This paper examined a problem-solving partnership where both partners actively participated and where there were conflicting points of view. After several weeks of observation of an entire fifth grade class working in small groups, a dyad of two boys was selected for closer examination. Nonroutine mathematics problems were used as tasks. A videotape taken while they were working both cooperatively and individually was analyzed for interaction patterns, partnership roles, and problem solving strategies. Individual problem solving behaviors of the two boys were analyzed; one was identified as a risk taker and the other as a strategist. The interaction behaviors of the boys were discussed in terms of roles, competitive collaboration, and strategy change by partner. (YP)
An Examination of a Problem Solving Partnership from a Social Interaction Perspective

Sharon B. Erle
Florida State University
Department of Curriculum and Instruction
Tallahassee, Florida 32306
Cooperative learning as described by Slavin (1987) refers to a set of instructional methods in which students are encouraged or required to work together on academic tasks. Slavin identifies two cooperative learning perspectives, the developmental and the motivational. The fundamental assumption of the developmental perspective is that interaction among children around appropriate tasks increases their mastery of critical concepts or skills, whereas motivationalists are concerned with the reward or goal structures under which group members operate.

The developmental perspective on cooperative learning takes the position that task-focused interaction enhances learning or "making sense". The fundamental assumption of this perspective is that of face to face interaction (Slavin, 1987). The developmental perspective is based on Piagetian and Vygotskian theories.

Vygotsky acknowledged that there might be a difference between individual and social problem solving when he developed his notion of the zone of proximal development. Vygotsky (1978, p. 86) defines the zone of proximal development as "... the distance between the actual development level as determined by independent problem solving and the level of potential development at determined through problem solving under adult guidance"
or in collaboration with more capable peers."

In what Graves and Graves (1985) call the ecological approach to cooperative learning, learning is considered to be inextricably linked to the total social context and involves a dynamic in which all who interact with a particular student are influenced by and in turn influence his or her activity. Graves and Graves posited that, "learning in general, but particularly cooperative small-group learning, which involves coordination of effort with other, emerges out of the total social and physical environment within which the person is immersed" (Graves & Graves, 1985, p. 403).

Piaget (1969) claimed that peer interaction and social experiences derive their importance from the influence they can exert on equilibration through the introduction of cognitive conflict. The Genevan researcher Perret-Clermont (1980) conducted a series of experiments to examine the effect of peer collaboration on logical reasoning skills. Perret-Clermont concludes that peer interaction enhances the development of logical reasoning when cognitive conflict induces active cognitive reorganization, consistent with Piaget's theory about the role of social interaction in giving rise to cognitive conflict. Perret-Clermont claims that this cognitive conflict is most likely to occur where children have discrepant points of view. It is not sufficient to
merely engage children in joint activity, but that there must be confrontation.

Slavin (1987) reports that experimental evidence has provided little support for the position that peer interaction in itself will facilitate student achievement. However, one might argue that since traditional curricula used in these studies presented knowledge as isolated sets of facts, skills and procedures the tasks simply were not problematic enough to cause disequilibrium.

In an examination of contextual perspectives, Cobb (1988) emphasizes that learning is an interactive as well as a constructive activity and that opportunities to construct mathematical knowledge arise from attempts to resolve conflicting points of view. In a study by Smith et al (1981) results indicate that controversy promotes higher achievement, retention, and a greater search for information.

Motivationalists emphasize the cooperative reward structure as the critical element of cooperative learning rather than the quality of interactions among students in collaborative activities (Slavin, 1987). Researchers taking the motivational perspective suggest that goal structures create a situation in which group members can attain their own personal goals and that these incentives for group learning efforts are crucial. Yackel, Cobb and
Wood (1988) argue that external reward structures are only essential if one's goal is "to find ways to coerce students to complete mathematical activities that are of limited interest and relevance to them" (p. 8).

Interactions among peers focused on intellectual content can be placed on a continuum (Forman & Cazden, 1985). At the extreme end one child knows more than the others and acts as a peer tutor. In contrast, at the other end of the continuum, knowledge is equal and peer collaboration is expected. Peer collaboration requires a task in which the partners work together to produce something that neither could have produced alone. In a study of collaborative problem solving Forman (1981) found that the sophisticated problem-solving strategies the collaborators displayed were not as apparent when partners were asked to work individually.

While research on small group cooperative learning has increased during recent years, few studies has focused on the interaction processes occurring within groups (Webb, 1982). Forman and Cazden (1985) suggest that the lack of research in this area may be partly due to the fact that peer collaboration requires an environment far from that of the traditional classroom.

The purpose of this research was to examine closely a problem-solving partnership where partners both actively participate and where there are conflicting
points of view. This close examination was done in an attempt to explore the following questions: What roles do the partners play in the interaction? Does the interaction impact individual partners problem solving strategies? Are there differences between problem-solving activity when working individually rather than cooperatively?

This study lies within what Slavin (1985) calls the "second generation" research on cooperative learning which focuses less on basic effects and more on investigating the processes involved in cooperative learning. This study is compatible with the developmental perspective of cooperative learning since it focuses on the quality of interactions among students and not a goal structure.

Method

Observation

After several weeks of observation of an entire fifth grade class working in small groups on nonroutine mathematics tasks, a partnership was selected for closer examination. The dyad consisted of two boys, Dan and Thomas. This particular dyad exhibited a high degree of conflict compared to other dyads and triads in the class. Neither partner took on the role of peer tutor, but rather each boy seemed equal in the partnership.

This partnership was observed over a period of six
months in several settings. In addition to observation of the boys in the mathematics classroom, they were observed working with LOGO on computers. The mathematics classroom observations were done biweekly for a period of three months. The subjects were observed once weekly for the second three months while working with LOGO on open ended projects.

**Problem Solving Sessions**

For a closer examination of the partnership the boys were videotaped in problem solving sessions while working both cooperatively and individually. In the individual setting session the students were given identical tasks.

The tasks used were nonroutine mathematics problems. Nonroutine tasks were chosen since the dyad selected demonstrated an intense interest in challenging problems rather than routine problems during the classroom observations. Students were provided with manipulatives if they chose to use them, as well as paper and pencil.

The videotapes were analyzed for interaction patterns, partnership roles, and problem solving strategies. Individual setting tapes were analyzed to determine if problem-solving activity differed from that used by partners in the cooperative setting.
Individual Characteristics of the Subjects

Dan: The Risk Taker

Dan has a very out-going extroverted personality. He is extremely likable and has a good sense of humor. He is usually on task when working on classroom tasks relating to any subject, particularly mathematics. Dan always makes an attempt at a problem even when the problem is not particularly challenging or interesting to him.

Dan does not get upset if he cannot find a solution or if he makes an error. He always attempts a problem even if he doesn't understand the task. He is a risk taker. His "you don't know unless you try" attitude is demonstrated in the following episode.

Problem: Can you construct a rectangle that's not a square using any five of these tangram pieces?

Response: When Dan was moving around some pieces in a somewhat consistent manner, Thomas related that he was sure it was not the way Dan was attempting to construct the rectangle. Dan firmly responded, "Well, you don't know unless you try!"

Dan's computational skills are quite good and he does arithmetic quickly in his head. He is constantly talking when solving problems. One might say that he is extremely interactive with the problem itself. Dan has no difficulty counting in sixs, elevens, or even fifteens. He does two digit addition problems
mentally, doubles numbers quickly, and counts money very easily. Dan does seem to have some difficulty with division, usually Thomas helps when Dan is dividing.

Dan is also interactive when working on the computer. He is constantly making changes, starting new projects, and expanding on old projects. When shown a program he wants to know why the program works and will explore the procedure. When shown more squares, a recursive program that draws squares inside of squares, he reproduced the program making squares to the left as well as the right, giving a 3-D effect. For this same program he experimented with the variables. Some of the other students were interested in the visual effect but not in the program itself, and none of the other students explored the variable or added to the program.

Dan tends to not be very planful or systematic. Although usually successful, Dan is unorganized in his problem-solving. Although he always tries something there is often little strategy or organization, resulting in him repeatedly making the same errors, getting aggravated and sometimes confused in the process. This is apparent in the following episodes, where Dan was working alone.

Problem 1: How many ways can you put 15 tigers in 4 cages, so that no two cages have the same number of tigers?

Response: He immediately gives a response without the aid of paper, pencil, or manipulatives. This
first solution had the same number of tigers in two of the cages. He then used 15 cubes and placed them in piles. He shuffled the cubes around in a nonsystematic manner arriving at the same solutions repeatedly. He often got solutions with the same number in two cages. You could see his frustration as he attempted to arrive at solutions he had not yet found, and instead either getting ones already found or ones not satisfying the constraints of the problem.

Problem 2: 12 sacks of corn and 15 sacks of beans weigh 2835 pounds. Each sack of beans weighs the same. Each sack of corn weighs 130 pounds. What is the weight of each sack of beans? (Subject was given a calculator to aid in computation)

Response: Did not write down numbers, but put them in calculator so he could remember them, but since he couldn't perform operations and keep numbers on calculator at the same time, he ended up repeatedly asking for the number of bags or the weight of the corn. He was hesitant as to whether to divide or multiply first, and finally divided 130 by 12. When I asked him what 10.3333 represented he realized his error and multiplied instead. He didn't write down the product he obtained (1560), and had to remultiply because he couldn't remember the product. He then, after some hesitation, divided 2835 by 1560. Then he multiplied this quotient by 15. When I asked what this result represented he responded that it was the weight of a sack of beans, but that he was going to multiply this result by 15 because he thought I wanted the total weight of beans. On several occasions during the course of finding his solution, Doug appeared confused about what he needed to do and about what he obtained at intermediate steps.

When not working with Thomas, Dan is less enthusiastic and persistent. He is not as successful at arriving at correct solutions and spends much more time finding a solution. During the individual taping he did not solve, or correctly solve several of the tasks. When presented with a task similar to one he and Thomas had
previously solved successfully the following occurred.

Task: Can you make a rectangle with 247 cubes?
Response: Very briefly he shuffled around two flats and four longs and then he gave up. I suggested he trade a flat for some longs, and he unenthusiastically shuffled those around and gave up. I suggested he try a smaller number like 91. He half-heartedly moved some cubes around, got a rectangle with 88 and quit. During most of the episode he was singing, and seemed not very interested.

Dan's actions on this task were strikingly different than those exhibited by him when he worked on similar tasks the week before in his class with his classmate Thomas.

Throughout the majority of the individual videotaping Dan was extremely subdued. He was not enthusiastically attempting any of the tasks until he was told that Thomas had obtained more solutions to a problem then he had. This episode also makes apparent the strong competition Dan has with Thomas. Dan's individual persistence during classroom observations when it existed, may be attributed to the fact that even though he was working alone, Thomas was usually in close proximity.

Problem: How many ways can you put 15 tigers in 4 cages, so that no two cages have the same number of tigers?
Response: After arriving at 4 solutions, I told Dan that Thomas had found more. Dan livened right up, convinced he could beat Thomas. When Dan found the sixth solution, the thought that if he could find one more solution he would beat Thomas thoroughly delighted him. When he commented that he was getting aggravated, I asked him if it was because Thomas found some solutions that
he had not found or because there might be more solutions. Dan responded without hesitation that it was because "Thomas got some I didn't". When I informed him that Thomas had in fact found the same six solutions, he commented that he felt he had done all the work for nothing, as if he was more interested in the competition with Thomas than the actual solving of the problem.

When working together this strong competition is not as apparent, and they work cooperatively.

Thomas: The Strategist

Thomas has a more introverted personality. He is not on task when tasks are not problematic for him. He is much quieter than Dan when working on tasks. Thomas's arithmetic skills are good, and he is quick to see relationships. He is an insightful mathematics student. He does so well in mathematics, that many of the students look up to him.

He gets very upset with himself if he does something incorrectly. He is more cautious than Dan when problem-solving. He is also more careful with his computations. For instance, although his arithmetic skills are good, he does not compute as quickly as Dan, and is more likely to use paper and pencil.

Thomas is planful and systematic in his problem solving. He listens carefully, makes sure he interprets the problem correctly before he starts, and tends to be organized. This was demonstrated in the following two problems given during the individual session with Thomas.
Problem 1: How many ways can you put 15 tigers in 4 cages so that no two cages have the same number of tigers?

Response: Thomas drew 15 marks on the paper and began circling groups of marks. He realized cubes would be easier to work with, so he used 15 cubes. He obtained six solutions by systematically moving between groups, or by leaving one group alone and then separating the other groups into three piles of different numbers of cubes. He obtained six solutions very quickly.

Problem 2: 12 sacks of corn and 15 sacks of bears weigh 2835 pounds. Each sack of beans weighs the same. Each sack of corn weighs 130 pounds. What is the weight of each sack of beans?

Response: He had a clear plan that he carried through. He first multiplied 130 by 12, subtracted this result from 2835, and then divided by 12. This was all done in a self-assured manner, confident that this was correct.

Thomas is reluctant to make an attempt when he does not know what to do and had to be coaxed by the interviewer to make guesses when interviewed alone. When he does not have a clear plan he tends to not be persistent. This was demonstrated in the Clear Lake problem.

Problem: The surface of Clear Lake is 35 feet above the surface of Blue Lake. Clear Lake is twice as deep as Blue Lake. The bottom of Clear Lake is 12 feet above the bottom of Blue Lake. How deep is Clear Lake?

Response: Uses lines on the paper for a scale, appearing confident in the beginning of attempt. But after drawing the bottoms of Clear Lake and Blue Lake 12 feet apart using his scale strategy, he seemed to lose his enthusiasm for the problem, perhaps realizing that scale would not give the solution. Even after I lead him through with a guess, he did not make another guess. I had to offer another "What if Clear Lake is ____ feet deep?" Finally, he abandoned the attempt.
Thomas's reluctance to take risks is exhibited in his computer activity also. He is cautious when working with LOGO. He much prefers the direct mode where he can have immediate feedback. Writing procedures is higher risk, a procedure may not do what you expect and require correction. When Thomas finally started working in the indirect mode he would write everything in direct mode first, and then go to the flip page and copy the same commands into a procedure. After two months of encouraging Thomas to try writing procedures, he wrote a program for a UFO, complete with blinking lights. What was important about this program was that Thomas was trying changes within the procedure. He was troubleshooting, and debugging and no longer felt a need to use the direct mode to try things first.

Thomas does not use the setpos command to position the turtle on the screen, he prefers to move the turtle with forward, backward, right and left commands. In this way he can always see where the turtle is in relation to where he wants it to be. The setpos command is much higher risk, the turtle could end up somewhere other than where he thought it would be.

Related to the risk factor is Thomas's tendency to get upset with himself when he can't remember what he's done or if he gets an incorrect result. His classroom teacher related that on occasion when Thomas did not perform
to his expectations he has cried. During the following episode Thomas spent much of the time muttering about an incorrect answer.

Problem: A hardware store sells both tricycles and bicycles. One day I counted 37 wheels in the store. How many tricycles and bicycles were in the store?

Solution: Thomas suggested 16 tricycles and 17 bicycles as a solution. When Dan responded with, "Huh? What is '6 x 3?'", Thomas slapped his face and said "Oh, I didn't get that right!". Throughout Dan's counting to get his first solution Thomas kept muttering, and after Dan got his solution, Thomas said "I don't know how I came up with that answer, that's what gets me." Even after reaching a correct solution he still seemed preoccupied with his first incorrect solution and the fact that he couldn't figure out how he got that answer.

The Interaction

Partnership Roles

During the course of the months of observation and examination of the interview sessions, some clear roles emerged. The behavior exhibited by the boys interacting with the computer was consistent with that exhibited in mathematics problem-solving.

Dan will solve problems in an unorganized, unsystematic manner, relying on arithmetic computations, doing arithmetic quickly until he finds solution. When working with Thomas he has a continuous dialogue of computations going on. Thomas in contrast will usually try by using a strategy of some sort after some thought. He most often is quiet while working on the task. For example, working together they obtained
solutions to the following problem by very different methods.

Problem: A hardware store sells both tricycles and bicycles. One day I counted 37 wheels in the store. How many tricycles and bicycles were in the store?

Dan: After just randomly trying some numbers, Dan started counting by twos until he reached 34, and then said 17 bikes and 1 trike. Later tried to find another solution by counting in groups of three. Even though Dan took the cubes he never used them. Thoughout the problem he had a continuous computation commentary.

Thomas: Obtained 8 bikes and 7 trikes by using cubes and grouping in 5's, where there were two groups of 5 or two of each kind on a length of ten cubes, until he reached thirty, and then grouped the 7 cubes that were left. Throughout most of the problem episode Thomas was quiet while working with the cubes.

In most problems the two boys took on roles of Dan being the calculator and Thomas being the strategist. Thomas is aware of how quickly Dan can compute and relies on him to do the computing. These roles are demonstrated in the following episod.--.

Problem: Mark gets $1.85 a week for an allowance. He gets 16 coins. He always gets only nickels, dimes, and quarters. How many of each does he get?

Solution: When the number of coins was correct but the amount of money incorrect, Thomas was making exchanges to change the number of coins but keep the same amount of money. While Thomas would count the number of coins, Dan counted the amount of money. Thomas would use his exchange strategy, and then Dan would immediately start counting money, almost as if he had received a cue.

Dan's calculator role was apparent throughout the videotape sessions. Thomas looked to him to do the
computing, and on some occasions when unsure of a product or sum, would rely on Dan for the result. Since Dan's quick calculations were sometimes wrong, Thomas would inform Dan of a questionable computation.

The roles that these two play in the partnership complement one another. Dan is not planful whereas Thomas is planful. While working together Thomas keeps Dan's unsystematic manner of solving problems in check. When working alone Dan's lack of planning causes him to get frustrated, the problem-solving becomes hectic and haphazard. Dan would spend more time and effort to solve a problem alone than with Thomas. For example, in both coin problems when Dan would want to abandon an attempt and start all over, Thomas would build on the closeness of the answer that they had obtained by making strategic exchanges.

The risk taking aspect of their collaboration is also complementary. Dan is a risk taker whereas Thomas is not. Dan is not afraid to make errors whereas Thomas is. Although Thomas would be a successful problem-solver without Dan, he would be a lot more cautious, and may not attempt to solve a problem if he does not have a plan. Where Thomas may not make an attempt if unsure of a strategy or method of solution, Dan would probably take the initiative and start doing something. Once Dan makes an attempt, I feel it's difficult for Thomas not to
become involved. For example, Thomas was relatively uninvolved in the following problem until Dan, through his persistence, made an important discovery to aid in the solution.

Problem: I built a fence around a square field using 48 posts. I placed the posts 5 meters apart. What is the area of the field bounded by the fence?

Response: While Dan was busily calculating and drawing, Thomas was quiet and appeared thoughtful but was not actively involved with the problem. He assembled cubes for Dan, but let Dan remain in control of the problem-solving. However, when Dan counted 13 on a side when they put two lengths of 12 together, Thomas became more involved. When Dan commented, "Oh, you can't count these twice", referring to the endposts, Thomas became a partner in the solution, offering suggestions, and conjectures. He was now working actively with Dan on finding the area.

A Competitive Collaboration

These two boys cooperate well together. Although it was apparent in the separate interview with Dan that he feels strongly competitive with Thomas, their collaboration is not argumentative. Their competition is cooperatively shared, they both take pleasure in arriving at the same number of solutions, and congratulate each other. In the following episode, they both found the same number of solutions after a lengthy effort at solving the problem.

Problem: Find the number of ways in which 20 coins consisting of quarters, dimes and nickels can have a value of $3.10.
Response: First Dan found a solution, by as he put it "getting lucky". Then Thomas found a solution. Dan found another solution. After the third solution, Dan seemed to be getting tired of the problem, so I asked if they wanted to quit. Apparently, Thomas was close to getting his second solution, so he did not want to stop. When he counted he had $3.00 and 19 coins, so Dan handed him a dime. With pride he commented, "We each got two!"

Sometimes Dan plays the dominant role in the partnership and other times Thomas will take over and conduct the problem-solving. During the course of a problem, the directorship may shift several times. In the above problem, first they worked together, then separately, then together, then separately. Although Dan has a more outgoing dominant personality, Thomas directs when necessary, especially when he knows he is correct.

Although sometimes when together as a team they work independently, they both seem more enthusiastic about tasks when together. This is particularly true of Dan. Dan is not only more successful when he works with Thomas, but he is more enthusiastic and persistent. One could argue that Dan has a certain dependence on Thomas, however, when working together Dan is in collaboration with Thomas and we do not see a dependence mathematically. The dependence is social in nature. Dan does not depend on Thomas to show him how to solve a problem. Perhaps the social interdependence has to do with the competition Dan feels with Thomas, and that he admires Thomas's mathematical abilities and in the partnership he can feel equal to Thomas.
Conclusions

The findings are consistent with Perret-Clermont's hypotheses regarding the occurrence of cognitive conflict as a result of conflicting perspectives. In the case of this peer interaction the roles taken on by the partners where not only contrasting but where complementary. Thomas is the strategist; Dan the calculator. Dan is not planful; Thomas is planful. Dan is a risk taker; Thomas is not a risk taker. The complementary nature of the roles allows the partners to solve the problems together before they could solve them alone.

Although Thomas is a successful problem-solver without Dan, he's a lot more cautious and without a plan does not attempt to solve a problem. Dan's "you don't you undless you try" attitude keeps Thomas's need to be successful in check.

Dan is more persistent and enthusiastic when working with Thomas. Although, they are both able mathematics students, Dan tends to be more successful when collaborating with Thomas. Dan gets more satisfaction from solving problems when collaborating with Thomas or when in competition with Thomas.

Dan also benefits from Thomas's systematic strategies. When Dan is tempted to abandon a solution, Thomas will build on the closeness of that solution. Dan picks up the use of strategies, such as the exchange of coins to make less
coins which have the same monetary value. This may prevent Dan from becoming frustrated and hectic during his problem-solving.

Since the classroom is not a social vacuum, it seems reasonable to assume that the social dynamics of the classroom play an important role in how students construct their mathematics. In this study a partnership was examined in which, at least for problem-solving, the social aspects and roles played by the partners were critical to the dynamics and energy of the partnership. The discrepant perspectives or complementary nature of the partnership roles contributed to the conflict and excitement.

The results suggest that placing students in groups where there will be conflicting perspectives may indeed lead to disequilibrium and cognitive reorganization and consequently learning. Further, the social interaction can impact each partner's strategies for carrying out the tasks themselves.
References


Yackel, E., Cobb, P. & Wood, T. (1988). Small group interactions as a source of learning opportunities in second grade mathematics. (NSF Grant No. MDR-874-0400), Purdue University, Education Department, West Lafayette, IN.