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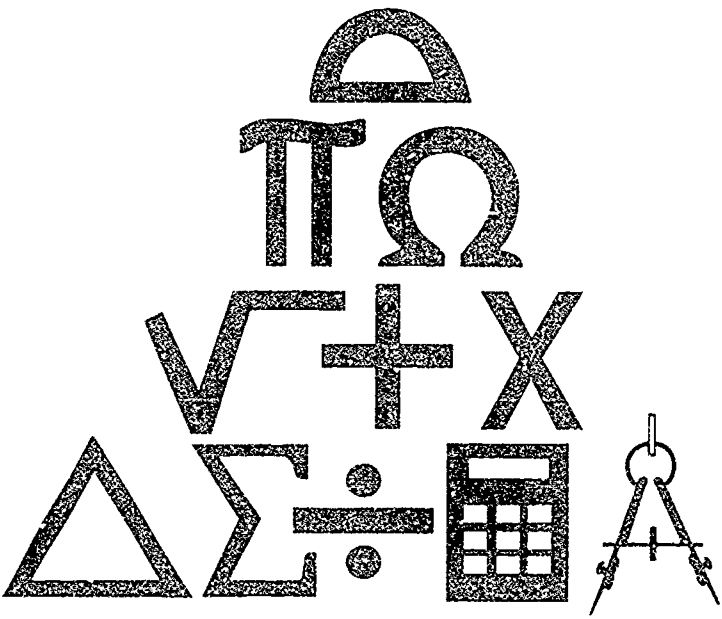
ABSTRACT

The National Assessment of Educational Progress (NAEP) reports on the status and progress of educational achievement in the United States. Based on its surveys, "The Nation's Report Card" provides comprehensive information about what students in the United States can do in various subject areas. The framework for the 1990 mathematics assessment is organized according to mathematical abilities and content areas for grades 4, 8, and 12. This document describes the organization of the 1990 effort and the construction of the instrument. The mathematical abilities to be assessed are conceptual understanding, procedural knowledge, and problem solving. The section on "Content Areas" contains assessments on: (1) "Numbers and Operations"; (2) "Measurement"; (3) "Geometry"; (4) "Data Analysis, Statistics, and Probability"; and (5) "Algebra and Functions." Abilities and subtopics are included under these headings. An appendix includes sample questions. (DC)

MATHEMATICS OBJECTIVES

1990 ASSESSMENT



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The Nation's Report Card,
The National Assessment of
Educational Progress (NAEP)

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Contents

6

Chapter one

Introduction

For almost 20 years, the National Assessment of Educational Progress (NAEP) has reported on the status and progress of educational achievement in the United States. Based on its surveys of students at the elementary, junior-high, and high-school levels, "The Nation's Report Card" provides comprehensive information about what students in the U.S. know and can do in various subject areas NAEP contributes information on students' strengths and weaknesses in basic and higher-order skills; provides data comparing groups of students by race/ethnicity, gender, type of community, and region; describes trends in performance across the years; and reports relationships between achievement and certain background variables.

Context for Planning the 1990 Mathematics Assessment

In 1988, Congress passed new legislation for NAEP which included — for the first time in the project's history — a provision authorizing voluntary state-by-state assessments on a trial basis, in addition to the national assessments that NAEP had conducted since its inception. Anticipating this legislation, the federal government arranged for a special grant from the

National Science Foundation and the Department of Education to the Council of Chief State School Officers (CCSSO) in mid-1987 to lay the groundwork for state comparisons. Providing recommendations for the 1990 mathematics assessment that reflect state-level concerns was the purpose of the National Assessment Planning Project, conducted under the auspices of the CCSSO.

The project had two primary responsibilities. The first was to recommend objectives for the state-level mathematics assessment, and the second was to make suggestions for reporting state results. Because the legislation specifying that the state trial mathematics assessment would be at the eighth-grade level had not been passed by Congress when the National Assessment Planning Project began its work, and because the objectives had to be coordinated for all grades assessed by NAEP — fourth, eighth, and twelfth — the project developed objectives for all three grades to be assessed in 1990.

Assessment Development Process

The National Assessment Planning Project patterned the development process for the 1990 mathematics objectives after the consensus process described in Public Law 98-511, Section 405 (E), which authorized NAEP through June 30, 1988. The law stated that "each learning area assessment shall have goal statements devised through a national consensus approach, providing for active participation of teachers, curriculum specialists, subject matter specialists, local school administrators, parents, and members of the general public." Because the 1990 mathematics assessment will produce state report cards in addition to the national report card, the development process was expanded to ensure careful attention to the formal mathematics objectives of states and of a sampling of local districts, and to the opinions of practitioners at the state and local levels as to what content should be assessed. Perhaps to a greater extent than any previous NAEP assessment, the design of the 1990 mathematics assessment — particularly the state trial assessment — depended on the involvement and support of diverse contributors.

To guide its efforts in developing recommendations for state-level assessments in mathematics, the National Assessment Planning Project's Steering Committee — whose members included policymakers, practitioners, and citizens nominated by 18 national organizations — adopted a policy statement on the purpose of state comparisons and the conditions that should be met. The statement follows.

The purpose of a state level student achievement comparison is to provide data on student performance to assist policymakers and educators to work toward the improvement of education. Such data can be useful by encouraging and contributing to a discussion of the quality of education and the conditions that determine it.

State comparative achievement data are useful if they:

★ *represent performance based on a consensus of what is important to learn;*

★ *use sound testing and psychometric practices;*

★ *take into account different circumstances and needs that states face; and*

★ *are associated with features of school systems that can be improved by policymakers and educators.*

A Mathematics Objectives Committee — comprised of a teacher, a school administrator, mathematics education specialists from various states, mathematicians, parents, and citizens — was created by the CCSSO to recommend objectives for the assessment based on these guidelines. The draft objectives, together with a set of sample questions, were distributed to the mathematics supervisor in the department of education in each of the 50 states. These specialists convened a panel that reviewed the draft objectives and returned comments and suggestions to the project staff. Copies of the draft were also sent to 25 mathematics educators and scholars for review. Following the incorporation of comments and revisions, the final recommendations of the Mathematics Objectives Committee were approved by the National Assessment Planning Project Steering Committee.[1] A list of participants in these stages of the assessment development process can be found on page 37.

The objectives were subsequently submitted to the National Center for Education Statistics (NCES), which forwarded them for review by NAEP's governing board, the Assessment Policy Committee (APC). The APC approved the recommendations of the National Assessment Planning Project with minor provisions about the feasibility of full implementation.[2] The objectives were further defined by NAEP's Item Development Panel, reviewed by the Task Force on State Comparisons, and resubmitted to NCES for peer review. The penultimate draft was

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[1] Council of Chief State School Officers, *Assessing Mathematics in 1990 by the National Assessment of Educational Progress*, Washington, DC: National Assessment Planning Project, March 1988.

[2] This action is contained in a statement issued by the Assessment Policy Committee's Executive Committee on April 29, 1988. The recommendations were ratified by the full committee on June 18, 1988, with two stipulations: first, that the objectives be so weighted as to permit reporting on trends in performance; and second, with regard to the use of calculator active items and open response questions, that the assessment be developed within the resources available for its administration.

then distributed to state representatives for comments. The final objectives, presented herein, provide specifications for the 1990 mathematics assessment at grades 4, 8, and 12. In addition, NAEP is preparing a second booklet describing specifications for the eighth-grade state trial assessment, to be published under separate cover.

Assessment Design Principles

Several principles emerged during the discussions of the Mathematics Objectives Committee and became the basis for structuring the framework for the 1990 assessment. One is that a national assessment designed to provide state-level comparisons should not be directed to the states' "least common denominator," measuring only those topics and skills in the objectives of all states and thereby discouraging desirable curriculum development. Nor can it be geared to the least common denominator of student preparation; therefore, the objectives provide for challenging questions at each grade level. For example, some questions at the grade 12 level may be appropriate only for college-bound students. The assessment must also address the concern that its objectives might be used to steer instruction toward one particular pedagogical or philosophical viewpoint to the exclusion of others that are widely held.

No assessment can measure everything. In deciding what topics and abilities should be measured, the committee responsible for developing the 1990 mathematics objectives was guided by several considerations: The assessment should reflect many of the states' curricular emphases and objectives; be inclusive of what various scholars, practitioners, and interested citizens believe should be in the curriculum, and maintain some of the content of prior assessments to allow reporting of trends in performance.

Accordingly, the committee gave attention to several frames of reference. First, it considered states' goals and concerns, as reflected through analyses of state mathematics curriculum guides and the recommendations of state mathematics specialists. A report on "Issues in the Field," based on telephone interviews with leading mathematics educators, and a draft assessment framework provided by a subcommittee of the Mathematics Objectives Committee also contributed to the development process.

In generating its recommendations, the committee drew upon the draft of the *Curriculum and Evaluation Standards for School Mathematics* developed by the National Council of Teachers of Mathematics. Produced through intensive work by leading mathematics educators in the United States, the report is a significant statement on what mathematics should be taught in the schools.[3]

In devising objectives for the 1990 assessment, the committee also gave close consideration to the design of the 1986 mathematics assessment.[4] The framework for the 1986 assessment had seven content and five process areas for a matrix of 35 cells, whose complexity militated against easy understanding. In addition, given resource limitations, the weightings assigned to various cells in the framework left too few questions in some cells to provide reliable measures of students' knowledge and skills. Therefore, it was decided that the outline or matrix guiding the development of the 1990 assessment needed to be simplified, and that necessary complexity could be reflected

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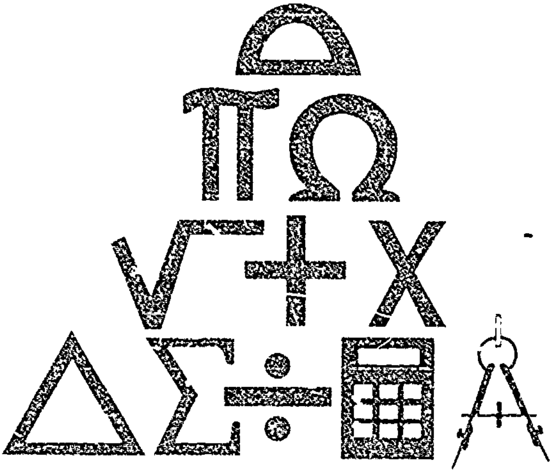
[3] National Council of Teachers of Mathematics, *Curriculum and Evaluation Standards for School Mathematics* (Reston, VA: National Council of Teachers of Mathematics, 1987).

[4] National Assessment of Educational Progress, *Mathematics Objectives, 1985-86 Assessment* (Princeton, NJ: National Assessment of Educational Progress, Educational Testing Service, 1987).

through the designation of specific abilities and topics in each content area.

A final principle recognized by the committee drafting the objectives was that the application of mathematics is far more holistic than a matrix implies. The mathematics content areas and ability categories described in these objectives are, therefore, not discrete or mutually exclusive, but rather highly integrated.



Chapter two

Framework for the Assessment

The framework for the 1990 mathematics assessment is organized according to mathematical abilities and content areas. The mathematical abilities assessed are **conceptual understanding**, **procedural knowledge**, and **problem solving**. Content is drawn primarily from elementary and secondary school mathematics up to, but not including, Calculus. The content areas assessed are **Numbers and Operations; Measurement, Geometry; Data Analysis, Statistics, and Probability**; and **Algebra and Functions**.

The framework for the 1990 mathematics assessment is provided in Figure 1, illustrating the relationship betwe› › the three categories of mathematical abilities and the five content areas included in the assessment. The weightings for these dimensions of the framework are pres ›ted in the latter part of this chapter. Descriptions of the two dimensions of the framework — mathematical abilities and content areas — are provided in Chapters 3 and 4, respectively

Figure 1

Framework for the 1990 Mathematics Assessment

Mathematical Ability	Content Area				
	Numbers & Operations	Measurement	Geometry	Data Analysis, Statistics & Probability	Algebra & Functions
Conceptual Understanding					
Procedural Knowledge					
Problem Solving					

Distribution of Assessment Questions

The assignment of percentages to various mathematical abilities and content areas is an important feature of assessment design because such "weighting" reflects the importance or value given to these areas at each grade level. Over the four previous mathematics assessments, the percentage distribution of questions in each area has changed, and these changes continue in the new assessment. For 1990, the CCSSO advisors were interested in creating an assessment that would be forward-thinking and could lead instruction; thus, more emphasis was given to problem solving than in previous assessments. Also, participants in the CCSSO process advised that greater emphasis be given to Geometry and Algebra and Functions, and less to Numbers and Operations than in the past. The approximate percentage distribution of questions by mathematical ability, content area, and grade is provided in Tables 1 and 2.

Table 1: Percentage Distribution of Questions by Grade and Mathematical Ability

Mathematical Ability	Grade 4	Grade 8	Grade 12
Conceptual Understanding	40	40	40
Procedural Knowledge	30	30	30
Problem Solving	30	30	30

Table 2: Percentage Distribution of Questions by Grade and Content Area

Content Area	Grade 4	Grade 8	Grade 12
Numbers and Operations	45	30	25
Measurement	20	15	15
Geometry	15	20	20
Data Analysis, Statistics and Probability	10	15	15
Algebra and Functions	10	20	25

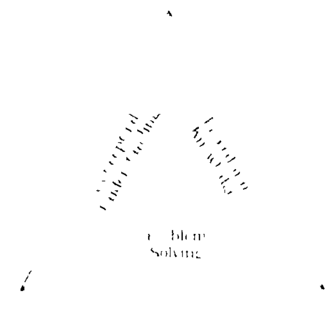
It should be emphasized that the percentage distributions presented here, and the lists of sample topics provided in later sections of this booklet, are not intended to prescribe curriculum standards, rather, they are designed for the purpose of constructing a complete and balanced assessment instrument at each grade level. An analysis of students' performance based on the entire set of items allows NAEP to report on average mathematics proficiency. In addition, analysis of performance on subsets of items, corresponding to the content areas of the assessment framework, permits reporting on patterns of achievement on five mathematics subscales: Numbers and Operations, Measurement, Geometry; Data Analysis, Statistics, and Probability and Algebra and Functions.

Chapter three

Mathematical Abilities

Students' mathematical abilities can be classified into three categories, conceptual understanding, procedural knowledge, and problem solving (see Figure 2). This classification is not meant to be hierarchical, in that questions within any of the three categories may be relatively complex or simple. Problem solving involves interactions between conceptual knowledge and procedural skills at any grade level, but what is considered complex problem solving at one grade level may be considered conceptual understanding or procedural knowledge at a different grade level. The same concept or skill can be assessed in a variety of representations, with tables, pictures, verbal descriptions, or other cues. The context of a question thus helps to determine its categorization.

Figure 2
Mathematical Abilities



Conceptual Understanding

Students demonstrate conceptual understanding in mathematics when they provide evidence that they can recognize, label, and generate examples and counterexamples of concepts; can use and interrelate models, diagrams, and varied representations of concepts; can identify and apply principles; know and can apply facts and definitions, can compare, contrast, and integrate related concepts and principles; can recognize, interpret, and apply the signs, symbols, and terms used to represent concepts; and can interpret the assumptions and relations involving concepts in mathematical settings. Such understandings are essential to performing procedures in a meaningful way and applying them in problem-solving situations.

Abilities:

1. Recognize, label, and generate examples and counterexamples of concepts
2. Use models, diagrams, and symbols to represent concepts
3. Identify and apply principles.
4. Know and apply facts and definitions
5. Make connections among different modes of representation of concepts.
6. Compare, contrast, and integrate concepts and principles
7. Recognize, interpret, and apply symbols to represent concepts.
8. Interpret assumptions and relations involving concepts.

Procedural Knowledge

Students demonstrate procedural knowledge in mathematics when they provide evidence of their ability to select and apply appropriate procedures correctly; verify and justify the correctness of a procedure using concrete models or symbolic methods; and extend or modify procedures to deal with factors inherent in problem settings.

Procedural knowledge includes the various numerical algorithms in mathematics that have been created as tools to meet specific needs in an efficient manner. It also encompasses the abilities to read and produce graphs and tables, execute geometric constructions, and perform noncomputational skills such as rounding and ordering.

Abilities:

1. Select and apply appropriate procedures correctly.
2. Verify and justify the correctness of applications of procedures.

Problem Solving

In problem solving, students are required to use their reasoning and analytic abilities when they encounter new situations. Problem solving includes the ability to recognize and formulate problems; determine the sufficiency and consistency of data; use strategies, data, models, and relevant mathematics; generate, extend, and modify procedures; use reasoning (i.e., spatial, inductive, deductive, statistical, and proportional); and judge the reasonableness and correctness of solutions.

Abilities:

1. Recognize and formulate problems
2. Understand data sufficiency and consistency
3. Use strategies, data, models, and relevant mathematics.
4. Generate, extend, and modify procedures.
5. Reason (spatially, inductively, deductively, statistically, and proportionally).
6. Judge the reasonableness and correctness of solutions.

Chapter four

Content Areas

To conduct a meaningful assessment of mathematics proficiency, it is necessary to measure students' abilities in various content areas. Classification of topics into these content areas cannot be exact, however, and inevitably involves some overlap. For example, some topics appearing under Data Analysis, Statistics, and Probability may be closely related to others that appear under Algebra and Functions. Context can also determine content area; for example, a question asking students to compute the area of a geometric figure may be considered either Measurement or Geometry, depending on the representation of the problem.

The following sections of this chapter provide a brief description of each content area with a list of topics and subtopics illustrative of those to be included in the assessment. Using the topics provided in the CCSSO report, the NAEP Item Development Panel generated lists of subtopics. This level of specificity was needed to guide item writers and ensure adequate coverage of the content areas and abilities to be assessed.

Numbers and Operations

This content area focuses on students' understanding of numbers (whole numbers, fractions, decimals, integers) and their application to real-world situations, as well as computational and estimation situations. Understanding numerical relationships as expressed in ratios, proportions, and percents is emphasized. Students' abilities in estimation, mental computation, use of calculators, generalization of numerical patterns, and verification of results are also included.

The grade 4 assessment should include questions requiring the manipulation of whole numbers, simple fractions, and decimals, using the operations of addition, subtraction, multiplication, and division. The grade 8 assessment should include questions using whole numbers, fractions, decimals, signed numbers, and numbers expressed in scientific notation. In addition to the operations included in the grade 4 assessment, students at grade 8 should be asked to demonstrate their ability to work with elementary powers and roots.

Students participating in the grade 12 assessment should demonstrate a detailed understanding of real numbers — including whole numbers, fractions, decimals, signed numbers, rational and irrational numbers, and numbers expressed in scientific notation — and a general understanding of complex numbers. The operations assessed at this grade level include addition, subtraction, multiplication, division, powers, and roots.

Topic: Numbers and Operations	Grade 4	Grade 8	Grade 12
1. Relate counting, grouping, and place value			
a. Whole number place value	•	•	•
b. Rounding whole numbers	•	•	•
c. Decimal place value	•	•	•
d. Rounding decimals	•	•	•
e. Order of magnitude (estimation related to place value)	•	•	•
f. Scientific notation		•	•

Topic: Numbers and Operations	Grade 4	Grade 8	Grade 12
2. Represent numbers and operations using models, diagrams, and symbols			
a. Set models such as counters	•	•	•
b. Number line models	•	•	•
c. Region models (two- and three-dimensional)	•	•	•
d. Other models (e.g., draw diagrams to represent a number or an operation; write a number sentence to fit a situation or describe a situation to fit a number sentence)	•	•	•
3. Read, write, rename, and compare numbers	•	•	•
4. Compute with numbers			
a. Basic properties of operations	•	•	•
b. Effect of operations on size and order of numbers	•	•	•
c. Features of algorithms (e.g., regrouping and partial products)	•	•	•
d. Selection of procedure (e.g., pencil and paper, calculator, mental arithmetic)	•	•	•
e. Applications	•	•	•
5. Make estimates appropriate to a given situation			
a. When to estimate	•	•	•
b. What form to use			
i. Overestimate	•	•	•
ii. Underestimate	•	•	•
iii. Range of estimate	•	•	•
c. Applications	•	•	•
d. Order of magnitude (scientific notation)		•	•
6. Verify solutions and determine the reasonableness of a result			
a. Abstract settings	•	•	•
b. Real-world situations	•	•	•
7. Apply ratios, proportions, and percents in a variety of situations			
a. Ratio and proportion			
i. Meaning of ratio and proportion		•	•
ii. Simple ratio		•	•
iii. Proportion		•	•
iv. Scale		•	•
v. Rate		•	•

Topic: Numbers and Operations	Grade 4	Grade 8	Grade 12
b. Percent			
i. Meaning of percent		•	•
ii. p% of q = r (find one given the other two)		•	•
iii. Percent change		•	•
iv. Percents greater than 100		•	•
v. Percents less than 1		•	•
vi. Applications such as interest, discounts, prices, rates		•	•
8. Use elementary number theory			
a. Odd and even	•	•	•
b. Multiples including LCM and divisors including GCD		•	•
c. Prime numbers		•	•
d. Factorization (includes prime factorization)		•	•
e. Divisibility		•	•
f. Remainders		•	•
g. Number patterns		•	•

Measurement

This content area focuses on students' ability to describe real-world objects using numbers. Students should be asked to identify attributes, select appropriate units, apply measurement concepts, and communicate measurement-related ideas to others. Questions should be included that require an ability to read instruments using metric, customary, or nonstandard units, with emphasis on precision and accuracy. Questions requiring estimates, measurements, and applications of measurements of length, time, money, temperature, mass/weight, area, volume, capacity, and angles are also included under this content area.

The measurement concepts to be considered in the grade 4 assessment are length (perimeter), area, capacity, weight and mass, angle measure, time, money, and temperature. At grades 8 and 12, these measurement concepts are length (perimeter and circumference), area and surface area, volume and capacity, weight and mass, angle measure, time, money, and temperature. At all three grades, students are asked to work with customary, metric, and nonstandard units.

Topic: Measurement	Grade 4	Grade 8	Grade 12
1. Compare objects with respect to a given attribute	•	•	•
2. Select and use appropriate measurement instruments			
a Ruler, meter stick, etc. (distance)	•	•	•
b Protractor	•	•	•
c Thermometer	•	•	•
d Scales for weight or mass	•	•	•
e Gauges	•	•	•
3. Select and use appropriate units of measurement			
a Type	•	•	•
b Size	•	•	•
4. Determine perimeter, area, volume, and surface area			
a Perimeter			
i Triangles	•	•	•
ii Squares	•	•	•
iii Rectangles	•	•	•
iv Parallelograms	•	•	•
v Trapezoids	•	•	•
vi Other quadrilaterals	•	•	•
vii Combinations	•	•	•
viii Other polygons	•	•	•
ix Circles		•	•
b Area			
i Squares	•	•	•
ii Rectangles	•	•	•
iii Triangles		•	•
iv Parallelograms		•	•
v Trapezoids		•	•
vi Other quadrilaterals		•	•
vii Circles		•	•
viii Combinations		•	•
ix Other polygons			•

Topic: Measurement	Grade 4	Grade 8	Grade 12
c. Volume			
i. Rectangular solids	●	●	●
ii. Cylinders		●	○
iii. Cones			●
iv. Pyramids			●
v. Prisms			●
vi. Combinations			●
d. Surface area			
i. Rectangular solids		○	●
ii. Cylinders		○	●
iii. Cones			●
iv. Pyramids			●
v. Prisms			●
vi. Combinations			●
5. Estimate the size of an object or a measurement	●	●	●
6. Apply common measurement formulas		○	●
7. Convert from one measurement to another within the same system		○	●
8. Determine precision, accuracy, and error			
a. Significant digits		●	●
b. Size of unit of measurement		●	●
c. Accuracy of measurement		●	●
d. Absolute and relative error			●
9. Make and read scale drawings			
a. Convert from scale to actual measurement		○	●
b. Convert from actual measurement to scale		●	●

Geometry

This content area focuses on students' knowledge of geometric figures and relationships and on their skills in working with this knowledge. These skills are important at all levels of schooling as well as in practical applications. Students need to be able to model and visualize geometric figures in one, two, and three dimensions and to communicate geometric ideas. In addition, students should be able to use informal reasoning to establish geometric relationships.

Topic: Geometry

	Grade 4	Grade 8	Grade 12
1. Describe, compare, and classify geometric figures			
a. Points, lines, segments, and rays in a plane and in space			
i. Parallel lines	•	•	•
ii. Perpendicular lines	•	•	•
iii. Skew lines		•	•
iv. Diagonals		•	•
v. Bisectors		•	•
vi. Radius		•	•
vii. Diameter		•	•
viii. Altitudes			•
ix. Medians			•
b. Angles in a plane			
i. In triangles and other polygons	•	•	•
ii. Supplementary		•	•
iii. Complementary		•	•
iv. In circles		•	•
v. Right angles		•	•
vi. Angle bisector			•
vii. Alternate interior and corresponding			•
viii. Vertical			•
c. Triangles			
i. General properties of triangles	•	•	•
ii. Acute, right, or obtuse		•	•
iii. Equilateral		•	•
iv. Isosceles		•	•
v. Scalene		•	•
d. Quadrilaterals			
i. Square	•	•	•
ii. Rectangle	•	•	•
iii. Parallelogram		•	•
iv. Trapezoid		•	•
v. Rhombus		•	•
e. Other polygons			
i. Regular, not regular		•	•
ii. Convex, concave			•
iii. Interior and exterior angle measures			•
f. Three-dimensional solids			
i. Rectangular solid	•	•	•
ii. Prism	•	•	•
iii. Pyramid	•	•	•
iv. Cylinder	•	•	•
v. Cone		•	•
vi. Sphere		•	•

26

Topic: Geometry	Grade 4	Grade 8	Grade 12
g. Circles			
i. General properties of circles	•	•	•
ii. Secants, tangents, chords, arcs, circumscribed and inscribed circles			•
2. Given descriptive information, visualize, draw, and construct geometric figures			
a. Draw or sketch a figure given a verbal description	•	•	•
b. Straightedge and compass constructions			
i. Angle bisector		•	•
ii. A line perpendicular to a given line that passes through a given point		•	•
iii. A line parallel to a given line that passes through a given point		•	•
c. Given a figure, write a verbal description of its geometric qualities		•	•
3. Investigate and predict results of combining, subdividing, and changing shapes (e.g., paper folding, dissecting, tiling, and rearranging pieces of solids)	•	•	•
4. Identify the relationship between a figure and its image under a transformation			
a. Motion geometry (informal: lines of symmetry, flips, turns, and slides)	•	•	•
b. Transformations (translations, rotations, reflections, dilations, symmetry)			•
5. Describe the intersection of two or more geometric figures			
a. Two-dimensional		•	•
b. Three-dimensional		•	•
6. Classify figures in terms of congruence and similarity, and informally apply these relationships		•	•
7. Apply geometric properties and relationships in solving problems			
a. Between, inside, on, and outside		•	•
b. Pythagorean relationship			
i. Special right triangles (e.g., 3-4-5, 30-60-90, 45-45-90)		•	•
c. Properties of similarity			
i. Ratio and proportion			•

Topic: Geometry	Grade 4	8	12
d. Prove congruence of triangles			•
e. Others		•	•
8. Establish and explain relationships involving geometric concepts			
a. Logic	•	•	•
b. Informal induction and deduction		•	•
9. Represent problem situations with geometric models and apply properties of figures	•	•	•
10. Represent geometric figures and properties algebraically using coordinates and vectors			
a. Distance formula			•
b. Slope			•
c. Parallel, perpendicular lines			•
d. Midpoint formula			•
e. Conic sections			•
f. Vectors			
i. Addition, subtraction			•
ii. Scalar multiplication, dot product			•

Data Analysis, Statistics, and Probability

This content area focuses on data representation and analysis across all disciplines, and reflects the importance and prevalence of these activities in our society. Statistical knowledge and the ability to interpret data are necessary skills in the contemporary world. Questions should emphasize appropriate methods for gathering data, the visual exploration of data, and the development and evaluation of arguments based on data analysis. For grade 4, students can be asked to make predictions from given results and explain their reasoning.

Topic: Data Analysis, Statistics, and Probability	Grade 4	Grade 8	Grade 12
1. Read, interpret, and make predictions using tables and graphs			
a. Read data	•	•	•
b. Interpret data	•	•	•
c. Solve problems and compute with data	•	•	•
d. Estimate using data	•	•	•
e. Interpolate or extrapolate		•	•
2. Organize and display data and make inferences			
a. Tables	•	•	•
b. Bar graphs	•	•	•
c. Pictograms	•	•	•
d. Line graphs	•	•	•
e. Circle graphs		•	•
f. Scattergrams		•	•
g. Other			
i. Stem and leaf plots		•	•
ii. Box and whisker plots		•	•
iii. Others		•	•
3. Determine the probability of a simple event			
a. Sample space	•	•	•
b. Definition of probability	•	•	•
c. Odds	•	•	•
d. Expected value	•	•	•
e. Counting principle (permutations and combinations)		•	•
f. Independent, dependent events		•	•
4. Compute measures of central tendency and dispersion			
a. Mean	•	•	•
b. Range		•	•
c. Median		•	•
d. Mode		•	•
5 Recognize sampling, randomness, and bias in data collection			
a. Given a situation, identify sources of sampling error		•	•
b. Describe a procedure for selecting an unbiased sample		•	•

Topic: Data Analysis, Statistics, and Probability	Grade 4	8	12
6. Recognize the use and misuse of statistics in our society			
a. Given certain situations and reported results, identify faulty arguments or misleading presentations of the data		•	•
b. Recognize appropriate uses of statistics . . .		•	•
7. Estimate probabilities by use of simulations . .		•	•
8. Design a statistical experiment to study a problem and communicate the outcomes . . .		•	•
9. Use formulas for combinations, permutations, and other counting techniques to determine the number of ways an event can occur			•
10. Fit a line or curve to a set of data and use this line or curve to make predictions about the data			
a. Curve fitting			•
b. Normal distribution			•
c. Frequency distribution			•
d. Binomial distribution			•
11. Apply the basic concept of probability, including independent/dependent events, simple/compound events, and conditional probability			•
12. Use measures of central tendency, correlation, dispersion, and shapes of distributions to describe statistical relationships			
a. Standard deviation . . .			•
b. Variance			•
c. Standard normal distribution .			•
d. Correlation coefficient . . .			•
e. Confidence level . . .			•
f. Degrees of freedom . .			•

Algebra and Functions

This content area is broad in scope, covering a significant portion of the grade 9-12 curriculum, including algebra, elementary functions (precalculus), trigonometry, and some topics from discrete mathematics. At the K-4 and 5-8 grade levels, algebraic and functional concepts are treated in more informal, exploratory ways. Proficiency in this content area requires both manipulative facility and conceptual understanding; it involves the ability to use algebra as a means of representation and algebraic processing as a problem-solving tool. Functions are viewed not only in terms of algebraic formulas, but also in terms of verbal descriptions, tables of values, and graphs.

Algebraic expressions included in the grade 8 assessment may be monomial, polynomial, or rational, and may involve one or more variables. They may include symbols for exponents, radicals, and absolute value. In the grade 12 assessment, algebraic expressions may also be monomial, polynomial, or rational. The coefficients of the algebraic expressions may be rational, irrational, or complex; they may involve one or more variables and include symbols for exponents, radicals, logarithms, and absolute value.

Topic: Algebra and Functions	Grade 4	Grade 8	Grade 12
1 Describe, extend, and create a wide variety of patterns and functional relationships			
a. Recognize patterns and sequences	•	•	•
b. Extend a pattern or functional relationship	•	•	•
c. Create an example of a pattern or functional relationship	•	•	•
d. Understand concept of variable		•	•
2. Interrelate symbolic expressions and verbal statements, between diagrams and models and verbal statements, between diagrams and models and symbolic relationships	•	•	•

Topic: Algebra and Functions	Grade 4	Grade 8	Grade 12
3. Use number lines and rectangular coordinate systems.			
a. Plot or identify points on a number line or in a rectangular coordinate system	•	•	•
b. Graph solution sets on the number line		•	•
c. Work with elementary applications using coordinates		•	•
4. Solve linear equations and inequalities. (Note: The complexity of equations and inequalities will vary depending on the coefficients, number of terms, operations, and solution set.)			
a. Solution sets of whole numbers	•	•	•
b. Solution sets of rational numbers		•	•
c. Solution sets of ordered pairs		•	•
d. Solution sets of real and imaginary numbers			•
5. Perform algebraic operations with real numbers and algebraic expressions.			
a. Addition, subtraction, multiplication, division		•	•
b. Powers and roots		•	•
c. Multiple operations (grouping and order of operations)		•	•
d. Substitution in expressions and formulas		•	•
e. Equivalent forms (simplify, combine, expand, and factor)		•	•
f. Solving a formula for one variable		•	•
6. Represent functions and relations by number sentences, verbal statements, models, tables, graphs, variables, algebraic expressions, and equations, and translate among modes. (Note: At the grade 4 level, algebraic and function concepts are treated in more informal, exploratory ways.)	•	•	•
7. Solve systems of equations and inequalities algebraically and graphically.		•	•
8. Use mathematical methods.			
a. Logic		•	•
b. Informal induction and deduction		•	•

32

Topic: Algebra and Functions	Grade 4	Grade 8	Grade 12
9. Represent problem situations with discrete structures			
a. Finite graphs			•
b. Matrices			•
c. Sequences			•
d. Series			•
e. Recursive relations			•
10. Solve polynomial equations with real and complex roots algebraically and graphically			
a. Factoring			•
b. Graphing			•
c. Factor Theorem			•
d. Synthetic and long division			•
e. Estimation of roots			•
f. Special techniques for quadratic equations (quadratic formula, completing squares)			•
11. Apply function notation and terminology			
a. Domain and range			•
b. Composite functions			•
c. Inverse			•
12. Compare and apply the numerical, algebraic, and graphical properties of functions			
a. Absolute value			•
b. Linear			•
c. Polynomial			•
d. Exponential			•
e. Logarithmic			•
f. Trigonometric			•
13 Apply trigonometric concepts			
a. Circular functions and their inverses			•
b. Radian measure			•
c. Trigonometric identities			•
d. Applications			
i. Geometric problems			•
ii. Periodic real-world phenomena			•

Chapter five

Development of Cognitive Questions

Question Format

In addition to multiple-choice questions, the 1990 mathematics assessment will include open-response questions designed to provide an extended view of students' mathematical abilities. Building on the recommendations from the CCSSO report, the NAEP Item Development Panel will create some open-response items to assess abilities that cannot be measured using multiple-choice questions. These may include the ability to articulate mathematical ideas, estimate, generate informal proofs, draw figures, or generalize relationships.

Some open-response questions will be designed to provide insight into the ways in which students think about mathematics, for example, students may be asked to write in their booklets the procedures they used to arrive at answers to selected problems. Although time-consuming to analyze, these descriptions can provide a better understanding of the ways in which students reach correct and incorrect answers.

At all three grades, questions allowing calculators will attempt to assess not only the correct use of a calculator but also the ability to choose the appropriate computational method; that is, to decide which is the most appropriate method for solving a given problem — calculator, paper and pencil, or mental arithmetic and estimation.

Use of Calculators

Because the calculator is a tool often used for numerical computations, mathematics assessments should reflect the use of calculators in the classroom and in society at large. Calculators have been included in the NAEP mathematics assessments since 1977-78. Items included in the 1990 assessment will be classified in three ways: calculator-inactive items, calculator-neutral items, and calculator-active items. Calculator-inactive items are those whose solution neither requires nor suggests the use of a calculator, in fact, a calculator would be virtually useless as an aid to solving the problem. Calculator-neutral items are those in which the solution to the question does not require the use of a calculator. Given the option, however, some students might choose to use a calculator to perform numerical operations. In contrast, items classified as "calculator-active" require calculator use; a student would likely find it almost impossible to solve the question without the aid of a calculator. Sample items representing these three categories of items are provided in the Appendix.

As currently planned, some fourth-grade students participating in the 1990 assessment will have the use of a four-function calculator and some eighth- and twelfth-grade students will have the use of a scientific calculator. NAEP will ensure that the calculators used by these students are comparable in design and function. In addition, because students may be accustomed to using calculators different from those used in the assessment, students assessed using calculators will be provided an orientation to calculator use.

Chapter six

Development of Background Questions

In addition to the cognitive questions, the 1990 mathematics assessment will include a set of general background questions and a series of subject specific background questions designed to gather contextual information on students' experiences in mathematics and their feelings toward the subject. Three categories of information will be represented in the five-minute section of mathematics background questions:

★ time spent studying mathematics,

★ instructional experiences in mathematics, and

★ attitudes toward mathematics

The number, content, and format of background questions in each of these categories varies according to grade level. A set of background questions will be repeated from previous assessments to permit an analysis of trends across time in students' exposure to mathematics, instructional experiences, and attitudes toward the subject

36

Time Spent Studying Mathematics. Time spent on task and mathematics coursework have been shown to be strongly related to mathematics achievement. Students participating in the 1990 assessment are asked to describe both the amount of instruction they receive in mathematics and the time spent on homework in the subject.

Instructional Practices. The nature of students' mathematics instruction is also thought to be related to achievement.[6] Students are asked to report their experience using various instructional materials in the mathematics classroom, including calculators, models, and manipulatives; in addition, they are requested to describe the instructional practices of their mathematics teachers. For example, students are asked how much time they spend in a typical mathematics class reading a textbook or listening to teachers' lectures compared with the amount of time they spend working on projects or engaging in other small group activities. Other questions ask students to describe to what extent they practice communicating mathematical ideas — such as writing out explanations, justifications, or proofs — in their mathematics classes.

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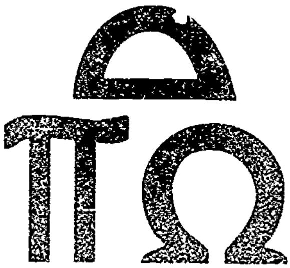
[5] Senta Raizen and Lyle Jones, Eds., *Indicators of Precollege Education in Science and Mathematics: A Preliminary Review* (Washington, DC: National Academy Press, 1985).

[6] National Assessment of Educational Progress, *The Mathematics Report Card: Are We Measuring Up?* (Princeton, NJ: National Assessment of Educational Progress, Educational Testing Service, 1988).

Attitudes Toward Mathematics. Students' enjoyment of and confidence in mathematics and their perceptions of the usefulness of the discipline to their present and future lives appear to be related to mathematics achievement.[7] Students are therefore asked to report their involvement in mathematics-related activities outside of school — for example, their participation in mathematics clubs, fairs, and competitions. The assessment also includes background questions designed to highlight gender stereotypes in relation to mathematics achievement.



Sheila Tobias, *Succeed with Math: Every Student's Guide to Conquering Mathematics Anxiety* (New York: The College Entrance Examination Board, 1987).

Participants in the development process

The National Assessment of Educational Progress appreciates the efforts of the many individuals who contributed to the development of the 1990 mathematics assessment.

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Wilmer Cody, director of the National Assessment Planning Project, and project associate Marlene Holayter are recognized for their contributions throughout the objectives development process. The project was conducted under the auspices of the Council of Chief State School Officers through its State Education Assessment Center, under the leadership of center director Ramsay Selden.

The assessment development process was managed by Ina Mullis, deputy director of the National Assessment of Educational Progress, and Walter MacDonald, NAEP's director of test development. Test development consultants from ETS College Board Test Development Division are gratefully acknowledged for their efforts in developing specifications for the assessment. Last but not least, NAEP appreciates the efforts of the Item Development Panel, whose members spent long hours writing and reviewing assessment questions.

The National Assessment of Educational Progress extends its deep appreciation to all participants.

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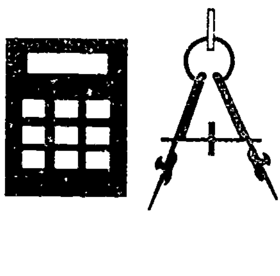
Appendix

Sample Questions

The first part of this Appendix contains sample questions classified by content area, mathematical ability, and grade level. These questions are in no way intended to represent the full range of content areas and mathematical abilities included in the assessment or to reflect the distribution of assessment items. The classifications of questions into the ability categories are a matter of the professional judgment of the CCSSO Mathematics Objectives Committee and the NAEP Item Development Panel; however, there may be differences of opinion as to how particular questions should be classified. It should also be noted that although a question may be appropriate for use at more than one grade level, its classification in terms of the abilities it requires may differ at each grade level.

The latter part of the Appendix contains a second set of sample questions, representing the three classifications for calculator usage described in Chapter 5.



Mathematical Ability:
Procedural Knowledge *

Grade Level: 4

88 + 112 + 6 =

(A) 196
*(B) 206
(C) 260
(D) 1,592

Grade Level: 8

Which is the closest to 7.82 × 5.09?

(A) 0.4
(B) 4
*(C) 40
(D) 400

Grade Level: 12

$$\frac{4 \times 10^{3}}{2 \times 10^{12}}$$

*(A) 2×10^{-9}
(B) 2×10^{-4}
(C) 2×10^{4}
(D) 2×10^{9}

*Correct answers for multiple choice items are indicated by an asterisk (*)

Mathematical Ability:
Procedural Knowledge *

Grade Level: 4

88 + 112 + 6 =

(A) 196
*(B) 206
(C) 260
(D) 1,592

Grade Level: 8

Which is the closest to 7.82 × 5.09?

(A) 0.4
(B) 4
*(C) 40
(D) 400

Grade Level: 12

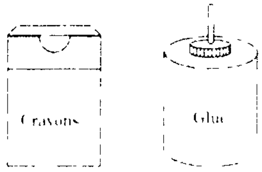
$$\frac{4 \times 10^{3}}{2 \times 10^{12}}$$

*(A) 2×10^{-9}
(B) 2×10^{-4}
(C) 2×10^{4}
(D) 2×10^{9}

*Correct answers for multiple choice items are indicated by an asterisk (*)

Mathematical Ability:
Problem Solving *

Grade Level. 4



The prices for crayons and glue are shown above Katie has $2.50 If Katie buys 3 boxes of crayons, what is the greatest number of jars of glue she can buy with the rest of her money?

Answer: Two

Grade Level. 8

A schedule of class periods is to be prepared for Pinecrest High School The school day will begin at 8 30 a m and end at 2 30 p m There will be 6 class periods all of the same length, and a lunch period that is 30 to 60 minutes long Ten minutes will be allowed for students to move from one class to the next or to move to or from lunch Prepare in the space below a schedule of times for the school day that meets all requirements given above.

Answer: The schedule shown below is one possible solution.

School Day Schedule

Period	*Time*
First	8 30 a m — 9 15 a m
Second	9 25 a m — 10 10 a m
Third	10 20 a m 11 05 a m
LUNCH	11.15 a m -- 11 45 a m
Fourth	11 55 a m 12 40 p m
Fifth	12 50 p m -- 1 35 p m
Sixth	1 45 p m 2 30 p m

Grade Level. 12

When a certain number is divided by 7 the remainder is 4 What is the remainder when 6 times that number is divided by 7?

(A) 2 *(B) 3 (C) 4 (D) 5

*Correct answers for multiple choice items are indicated by an asterisk (*)

Measurement

Mathematical Ability:
Conceptual Understanding *

Grade Level: 4

Which of the following units would be best for measuring the length of a pencil?

*(A) Inches
(B) Feet
(C) Yards
(D) Miles

Grade Level 8

The average height of the girls in a certain eighth grade class could be

(A) 60 centimeters
*(B) 160 centimeters
(C) 300 centimeters
(D) 500 centimeters

Grade Level. 12

A floor plan of a house that has maximum length of 64 feet and width of 44 feet is to be drawn to scale on an 8-inch by 11 inch grid. Which of the following scales will give the largest possible scale drawing of the house on the grid?

(A) $\frac{1}{16}$ inch = 1 foot

(B) $\frac{1}{8}$ inch = 1 foot

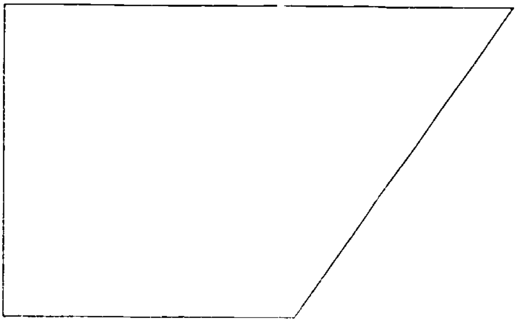
*(C) $\frac{1}{6}$ inch = 1 foot

(D) $\frac{1}{4}$ inch = 1 foot

*Correct answers for multiple choice items are indicated by an asterisk (*)

Mathematical Ability:
Procedural Knowledge *

Grade Level: 4



Note Ruler will be provided

Using the ruler you have been given, find the distance, in centimeters, around the figure shown above

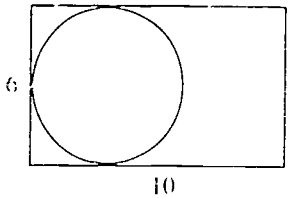
Answer: 20 centimeters

Grade Level. 8

If each edge of a cube has length 5 centimeters what is its volume in cubic centimeters?

(A) 15
(B) 25
*(C) 125
(D) 150

*Correct answers for multiple choice items are indicated by an asterisk (*)



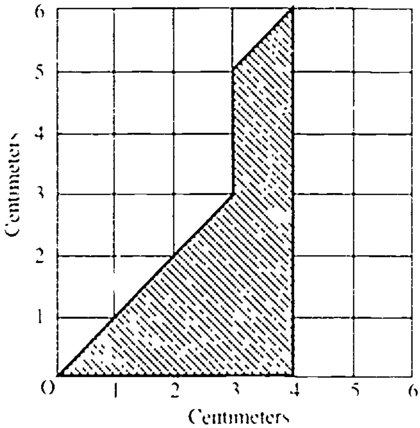
If the width and length of the rectangle shown above are 6 and 10, which of the following is closest to the circumference of the circle?

*(A) 18
(B) 27
(C) 36
(D) 60

*Correct answers for multiple choice items are indicated by an asterisk (*)

Mathematical Ability:
Problem Solving *

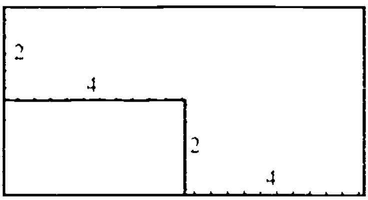
Grade Level: 4



What is the area of the shaded figure shown above?

(A) 8 square centimeters
*(B) 10 square centimeters
(C) 12 square centimeters
(D) 24 square centimeters

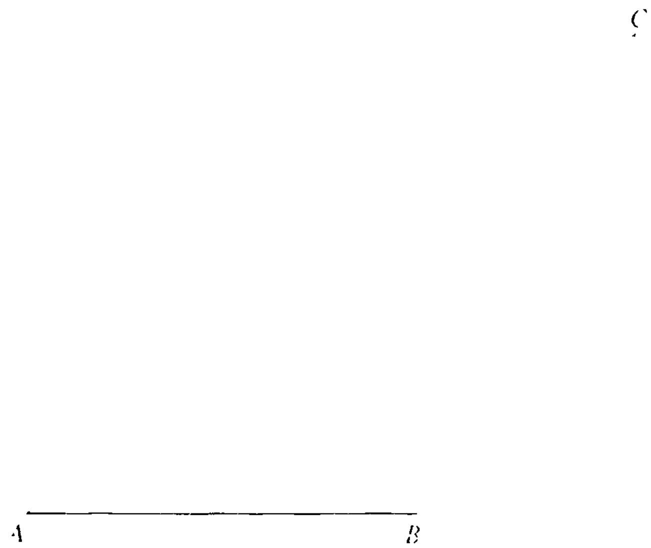
Grade Level. 8



The figure above shows a piece of construction paper. If the unshaded portion of the paper is removed, the area of the remaining shaded portion will be how many times as large as the area of the portion removed?

(A) 2 *(B) 3 (C) 4 (D) 6

*Correct answers for multiple choice items are indicated by an asterisk (*)



Note Ruler will be provided

Use the ruler provided to find the area, in square centimeters, of triangle *ABC* shown above

Answer: 17.3 (rounded to the nearest tenth) square centimeters

Geometry

Mathematical Ability:
Conceptual Understanding *

Grade Level: 4

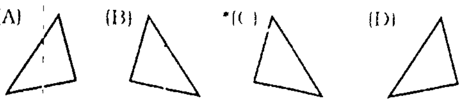
In the space below draw a circle inside a triangle.

Answer: The figure shown below is one possible solution.

Grade Level: 8

If the triangle above is reflected through the dotted line, which of the following shows the reflection of the triangle?

(A) (B) *(C) (D)

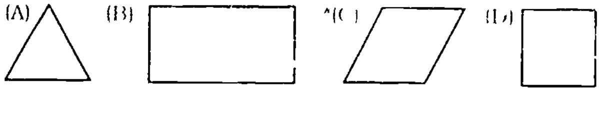


Grade Level: 12

Which figure could be used to prove that the following statement is NOT true?

"If all the sides of a figure have equal lengths, then all of the interior angles have equal measures."

(A) (B) *(C) (D)

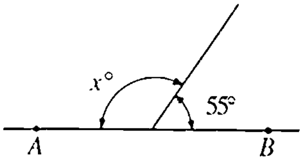


*Correct answers for multiple choice items are indicated by an asterisk (*).

Mathematical Ability:

Procedural Knowledge *

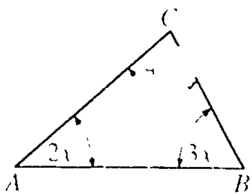
Grade Level: 8



In the figure shown above, AB is a line segment. What is the value of x?

* (A) 125
(B) 115
(C) 45
(D) 35

Grade Level: 12



In triangle ABC shown above, what is the degree measure of $\angle A$?

(A) 20
* (B) 40
(C) 60
(D) 80

*Correct answers for multiple-choice items are indicated by an asterisk (*)

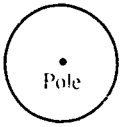
Mathematical Ability:

Problem Solving *

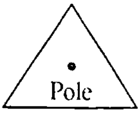
Grade Level: 4

A dog walks on a path that is always 20 feet from a pole. Which of the following could be a drawing of the path?

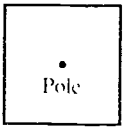
*(A)



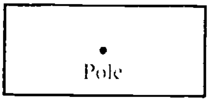
(B)



(C)



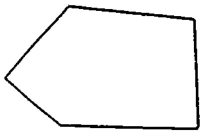
(D)



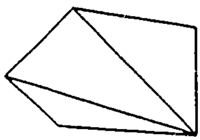
*Correct answers for multiple choice items are indicated by an asterisk (*)

Grade Level. 8

The sum of the degree measures of the interior angles of a triangle is 180. Use this fact to find the sum of the degree measures of the interior angles in the figure shown below.



Answer: 540 . since the figure can be sub-divided into three triangles as illustrated below.



Grade Level: 12

What is the maximum area of a rectangle with perimeter 36?

(A) 36
*(B) 81
(C) 324
(D) 1 296

*Correct answers for multiple choice items are indicated by an asterisk (*)

Data Analysis, Statistics, and Probability

Mathematical Ability:
Conceptual Understanding *

Grade Level 8

Three fair coins are tossed at the same time. What is the probability that one of the coins is a head and the other two coins are tails?

(A) $\frac{1}{8}$

(B) $\frac{1}{3}$

*(C) $\frac{3}{8}$

(D) $\frac{1}{2}$

Grade Level 12

There are 5 Democrats and 4 Republicans on a Senate committee. What is the greatest number of ways that a subcommittee can be formed that consists of 2 Democrats and 2 Republicans?

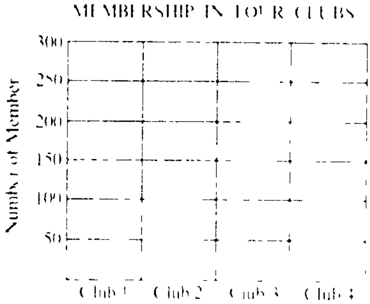
(A) 2
(B) 20
*(C) 60
(D) 80

*Correct answers for multiple choice items are indicated by an asterisk (*).

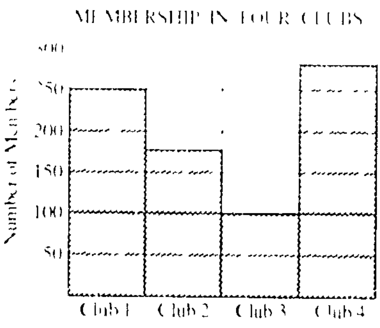
Mathematical Ability:
Procedural Knowledge *

Grade Level 4

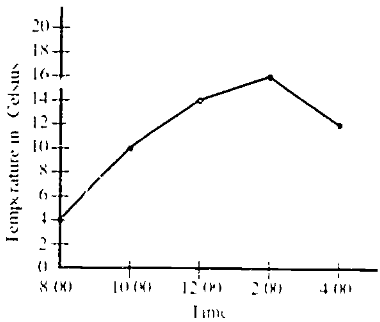
Club 1 has 250 members. Club 2 has 175 members. Club 3 has 100 members, and Club 4 has 275 members. On the grid below, fill in a bar graph that shows the membership of the clubs.



Answer:



HOURLY TEMPERATURES



According to the graph shown above at what time was the temperature the highest?

(A) 8 00 (B) 12 00 *(C) 2 00 (D) 4 00

Grade Level: 8

What is the average (arithmetic mean) of 4, 8, 12, and 20?

Answer: 11

Grade Level: 12

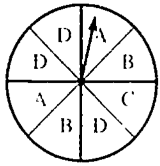
A local newspaper publishes a weekly comparison of the total cost for 20 grocery items at 7 supermarkets. Following are the costs at the 7 supermarkets for one week: $18.48, $17.03, $20.17, $16.74, $19.11, $17.03 and $20.92. Which of the following is NOT true for these data?

(A) The range is $4.18
(B) The mode is $17.03
(C) The median is $18.48
*(D) The arithmetic mean is $17.64

*Correct answers for multiple-choice items are indicated by an asterisk (*)

Mathematical Ability:
Problem Solving *

Grade Level: 4



When the arrow shown above is spun, what are the chances that the arrow will stop on a region labeled with the letter D?

Answer: 3 out of 8

Grade Level 8

Tax Rates for Country A

If your taxable income is

over	but not over	the tax is
0	$2 000	3% of net taxable income
$2,000	$4 000	$60 + 4% of amount over $2 000
$4,000	$6 000	$140 + 5% of amount over $4 000
$6 000	$10 000	$240 + 6% of amount over $6 000
$10 000		$480 + 7% of amount over $10 000

How much should be paid in taxes in Country A for a taxable income of $3 000?

Answer: $100

Grade Level 12

Let *P* be the vertex of a regular 7 sided polygon. What is the probability that a diagonal drawn at random from *P* will form a triangle with two sides of the polygon?

(A) $\frac{1}{3}$

(B) $\frac{2}{7}$

(C) $\frac{2}{5}$

*(D) $\frac{1}{2}$

*Correct answers for multiple choice items are indicated by an asterisk (*)

Algebra & Functions

Mathematical Ability:
Conceptual Understanding *

Grade Level: 4

$4 + 7 + 2 = 2 + \square$

Which number, when placed in the box shown above, will make the number sentence true?

(A) 9
*(B) 11
(C) 13
(D) 15

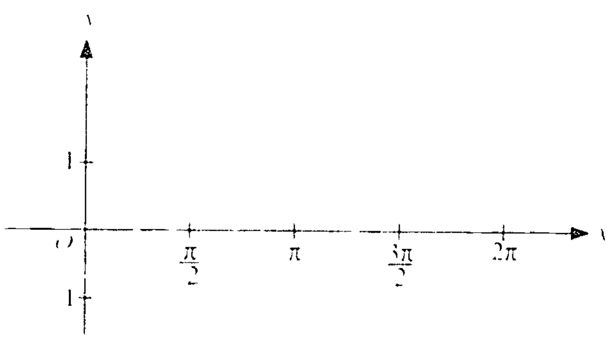
Grade Level: 8

If George has x ten-dollar bills and y five-dollar bills, which of the following gives the total amount of money in dollars that George has?

(A) $50xy$
(B) $15xy$
(C) $15(x + y)$
*(D) $10x + 5y$

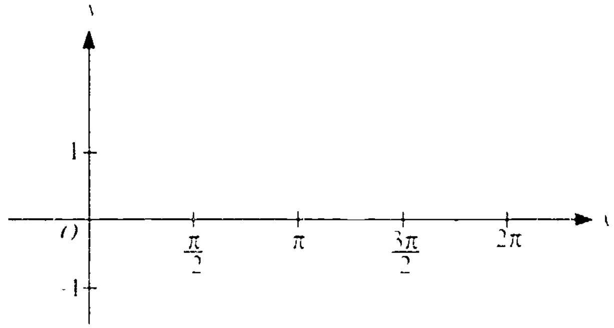
Grade Level: 12

On the axes provided below, sketch the graph of $y = \cos(2x)$ from $x = 0$ to $x = 2\pi$.

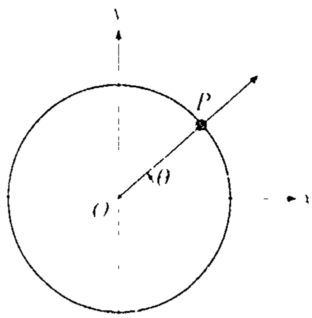


*Correct answers for multiple-choice items are indicated by an asterisk (*).

Answer:



The following question is to be solved with the aid of a nonprogrammable scientific calculator

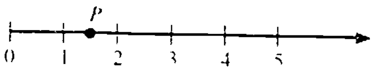


In the figure above, if point P is $(6, 5)$, find θ in radian measure

Answer. 0 695 radians (rounded to 3 decimal places)

Mathematical Ability:
Procedural Knowledge *

Grade Level: 4



On the number line shown above, point P is located at what number?

*(A) $1\frac{1}{2}$ (B) $2\frac{1}{2}$ (C) $3\frac{1}{2}$ (D) $4\frac{1}{2}$

Grade Level: 8

If $7 + 5x = 20$, then $x =$

Answer: $\frac{13}{5}$ **or 2.6**

Grade Level: 12

If $f(x) = 2x^2 - 1$, then $f(x + 3) =$

(A) $2x^2 + 2$
(B) $2x^2 + 16$
(C) $2x^2 + 6x + 8$
*(D) $2x^2 + 12x + 17$

The following question is to be solved with the aid of a nonprogrammable scientific calculator.

If $x^{\frac{9}{2}} = 20$, find x rounded to the nearest thousandth.

Answer: 1.946

*Correct answers for multiple choice items are indicated by an asterisk (*).

62

Mathematical Ability:
Problem Solving *

Grade Level. 4

On the first day Joe reads 1 page of a book, on the second day he reads 2 pages, on the third day he reads 4 pages, and on the fourth day he reads 7 pages. If Joe continues to read the book following this pattern, how many pages will he read on the sixth day?

Answer: 16 pages

Grade Level. 8

x	y
0	−7
1	−5
2	−3
3	−1
4	1
5	3

If x and y are related as shown in the table above, write an algebraic rule that shows the relationship between x and y.

Answer. $y = 2x - 7$

Grade Level. 12

During the first 3 hours of a 3,000 mile trip, a plane is flown at an average speed of x miles per hour. At what average speed, in miles per hour, must the plane be flown for the remainder of the distance if the entire trip takes 2 more hours?

*(A) $1{,}500 - \frac{3x}{2}$

(B) $\frac{1}{1{,}500} - \frac{2}{3x}$

(C) $\frac{3x}{2} - 1{,}500$

(D) $\frac{2}{3x} - \frac{1}{1{,}500}$

*Correct answers for multiple choice items are indicated by an asterisk (*)

CALCULATOR CLASSIFICATIONS

Sample Questions

As described in Chapter 5, items may be classified as "calculator-inactive," "calculator-neutral," or "calculator-active." The solution to a calculator-inactive item neither requires nor suggests the use of a calculator. The solution to a calculator-neutral item does not require the use of a calculator; given the option, however, some students might choose to use a calculator to perform numerical operations. The solution to a calculator-active item requires the use of a calculator. The following sample items provide examples of each type of calculator classification.

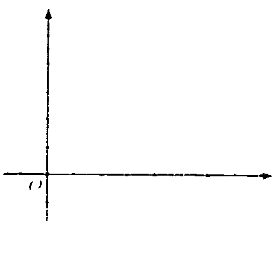
Grades 8 and 12

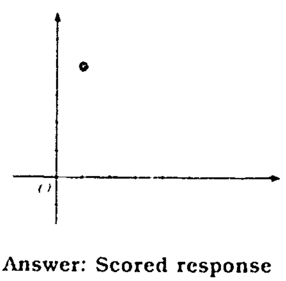
Classification: Calculator-inactive

The number of minutes in 2 days could be determined by performing which of the following operations?

(A) Multiplying 60 by 2
(B) Dividing 60 by 24
(C) Multiplying 60 by 24
(D) Dividing 120 by 24
*(E) Multiplying 60 by 48

Plot the point (1, 4) on the rectangular coordinate grid shown below.





Answer: Scored response

*Correct answers for multiple-choice items are indicated by an asterisk (*).

Grades. 4 and 8

Classification: Calculator-neutral

Multiply

$$\begin{array}{r} 14 \\ \times\ 28 \\ \hline \end{array}$$

(A) 42
(B) 362
(C) 372
*(D) 392
(E) 522

Grades. 8 and 12

Classification: Calculator-neutral

The fat content of a certain food product is known to be approximately 3 percent by weight. Approximately how many *ounces* of fat are in 2 1/2 pounds of this product? (16 ounces = 1 pound)

(A) 0.075
(B) 0.12
*(C) 1.2
(D) 7.5
(E) 12.0

Grades. 8 and 12

Classification: Calculator-active

What is the product of 42.67 and 5 to the nearest whole number?

Answer: 26,669 (scored response)

Which of the following best approximates the radius of a circle with circumference 30?

(A) 3.87
*(B) 4.77
(C) 5.48
(D) 9.55
(E) 15.00

*Correct answers for multiple choice items are indicated by an asterisk (*)