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ABSTRACT

Fifty-five college students participated in a study investigating the effect of field dependence and time limitations on the solution of mathematics word problems and visual interpretation problems. Field-independent (n=25) and field-dependent (n=30) subjects represented the upper and lower thirds on the Hidden Figures Test. Field-independent subjects scored higher than did field-dependent subjects on both base and complex word problems, but there was no difference between the groups on visual problems. Although effects for time were found, they appeared to be artifactual. Results are discussed in terms of the relationship between the demands of the verbal and visual problems, and differences in restructuring abilities of field-independent and field-dependent learners. Two tables and three graphs contain study data. (Author/SLD)

Cognitive Style and Competence in Mathematics Problem Solving

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Abstract

Fifty-five college students participated in a study investigating the effect of field dependence and time limitations on the solution of mathematics word problems and visual interpretation problems. Field-independent (N=25) and field-dependent (N=30) represented the upper and lower thirds on the Hidden Figures Test. Field-independent subjects scored higher than field-dependent subjects on both Base and Complex word problems, but there was no difference between the groups on Visual problems. Although effects for Time were found, they appeared to be artifactual. Results are discussed in terms of the relationship between the demands of the verbal and visual problems, and differences in restructuring ability of field-independent and field-dependent learners.

Introduction

Cognitive styles has been described as "consistent individual differences in ways of organizing and processing information and experience" (Messick, 1982, p. 3). The usefulness of a cognitive styles approach to education depends on its ability to account for differences in performance that are related to differences in cognitive style. Research has consistently found a relationship between field dependence and performance on a variety of academic tasks including concept learning (Frank, & Davis, 1979), reading comprehension (Blaha, 1982; Buriel, 1978; Clark, Ward, Forlizzi, & Brubaker, 1986), identification of main ideas in text (Annis, 1979), learning information from lectures (Frank, 1984; Ward & Clark, 1988), and performance on intelligence tests (Clark & Roof, 1988; Linn & Kyllonen, 1981). One explanation for these performance differences is that field-dependent and field-independent learners tend to approach tasks differently (Clark & Roof, 1987; Pascual-Leone, Ammon, Goodman, & Subelman, 1978). Field-independent learners have been characterized as relatively more analytic, better able to identify relevant problem components, and more sensitive to part-whole relationships; field-dependent learners have been characterized as relatively more passive and global in their approach to problem solving (Clark, & Roof, 1988; Goodenough, 1976; Pascual-Leone, et al., 1978; Witkin & Goodenough, 1981).

The ability to analyze problems, and identify relevant problem components, may be particularly important in solving certain types of mathematics problems. Studies of the relationship between field dependence and mathematics achievement have found that field-independent learners perform better than field-dependent learners in computation, knowledge of mathematics concepts, and application of mathematical principles (Bieri,

Bradburn, & Galinsky, 1958; Blaha, 1982; Buriel, 1978; Satterly, 1976; Vaidya & Chansky, 1980). However, relatively little research has addressed the relationship between field dependence and solution of word problems and visual interpretation problems. These types of problems may place different types of cognitive demands on learners. Solution of word problems often requires identifying relevant information embedded in the problem statement, and specifying the relationship among the relevant elements, before carrying out the required computation. Field-dependent learners, who have difficulty identifying key ideas in verbal information (Annis, 1979), might be expected to score lower on mathematics word problems than field-independent learners with comparable computational ability. One study that has addressed this issue (Bien, 1974) found that field-independent learners solved basic word problems better than field-dependent learners, but when the important pieces of information in the problems were circled, performance differences disappeared. This suggests that the initial differences may have been related more to the ability to extract information from the text of the problem than to ability to execute the required procedures. An implication of this is that field-dependent learners might have particular trouble disembedding relevant information from complex word problems containing distracting information not required for problem solution.

Visual problems often involve locating and interpreting information in charts, diagrams or tables. When visual information is embedded in a complex or contradictory context (as in the Hidden Figures Test), field-independent learners perform better than field-dependent learners. However, when the information is readily available, and learners are not required to disembed the information or analyze the visual field, then there is no reason to believe that one group should have an advantage.

One purpose of this research was to investigate the solution of verbal and visual

mathematics problems by field-dependent and field-independent college students. Two levels of complexity of word problems were used, one containing only information relevant to problem solution (Base problems), and the other containing additional irrelevant numerical information (Complex problems). It was anticipated that field-independent learners would score higher than field-dependent learners on both levels of word problems, and that the difference would be greatest on Complex problems. No difference between the groups was anticipated on visual problems.

A second purpose of this research was to explore the effect of limiting solution time on field-dependent and field-independent subjects. Since the analytic approach of field-independent learners tends to be more efficient (Pascual-Leone, et al., 1978), it was anticipated that field-independent subjects would be less affected by time constraints than would field-dependent subjects.

Method

Subjects: Fifty-five subjects were selected from an initial pool of eighty-eight students enrolled in an introductory class in educational psychology. The Hidden Figures Test (HFT) and a mathematics computation test were administered to all 88 students in a group session. The coefficient alpha reliability of the HFT was .84. Students whose scores were in the upper third (M=17.32) and lower third (M=6.91) on the HFT were selected for the study. Two students were subsequently eliminated from each group when they scored below the minimum on the mathematics computation test. This resulted in a final sample of 55 (25 field-independent and 30 field-dependent) subjects. Subjects were awarded extra credit applicable to their final course grades.

Instruments. A 10-item test of basic computation ability was adapted from the university's mathematics placement test. This test assessed skill in the types of operations required to solve the word problems used in the study. The coefficient alpha reliability of the computation test with the initial sample (N=88) was .73. Students who scored less than 70% on the computation test were eliminated from the study.

The experimental materials consisted of 24 word problems. and 10 visual problems adpated from the university's placement test in mathematics, and from materials used by the remedial mathematics program. These final set of problems was selected from a pool of 41 word problems and 15 visual problems following pilot testing with a group of students similar to those in the study. Two forms of each of the word problems were developed, one containing only information necessary for solving the problem (Base problems), and the other containing two items of irrelevant numerical information (Complex problems). An example of each type of problem is presented in Table 1.

The word problems were separated into two groups of 12 problems, with each group containing only Base or Complex problems. Problems were typed three to a sheet, with work space provided under each problem. Each visual problem was placed on a separate sheet. Problems were then grouped, with the word problems preceding the visual problems. Each subject received all 24 word problems, 12 Base and 12 Complex, and the 10 visual problems. Order and version of the word problems were balanced across groups. The coefficient alpha reliabilities for the Base, Complex, and Visual problems were .71, .68 and .84 respectively.

Insert Table 1 About Here

<u>Procedures.</u> Subjects participated in one of two sessions run on consecutive nights. Upon arrival, subjects were randomly assigned, within field dependence group, to either a timed or untimed condition, and directed to separate rooms based on condition. Subjects arriving late were placed in the untimed condition. This resulted in uneven numbers of subjects in the two conditions. Subjects in each room were given a set of problems and instructions appropriate to their condition, and were asked to begin work. No calculators were allowed. In the timed condition, students were given 15 minutes to work on each set of 12 word problems, and 10 minutes for the visual problems. These time limits represented approximately 80% of the average solution time required by the pilot sample. In the untimed condition, subjects were allowed to work on the problems for as long as they wished.

<u>Scoring</u>: Word problems were scored for both correct answers and correct problem set-ups. Due to the high correspondence between the two (.97), only correct answers were analyzed. Visual problems were scored as correct or incorrect.

Results

Data were analyzed using a 2 (Field Dependence) X 2 (Time Condition) X 3 (Problem Type) mixed analysis of variance. Since the number of items was not identical for each problem type, scores were converted to percentages prior to analysis. Means and standard deviations of the percentage scores are provided in Table 2. Main effects were found for Field Dependence ($F(1, 51)=28.57$, $p<.001$), Time ($F(1, 51)=6.45$, $p<.02$), and Problem Type ($F(2,102)=28.34$, $p<.001$. In addition, significant interactions were found between Field Dependence and Problem Type ($F(2, 102)=8.94$, $p<.001$), and between Time and Problem Type ($F(1, 102)=6.16$, $p<.01$). Since each of the main effects was involved in an interaction, only the interactions will be discussed.

Insert Table 2 About Here

For the Field Dependence X Problem Type interaction, Newman-Keuls contrasts indicated that field-independent subjects scored higher than field-dependent subjects on both Base and Complex word problems, but that there was no difference between the two groups on the Visual problems. In addition, both groups had lower score on Complex word problems than on Base problems. This interaction is presented in Figure 1.

Insert Figure 1 About Here

For the Time X Problem Type interaction, Newman-Keuls contrasts indicated no difference between the timed and untimed conditions on word problems, although scores on the Base problems were again higher than scores on Complex problems. Subjects in the untimed condition scored higher on the visual problems than subjects in the untimed condition. This interaction is presented in Figure 2.

Insert Figure 2 About Here

Discussion

As predicted, field-independent subjects scored higher than field-dependent subjects on both Base and Complex word problems. Since the groups were comparable on required computational

skills, performance differences are likely to be the result of differences in approaching the task. Solution of word problems requires some problem restructuring prior to computation. Central to problem restructuring would be disembedding relevant information from the problem statement, and specifying a relationship among the relevant pieces of information. The higher scores of the field-independent subjects are likely to be the result of their superior ability to disembed relevant information.

The absence of an interaction between field dependence and problem complexity can be attributed primarily to lower than expected performance by field-independent subjects on the Complex problems. The use of multiple pieces of distracting numerical information may have added too much complexity to the task of disembedding relevant problem components. Additional research needs to be done to clarify the relationship between disembedding ability and problem complexity.

Although prior research has found that field-independent learners perform better than field-dependent learners on visual tasks that require subjects to overcome distracting perceptual cues (Karp, 1976; Witkin & Goodenough, 1977), no difference between the two groups was found on the current visual problems. This may be a function of the demands of the visual task. The visual problems required subjects to locate and interpret information presented in a visual context, but the information was not embedded in a distracting perceptual field. Thus, while solution of the word problems was facilitated by an analytic approach, solution of the visual problems did not require a high degree of restructuring.

Although significant effects for Time were found, none involved field dependence. This suggests that in the absence of any cues facilitating problem solution, allowing additional solution time did not help field-dependent subjects. The Time X Problem Type interaction was centered in

the higher solution rate for visual problems in the untimed condition. The most parsimonious explanation for this is that it represents an artifact of the particular time limits placed on the task.

Results of the present investigation are consistent with prior research in finding that field dependence is an important component in successful solution of mathematics problems. The nature of the effect, however, depends on the demands of the task. Field-independent learners appear to have an advantage on tasks which require disembedding information from a complex context prior to solution. However when information is readily available, and an analytic approach is not required for solution, field-dependent learners perform as well as field-independent learners. While some research exists suggesting ways to help field-dependent learners compensate for their difficulty in disembedding information from verbal materials (Bien, 1974; Frank, 1984; Ward & Clark, 1988), more research is needed in applying these techniques to school-based tasks. In the meantime, educators need to be sensitive to the effect of field dependence in interpreting performance on tasks with different analytic demands.

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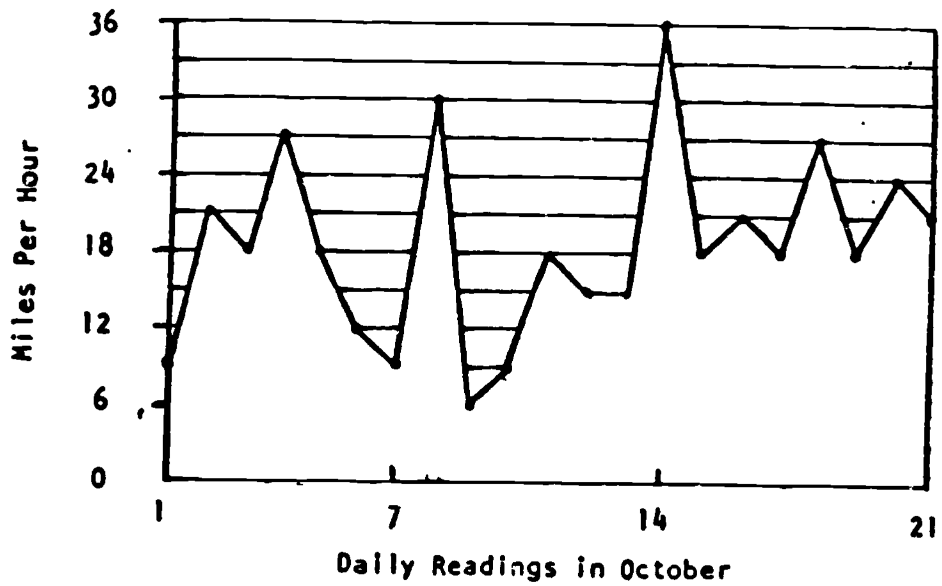
Table 1
Sample Problems

Base Word Problem: The East German olympic crew team left the boat house and rowed upstream at a rate of 8 mph. They returned downstream at a rate of 16 mph. If the trip took 1 1/2 hours, how far upstream did they row?

Complex Word Problem: The East German olympic crew team left the boat house and rowed upstream at a rate of 8 mph., a rate they can maintain for up to 10 miles. They returned downstream at a rate of 16 mph. with the help of a 7 mph. current. If the round trip took 1 1/2 hours, how far upstream did they row?

Visual Problem:



WIND SPEED READ DAILY AT 2:00 P.M. AT BEAVER STADIUM

Using the above graph, determine between which two consecutive recordings t e greatest change in wind speed occurred.

A) between Oct. 7 and Oct. 8

B) between Oct. 8 and Oct. 9

C) between Oct. 9 and Oct. 10

D) between Oct. 13 and Oct. 14

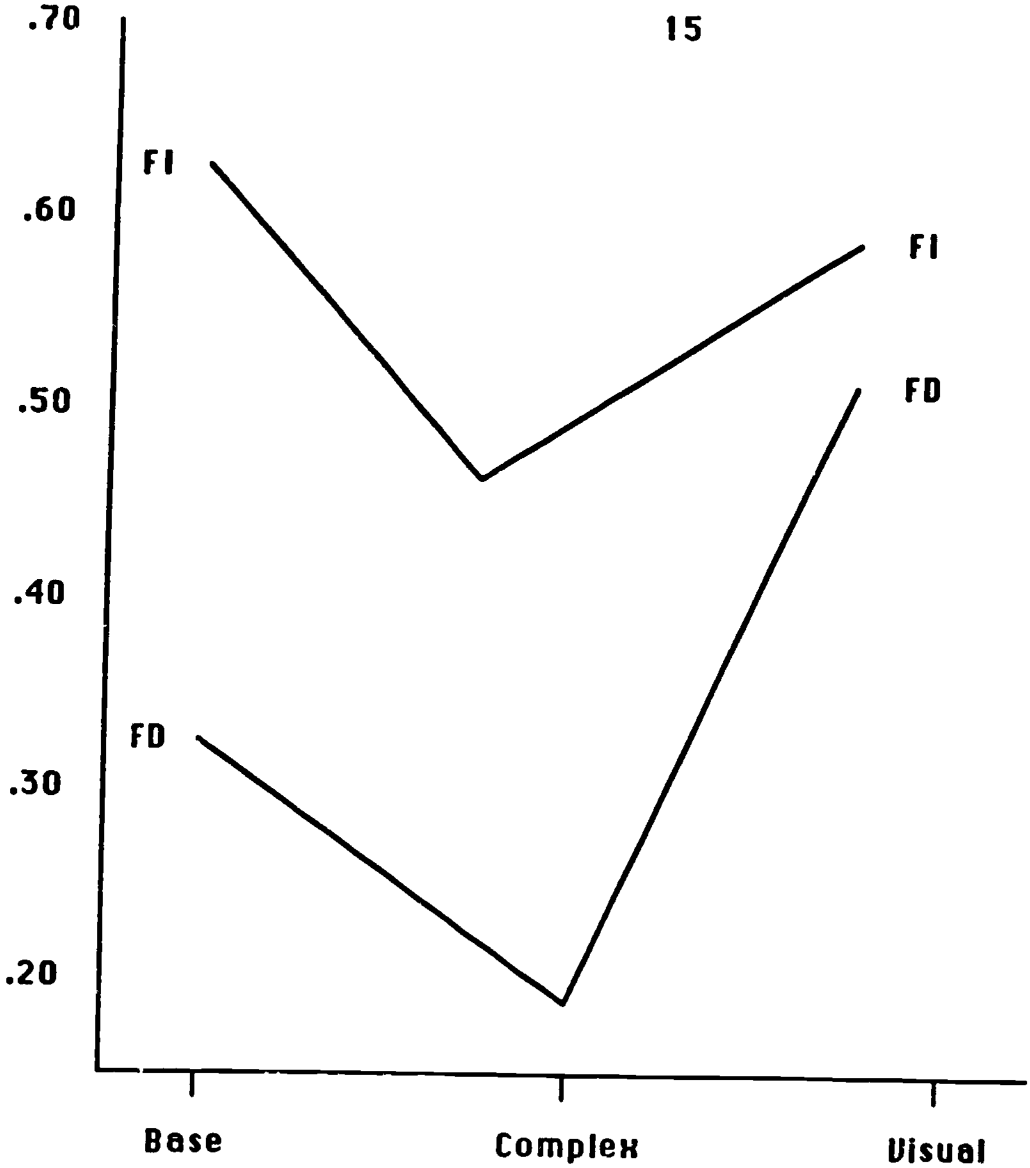
E) between Oct. 14 and Oct. 15

Table 2

Means and SD's of Percentage Scores on Base, Complex, and Visual Problems by Field Dependence and Time Condition

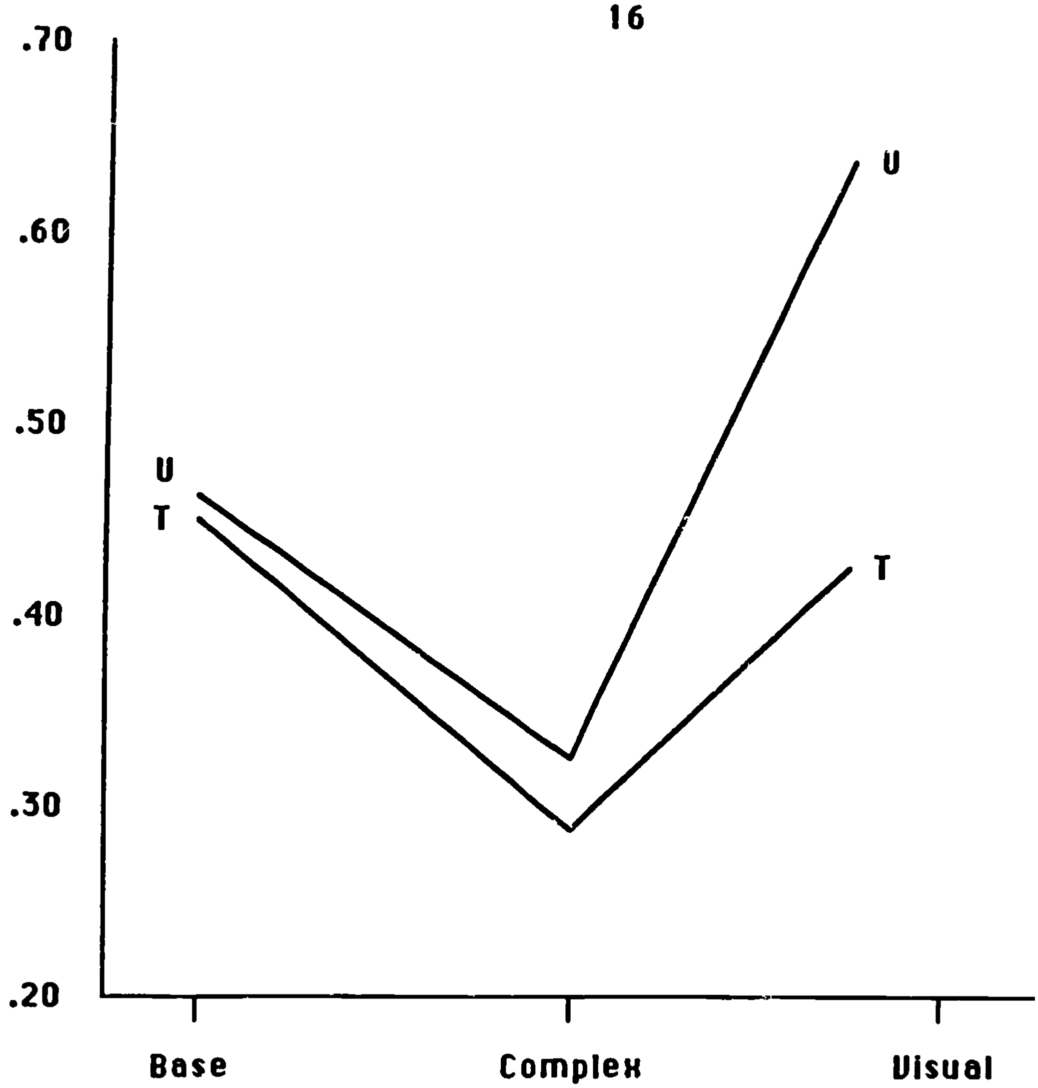
	Field-Dependent		Field-independent	
	Timed N=11	Untimed N=19	Timed N=10	Untimed N=15
Base	.348 (.139)	.305 (.168)	.567 (.245)	.664 (.166)
Complex	.174 (.102)	.195 (.147)	.408 (.154)	.492 (.199)
Visual	.416 (.243)	.579 (.221)	.429 (.178)	.695 (.194)

Note: Standard deviations are in parentheses.



Interaction between Field Dependence and Problem Type

Figure 1



Interaction between Time Condition and Problem Type

Figure 2