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ABSTRACT

For almost 20 years, student achievement in the United States has been measured by the National Assessment of Educational Progress (NAEP) reported in recent years as "The Nation's Report Card." Based on a sample size of 25,000 students at each of three grade levels, it has provided the only data that reflect in a comprehensive way what students in the United States know and can do in various subject areas. It has provided: (1) descriptive information about student strengths and weaknesses in basic and higher order skills; (2) data comparing groups of students by race and ethnicity, gender, type of community, and region; (3) trend data reflecting the ups and downs of performance over the years; and (4) correlations between achievement and some student experience variables. It has been recommended that this assessment be expanded by NAEP to provide state-by-state comparisons. The purpose of the National Assessment Planing Project is to: (1) provide a general design for national assessment; (2) make recommendations which address state and local questions; and (3) describe the mathematical abilities and content topics on the 1990 National Assessment in Mathematics. (CW)

ED309990

ASSESSING MATHEMATICS IN 1990 BY THE NATIONAL ASSESSMENT OF EDUCATIONAL PROGRESS

CHIEF STATE SCHOOL OFFICERS • COUNCIL •

CCSSO

RECOMMENDATIONS FROM THE
NATIONAL ASSESSMENT PLANNING PROJECT
COUNCIL OF CHIEF STATE SCHOOL OFFICERS
379 HALL OF THE STATES
400 NORTH CAPITOL STREET, NW
WASHINGTON, DC 20001

MARCH 1988

SE 050 832

The Council of Chief State School Officers (CCSSO) is a nationwide non-profit organization of the 56 public officials who head departments of public education in every state, U.S. Territory, and the District of Columbia. CCSSO seeks its members' consensus on major education issues and expresses their views to civic and professional organizations, to federal agencies, to Congress, and to the public. Through its structure of standing and special committees, the Council responds to a broad range of concerns about education and provides leadership on major education issues.

Because the Council represents the chief education administrator in each state and territory, it has access to the educational and governmental establishments in each state, and the national influence that accompanies this unique position. CCSSO forms coalitions with many other educational organizations, and is able to provide leadership for a variety of policy concerns that affect elementary and secondary education. Thus, CCSSO members are able to act cooperatively on matters vital to the education of America's young people.

The State Education Assessment Center was founded by CCSSO in 1985 to provide a locus for leadership by the states to improve the monitoring and assessment of education. This is a report of the Assessment Center's National Assessment Planning Project.

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The National Assessment Planning Project, conducted under the auspices of the Council of Chief State School Officers (CCSSO), was created by a consortium of eighteen national organizations interested in education and in exploring the feasibility of expanding the National Assessment of Educational Progress (NAEP) in order to produce state-by-state comparisons of student achievement.

The project is governed by a Steering Committee (Appendix A). Each member was appointed on the recommendation of an organization in the consortium. This publication was conveyed to the Department of Education (ED) and to the National Assessment of Educational Progress on the review and approval of the Steering Committee. The publication, however, does not necessarily reflect the views of each of the associations in the consortium.

The project was supported by Grant No. SPA-1549 from the National Science Foundation (NSF) with funds partly from NSF and partly from the Department of Education through an inter-governmental transfer from ED to NSF. The mutual interest of the two agencies in this project and their willingness to provide joint support made the project possible. This publication, however, does not necessarily reflect the views of either agency.

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I. INTRODUCTION

PURPOSE/HISTORICAL PERSPECTIVE

For almost twenty years, student achievement in the United States has been measured by the National Assessment of Educational Progress (NAEP) reported in recent years as "The Nation's Report Card." Based on a sample size of 25,000 students at each of three grade levels, it has provided the only data that reflect in a comprehensive way what students in the U.S. know and can do in various subject areas. It has provided descriptive information about student strengths and weaknesses in basic and higher order skills; it has provided data comparing groups of students by race and ethnicity, gender, type of community and region; it has provided trend data reflecting the ups and downs of performance over the years; it has reported correlations between achievement and some student experience variables.

In 1990, a large number of states will be able to participate in a state-level assessment that will provide "state report cards" which will make state-by-state or state-to-nation comparisons of student achievement in mathematics. Interest in state-level comparisons is rising in many quarters. In 1984, a majority of CCSSO members supported the development of a system of student assessment that would provide state comparisons, and in 1985, the members endorsed the expansion of NAEP as the most feasible vehicle. During the 1986 national assessment, two individual states, Wyoming and Georgia, contracted with NAEP to conduct an in-state concurrent assessment and provide state-to-nation comparisons. Also in 1986, and 1987, groups of southern states, in a project coordinated by the Southern Region Education Board, contracted with NAEP to conduct an assessment in mathematics and provide a report comparing achievement among those states. Some of the rising interest can be attributed to governors. The 1987 report from the National Governors' Association, called Results in Education, presented a number of education comparison indicators and displayed a blank column for achievement, clearly expressing the intent to fill in that column in future years with achievement data. In a 1987 report, a national group appointed by the Secretary of Education, William Bennett, and chaired by the former governor of Tennessee, Lamar Alexander, made a series of recommendations on the future of NAEP. A major one was that NAEP should be expanded to provide state-by-state comparisons.

This rising interest is not without its critics. Some worry that Federal, state and local policymakers may misuse the data, drawing inappropriate inferences and unwarranted cause and effect relationships. Some fear that the test will be very influential, and that such influence will foster a national curriculum. Still others fear that the compromises made on test objectives will result in an assessment that measures the least common denominator and discourages badly needed curriculum reform.

Providing a general design for a national assessment that would not only be constructive but also minimize potential disadvantages is the purpose of this National Assessment Planning Project. The project is to make recommendations that answer questions of major interest to state and local educators and policymakers, who at some time prior to 1990, will be asked whether they want a report card for their state. They will likely want to know what educational content topics the assessment will measure so they can determine whether those content topics are more or less compatible with what the schools in their states are trying to teach. This report attempts to answer that question by describing what mathematical abilities and content topics the 1990 National Assessment in Mathematics will measure.

DEVELOPMENT PROCESS FOR THE 1990 MATHEMATICS ASSESSMENT

This report contains the educational mathematical abilities and content topics for the 1990 national assessment in mathematics. In part, the development process which resulted in this report was patterned after the consensus process that evolved over the years in planning prior national assessments in response to the language in Public Law 98-511, Section 405 (E) authorizing NAEP. It requires that "each learning area assessment shall have goal statements devised through a national consensus approach, providing for active participation of teachers, curriculum specialists, subject matter specialists, local school administrators, parents and members of the general public." In addition, however, this report was developed on the premise that the 1990 assessment in mathematics will provide state-by-state comparisons of student achievement. Because state report cards, as well as "The Nation's Report Card," will result from this assessment, the process was expanded considerably beyond recent practice to ensure that careful attention was given to the formal mathematics objectives of states and a sampling of local districts, and to the opinions of practitioners at the state and local levels of what should be assessed.

The process, carried out between August 1987 and March 1988, had the following features:

A Steering Committee with members recommended by each of the 18 national organizations representing policymakers, practitioners and citizens met, modified and approved the overall plan of work (Appendix A).

A Mathematics Objectives Committee was created to draft the report. Its membership consisted of a teacher, a local school administrator, state mathematics education specialists, mathematicians, parents and citizens (Appendix B). It met once for preliminary planning to consider and determine what information it would need prior to its major work session in December. For its review and consideration, the committee was provided the following:

- Math 1985-86 Assessment Objectives (NAEP Booklet).
- Curriculum and Evaluation Standards for School Mathematics (NCTM Draft).
- Content analyses of state and local mathematics guides.
- Suggestions solicited from state mathematics specialists.
- Report on "Issues in the Field" based on telephone interviews with leading mathematics educators.
- Draft framework provided by a subcommittee.

During a December meeting, the Mathematics Objectives Committee revised the sub-committee's draft. In subsequent weeks, the draft report was edited for form, sample questions were added and a copy was mailed to each state department of education mathematics specialist in the fifty states. These individual specialists were requested to convene a committee of state and local people (Appendix C). Those committees reviewed the draft report and returned comments and suggestions to the project staff. A copy of the draft report was also sent to and comments received from twenty-five mathematics educators and mathematics scholars (Appendix D). The reactions were given to the Mathematics Objectives

Committee which met a third time in late February. Modifications were made in response to the comments and the final draft report was reviewed, modified and approved by the Steering Committee in mid-March.

STEERING COMMITTEE POLICY

At its initial meeting in March, the Project's Steering Committee adopted a policy statement on the purpose of state comparisons and the conditions that should be met as follows:

> The purpose of State Level Student Achievement Comparison is to provide data on student performance to assist policymakers and educators to work toward the improvement of education. Such data can be useful by encouraging and contributing to a discussion of the quality of education and the conditions that determine it.
>
> State-comparative achievement data are useful if they:
>
> - o Represent performance on a consensus of what is important to learn;
> - o Use sound testing and psychometric practices;
> - o Use procedures that minimize intrusion into instructional time;
> - o Take into account different circumstances and needs that the states face; and
> - o Are associated with features of the school systems that can be improved by policymakers and educators.

ASSESSMENT DESIGN PRINCIPLES

Several major "design principles" emerged during the discussions of the Mathematics Objectives Committee and became the basis for making decisions. One is that a national assessment that will provide state-level comparisons must be comprehensive or inclusive in the scope of its mathematical abilities and content topics. The design of an assessment that produces state report cards must not be a "least common denominator" of only that which is in the objectives of all the states and thereby discourage desirable curriculum development. Nor is it a "least common denominator" of individual student preparation; therefore the assessment includes some items at the grade 12 level appropriate only for college-bound students. It must allay the fear that the assessment might be used to steer instruction toward one particular pedagogical or philosophical viewpoint to the exclusion of other viewpoints that are widely held. A national assessment cannot measure everything. It can embrace different views when they are prominent. The content topics of the assessment need to be inclusive of the curriculum areas of the various states, inclusive of what various scholars, practitioners, and interested citizens believe should be in the curriculum, and inclusive of some of the objectives that were tested in previous assessments so trend data can be provided.

It is appropriate to acknowledge that as efforts were made to produce an inclusive set of content topics, major attention was given to three frames of reference. One was the objectives of the states as reflected through several analyses of state mathematics curriculum guides and through the recommendations of state department of education mathematics specialists and the state committees they convened to react to a draft document. A second frame of reference was the recently issued draft of the Curriculum and Evaluation Standards for School Mathematics developed by the National Council of Teachers of Mathematics. Produced through intensive work by many leading mathematics educators in the United States, it is a significant statement on what mathematics should be taught in the schools. The third major frame of reference was the set of objectives on which the 1986 NAEP in mathematics was based.

A second design principle was that the overall outline or matrix needed to be relatively simple. Complexity could be reflected through definitions and through specific topics in content areas. The 1986 NAEP in mathematics had seven content and five process areas for a matrix of thirty-five cells. Its complexity mitigated against easy understanding. In addition, the weighting assigned to various cells resulted in too few exercises in some of the cells to provide reliable measure of student knowledge and skills.

A third design principle, addressed to those who wrote and field tested the exercises, was that the content areas and mathematical ability categories are not discrete, or mutually exclusive. In application, one mathematical ability is integrated with another mathematical ability and one content area is integrated with another content area. The application of mathematics is far more holistic than a matrix implies. Of necessity, some items will be multi-dimensional; by intent, some items should be multi-dimensional and designed to cut across content columns.

MATHEMATICS ASSESSMENT

THE FRAMEWORK

The mathematics assessment framework is organized by mathematical abilities and content areas. The mathematical abilities assessed are:

Conceptual Understanding
Procedural Knowledge
Problem Solving

Content is primarily from elementary and secondary school mathematics up to but not including calculus. The mathematics content areas assessed are:

Numbers and Operations
Measurement
Geometry
Data Analysis, Statistics, and Probability
Algebra and Functions

Framework for the Fifth Assessment

CONTENT AREAS

MATHEMATICAL ABILITIES	Numbers & Operations	Measurement	Geometry	Data Analysis Statistics & Probability	Algebra & Functions
Conceptual Understanding					
Procedural Knowledge					
Problem Solving					

Figure 1.

The framework for the Fifth Mathematics Assessment is given by the matrix in Figure 1, which shows the relationship between the three mathematical abilities and the five content areas. The weighting and detailed descriptions of the individual mathematical abilities and content areas are given later in this document.

WEIGHTING

The percentage of items in an assessment that address the various mathematical abilities and content areas is an important feature of assessment design because such "weighting" reflects the importance or value assigned to each area for the different grade levels. Over the four previous mathematics assessments, the percentage distribution of items in each area has changed. The approximate percentage distribution of items for the 1990 assessment also reflects changes. More emphasis is given to problem solving and less to procedural knowledge. More emphasis is given to geometry, data analysis, and algebra and functions, and less to numbers and operations. While numbers and operations is to continue to be the most heavily "weighted" content area for grades 4 and 8, numbers and operations has been replaced by algebra and functions as the area with the highest percentage of items for grade 12 in the 1990 assessment. The approximate percentage distribution of exercises by mathematical abilities, content and grade is in the following tables.

It is important to remember this weighting while reading the individual content area descriptions. Although these descriptions all have similar lengths and approximately the same number of sample questions are given for each area, on any test there maybe many more questions in one area than in another. For example, at grade 12 there would be almost four times as many items classified "algebra and functions" as "measurement."

TABLE 1: PERCENTAGE DISTRIBUTION OF EXERCISES BY GRADE AND MATHEMATICAL ABILITIES

Mathematical Abilities	Grade 4	Grade 8	Grade 12
Conceptual Understanding	40	40	40
Procedural Knowledge	30	30	30
Problem Solving	30	30	30

TABLE 2: PERCENTAGE DISTRIBUTION OF EXERCISES BY GRADE AND CONTENT AREA

Content	Grade 4	Grade 8	Grade 12
A. Numbers and Operations	45	30	10
B. Measurement	20	15	10
C. Geometry	15	20	25
D. Data Analysis, Statistics & Probability	10	15	20
E. Algebra & Functions	10	20	35

MATHEMATICAL ABILITIES

Most aspects of students' mathematical abilities can be classified into three categories: conceptual understanding, procedural knowledge, and problem solving (see Figure 2). This classification is not meant to be hierarchical; questions within any of the three categories may be complex or relatively simple. Problem solving is an interaction of conceptual knowledge and procedural skills at a given level, but a complex problem at one grade level could well be considered conceptual understanding or procedural knowledge for a different level. The same concept or skill can be tested in a variety of different representations, with or without tables, pictures, verbal descriptions, or other cues. The context of a question thus determines its category area in subtle ways.

CONCEPTUAL UNDERSTANDING

Students demonstrate conceptual understanding in mathematics when they provide evidence of their ability to label and define concepts; recognize and generate examples and counter examples; use models, diagrams and symbols to represent concepts; translate from one mode of representation to another; compare, contrast and integrate related concepts; interpret assumptions and relationships and communicate the meanings of these concepts to others. Conceptual understanding is essential to performing procedures in a meaningful way and to applying them to the solutions of problems.

PROCEDURAL KNOWLEDGE

Students demonstrate procedural knowledge in mathematics when they provide evidence of their ability to select the appropriate procedure and apply it correctly, verify results, give reasons for steps in a procedure, and extend or modify a given procedure to meet special needs. Procedural knowledge includes the many algorithms in mathematics that have been created or generated as tools to meet specific needs in an efficient manner. For a given procedure it also involves understanding the logic in the sequence of the steps, purpose, scope of applicability and its relation to other procedures. It requires that students know the signs, symbols, and terms commonly used with respect to a particular content domain and to the rules of operation commonly used to manipulate those symbols.

PROBLEM SOLVING

In problem solving, students are required to utilize their reasoning and analytic skills when they encounter new or unexpected situations. Students demonstrate an ability to use mathematical knowledge for problem solving when they can recognize and solve problems by identifying the key features and questions in a given situation; solve problems by selecting and using appropriate strategies, models and relevant mathematics; and verify and generalize solutions to solve a problem for which they do not have a simple procedural algorithm. Problem solving involves selecting appropriate strategies and applying conceptual understanding and procedural knowledge to produce an answer.

Mathematical Abilities

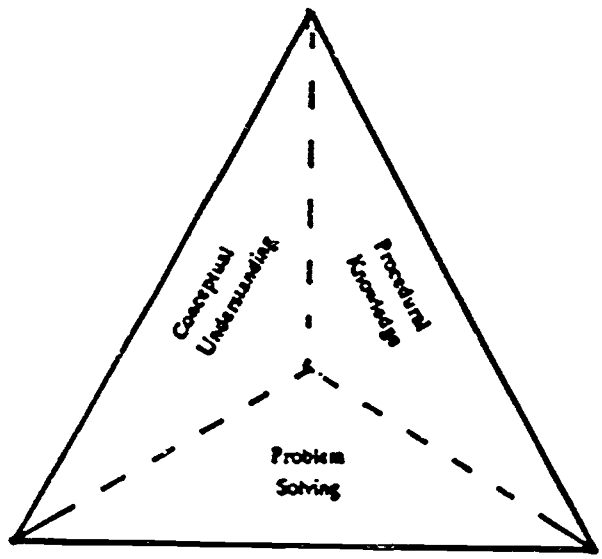


Figure 2.

CONTENT AREAS

It is necessary to assess students' mathematical abilities in each content area. However, classification into the content area cannot be exact and it inevitably involves some overlap. For example, Discrete Mathematics, which was a separate content area on previous NAEP objective descriptions, has topics appearing under both "Data Analysis, Statistics and Probability" and "Algebra and Functions." Individual questions will cut across content areas. Context can also determine content area; a question asking for the area of some geometric figure can be considered "Measurement" as easily as "Geometry," depending on the representation of the problem.

Even the classification into grade levels has some latitude. Mathematics is vertically structured; a grade 12 question can draw on grade 4 skills. In addition, some items may even appear on more than one grade level test. For example, there could be a question that may be answered correctly by 20% of the grade 4 population and 50% of the grade 8 population. Such content area questions are important for analyzing trends in the development of students' mathematical abilities.

A. NUMBERS AND OPERATIONS

This content area focuses on understanding of numbers (whole numbers, fractions, decimals, integers) and their application to the real world, as well as on computational and estimation situations. Understanding of numerical relationships as expressed in ratios, proportions and percents is also emphasized. Students' abilities in estimation, mental computations, use of calculators, generalization of numerical patterns, and verification of results are assessed.

TOPICS	GRADES 4	8	12
1. Relate counting, grouping, and place value.	●	●	●
2. Represent numbers and operations using models, diagrams, and symbols.	●	●	●
3. Read, write, rename, and compare whole numbers, decimals, fractions.	●	●	●
4. Compute with whole numbers.	●	●	●
5. Make estimates appropriate to a given problem.	●	●	●
6. Verify solutions and determine the reasonableness of a result.	●	●	●
7. Compute with fractions, signed numbers, decimals, and numbers expressed in scientific notation.		●	●
8. Apply ratios, proportions, and percents in a variety of situations.		●	●
9. Use elementary number theory including divisibility and factorization.		●	●

B. MEASUREMENT

This content area focuses on the ability of students to describe real world objects using numbers. Students will identify attributes, select appropriate units, and apply measures to communicate ideas so that they are understandable to others. Students will be required to read instruments with emphasis on precision and accuracy using metric, customary or non-standard units. Estimates, measurements, and applications of measurements of length, time, money, temperature, mass/weight, area, volume, capacity, and angles are included.

TOPICS		GRADES 4	8	12
1.	Compare objects with respect to a given attribute.	●	●	●
2.	Select and use appropriate measurement instruments.	●	●	●
3.	Select and use appropriate units of measurement.	●	●	●
4.	Determine perimeter, area, volume, and surface area.	●	●	●
5.	Estimate the size of an object or of a measurement.	●	●	●
6.	Apply common measurement formulas.		●	●
7.	Convert from one measurement to another within the same system.		●	●
8.	Determine precision, accuracy, and error of measurement.		●	●
9.	Make and read scale drawings.		●	●

C. GEOMETRY

This content area focuses on geometric knowledge, relationships, and skills which are important at all levels of schooling as well as in the real world. Students need to be able to model and visualize geometric figures in one, two, and three dimensions as well as communicate geometric ideas.

	TOPICS	GRADES 4	8	12
1.	Describe, compare, and classify geometric figures.	●	●	●
2.	Given descriptive information, visualize, draw and construct geometric figures.	●	●	●
3.	Investigate and predict results of combining, subdividing, and changing shapes.	●	●	●
4.	Identify the relationship between a figure and its image under transformation.	●	●	●
5.	Describe the intersection of two or more geometric figures.		●	●
6.	Classify figures in terms of congruence and similarity and apply these relationships.		●	●
7.	Apply geometric properties and relationships in solving problems.		●	●
8.	Establish relationships involving geometric concepts.		●	●
9.	Represent geometric figures and properties algebraically using coordinates and vectors.			●
10.	Represent problem situations with geometric models and apply properties of figures.			●

D. DATA ANALYSIS, STATISTICS, AND PROBABILITY

This content area focuses on the importance of data analysis and the representation of data across all disciplines, and reflects the prevalence of these activities in our society. Statistical knowledge and the ability to interpret data are necessary qualities for students to possess in the contemporary world.

TOPICS		GRADES 4	8	12
1.	Read, interpret and make predictions using tables and graphs.	•	•	•
2.	Organize and display data and make inferences.	•	•	•
3.	Determine the probability of a simple event.	•	•	•
4.	Compute measures of central tendency and dispersion.		•	•
5.	Recognize sampling, randomness, and bias in data collection.		•	•
6.	Recognize the use and misuse of statistics in our society.		•	•
7.	Estimate probabilities by use of simulations.		•	•
8.	Design a statistical experiment to study a problem and communicate the outcomes.		•	•
9.	Use formulas for combinations, permutations, and other counting techniques to determine the number of ways an event(s) can occur.			•
10.	Fit a line or curve to a set of data and use this line or curve to make predictions about the data.			•
11.	Apply the basic concepts of probability, including independent/dependent events, simple/compound events, and conditional probability.			•
12.	Use measures of central tendency, correlation, dispersion, and shapes of distributions to describe statistical relationships.			•

E. ALGEBRA AND FUNCTIONS

This content area is broad in scope, covering a significant portion of the 9-12 curriculum including algebra, elementary functions (pre-calculus), trigonometry, and some topics from discrete mathematics. At the K-4 and 5-8 grade levels, algebraic and function concepts are treated in more informal, explorative ways. This area includes not only manipulative facility but also an emphasis on conceptual understanding, on algebra as a means of representation, and on algebraic processing as a problem-solving tool. Functions are viewed not only in terms of algebraic formulas but also in terms of verbal descriptions, tables of values, and graphs.

TOPICS		GRADES 4	8	12
1.	Describe, extend, and create a wide variety of patterns and functional relationships.	●	●	●
2.	Represent functions and relations by number sentences, equations, tables, and graphs.	●	●	●
3.	Use number lines and rectangular coordinate systems.	●	●	●
4.	Solve linear equations and inequalities	●	●	●
5.	Perform algebraic operations with real numbers and algebraic expressions.		●	●
6.	Represent functions and relations by graphs, variables, algebraic expressions, and equations.		●	●
7.	Solve systems of equations and inequalities algebraically and graphically.		●	●
8.	Represent problem situations with discrete structures including finite graphs, matrices, sequences, series and recursive relations.			●
9.	Solve polynomial equations with real and complex roots algebraically and graphically.			●
10.	Apply function notation and terminology including domain, range, composition, and inverse.			●
11.	Compare and apply the numerical, algebraic, and graphical properties of various kinds of functions: absolute value, linear, polynomial, exponential, logarithmic, and trigonometric.			●
12.	Apply trigonometric concepts including circular functions, radian measure, and fundamental identities, and apply trigonometry to the solution of geometric problems involving triangles and the modeling of periodic real-world phenomena.			●

III. ASSESSMENT EXERCISES

FORMS OF EXERCISES

There will be open-ended questions to expand the types of problems or exercises presented to students beyond that which is possible with multiple-choice or other closed-ended questions. It is also intended, with a few selected questions, to provide insights into the different ways in which students think about mathematics in responding to a question. In addition to providing an answer, students will sometimes be asked to write (in their booklets) the calculations or procedures they used to arrive at an answer. Although time consuming to analyze, such descriptions can provide a better understanding of the ways in which students reach correct and incorrect answers.

USE OF CALCULATORS

The use of the calculator has been included in the National Assessment of Education Progress since the second mathematics assessment conducted in 1977-78. The calculator is a tool used by nearly everyone in numerical computations. Mathematics assessment must reflect the use of calculators in the classroom, society, engineering and business.

In the 1990 assessment, all fourth grade students tested will have the use of a four-function calculator and all eighth and twelfth grade students will have the use of a scientific calculator. NAEP will ensure that the calculators used by the students are the same or comparable in design and function. Calculators are allowed on a significant portion of the test and will be necessary on some items. On another portion of the test, calculators will not be allowed in order to assess estimation, mental arithmetic, and pencil-and-paper abilities and in order to maintain trend analyses into previous NAEP assessments. NAEP will ensure that all students assessed will be provided an orientation in the use of the calculators provided.

At all grade levels, items allowing calculators will assess not only the correct usage of a calculator but also the ability to choose the appropriate computational method (calculator, paper-and-pencil, or mental arithmetic and estimation).

SAMPLE QUESTIONS

The following are samples of questions classified by mathematical abilities, content areas, and grade levels. These samples in no way represent the full range of mathematical abilities and content areas. In addition, the classifications cannot be exact, and there will inevitably be differences of opinion especially as to the mathematical abilities categories. It should also be remembered that a question classified at one grade level may be appropriate for use at another grade level; however, the mathematical abilities classification may change.

NAEP - 1990 ASSESSMENT OF MATHEMATICS

GRADE FOUR SAMPLE QUESTIONS (1-14)

1. 88 + 112 + 6 =

    (A) 196

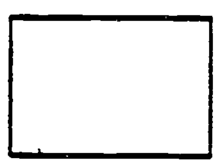
    (B) 206

    (C) 260

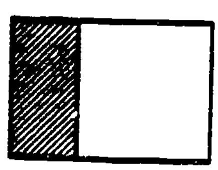
    (D) 1,592

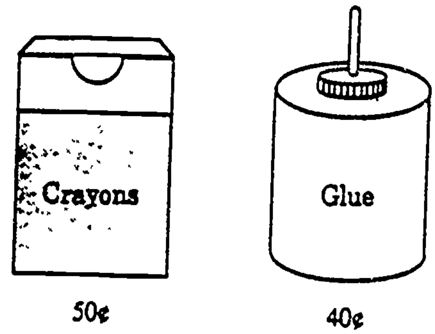
Content: Numbers and Operations
Ability: Procedural Knowledge
Answer: B

2. Shade $\frac{1}{3}$ of the rectangle shown below.



Content: Numbers and Operations
Ability: Conceptual Understanding
Answer: The figure shown below is one possible solution.





3. The prices for crayons and glue are shown above. Katie has $2.50. If Katie buys 3 boxes of crayons, what is the greatest number of jars of glue she can buy with the rest of her money?

Content: Numbers and Operations
Ability: Problem Solving
Answer: Two

4. Which of the following units would be best for measuring the length of a pencil?

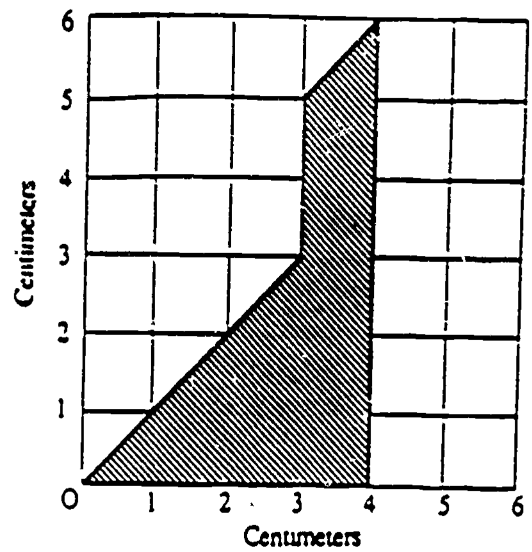
(A) Inches

(B) Feet

(C) Yards

(D) Miles

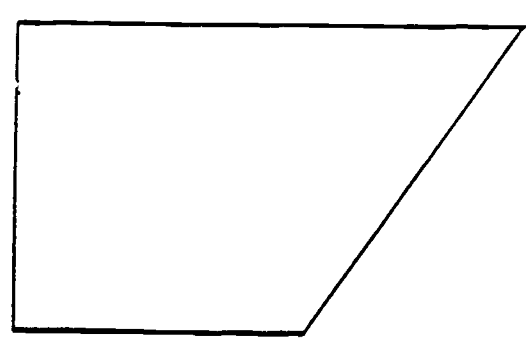
Content: Measurement
Ability: Conceptual Understanding
Answer: A



5. What is the area of the shaded figure shown above?

(A) 8 square centimeters
(B) 10 square centimeters
(C) 12 square centimeters
(D) 24 square centimeters

Content: Measurement
Ability: Problem Solving
Answer: B



Note: Ruler will be provided

6. Using the ruler you have been given, find the distance, in centimeters, around the figure shown above.

Content: Measurement
Ability: Procedural Knowledge
Answer: 20 centimeters

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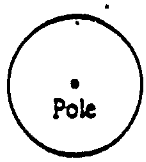
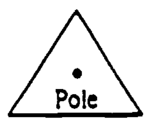
7. A dog walks on a path that is always 20 feet from a pole. Which of the following could be a drawing of the path?

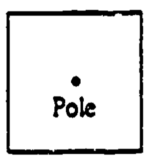
(A)



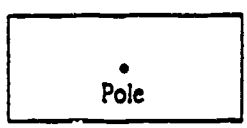
(B)



(C)



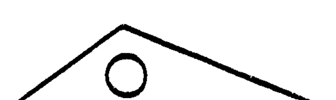
(D)

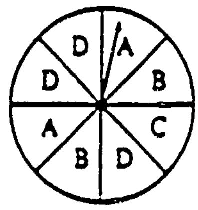


Content: Geometry
Ability: Problem Solving
Answer: A

8. In the space below draw a circle inside a triangle.

Content: Geometry
Ability: Conceptual Understanding
Answer: The figure shown below is one possible solution



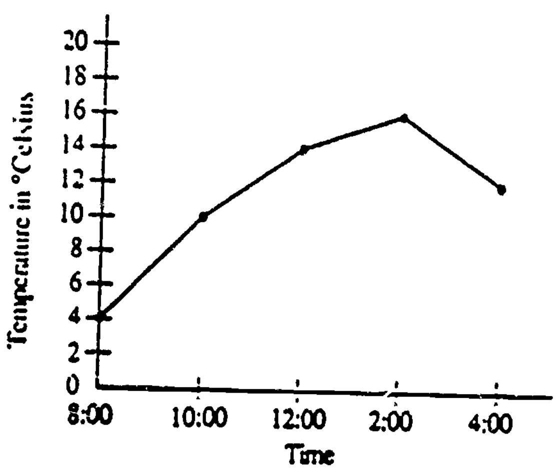


9. When the arrow shown above is spun, what are the chances that the arrow will stop on a region labeled with the letter D?

Content: Data Analysis, Statistics, and Probability
Process: Problem Solving
Answer: 3 out of 8

Question 10 refers to the following graph.

HOURLY TEMPERATURES



10. According to the graph shown above, at what time was the temperature the highest?

(A) 8:00

(B) 12:00

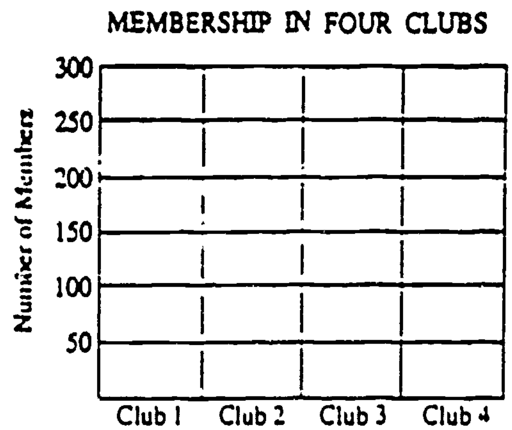
(C) 2:00

(D) 4:00

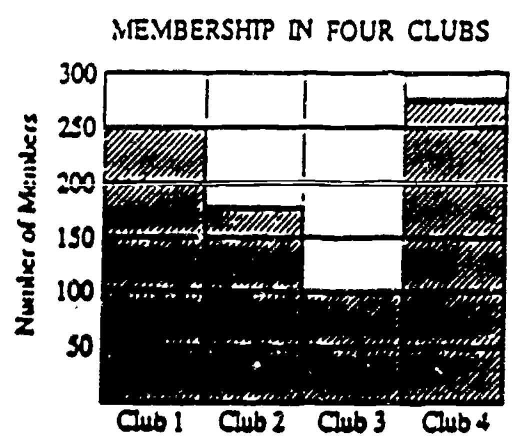
Content: Data Analysis, Statistics, and Probability
Ability: Conceptual Understanding
Answer: C

25

11. Club 1 has 250 members, Club 2 has 175 members, Club 3 has 100 members, and Club 4 has 275 members. On the grid below, fill in a bar graph that shows the membership of the clubs.



Content: Data Analysis, Statistics, and Probability
Ability: Procedural Knowledge
Answer:



26

$4 + 7 + 2 = 2 + \square$

12. Which number, when placed in the box shown above, will make the number sentence true?

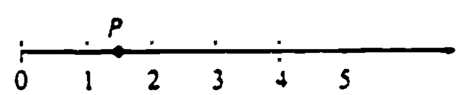
(A) 9

(B) 11

(C) 13

(D) 15

Content: Algebra and Functions
Ability: Conceptual Understanding
Answer: B



13. On the number line shown above, point *P* is located at what number?

(A) $1\frac{1}{2}$

(B) $2\frac{1}{2}$

(C) $3\frac{1}{2}$

(D) $4\frac{1}{2}$

Content: Algebra and Functions
Ability: Procedural Knowledge
Answer: A

14. On the first day Joe reads 1 page of a book, on the second day he reads two pages, on the third day he reads 4 pages, and on the fourth day he reads 7 pages. If Joe continues to read the book following this pattern, how many pages will he read on the sixth day?

Content: Algebra and Functions
Ability: Problem Solving
Answer: 16 pages

END OF GRADE FOUR SAMPLE QUESTIONS.

NAEP 1990 ASSESSMENT OF MATHEMATICS

GRADE EIGHT - SAMPLE QUESTIONS (1-15)

1. Which is the closest to 7.82 x 5.09?

   (A) 0.4

   (B) 4

   (C) 40

   (D) 400

Content: Numbers and Operations
Ability: Procedural Knowledge
Answer: C

2. If 20 percent of the marbles in one bag are red and 30 percent of the marbles in another bag are red, what percent of the total number of marbles in both bags are red?

   (A) 10%

   (B) 25%

   (C) 50%

   (D) It cannot be determined from the information given.

Content: Numbers and Operations
Ability: Conceptual Understanding
Answer: D

3. A schedule of class periods is to be prepared for Pinecrest High School. The school day will begin at 8:30 a.m. and end at 2:30 p.m. There will be 6 class periods, all of the same length, and a lunch period that is 30 to 60 minutes long. Ten minutes will be allowed for students to move from one class to the next or to move to or from lunch. Prepare in the space below a schedule of times for the school day that meets all requirements given above.

Content: Numbers and Operations
Ability: Problem Solving
Answer: The schedule shown below is one possible solution.

School Day Schedule

Period	Time
First	8:30 a.m. - 9:15 a.m.
Second	9:25 a.m. - 10:10 a.m.
Third	10:20 a.m. - 11:05 a.m.
LUNCH	11:15 a.m. - 11:45 a.m.
Fourth	11:55 a.m. - 12:40 p.m.
Fifth	12:50 p.m. - 1:35 p.m.
Sixth	1:45 p.m. - 2:30 p.m.

4. The average height of the girls in a certain eighth grade class could be

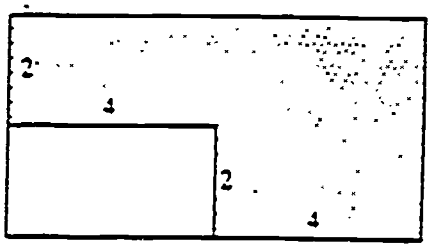
(A) 60 centimeters

(B) 160 centimeters

(C) 300 centimeters

(D) 500 centimeters

Content: Measurement
Ability: Conceptual Understanding
Answer: B



5. The figure above shows a piece of construction paper. If the unshaded portion of the paper is removed, the area of the remaining shaded portion will be how many times as large as the area of the portion removed?

(A) 2

(B) 3

(C) 4

(D) 6

Content: Measurement
Ability: Problem Solving
Answer: B

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6. If each edge of a cube has length 5 centimeters, what is its volume in cubic centimeters?

(A) 15

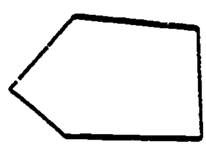
(B) 25

(C) 125

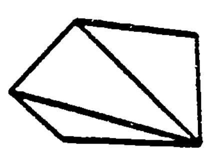
(D) 150

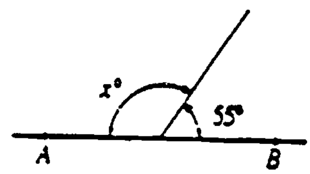
Content: Measurement
Ability: Procedural Knowledge
Answer: C

7. The sum of the degree measures of the interior angles of a triangle is 180°. Use this fact to find the sum of the degree measures of the interior angles in the figure shown below.



Content: Geometry
Process: Problem Solving
Answer: 540°, since the figure can be sub-divided into three triangles as illustrated below.





8. In the figure shown above, AB is a line segment. What is the value of x?

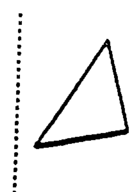
(A) 125

(B) 115

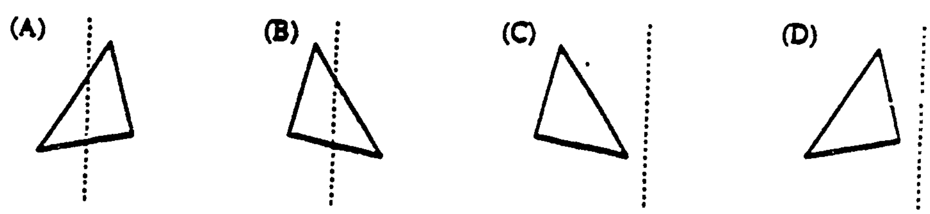
(C) 45

(D) 35

Content: Geometry
Ability: Procedural Knowledge
Answer: A



9. If the triangle above is reflected through the dotted line, which of the following shows the reflection of the triangle?



Content: Geometry
Ability: Conceptual Understanding
Answer: C

10. What is the average (arithmetic mean) of 4, 8, 12, and 20?

Content: Data Analysis, Statistics, and Probability
Ability: Procedural Knowledge
Answer: 11

Question 11 refers to the following information.

Tax Rates for Country *X*

If your taxable income is

over	but not over	the tax is
0	$2,000	3% of net taxable income
$2,000	$4,[illegible]	$60 + 4% of amount over $2,000
$4,000	$6,000	$140 + 5% of amount over $4,000
$6,000	$10,000	$240 + 6% of amount over $6,000
$10,000		$480 + 7% of amount over $10,000

11. How much should be paid in taxes in Country *X* for a taxable income of $3,000?

Content: Data Analysis, Statistics, and Probability
Ability: Problem Solving
Answer: $100

12. Three fair coins are tossed at the same time. What is the probability that one of the coins is a head and the other two coins are tails?

(A) $\frac{1}{8}$

(B) $\frac{1}{3}$

(C) $\frac{3}{8}$

(D) $\frac{1}{2}$

Content: Data Analysis, Statistics, and Probability
Ability: Conceptual Understanding
Answer: C

13. If George has x ten dollar bills and y five dollar bills, which of the following gives the total amount of money in dollars that George has?

(A) $50xy$

(B) $15xy$

(C) $15(x + y)$

(D) $10x + 5y$

Content: Algebra and Functions
Ability: Conceptual Understanding
Answer: D

14. If $7 + 5x = 20$, then $x =$

Content: Algebra and Functions
Ability: Procedural Knowledge

Answer: $\frac{13}{5}$ or 2.6

x	y
0	-7
1	-5
2	-3
3	-1
4	1
5	3

15. If x and y are related as shown in the table above, write an algebraic rule that shows the relationship between x and y.

Content: Algebra and Functions
Ability: Problem Solving
Answer: $y = 2x - 7$

END OF GRADE EIGHT SAMPLE QUESTIONS.

NAEP 1990 ASSESSMENT OF MATHEMATICS

GRADE TWELVE - SAMPLE QUESTIONS (1-17)

1. $\frac{4 \times 10^3}{2 \times 10^{12}} =$

   (A) 2×10^{-9}

   (B) 2×10^{-6}

   (C) 2×10^6

   (D) 2×10^9

Content: Numbers and Operations
Ability: Procedural Knowledge
Answer: A

2. In a certain country, 0.4% of the population is at least 90 years old. If the population is 102,840,000, then the number of people who are at least 90 years old is approximately

   (A) 40,000

   (B) 400,000

   (C) 4,000,000

   (D) 40,000,000

Content: Numbers and Operations
Ability: Conceptual Understanding
Answer: B

3. When a certain number is divided by 7, the remainder is 4. What is the remainder when 6 times that number is divided by 7?

(A) 2

(B) 3

(C) 4

(D) 5

Content: Numbers and Operations
Ability: Problem Solving
Answer: B

4. A floor plan of a house that has maximum length of 64 feet and width of 44 feet is to be drawn to scale on an 8-inch by 11-inch grid. Which of the following scales will give the largest possible scale drawing of the house on the grid?

(A) $\frac{1}{16}$ inch = 1 foot

(B) $\frac{1}{8}$ inch = 1 foot

(C) $\frac{1}{6}$ inch = 1 foot

(D) $\frac{1}{4}$ inch = 1 foot

Content: Measurement
Process: Conceptual Understanding
Answer: C

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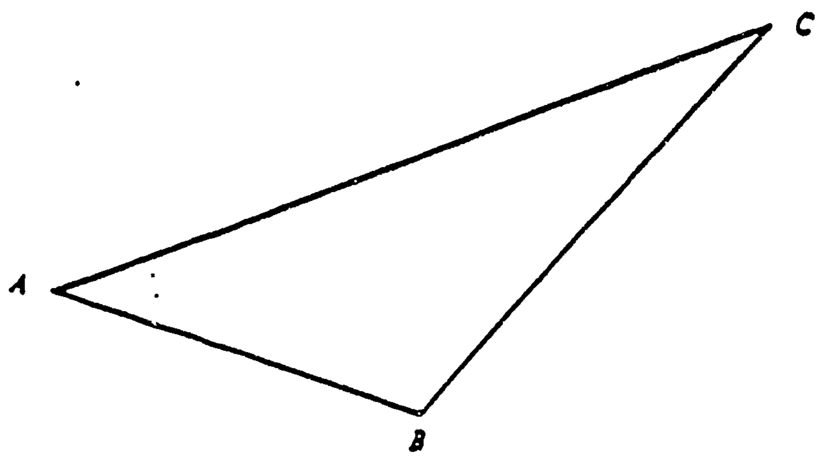
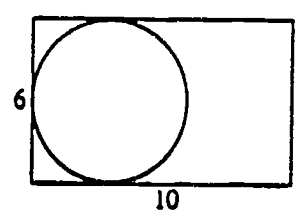


Note: Ruler will be provided.

5. Use the ruler provided to find the area, in square centimeters, of triangle *ABC* shown above.

Content: Measurement
Ability: Problem Solving
Answer: 15 square centimeters



6. If the width and length of the rectangle shown above are 6 and 10, which of the following is closest to the circumference of the circle?

(A) 18

(B) 27

(C) 36

(D) 60

Content: Measurement
Process: Procedural Knowledge
Answer: A

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7. What is the maximum area of a rectangle with perimeter 36?

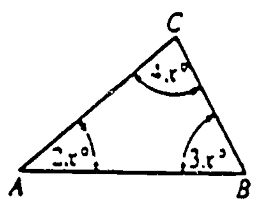
(A) 36

(B) 81

(C) 324

(D) 1,296

Content: Geometry
Ability: Problem Solving
Answer: B



8. In triangle *ABC* shown above, what is the degree measure of $\angle A$?

(A) 20

(B) 40

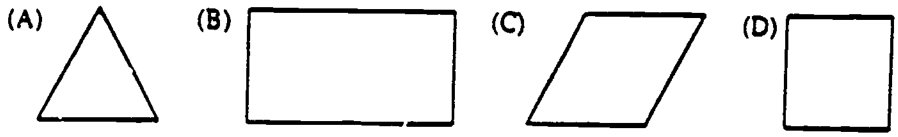
(C) 60

(D) 80

Content: Geometry
Ability: Procedural Knowledge
Answer: B

9. Which figure could be used to prove that the following statement is NOT true?

"If all the sides of a figure have equal lengths, then all of the interior angles have equal measures."



Content: Geometry
Ability: Conceptual Understanding
Answer: C

10. A local newspaper publishes a weekly comparison of the total cost for 20 grocery items at 7 supermarkets. Following are the costs at the 7 supermarkets for one week: \$18.48, \$17.03, \$20.17, \$16.74, \$19.11, \$17.03, and \$20.92. Which of the following is NOT true for these data?

(A) The range is \$4.18.

(B) The mode is \$17.03.

(C) The median is \$18.48.

(D) The arithmetic mean is \$17.64.

Content: Data Analysis, Statistics, and Probability
Ability: Procedural Knowledge
Answer: D

11. Let P be the vertex of a regular 7-sided polygon. What is the probability that a diagonal drawn at random from P will form a triangle with two sides of the polygon?

(A) $\frac{1}{3}$

(B) $\frac{2}{7}$

(C) $\frac{2}{5}$

(D) $\frac{1}{2}$

Content: Data Analysis, Statistics, and Probability
Ability: Problem Solving
Answer: D

12. There are 5 democrats and 4 republicans on a senate committee. What is the greatest number of ways that a sub-committee can be formed that consists of 2 democrats and 2 republicans?

(A) 2

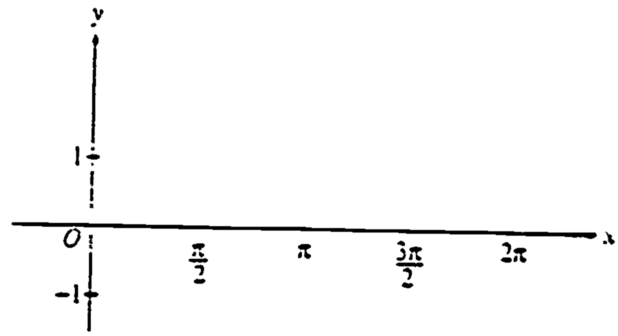
(B) 20

(C) 60

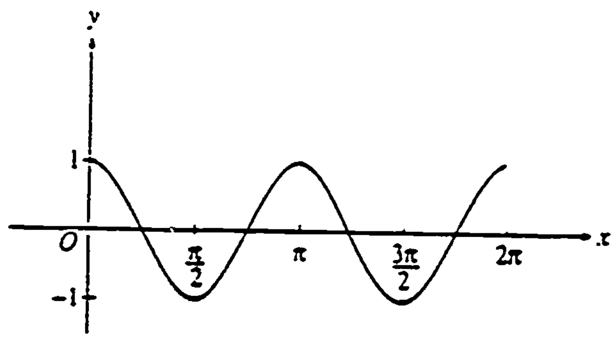
(D) 80

Content: Data Analysis, Statistics, and Probability
Ability: Conceptual Understanding
Answer: C

13. On the axes provided below, sketch the graph of $y = \cos(2x)$ from $x = 0$ to $x = 2\pi$.



Content: Algebra and Functions
Ability: Conceptual Understanding
Answer:



14. If $f(x) = 2x^2 - 1$, then $f(x + 3) =$

(A) $2x^2 + 2$

(B) $2x^2 + 16$

(C) $2x^2 + 6x + 8$

(D) $2x^2 + 12x + 17$

Content: Algebra and Functions
Ability: Procedural Knowledge
Answer: D

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15. During the first 3 hours of a 3,000 mile trip, a plane is flown at an average speed of x miles per hour. At what average speed, in miles per hour, must the plane be flown for the remainder of the distance if the entire trip takes 2 more hours?

(A) $1,500 - \frac{3x}{2}$

(B) $\frac{1}{1,500} - \frac{2}{3x}$

(C) $\frac{3x}{2} - 1,500$

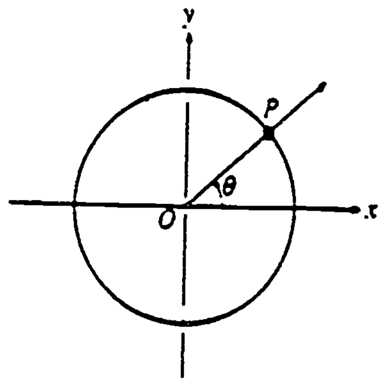
(D) $\frac{2}{3x} - \frac{1}{1,500}$

Content: Algebra and Functions
Ability: Problem Solving
Answer: A

Questions 16 and 17 that follow are to be solved with the aid of a nonprogrammable scientific calculator.

16. If $x^{\frac{9}{2}} = 20$, find x rounded to the nearest thousandth.

Content: Algebra and Functions
Process: Procedural Knowledge
Answer: 1.946



17. In the figure above, if point P is (6,5), find θ in radian measure.

Content: Algebra and Functions
Ability: Conceptual Understanding
Answer: 0.695 radians (rounded to 3 decimal places)

END OF GRADE TWELVE SAMPLE QUESTIONS.

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ACKNOWLEDGMENTS

The National Assessment Planning Project (NAPP) for the 1990 National Assessment of Educational Progress (NAEP) in mathematics would not have been possible without the generous time and support given by the committees, sub-committees, state mathematics specialists and consultants. Since the beginning of the project in August 1987, an extensive effort to complete the many tasks would not have been accomplished without the expertise and support of all who assisted.

We wish to pay special tribute to the Committee members, to the Steering Committee for overseeing progress and to the Mathematics Objectives Committee which became an integral part of the total process and which guided the project to its completion. State mathematics specialists and consultants provided constructive suggestions and support through an important communication review process. In most cases, state mathematics specialists convened state level meetings consisting of teachers, professors, testing specialists, mathematics coordinators, mathematics supervisors, and school administrators. The efforts of all who were involved resulted in a product that reflects the concerns about what students should know in mathematics for assessment purposes.

APPENDIX A.

NATIONAL ASSESSMENT PLANNING PROJECT (NAPP)

STEERING COMMITTEE

American Association of School Administrators
James E. Morrell - Superintendent of Public Schools, Muhlenberg, Pennsylvania

American Federation of Teachers
Antonia Cortese - First-Vice-President State United Teachers, New York

Association of State Assessment Programs
Thomas Fisher - State Assessment Administrator, Florida

Association for Supervision and Curriculum Development
Alice Houston - Assistant Superintendent, Seattle Public Schools, Washington

Council for American Private Education and National Association of Independent Schools - Glenn Bracht - Director American Lutheran Church, Minnesota

Council of Chief State School Officers
Richard A. Boyd - Superintendent of State Department of Education, Mississippi

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Directors of Research and Evaluation
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National Association of Elementary School Principals
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National Association of Secondary School Principals
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National Association of State Boards of Education
Barbara Roberts Mason - President, State Board of Education, Michigan

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APPENDIX A (Cont'd)

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National Council of State Legislators
Wilhelmina Delco - Congresswoman, Texas House of Representatives, Texas

National Education Association
Robert Astrup - President, Minnesota Education Association, Minnesota

National School Board Association
William M. Soult - Director, St. Urain Valley Board of Education, Colorado

National Governors' Association
Nancy DiLaura - Education Assistant, State House, Indiana

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Strong, Dorothy - Director of Mathematics, Chicago Public Schools, Chicago, Illinois

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Watson, Charles - Mathematics Specialist, Arkansas Department of Education, Little Rock, Arkansas

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APPENDIX C.

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Gay, Susan - Mathematics Specialist, Oklahoma

Hanna, Karen - Mathematics Specialist, Tennessee

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Meeks, Wendell - Education Consultant, Illinois

Mitchell, Jacqueline P. - Mathematics Consultant, Maine

Nishimura, Kathleen - Mathematics Specialist, Hawaii

Paul, Fredric - Chief of Bureau Mathematics, New York

Peeler, Lane E. - Mathematics Specialist, South Carolina

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Prevost, Fernand J. -Mathematics Consultant, New Hampshire

Reeves, Andy - Mathematics Specialist, Florida

Vice, Sheila - Mathematics Consultant, Kentucky

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Wiley, Larry - Mathematics Curriculum Specialist, New Jersey

Wilson, Ruth - Mathematics Specialist, Nevada

Wilson-Hegg, Martha - Mathematics Consultant, Indiana

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Kilpatrick, Jeremy - University of Georgia

Kouba, Vicki - University of Albany

Lappan, Glenda - Michigan State University

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Porter, Andrew - Michigan State University

Ralston, Anthony - University of New York

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Rosen, Linda - Mathematics Sciences Education Board

Schwartz, Judah L. - Harvard Graduate School of Education

Steen, Lynn Arthur - St. Olaf College

Stiff, Lee - North Carolina State University

APPENDIX D (Cont'd)

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Usiskin, Zalman - University of Chicago