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ABSTRACT

Preschool children's problem-solving strategies on structured and unstructured number tasks were compared in this study. Participants were 20 four-years-olds and 20 five-year-olds. Each child was administered a number conservation, addition and subtraction, and division task. The addition and subtraction task contained trials which were designed to classify children into one of three levels of addition and subtraction understanding: primitive, qualitative, and quantitative. In the division task, subjects were asked to allocate equal numbers of cardboard "cookies" to two "cookie monsters." Subjects in the qualitative and quantitative levels on the addition and subtraction task had different levels of success on the division trials. Findings indicated that qualitative children relied on perceptual strategies to make judgments. Strategies children used on a large, odd number division task included one-one correspondence, many-many correspondence, subtraction, counting, and miscellaneous distribution. Both qualitative and quantitative children used all types of strategies, but differed in the frequency with which they used miscellaneous distribution and subtraction. The qualitative children used these two strategies significantly more than did quantitative children. It is concluded that the three-level model of addition and subtraction understanding has applicability beyond addition and subtraction problems. (RH)

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# Connections Between Addition and Subtraction Reasoning and the Use of Quantifiers

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# Connections Between Addition and Subtraction Reasoning and the Use of Quantifiers

Many studies of number development focus on children's competence in solving problems in a variety of tightly constrained situations. Fewer of these studies focus on whether children use this competence when they solve everyday number problems. Information concerning the spontaneous strategies children use to solve number problems is of practical importance since it indicates what children are actually doing on a day to day basis and it is also of theoretical importance since it relates directly to the relationship between competence and performance, an issue which has recently been given new life by Gelman, Meck, & Merkin (1986).

The purpose of this study was to compare preschool children's problem solving strategies on a relatively unstructured number task, with their performance on a more structured task. The more structured task has been used in previous number studies and is similar to many number tasks in that clues about number are controlled by the experimenter. I assumed that this task was a more sensitive measure of children's capacity, and would therefore come closer to being an indication of their competence than the less structured task. The less structured task was assumed to be similar to problems children encounter everyday and performance factors should have more influence on this task. I chose an addition and subtraction task used in several previous studies as the more structured task (Cooper, 1984; Cooper, Starkey, Blevins, Goth, & Leitner, 1978). I classified it as the

more structured task because the addition and subtraction problems are solved by using one-to-one correspondence cues which the experimenter makes obvious to the child. Children are not allowed to count or estimate the objects so they have to rely on these one-to-one correspondence cues to solve the problems. I used a division task that can be solved in a variety of ways for the less structured task. In this task children are told to divide a group of objects in half and are given no hints or help.

Twenty 4-year-olds and twenty 5-year-olds participated in the study. Each child received a number conservation task, an addition and subtraction task, and a division task. Only the addition and subtraction and division tasks will be discussed. Children were tested across two days. The addition and subtraction task was given on day 1 and the division task on day 2.

The addition and subtraction task contained trials which were designed to classify children into one of three levels of addition and subtraction understanding, the primitive level, the qualitatitive level, or the quantitative level (Table 1). In each trial the child was asked to make relative numerosity judgements about two sets of objects which the experimenter established by placing one object simultaneously into each of two cups until the sets were established. As the objects were placed into the cups the experimenter said, "I'm putting one here and one here." If the initial arrays were unequal the experimenter continued to place the objects one at a time into a cup until it contained the correct number.

The objects were placed into cups so that the final arrangement was not visible to the child. This helped to prevent solutions based on estimation. If children tried to count they were stopped and asked to do the problems without counting.

The division problem contained the trials listed in Table 2. On each trial children were shown two cookie monsters and a pile of round cardboard cookies, randomly arranged, and told to give both cookie monsters the same number of cookies to eat. When they were finished they were asked whether the two cookie monsters did in fact have the same number to eat.

Children were classified into one of the three addition and subtraction levels based on their performance. There were 5 in the primitive level, 23 in the qualitative level, and 11 in the quantitative level (Table 3). Because there were so few children in the primitive level only the performance of children in the qualitative and quantitative levels will be discussed.

Children's performance on the division problem was scored on the basis of whether they divided the cookies correctly and in terms of the strategies they used. On the even number division trials children were scored as correct if they used all the cookies and put the same number on each plate. On the odd number trials they were correct when they used all but one cookie and put the same number on each plate. The qualitative and quantitative children had different levels of success on the division trials. As you can see by looking at Table 4 the qualitative children

found the 4 cookie trial to be the easiest, followed by the 3 and 10 cookie trials, and then the 11 cookie trial (Newman Keuls tests, $p < .05$). For the quantitative children the 4 and 10 cookie trials were easier than the 3 and 11 cookie trials (Newman Keuls tests, $p < .05$). The quantitative children performed significantly better than the qualitative children on the 11 cookie trials ($z = 2.72$, $p < .05$). However, there are more differences between the qualitative and quantitative children than this percent correct data indicates. The qualitative children were incorrect on the small and large number odd trials for different reasons. On the 3 cookie trial the majority of qualitative children, who were incorrect, knew the 2 plates did not have the same number of cookies, but they said they could not make them have the same number. On the 11 cookie trial the majority of the qualitative children, who were incorrect, said both plates had the same number. It appears that the qualitative children were relying on perceptual strategies to make their judgements on the 11 cookie trials. When it looked like both plates had the same number, the children claimed that they did. This interpretation is supported by the fact that the majority of children who were incorrect had put 6 on one plate and 5 on the other plate. On both the 3 and 11 cookie trials, quantitative children who were incorrect knew the two plates were not the same. Some said they could not fix them, and others said they needed 1 more cookie. On the 11 cookie trial none of the children said the 2 arrays were the same.

Because children had to use some type of quantitative information to be correct on the 11 cookie trials, children's strategies on these trials were examined. Only children who were correct were included in the analysis. The strategies children used were classified into the following groups (see Table 5): one-one correspondence (putting one cookie on a plate and one cookie on the other plate), many-many correspondence (puting some on one plate and the same number on the other plate), subtraction (removing one or more cookies from a plate), counting (any form of counting), miscellaneous distribution (putting some on one plate and then putting a different number on the other plate). The frequency of use of these strategies by qualitative and quantitative children is presented in Table 6. Children often used more than one strategy on a trial, however, regardless of how many times it was used on a trial, each strategy was only counted once per trial.

Both qualitative and quantitative children used all types of strategies, but there was a difference in the frequency of use of miscellaneous distribution and subtraction. The qualitative children used these strategies significantly more than the quantitative children. The use of these strategies fits with the intrepretation given above, that the qualitative children put the cookies on the plates, looked at them to see if they looked equal, and if they did not they adjusted the cookies, using subtraction. On the other hand, the quantitative children were less likely to make adjustments to the cookies because they were more likely to stop

dividing the cookies once they had 5 on each plate. This suggests that the quantitative children were relying on quantitative information to guide their performance.

These results indicate that the three level model of addition and subtraction understanding has applicability beyond addition and subtraction problems. Children in the quantitative level used their quantification skills not only on the relatively structured addition and subtraction task, but also on the less structured division task. It is interesting that the qualitative and quantitative children use the one-one correspondence, many-many correspondence, and counting strategies with approximately the same frequency, but that the quantitative children are more successful than the qualitative children. It may be the case that the quantitative children attend to the quantitative information provided by the strategies more than the qualitative children.

The results indicate then that there are systematic relationships between performance on the more structured addition and subtraction task and the less structured division task. Using Gelman, Merkin, & Meck's (1986) terminology, performance on the addition and subtraction task can be taken as an indication of conceptual competence and performance on the division task can be taken as an indication of both utilization and procedural competence. Since most, but not all, of the quantitative children rely on strategies that can provide accurate information about relative numerosity, the results demonstrate in this case that changes in conceptual compentence precede changes in utilization and procedural competence.

# References

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## CONNECTIONS BETWEEN ADDITION AND SUBTRACTION REASONING AND THE USE OF QUANTIFIERS

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### TABLE 1

**Predicted Responses to Trials by Addition and Subtraction Reasoning Levels**

| *TRIALS* | | | *REASONING LEVELS* | | |
|---|---|---|---|---|---|
| **Initial State** | **Transformation** | **Final State** | **Primitive** | **Qualitative** | **Quantitative** |
| N* | +1 | N+1 | correct | correct | correct |
| N | +0 | N | | | |
| N | +1 | N+1 | N+1>N+1+0 | correct | correct |
| N+1 | +0 | N+1 | | | |
| N | +1 | N+1 | N+1>N+2+0 | N+1=N+2 | correct |
| N+2 | +0 | N+2 | | | |

* N means that arrays are initially equal.
N

## TABLE 2

### Trials Used in Division Task

| Total number of cookies | Number of trials |
|---|---|
| 3 | 2 |
| 4 | 2 |
| 10 | 2 |
| 11 | 2 |

## TABLE 3

### Number of Children in Each Addition and Subtraction Reasoning Level

| | *ADDITION AND SUBTRACTION REASONING LEVEL* | | |
|---|---|---|---|
| Age | Primitive | Qualitative | Quantitative |
| 4 | 3 | 14 | 2 |
| 5 | 2 | 9 | 9 |

## TABLE 4

### Percent Correct by Addition and Subtraction Reasoning Level and Trial Type on the Division Trials

| | *TRIAL TYPE* | | | |
|---|---|---|---|---|
| Addition/Subtraction Reasoning Level | 3 | 4 | 10 | 11 |
| Qualitative | $69^{ad}$ | $100^{ab}$ | $73^{bc}$ | $28^{acd}$ |
| Quantitative | $63^{eg}$ | $100^{ef}$ | $95^{gh}$ | $68^{fh}$ |

Note. Trials with the same superscripts are significantly different at $p < .05$.

## TABLE 5

**Types of Strategies Used in the Division Task**

| | |
|---|---|
| *One-one Correspondence -* Put one cookie on a plate and one cookie on the other plate | |
| *Many-many Correspondence -* Put some on one plate and the same number on the other plate | |
| *Subtraction -* Remove one or more cookies from a plate | |
| *Counting -* Any form of counting | |
| *Miscellaneous Distribution -* Put some on one plate and then put a different number on the other plate | |

## Table 6

### Percent of Children Using Division Strategies on the 11 Cookie Trials

| Strategy | Addition/Subtraction Level | TRIAL: 11 cookies (Trial 1) | TRIAL: 11 cookies (Trial 2) |
|---|---|---|---|
| One-one | Qualitative | 30 (N=8) | 67 (N=6) |
| | Quantitative | 57 (N=7) | 38 (N=8) |
| Many-many | Qualitative | 50 | 33 |
| | Quantitative | 29 | 13 |
| Counting | Qualitative | 50 | 33 |
| | Quantitative | 57 | 63 |
| Miscellaneous Distribution | Qualitative | 75* | 50** |
| | Quantitative | 28* | 13** |
| Subtraction | Qualitative | 88*** | 67 |
| | Quantitative | 43*** | 38 |

Note. Only the performance of children who were correct on the 11 cookie trials is represented in this table.
*a significant decrease ($z = 2.93, p < .05$)
**a significant decrease, ($z = 2.31, p < .05$)
***a significant decrease ($z = 3, p < .05$)