English course).

2. ACT test scores and self-reported high school grades provide either direct measurements or indirect indicators of the required skills.

Note that no claim is made that prior educational achievement is the only determinant of student performance in college, or the only practical basis for placing students. Other student characteristics could also be important (or conceivably, more important) and could be included in the validity argument by making additional assumptions. The simple argument described here is a foundation on which to construct a justification for using achievement-oriented tests for placement. A different argument would, of course, be required for using aptitude-oriented tests for placement.

Plausibility of Assumption 1

It is difficult to conceive of any college level course in English, mathematics, social studies, or natural sciences for which the first assumption is not true; indeed, the structure of all educational systems seems
to take this as a given. If, though, a college-level course of the type we are considering did not require previously acquired academic skills, then there would be no need for placement. More typically, the need for a placement system results from a practically significant number of students not possessing these skills.

**Plausibility of Assumption 2 (ACT test scores)**

Before using ACT test scores for placement, college staff should review the contents of the tests (ACT, 1986) to determine their relationship with those of the standard college course. Following is a general discussion of the contents of the ACT tests and their relationship to typical college courses.

The ACT Assessment cognitive tests were designed to measure directly the academic skills needed to do college-level work in English, mathematics, social studies, and natural sciences. The tests are oriented toward the content of secondary and postsecondary programs in these fields, rather than toward a factorial definition of various dimensions of intelligence, and are intended to have a direct and obvious relationship to students' academic development. The tasks included in the tests are representative of academic skills typically taught in high schools and needed for postsecondary work; they are intended to be comprehensive in scope and educationally significant, rather than narrow or artificial. They rely partly on students' reasoning abilities and partly on their knowledge of the subject matter fields, and emphasize the use of reasoning skills in conjunction with knowledge of subject matter.

It is unlikely that the ACT Assessment (or any other battery of multiple choice tests with similar content, length, and breadth) measures all the academic skills required for a particular college course. It is likely,
though, that for many freshman courses the ACT Assessment directly measures many of the important required skills. Because better prepared students will probably learn more in a college course than less well prepared students (Assumption 1), one can reasonably expect that ACT test scores would be statistically related to students' performance in such courses.

In addition, some college courses may require skills that are not directly measured by the ACT Assessment, but that are closely related to skills the ACT Assessment does measure. A good example of this is writing skills, which are obviously necessary for many college courses. The ACT Assessment does not provide a direct measurement of students' writing skills, as for example, from a writing sample. The ACT English Usage test does, however, measure students' skills in editing, which are closely related, both conceptually and statistically, to their writing skills.

**Plausibility of Assumption 2 (High School Grades)**

High school grades are another traditional measure of students' readiness for college level study. To use them for placement, college staff should, at least in principle, review the contents of individual courses at particular high schools (as they would the contents of ACT tests) to determine their relevance to the college course. In practice, this is not usually feasible; but, if the assumed contents of the high school courses required for admission to the institution have a plausible relationship to the college course, then one may reasonably assume that students' grades in these courses will be related to their readiness for the course.

High school grades probably also measure students' socialization, motivation, work habits, and study skills, as well as their academic skills and knowledge. In the simple validity argument based solely on prior achievement, these other factors are irrelevant. On the other hand, the amount students learn in a college course may well be related to these
factors, and high school grades may provide a broader perspective on the likely benefit to students in taking the course. In this case, the validity argument could be expanded to take into account factors other than achievement.

In the ACT Assessment, high school grades are self-reported. Sawyer, Laing, and Houston (1988) compared the grades reported by students in 30 standard high school courses with corresponding grades from the students' transcripts. They found that in typical courses, 71% of students reported their grades exactly; and that 97% reported their grades accurately to within 1 grade unit. Moreover, specific course grade predictions based on self-reported high school grades are almost as accurate as predictions based on ACT test scores; and predictions based on both high school grades and test scores combined are more accurate than predictions based on either alone (Noble and Sawyer, 1987). One can conclude that some individuals' self-reported grades may be of doubtful accuracy, but that for most students, self-reported grades are useful in placement.

**Selection Rules Based on Grade Predictions---An Additional Assumption**

Determining the content "fit" among ACT test scores, high school grades, and a particular course at a given postsecondary institution must, of course, be done by individuals who know the course content at the institution. If the fit of ACT tests and high school courses to the college course is good, it is reasonable to expect that students with higher ACT test scores and high school grades will outperform students with lower test scores and high school grades. It is therefore appropriate to consider using these two kinds of measures for course placement.

Practical implementation of a course placement procedure requires that some decision rule be formulated in terms of critical values of the test.
scores and high school grades. In principle, one could determine the critical values on the basis of expert judgment about test content and high school curriculum, and about their overlap with the content of specific college courses. Jaeger (1989) provides an overview of standard setting procedures based on judgments. Making such judgments would likely be very difficult, especially if the placement rule were based on more than one variable.

The easiest and most common way to implement a placement decision rule based on multiple measures of ability is through a prediction equation for course grades, such as Equation (1). Placement decisions are made on the basis of a critical value, either for the predicted grade or for its transformation to a grade expectancy. Moreover, measures of the strength of the statistical relationships between placement variables and course grades are routinely provided by the software that calculates prediction equations; these summary measures provide additional support of the plausibility of Assumption 2. The appropriateness of grade predictions, however, depends on an additional assumption:

3. The course grades being predicted are valid measures of the academic skills learned in the course, rather than measures of random or irrelevant factors.

If this assumption is not met, then it would be inappropriate to base placement decisions on grade predictions, and data on the statistical relationship between placement variables and course grades would be irrelevant to the validity argument. Of course, if this assumption is not true, then an institution has a much more serious problem than validating placement rules!

Lesser reliability in the course grade will (other things being equal) always result in lesser prediction accuracy, as indicated by smaller multiple correlations and larger standard errors of estimate. Assuming that one has
determined that the predictor variables are related to course content, then a reliable but invalid course grade would also generally result in smaller multiple correlations and larger standard errors.

One could also hypothesize a situation in which the predictors and a highly reliable course grade were all unrelated to mastery of course content, but were related to the same irrelevant factor. In this situation, course grade prediction equations could have respectable multiple correlations and standard errors, but still be inappropriate for use in placement.

Following is an example of potential "shared invalidity". Both high school grades and college course grades are ratings by individual instructors, and both may be influenced by factors other than their students' academic accomplishments. For example, some instructors may be more likely to give higher grades to students who attend every class, who turn in assignments punctually, or who are courteous, well-groomed, and appear to be interested in what the instructors have to say, than they are to students who are not so well socialized. Such grading practices may or may not be appropriate, depending on the values of the instructor and the policies of the high school and college, but to the extent that they do occur, the statistical relationship between high school grades and college course grades will be less relevant to validating a placement rule based on educational achievement.

The reliability and validity of course grades must, of course, be determined at each individual institution. How to make this determination is a difficult psychometric problem, and is beyond the scope of this study. The few published results available are briefly summarized here.

Etaugh, Etaugh, and Hurd (1972) obtained an estimated single course grade reliability of .44 using the single course grades for each student as repeated measurements. Schoenfeldt and Brush (1975) adapted the procedure of Etaugh,
et al. to estimate reliabilities in 12 different course areas; their estimates ranged from .39 to .76. Although both estimation procedures may be open to question, the results suggest that specific course grades are not as reliable as ACT test scores, for which parallel form reliabilities approaching .9 are typically obtained (ACT, 1987). Assuming a reliability of .9 for ACT tests and .4 for specific course grades, one cannot expect correlations between single ACT test scores and course grades to exceed .6, since

\[ r_{xy} \leq \sqrt{r_{xx} \cdot r_{yy}}. \]

Duke (1983) found that the distribution of grades awarded at a particular university varied markedly by department, and that students who earned high grades in departments with easy grading earned average or low grades in other departments with stringent grading. He concluded that GPA is made up of components that are not equivalent among students. His findings also suggest that grades in some departments measure, to a significant extent, characteristics other than students' academic achievement. Although Duke investigated the grading practices of only one institution, he believed his results were typical of those in higher education generally. I shall assume, for the purpose of this discussion, that the course grade being predicted measures academic achievement, and is therefore relevant to the goals of the placement function. Duke's results indicate, though, that this assumption needs to be justified in particular applications.

A Decision Model for Validating Placement Rules

Given that the contents of ACT tests and high school courses are reasonably (but less than perfectly) congruent with the skills required in the college course, and that the course grade is reasonably (but less than perfectly) reliable and valid, one can expect there to be a (less than perfect) statistical relationship between ACT test scores, high school grades,
and college course grades. This expectation can be tested against data collected by the institution, and if not borne out, would lead one to reconsider the plausibility of the assumptions in the validity argument.

But, assuming that the expectation of a statistical relationship is borne out, how does one quantify the usefulness of placement decisions based on the grade predictions made possible by this relationship? The answer, in general, is that the validity argument must be augmented with additional assumptions about the benefits of student achievement and the costs of providing instruction. These assumptions would need to address all the important outcomes of the placement system, such as the performance of students who are placed in the remedial course, as well as that of students who are placed in the standard course. These assumptions can then be related to the statistical relationships estimated from the data to produce a summary of the usefulness of the placement system as a whole. Note that at this point we are making inferences about an entire placement system, of which the placement test is but one component.

Statistical decision theory has been proposed by several writers, including Cronbach and Gleser (1965) and Petersen and Novick (1976), as a useful means for analyzing educational selection problems. Validating placement systems, in their full generality, through decision theory is a complex undertaking. To structure the discussion, let us first consider the requirement that a placement system accurately identify high risk students.

Suppose we could collect test scores, high school grades, and course grades for a representative sample of students who have all taken the standard course without any prior remediation. Some of these students would be successful, as measured by their course grades; others, presumably, would be
unsuccessful. The students' predicted and actual grades could then be compared and a numerical value could be assigned to each outcome.

Table 1
Outcomes Among a Group of Students
Who Take the Standard Course

<table>
<thead>
<tr>
<th>Performance in standard course</th>
<th>Predicted grade</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Below critical value</td>
</tr>
<tr>
<td>Successful</td>
<td>D</td>
</tr>
<tr>
<td>Unsuccessful</td>
<td>C</td>
</tr>
</tbody>
</table>

An example of a simple decision model is given in Table 1, in which there are four possible outcomes. Outcome A is called a "true positive"; outcome B, a "false positive"; outcome C, a "true negative"; and outcome D, a "false negative". Let \( f(A) \), \( f(B) \), etc., denote the frequencies for outcomes A, B, etc. in the group of students. Then \( f(A) + f(C) \) is the number of students for whom correct placement decisions would have been made had the placement procedure been used; and \( f(B) + f(D) \) is the number of incorrect decisions. The overall usefulness of the predictions would then be evaluated in terms of the benefits of correct placement decisions and the losses resulting from incorrect placement decisions. A function that assigns a value to outcomes such as these is called a utility function. For this model, in which outcomes for groups of students are considered, a simple type of utility function would be the frequency or proportion of correct placement decisions; according to such a utility, every correct decision results in the same positive benefit and every incorrect decision results in zero benefit. A more complex utility function would assign different values to each outcome: \( k_A f(A) + k_B f(B) + k_C f(C) + k_D f(D) \), where \( k_A, k_B, k_C, k_D \) are constants. Such a function would quantify the different benefits of the true positives and true negatives, as well as the different costs of false positives and false negatives.
The utility function of this model superficially resembles the expected utility function (see below) in the decision model developed by Petersen and Novick (1976). Their "threshold" utility function, however, pertained to the possible outcomes for an individual student, rather than for a group of students. By considering outcomes for groups of students, this model can directly address an institution's utilities for the results of its placement system. For simple utility functions like the one described above, in which an institution's utility is the sum of utilities for individual students, the two approaches amount to the same thing. The group model, however, by considering placement outcomes from an institutional, rather than individual perspective, permits one to consider the more complex outcomes an institution, rather than an individual, must consider.

For example, an institution's utility function should consider the costs of providing remedial instruction to high-risk students. With each additional student placed in the remedial course, one can expect a small increment in cost until a new instructor must be hired and an additional classroom provided. At this point, the total cost jumps by a large amount, and the per-student cost must be recomputed. Therefore, an institution's utility function cannot be represented as a simple sum of utilities for individual students; it must take into account the total number of students assigned to remedial instruction. One might approximate such a step function by a linear function, which could be used in the individual level model, but the structure of the decision problem is more easily conceived in the group model.

An institution's utility function should also take into account the cost of testing. For example, a placement rule could be based on using a general purpose, relatively low cost battery like the ACT Assessment for most students, and a custom designed, higher cost, local placement test for
students with ACT scores near the critical score level. Such a placement rule could be more advantageous to an institution than one based on either test alone, when both costs and accuracy of placement are considered. Cronbach and Gleser (1965) discussed two-stage testing in the context of a utility function that is assumed to be linearly related to the scores on the two tests and that can be related to the cost of testing. They showed how, given these assumptions, the critical scores on the two tests and the resulting gain in efficiency can be determined.

A difficult practical problem in implementing decision models is relating an institution's costs to its students' performance. Essentially, an institution must determine how much, in dollars, any given level of student performance is worth. Although institutions implicitly make such judgments whenever they decide on how to allocate their resources, explicitly declaring their values by way of a utility function is difficult, both technically and politically. (Cronbach and Gleser called it the "Achilles' heel of decision theory".) Reilly and Smither (1985) studied various methods that have been proposed in employment testing for linking criterion variables to a dollar scale. They found the methods effective in artificial situations where much information was available, but they were less sure about how effective the methods would be in more general situations. Clearly, much work needs to be done before utility functions can be routinely developed for educational testing applications like placement.

An institution's utility function reflects its concerns, which do not necessarily coincide with those of individual students; Whitney (1989) described ways in which institutions' and students' concerns typically differ. An institution must take care in formulating its utility function that the rights of individual students are respected; in particular, the...
resulting placement policies should be consistent with the Standards for Educational and Psychological Testing (APA, 1985).

Expected Utilities

In general, a utility function can not be directly computed for future individuals because their actual outcomes are not known. In the decision model described in Table 1, for example, the actual frequencies \( f(A) \), \( f(B) \), etc., for future students are not known. These frequencies must instead be estimated under the assumption that future students will be like, at least in some ways, the students for whom there are data. A utility function like the one considered here, when expressed in terms of estimated cell frequencies or proportions, is called an expected utility; it is from the expected utility function that decisions on the effectiveness of a placement rule can be made.

More precisely, an expected utility in a Bayesian decision model is the mean of a utility function with respect to the subjective probability distribution of the unknown parameters in the function. In the group decision model described by Table 1, the unknown parameters are the frequencies \( f(A) \), \( f(B) \), etc. for future students. These parameters could be estimated by modelling them directly with a multinomial distribution, or, alternatively, they could be inferred from a model of the joint density of course grades and test scores. In the example given later in this paper, statistical inferences are grounded in classical (frequency) probability, but Bayesian (subjective) probability models are being developed (Houston, 1988). The merits of alternative statistical techniques for computing expected utilities, while important, are not considered in this paper. Rather, I have chosen to minimize issues of statistical inference and to emphasize how expected utilities can be used to evaluate placement rules.
Note that expected utilities can be used not only to evaluate an existing critical score for placing students in the remedial course, but also to determine the optimal critical score. In the model described by Table 1, for example, each potential critical score is associated with a potentially different expected utility; the critical score level can be selected that maximizes the expected utility.

Clearly, the components of a utility function, such as the benefits of correct placement decisions and the costs of incorrect ones, vary among institutions. Furthermore, the statistical characteristics of grades and test scores will be unique to each institution. Therefore, computing an expected utility requires the involvement of local institutional staff, as does constructing the other components of a validity argument.

**Traditional Predictive Validity Statistics**

In this section the properties of two statistics commonly reported in predictive validity studies, the multiple correlation and the standard error of estimate, are discussed. It is shown that the multiple correlation can be related to the placement outcomes described in the four-fold table discussed previously. It is assumed, as before, that no prior selection of students has occurred, so that standard course grades are obtainable for all students. Adjustments are later described that take into account prior selection.

**Multiple Correlation**

The multiple correlation coefficient is probably used more often than any other statistic to summarize the results of predictive validity studies. As a measure of the strength of the statistical relationship between standard course grades and placement test scores, it can lend credibility to the
validity argument based on content fit. It does not, however, measure the value of a placement decision rule for identifying high risk students.

With additional assumptions about the joint distribution of the predicted and earned course grades, a multiple correlation coefficient can be related to the expected utilities previously described. The most straightforward distributional assumption is that of bivariate normality. The bivariate normal distribution is never an accurate representation of the relationship between predicted and earned grades because, among other things, it attempts to represent a 5-value grade scale as a continuum. At many colleges, though, it is a useful approximation for most students. The principal departure from bivariate normality typically occurs in data from students who are predicted to do well, but who have very low grades, usually Fs. Less frequently, a student with a low predicted grade will earn a high grade, or a student will have an unusual combination of test scores and high school grades. When such outlier observations are removed, and when the remaining data are not markedly skewed, the bivariate normal distribution is a reasonable approximation. Of course, any inferences based on such an approximation would be applicable only to the non-outlier portion of the population.

Cronbach and Gleser (1965), expanding on the work of Brogden (1946), investigated a utility that, in the context of placement, has a linear regression on test score among students enrolled in the standard course, and that is zero for students assigned to the remedial course. Cronbach and Gleser showed that if test scores are normally distributed in the unselected population, then the average gain in utility that results from using the test to screen out high risk students is linearly related to the product $r \sigma$, where $r$ is the correlation between test score and the utility and $\sigma$ is the standard deviation of the utility.
**Relationship to placement outcomes.** The correlation coefficient can also be related to the utility function of the decision model described by Table 1. Figure 1 shows elliptical probability contours of two bivariate normal distributions. Superimposed on the contours are two perpendicular lines that divide the plane into four regions. Each region in the plane corresponds to one of the outcomes in Table 1; moreover, the probabilities of the outcomes are indirectly related to the areas enclosed in the four regions. Therefore, the probability of a true positive or true negative is greater for the distribution with the narrow contour than for the distribution with the wide contour. Because narrow contours are associated with large correlations, Figure 1 shows that increasing the correlation increases the hit rate.

Table 2 shows the outcome probabilities associated with a variety of assumed failure rates in the standard course (failure rate), proportions of students placed in the remedial course (selection rate), and multiple correlation coefficients, assuming bivariate normality. To simplify the table, the failure rate has been set equal to the selection rate, but there is no reason why this would have to occur in practice.

Table 2 shows that when used for placement, even prediction equations with rather low multiple correlations substantially decrease the probability of students' (immediate) failing. For example, when the failure rate and selection rate are .40 and the correlation is .30, the probability of a false positive is .19, which is less than half the failure rate in the unselected population. This is the phenomenon described by Taylor and Russell (1939).

Note also that for any given combination of overall failure rate and selection rate, the proportion of true positives increases as the correlation increases. Curtis and Alf (1969) showed that the true positive rate is very nearly a linear function of the correlation.
Figure 1. Placement Outcomes Related to the Bivariate Normal Distribution
Table 2

Probabilities Associated with Outcomes of Placement by Standard Course Failure Rate, Remedial Course Selection Rate, and Correlation Coefficient, Assuming Multivariate Normality

<table>
<thead>
<tr>
<th>Standard course failure rate in entire population</th>
<th>Remedial course selection rate</th>
<th>Correlation coefficient in entire population</th>
<th>Outcome of placement decision</th>
</tr>
</thead>
<tbody>
<tr>
<td>.40</td>
<td>.40</td>
<td>.30</td>
<td>True pos (A) .41 False pos (B) .19 True neg (C) .21 False neg (D) .19 Hit K'ss .62 .38</td>
</tr>
<tr>
<td>.40</td>
<td>.40</td>
<td>.30</td>
<td>True pos (A) .42 False pos (B) .18 True neg (C) .22 False neg (D) .18 Hit K'ss .64 .36</td>
</tr>
<tr>
<td>.50</td>
<td>.40</td>
<td>.44</td>
<td>True pos (A) .46 False pos (B) .14 True neg (C) .26 False neg (D) .14 Hit K'ss .72 .28</td>
</tr>
<tr>
<td>.60</td>
<td>.50</td>
<td>.46</td>
<td>True pos (A) .46 False pos (B) .14 True neg (C) .26 False neg (D) .14 Hit K'ss .72 .28</td>
</tr>
<tr>
<td>.20</td>
<td>.20</td>
<td>.30</td>
<td>True pos (A) .67 False pos (B) .13 True neg (C) .07 False neg (D) .13 Hit K'ss .74 .26</td>
</tr>
<tr>
<td>.40</td>
<td>.20</td>
<td>.68</td>
<td>True pos (A) .68 False pos (B) .12 True neg (C) .08 False neg (D) .12 Hit K'ss .76 .24</td>
</tr>
<tr>
<td>.50</td>
<td>.40</td>
<td>.69</td>
<td>True pos (A) .69 False pos (B) .11 True neg (C) .09 False neg (D) .11 Hit K'ss .78 .22</td>
</tr>
<tr>
<td>.60</td>
<td>.50</td>
<td>.70</td>
<td>True pos (A) .70 False pos (B) .10 True neg (C) .10 False neg (D) .10 Hit K'ss .80 .20</td>
</tr>
<tr>
<td>.10</td>
<td>.10</td>
<td>.30</td>
<td>True pos (A) .82 False pos (B) .08 True neg (C) .02 False neg (D) .08 Hit K'ss .84 .16</td>
</tr>
<tr>
<td>.40</td>
<td>.10</td>
<td>.83</td>
<td>True pos (A) .83 False pos (B) .07 True neg (C) .03 False neg (D) .07 Hit K'ss .86 .14</td>
</tr>
<tr>
<td>.50</td>
<td>.40</td>
<td>.83</td>
<td>True pos (A) .83 False pos (B) .07 True neg (C) .03 False neg (D) .07 Hit K'ss .86 .14</td>
</tr>
<tr>
<td>.60</td>
<td>.50</td>
<td>.84</td>
<td>True pos (A) .84 False pos (B) .06 True neg (C) .04 False neg (D) .06 Hit K'ss .88 .12</td>
</tr>
</tbody>
</table>
One must, of course, consider the other kind of placement error, the false negative (D). When the failure rate and selection rate are equal (as in Table 2), the probability of a false negative is equal to the probability of a false positive (B). The sum of these two probabilities is the overall error rate (or "miss rate"), and is shown in the last column of Table 2. Note that the miss rate decreases as the failure rate and selection rate decrease and as the correlation increases. Note also that for low failure rates and selection rates, the miss rate exceeds the failure rate, even if the correlation is moderately high. For example, when the failure rate and selection rate in the entire population are .10, and the correlation is .60, the rate of false positives and the rate of false negatives are each .06, and so the miss rate is .12. Thus, the placement system lowers the probability of students' failing from .10 in the unselected population to .06, but at the cost of requiring .06 of the students to take a course at a lower level than was appropriate for them.

Correction for prior selection. The interpretations just described assume that data have been collected for students without having made any prior placement intervention. This would be feasible at a postsecondary institution that did not already have a placement program, but that did collect students' ACT scores, high school grades, and college course grades. Most institutions that would consider doing this research, though, already have some kind of placement program, and it is unlikely that they would (or should) suspend their placement programs to collect the needed data. Suspending a placement program would, for example, deny to those students who were apparently at risk of failure the information that would enable them to improve their academic skills by enrolling in a remedial course. It would be ethically questionable to do this solely to conduct a validity study.
Most institutions doing predictive validity research for course placement must therefore deal with the statistical problem of prior selection of students---students for whom no data are available because they were selected for the remedial course. Correlations computed from data that have been subject to prior selection tend to underestimate the corresponding correlations in the unselected population. Suppose selection is done explicitly on the basis of a single predictor variable X. If the regression of the standard course grade Y on X is the same linear function of X both for those who actually enrolled in the standard course and for those who did not, and if the conditional variance of Y on X is constant for all values of X, then (Lord & Novick, 1968, p. 143):

\[ \rho_{XY} = \frac{1}{\sqrt{1 + \frac{\sigma_{X*}^2}{\sigma_X^2} \left[ \frac{1}{\rho_{X*Y*}^2} - 1 \right]}} \]

where \( \rho_{XY} \) is the correlation between X and Y in the entire population,
\( \rho_{X*Y*} \) is the correlation between X and Y among students who enrolled in the standard course
\( \sigma_X^2 \) is the variance of X in the entire population, and
\( \sigma_{X*}^2 \) is the variance of X in the standard course.

The more effective a placement system is in screening high risk students from the standard course, the smaller the variance \( \sigma_{X*}^2 \) for the standard course grades will be. A smaller variance \( \sigma_{X*}^2 \) in Formula (2) implies a larger adjusted correlation \( \rho_{XY} \). Therefore, more effective screening results in larger adjusted correlations. The result can also be stated another way: For a given correlation \( \rho_{XY} \) in the unselected population, the more effective placement system is in screening high risk students from the standard course, the smaller the observed correlation \( \rho_{X*Y*} \) will be.
Values of $p_{XY}$ for selected values of $p_{X^*Y^*}$ and $\sigma^2_{X^*}/\sigma^2_X$ that one might expect to encounter in predicting specific course grades are presented in Table 3. For example, if the multiple correlation for students in the standard course is .40, and the ratio of test score variance in this group to that in the entire population is .70, then the multiple correlation in the entire population would be .49; it should be noted that this adjusted correlation pertains only to the group of students who might have taken the standard course, not to some more general population. There is an analogous adjustment to the correlation between $X$ and $Y$ when selection is made explicitly on the basis of another variable $W$ related to $X$ (Lord & Novick, 1968, p. 144).

Lord and Novick caution that these formulas may undercorrect the correlation when $\sigma^2_{X^*}/\sigma^2_X < .7$, but the data in Table 3 are a useful approximation of what to expect in typical situations. The increase in correlation for the examples given in the table ranges from .02 to .13.

It should be noted that this statistical problem is caused by lack of data, rather than by the choice of statistic to be reported. Prior selection is, for example, just as much a problem in directly estimating the expected utilities on the decision model described by Table 1 as in estimating correlation coefficients. In either case, we do not have data for a certain segment of the population, and must estimate relationships by extrapolation from the segment of the population for which we do have data.

**Standard Error of Estimate**

The standard error of estimate (SEE) is the square root of the average squared difference between actual and predicted course grades; smaller values of SEE indicate more accurate prediction. (Actually, SEE is calculated by dividing the sum of squared differences by $N - p - 1$, where $N$ is the sample size.)
Table 3

Correlation Coefficients in Unselected Bivariate Normal Population, Corrected for Effects of Selection

<table>
<thead>
<tr>
<th>Correlation in selected population</th>
<th>Variance ratio</th>
<th>Correlation in unselected population</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_{X<em>Y</em>}$</td>
<td>$\sigma^2_{X*}/\sigma^2_X$</td>
<td>$\rho_{XY}$</td>
</tr>
<tr>
<td>.30</td>
<td>.90</td>
<td>.32</td>
</tr>
<tr>
<td>.80</td>
<td>.90</td>
<td>.35</td>
</tr>
<tr>
<td>.70</td>
<td>.90</td>
<td>.38</td>
</tr>
<tr>
<td>.60</td>
<td>.90</td>
<td>.42</td>
</tr>
<tr>
<td>.40</td>
<td>.90</td>
<td>.43</td>
</tr>
<tr>
<td>.80</td>
<td>.80</td>
<td>.45</td>
</tr>
<tr>
<td>.70</td>
<td>.80</td>
<td>.49</td>
</tr>
<tr>
<td>.60</td>
<td>.80</td>
<td>.53</td>
</tr>
<tr>
<td>.50</td>
<td>.90</td>
<td>.53</td>
</tr>
<tr>
<td>.80</td>
<td>.70</td>
<td>.56</td>
</tr>
<tr>
<td>.70</td>
<td>.70</td>
<td>.59</td>
</tr>
<tr>
<td>.60</td>
<td>.70</td>
<td>.63</td>
</tr>
<tr>
<td>.60</td>
<td>.60</td>
<td>.63</td>
</tr>
<tr>
<td>.80</td>
<td>.60</td>
<td>.65</td>
</tr>
<tr>
<td>.70</td>
<td>.60</td>
<td>.68</td>
</tr>
<tr>
<td>.60</td>
<td>.60</td>
<td>.71</td>
</tr>
</tbody>
</table>
size and \( p \) is the number of predictors, rather than dividing by \( N \). Division by \( N - p - 1 \) gives SEE the statistical property of being "unbiased").

SEE is associated with the squared error loss function, rather than the four-fold utility function described previously. The squared error loss function places high value on accuracy of grade prediction, without regard to overall student performance; like the multiple correlation, therefore, SEE is not very useful in measuring the effectiveness of a placement decision rule to an institution. On the other hand, because SEE is a measure of how close, on average, predicted grades are to earned grades, it maybe useful for providing to individual students information about the accuracy of their predicted grades. A statistic that is more intuitively appealing to students is mean absolute error (MAE), the average of the absolute differences between predicted and earned grades. A MAE of .5, for example, means that on average, the predicted grade differs from the earned grade by .5 grade units, whether higher or lower. MAE is not usually calculated by regression programs, but when the predicted and earned grades have an approximate bivariate normal distribution, then MAE is approximately equal to 0.8*SEE.

Unlike the multiple correlation, SEE is not affected by prior selection when predicted and earned grades have a bivariate normal distribution. The reason for this is that the bivariate normal distribution has "homoscedasticity of errors", i.e., the conditional variance of earned grades is constant for all predicted grades. If this were not true (for example, if grade predictions were markedly more accurate for low predicted grades than for high predicted grades), then prior selection would also affect SEE.
Determining the Effectiveness of Remediation

With an appropriate placement rule, students who are likely to be unsuccessful in the standard course can be identified and placed in a remedial course. Although one reduces the likelihood of immediate failure by such intervention, the question remains whether students placed in the remedial course will later succeed in the standard course. To extend the validity argument previously discussed, one then needs to examine the plausibility of the following additional assumption:

4. The remedial course provides, in a cost-effective way, the academic skills that students previously identified as high risk need to succeed in the standard course.

To establish the plausibility of this assumption, it is clearly appropriate to examine the syllabus for the remedial course: There should be a fit between the remedial course contents and the academic skills needed to succeed in the standard course, as previously identified in justifying Assumption 2.

Given the other assumptions in the validity argument, it is also appropriate to examine separately for remedial course students the relationship between their placement test scores and the grades they finally earn in the standard course. If this relationship is the same as that for students who enrolled directly in the standard course, then the remedial course is of no benefit---students with similar predicted grades from the placement test have the same outcome (i.e., they tend to earn unsatisfactory grades in the standard course), whether or not they take the remedial course. For the placement system as a whole to be successful, the students placed in the remedial course should, on average, have higher grades in the standard course than they would have if they had not been placed in the
remedial course. Note that this concept relates to the placement system as a whole, and not just to one component (such as the placement test).

If a decision model is to be an effective part of the validity argument concerning remediation, then it must consider the costs, as well as the benefits, of differential treatment. The utility function in such a model would place a value on the ultimate performance of every student in the standard course, and it would take into account the extra cost incurred when students first enroll in the remedial course. An institution would need to verify that low scoring students who enroll in the remedial course have a higher expected utility than do low scoring students who enroll directly in the standard course. The institution would also need to verify that high scoring students who enroll in the remedial course have a lower expected utility than do high scoring students who enroll directly in the standard course. This relationship between expected utility, test score, and treatment is an example of a statistical interaction, and is illustrated in Figure 2. Note that the vertical axis in this figure is a utility function that takes into account the extra cost associated with providing additional instruction to the remedial students.

If the vertical axis in Figure 2 represented the standard course grade instead of a utility function, then the two lines might not intersect or might intersect at a different point. Cronbach and Snow (1977, pp. 32-33) provide a discussion and illustration of this phenomenon. Moreover, if the regression slope of course grade on test score is the same for both treatment groups, (and if utility is linearly related to course grade), then there can be no interaction in the regression of utility on test score. The reason is that the cost of providing remedial instruction is not a function of test score; therefore, the regression line for utility can differ from the regression line
Figure 2. Relationship Between Expected Utility Function and Test Score, by Treatment Group
for course grade in intercept only. If the regression slopes for the two
groups' course grades are equal (or nearly equal), then the regression lines
for their utility functions will not intersect, and one treatment will always
be judged superior to the other. Therefore, the slopes of the regression
functions for course grades can be used to make inferences about the presence
of interactions in the utility function regressions.

In theory, treatment effects can be determined by randomly allocating
students either to the remedial course (treatment group) or the standard
course (no-treatment group), then studying the relationship between standard
course grade (or utility) and test score for the two groups. In practice,
students are not randomly allocated to the remedial and standard courses, and
this considerably complicates proper interpretation of treatment effects. At
an institution where there is an existing placement system, low scoring
students do not take the standard course before they have completed the
remedial course; and high scoring students do not take the remedial course.
Therefore, such an institution can ordinarily estimate the relationship
between course grade (or utility) and test score for the no-treatment group
only in the upper range of test scores. In an institution without a validated
placement system, it might be possible to assign low scoring students randomly
to the treatment and no-treatment groups until the system could be
validated. Otherwise, estimates of treatment effects must usually be based on
the extrapolation of the no-treatment group regression to the lower range of
scores (Cronbach and Snow, 1977, pp. 48-49).

When the placement test is but one component of the placement decision,
as in a voluntary system, there will be greater variation in the test scores
of each group, and the need for extrapolation would appear to be reduced.
Unfortunately in this situation, the differences in the regression lines are
confounded with whatever variables were used to make the placement decision. It is impossible to conjecture what effects the confounding has, in general, though it might be possible to do so in a particular placement system. For example, if the other variables can be quantified, they can be incorporated into a model with test score, treatment effect, and score by treatment interaction.

**Example**

This example is based on the ACT scores, self-reported high school grades, and freshman English course grades of students who enrolled in a medium size public university between summer, 1985 and fall, 1986. This institution encourages students with ACT English Usage scores between 1 and 15 to enroll in a remedial course, those with scores between 16 and 25 to enroll in a standard course, and those with scores of 26 and higher to enroll in a more advanced course. Placement is not determined solely by this rule, however; some students enroll in courses at a higher or lower level than recommended, and some students do not even have ACT scores. To simplify the analyses and the discussion, I retained only records with complete data and with enrollment patterns consistent with the placement rule just described. (Of 6,356 records in the total data set, 5,609 (or 88%) met these criteria.) For students who enrolled in the remedial course, both the grade in the remedial course and the eventual grade in the standard course were recorded. Records of students who enrolled in the advanced course were not analyzed.

According to the catalog for this institution, the standard English course teaches students to explore and communicate ideas through writing, and emphasizes various prose patterns and techniques. The ACT English Usage test measures students' skills in the mechanics and style of written English. The
test consists of passages with certain segments underlined; students are then asked, in multiple choice items, to indicate whether or how the underlined segments could be improved. The contents of the test would therefore appear to be relevant to the requirements of the course, and Assumption 2 of the validity argument would be plausible.

The test scores used in this example are from the version of the ACT Assessment administered prior to October, 1989. Effective in October, 1989, a new version of the ACT Assessment will be implemented (ACT, 1989). The new version will also contain an English test; its contents will be similar to those of the current English Usage test, but will incorporate recent changes in secondary and postsecondary curricula. It is therefore likely that Assumption 2 will be at least as plausible for the new version of the ACT Assessment as for the ACT Assessment used before October, 1989.

With regard to Assumption 3, one would need to know what aspects of students' performance were graded, and whether different instructors used the same grading standards. Unfortunately, no information was available on the grading methods used, and therefore, the plausibility of Assumption 3 can not be readily determined. I shall assume, in order to continue the discussion, that the grades predominantly measure aspects of students' performance related to the academic skills acquired in the course, and that different instructors had consistent grading standards.

Summary statistics for the ACT English Usage score and the English course grades of the remedial and standard groups are given in Table 4. Note that 736 of the original 951 students in the remedial course persisted through the standard course, and their average grade in the standard course was .45 grade units lower than the average grade of the 4,463 students who initially enrolled in the standard course.
Table 4

Summary Statistics for ACT English Usage Scores and English Course Grades, by Placement Group

<table>
<thead>
<tr>
<th>Placement group</th>
<th>N</th>
<th>ACT English Usage score</th>
<th>Remedial course grade</th>
<th>Standard course grade</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Mean</td>
<td>SD</td>
<td>Mean</td>
</tr>
<tr>
<td>Remedial</td>
<td>951</td>
<td>12.3</td>
<td>5.9</td>
<td>2.74</td>
</tr>
<tr>
<td>Standard</td>
<td>4463</td>
<td>20.4</td>
<td>6.0</td>
<td>----</td>
</tr>
</tbody>
</table>

*Based on N = 736 records.*
Table 5
Regression Statistics Associated with Predicting End of Term Grade in Standard English Course

<table>
<thead>
<tr>
<th>Model</th>
<th>Statistic</th>
<th>Total group</th>
<th>Outliers deleted</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I</td>
<td>N</td>
<td>4463</td>
<td>4318</td>
</tr>
<tr>
<td>(1 predictor)</td>
<td>Regression Coefficients</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Intercept</td>
<td>1.15</td>
<td>1.43*</td>
</tr>
<tr>
<td></td>
<td>ACT English Usage</td>
<td>.085*</td>
<td>.075*</td>
</tr>
<tr>
<td></td>
<td>R</td>
<td>.27</td>
<td>.29</td>
</tr>
<tr>
<td></td>
<td>SEE</td>
<td>.73</td>
<td>.61</td>
</tr>
<tr>
<td>II</td>
<td>N</td>
<td>3951</td>
<td>3793</td>
</tr>
<tr>
<td>(8 predictors)</td>
<td>Regression coefficients</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Intercept</td>
<td>.64*</td>
<td>.85*</td>
</tr>
<tr>
<td></td>
<td>ACT English Usage</td>
<td>.057*</td>
<td>.056*</td>
</tr>
<tr>
<td></td>
<td>ACT Mathematics Usage</td>
<td>.002</td>
<td>.003</td>
</tr>
<tr>
<td></td>
<td>ACT Social Studies Reading</td>
<td>.006</td>
<td>.008*</td>
</tr>
<tr>
<td></td>
<td>ACT Natural Science Reading</td>
<td>-.008</td>
<td>-.007*</td>
</tr>
<tr>
<td></td>
<td>HS Eng. sh</td>
<td>.155*</td>
<td>.136*</td>
</tr>
<tr>
<td></td>
<td>HS mathematics</td>
<td>.059*</td>
<td>.035*</td>
</tr>
<tr>
<td></td>
<td>HS social studies</td>
<td>.099*</td>
<td>.075*</td>
</tr>
<tr>
<td></td>
<td>HS natural science</td>
<td>.040</td>
<td>.040*</td>
</tr>
<tr>
<td></td>
<td>R</td>
<td>.38</td>
<td>.40</td>
</tr>
<tr>
<td></td>
<td>SEE</td>
<td>.70</td>
<td>.60</td>
</tr>
</tbody>
</table>

* Denotes regression coefficients significant at the .01 level.
Table 5 contains statistics associated with the prediction of the standard course grade for the 4,463 students who initially enrolled in the standard course. Statistics are presented for predictions based on ACT English Usage score alone (Model I) and for predictions based on the four ACT scores and four high school grades (Model II). Moreover, statistics were calculated both from the total group of cases with valid values and from a truncated data set with outlier cases removed (see previous discussion about correlation coefficients). Outliers were defined as those for which either the studentized residual or the leverage statistic (Belsley, Kuh, and Welsch, 1980) exceeded the 99th percentiles of their distributions, assuming a normal distribution of errors.

Note in Table 5 that Model II had larger correlations and smaller standard errors of estimate than did Model I. An anomalous result in Model II is the presence of a negative regression weight corresponding to ACT Natural Sciences Reading; fortunately, the magnitude of this coefficient is not large enough to have much practical effect on the predicted English grade. Finally, note that removing the outlier cases tended to increase the correlations only slightly, but considerably reduced the standard errors of estimate.

The correlations associated with the truncated data sets (.29 and .40) were adjusted for prior selection, using the procedures discussed previously. In the calculation for Model I, the adjusted correlation was .41. For Model II, ACT English Usage (X) was considered the explicit selection variable and the predicted English course grade (Y HAT) was considered to be a proposed selection variable. Let Y denote the actual English course grade. By taking into account the correlation between X and Y, the correlation between X and Y HAT, the correlation between Y HAT and Y in the selected population, and the variance ratio $\sigma^2_{X}/\sigma^2_{X}$, an adjusted multiple correlation of .55 was obtained.
Table 6 contains estimated cell probabilities associated with the "pass" criteria of C or better and B or better. Two of the estimates are based on the assumption that YHAT and Y have approximate bivariate normal distributions with the correlations just derived. Observe that the estimated hit rate for C or better using placement based on Model I is .78 + .02 = .80, and the corresponding hit rate from Model II is .81. In this particular example, placement based on the single variable ACT English Usage is almost as effective as placement based on all four ACT scores and all four high school grades. Placement based on eight variables was, however, somewhat more effective than placement based on ACT English Usage alone, as judged by the hit rates for B or better (.74 versus .70).

Note also that using ACT English Usage as a placement test reduced the failure rate, as determined by the standard of C or better, from .04 in the entire population to .02 in the selected population, though at the cost of a false negative rate of .18. By the standard of B or better, the failure rate was reduced from .32 to .21, and the false negative rate was .09.

Table 6 contains another set of estimated cell probabilities associated with placement using the ACT English Usage test. These estimates are based on logistic regression, which is in several ways more direct and straightforward than the linear regression/normal theory methodology discussed so far. In logistic regression, a dichotomous outcome (such as C or better) is modeled directly from the predictor variable:

\[ P[W=1 \mid X=x] = \frac{1}{1 + \exp(-a-b^*x)}, \]

where \( W=1 \) if a student's grade exceeds a certain threshold, and \( W=0 \) otherwise. Using iterative estimation techniques (such as Gauss-Newton), it
Table 6

Estimated Cell Probabilities Associated with Grades in Standard English Course

<table>
<thead>
<tr>
<th>&quot;Pass&quot; criterion</th>
<th>Outcome</th>
<th>Prediction model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Bivariate normal, 1 predictor</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C or better</td>
<td>True positive</td>
<td>.78</td>
</tr>
<tr>
<td></td>
<td>True negative</td>
<td>.02</td>
</tr>
<tr>
<td></td>
<td>False positive</td>
<td>.02</td>
</tr>
<tr>
<td></td>
<td>False negative</td>
<td>.18</td>
</tr>
<tr>
<td>B or better</td>
<td>True positive</td>
<td>.59</td>
</tr>
<tr>
<td></td>
<td>True negative</td>
<td>.11</td>
</tr>
<tr>
<td></td>
<td>False positive</td>
<td>.21</td>
</tr>
<tr>
<td></td>
<td>False negative</td>
<td>.09</td>
</tr>
</tbody>
</table>
is possible to find the constants a and b for which the fitted conditional
probabilities are closest to the observed outcomes, in the sense of weighted
least squares. In general, it is more difficult to compute parameter
estimates for a nonlinear model like this one than for a linear model. Once
parameter estimates have been computed, though, estimated cell probabilities
can easily be obtained by averaging the fitted conditional probabilities over
the relevant values of x. For example, the rate of true positives can be
estimated as:

\[
\hat{P}[W=1 \mid X \geq 16] = \frac{\sum_{x=16}^{33} \hat{P}[W=1 \mid X=x] n(x)}{\sum_{x=1}^{33} n(x)}
\]

where \( \hat{P}[W=1 \mid X=x] \) is the conditional probability estimated from the logistic
model (3), and \( n(x) \) is the number of observations of x. One could also assume
a statistical model for the marginal distribution of X, and adapt the
empirical frequencies \( n(x) \) accordingly (W. M. Houston, personal communication, 1989).

Estimates of the parameters a and b in equation (3) were computed using
the SAS LOGIST procedure (SAS, 1986). The estimates were -3.36 and .24,
respectively, for B or better; for C or better, they were -.36 and .19,
respectively. Both estimated b coefficients were statistically significantly
different from 0 (p < 10^-4).

The cell probabilities estimated from the fitted logistic curves are
shown in the right-most column of Table 6. Note that they are very similar to
the cell probabilities based on adjusted correlations and the bivariate normal
distribution. This result illustrates the point made earlier that while the
correlation coefficient can be made relevant to placement validity issues, given certain assumptions, it is not essential.

Estimated B-or-better cell probabilities were also computed for several hypothetical critical scores near the actual critical score of 16 (described previously). The hit rates associated with these estimates provide one means of judging the suitability of different critical scores. It turned out that the largest estimated hit rate (.73) was associated with the actual critical score of 16, although the critical scores of 15 and 17 had hit rates very nearly as large. Thus, when judged by the standard of the hit rate for B or better, 16 is the optimal critical score.

Finally, let us consider the issue of treatment effects in the remedial course. Table 7 contains statistics for the regression of standard course grade on ACT English Usage score. These statistics are based on the records both of students who first enrolled in the remedial course (Group A) and of students who enrolled in the standard course to begin with (Group B). Table 7 is based on the full data set, rather than on the edited data set used to compute Tables 5 and 6, so as to reduce the amount of extrapolation. Outliers were deleted from the full data set according to the procedures followed in calculating correlation coefficients. The difference in the regression slopes for Groups A and B is statistically significant (p < .001), and allows the possibility of an interaction effect in the regression of the utility function on test score.

Results are also displayed graphically in Figure 3. The thick solid lines in the graphs pertain to predicted grades calculated over the central 80% of the cases; the thinner solid lines pertain to the predicted grades of the outermost 20% of the cases; and the dashed lines indicate extrapolations of the predictions to the remainder of the ACT English Usage test score.
Table 7
Statistics Related to the Regression of English Course Grade on ACT English Score, by Treatment Group

<table>
<thead>
<tr>
<th>Group</th>
<th>N</th>
<th>Intercept</th>
<th>Slope</th>
<th>R</th>
<th>SEE</th>
<th>Average predicted grade</th>
<th>Estimated proportion C or better</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Group A equation</td>
<td>Group B equation</td>
</tr>
<tr>
<td>A. Treatment (Remedial)</td>
<td>863</td>
<td>1.86</td>
<td>.049</td>
<td>.76</td>
<td>.63</td>
<td>2.53</td>
<td>2.47</td>
</tr>
<tr>
<td>B. No treatment (Standard)</td>
<td>4553</td>
<td>1.46</td>
<td>.073</td>
<td>.32</td>
<td>.61</td>
<td>----</td>
<td>----</td>
</tr>
</tbody>
</table>

a All estimates are based on data sets with outliers deleted.
Figure 3. Predicted Grades in Standard English Course, for Students in Remedial and Standard Classes
Note that the lines for the remedial and standard course students intersect near the ACT English Usage score of 16, indicating an interaction. If the vertical axis in Figure 3 were an expected utility that incorporated the extra costs associated with providing remedial instruction, then the intersection point would have been at a lower ACT English Usage score.

Two other statistics, pertaining to the students in the remedial course, are shown in Table 7. One statistic is the average predicted grade, and the other is the proportion of students whose predicted grades are 2.0 or higher. Each statistic was computed both from the prediction model in Group A and from the prediction model in Group B. These statistics suggest that students who enrolled in the remedial course increased their grades by an average of .06 grade units, and that the proportion of them with a C or better increased by .03. This is a modest benefit; indeed, these results indicate that nearly all remedial course students would have earned a C or better even if they had enrolled in the standard course to begin with. Moreover, the statistics do not take into account either the extra cost of providing remedial instruction or the fact that about 7% of the remedial course students dropped out before completing the standard course. A decision model incorporating these factors would therefore suggest lowering the critical score for placement in the standard course. When interpreting these statistics, of course, one should remember that they are confounded with whatever other variables were used in making placement decisions, and are based on extrapolations on the test score scale.

Conclusions

An argument for the validity of a placement rule defined in terms of ACT Assessment test scores can be based on the fit between the skills measured by
the test battery and the skills required for success in a course. Demonstrating statistical relationships between test scores and performance in the course does not by itself provide a logical justification for the placement system, though it lends credibility to an argument based on content fit.

Statistical decision theory provides a means for an institution to evaluate the benefits and costs of a placement system. A group decision model, based on the outcomes of an entire group of students at an institution, is a natural way to express the concerns that the institution, rather than an individual, might have.

Traditional validity statistics, such as the correlation coefficient and the standard error of estimate, do not provide information on the practical effectiveness of a placement test in identifying high risk students. Utility-based statistics, developed in the context of a decision model, can provide such information. Given the right assumptions, correlation coefficients can be related to estimated hit rates, which are associated with certain utility functions. Correlation coefficients are not, however, essential in measuring the practical effectiveness of a placement test in identifying high risk students. Other techniques, such as estimating conditional success rates through nonlinear regression models, are conceptually simpler, though computationally more complex.
REFERENCES


