Children's nonschool learning provides a foundation for school instruction and problem-solving. Examples of nonschool learning are taken from various cultures. This study examines what young children know about written numerals in their everyday environment, how they come to understand the meaning of written numerals, and how they use this knowledge to reason and interpret nonschool situations. Most of the data are from interviews with the same children in the middle of kindergarten and again at the end of second grade. This is supplemented with test data obtained from the same children when they began kindergarten. Methods and results are discussed. Includes 43 references. (DC)
CHILDREN MAKE SENSE OF NUMBERS: THE DEVELOPMENT OF IDEAS ABOUT WRITTEN NUMERALS

by

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INTRODUCTION

There has long been a distinction made between the cognitive skills learned in school and those learned outside of school. Although anthropologists have established that there are cultures which lack formal schooling but possess extensive bodies of knowledge which require complex cognitive skills to be utilized, educators and cognitive scientists still tend to view nonschooled knowledge and reasoning as secondary and less powerful than school-based cognition. However, there is increasing evidence that effective school instruction depends upon the meshing of school learning with the knowledge that children learn outside of school. Not only can everyday learning provide a foundation for school instruction, the integration of everyday reasoning skills into the school curriculum may actually help students to become better problem solvers within academic subject matter domains.

Despite the growing acknowledgement of the importance of everyday reasoning, there is still little known about young children’s nonschool knowledge of academic subjects, particularly mathematics. This study examines what children know about written numerals in their everyday environment and how they use this knowledge to reason about different situations. It will be shown that children begin school with substantive knowledge of the meanings represented by numerals and that this knowledge changes qualitatively over the first three years of school. This growth in knowledge is a result of children’s active reasoning and construction of meaning rather than direct instruction from parents or teachers. One way that children construct their understanding of numerals is through the analysis of the functional use of numerals in a setting and transfer of meaning from more familiar settings where the numbers have a similar functional use. Children also know that settings support a
variety of quantitative activities. When faced with an unfamiliar numeral, they will
interprets its meaning in terms of these quantitative activities rather than through
analysis of the numeral or pattern of numerals in the setting. Thus it might be said that
they often fail to extend their understanding of everyday settings through utilization of
mathematical principles taught in school. On the other hand, children use good
problem solving skills such as judging the reasonableness of quantities and making
analogies to other similiar 'problems', skills which are all too often lacking when
children solve problems in school.

Background

In American society, the school system has been delegated the task of teaching
children particular domains of knowledge. Most prominent among these are literacy
skills, mathematics and science. Concomitant with this skill learning, children
reputedly also learn powerful, more generalized thinking skills that enable them to
apply these skills to a wide variety of problems (Scribner and Cole 1973). In contrast,
skills learned in nonschool settings have been characterized as particularistic,
concrete and limited in application. However, the pre-eminence of schools for
teaching academic and cognitive skills has been challenged on a number of grounds,
and nonschool learning experiences are being considered more seriously as
important sources of cognitive growth.

One challenge to the special status accorded formally schooled cognition is that
schools are not the only places in which people learn about literacy and math.
Studies of nonwestern literacies such as the Vai script in Liberia (Goody 1977;
Scribner and Cole 1981) have demonstrated that functional literacy can be transmitted
and used without the support of any institutionalized form of education. Similarly
arithmetic skills are learned by nonschooled persons in the course of craft
apprenticeship in Liberia (Lave 1977), while selling food and cloth in Brazil and the
Ivory Coast (Carraher, Carraher & Schliemann 1985; Pet:to 1982; Saxe ip) and while
participating in the money economy in Papua New Guinea (Saxe 1981, 1982a, 1982b). Within the United States the importance of nonschooled knowledge is increasingly acknowledged by educational researchers. Although debate continues about whether to characterize naive physics knowledge as theoretical or particularistic, unschooled ideas about physics are robust and well-developed to such an extent that they interfere with formal instruction in physics well into the college years (diSessa 1982, 1983, ip; McCloskey et al. 1980). On the more positive side, preschool age children develop early literacy skills--including concepts of letter-sound correspondence, the structure of narratives and the basic word recognition--which enable them to more easily benefit from literacy instruction in school (Hiebert 1982; Kontos 1986; Mason 1980).

In addition, the skills learned in school are not necessarily more powerful nor more easily generalized to a wider range of applications. Lave (1977) compared how well tailors and their apprentices were able to solve novel math problems which varied in how closely the problems resembled problems found in school and those found in tailoring practice. Both experienced tailors and schooled apprentices were able to solve unfamiliar problems (although to a limited degree), and both groups exceeded the problem solving success of people with little experience in school or tailoring. There is also substantial evidence that school learning does not transfer well to many everyday and job related contexts. Not only is school math seldom seen in everyday problem solving, people are quite successful using their own invented solutions to math problems that occur in everyday life. However, they are notably unsuccessful when similar problems or identical problems are presented to them in the context of a written math test (Carraher, Carraher and Schliemann 1985; de la Roche 1985; Lave, Murtaugh & de la Rocha 1984; Lave 1988). Conversely, formally instructed skill in such diverse areas as basic math, reading x-rays and electronic troubleshooting has been shown to have little application in the work settings of dairies (Scribner 1984), hospitals...
Just as school skills are often not exported to nonschool settings, problem solving in schools suffers from the lack of application of everyday reasoning skills. Word problems are included in the math curriculum in order to give students the opportunity to apply their new skills to real situations, but students have great difficulty doing this. A frequently cited example comes from the Third National Assessment of Educational Progress (Carpenter et al. 1980). The problem is "An army bus holds 36 soldiers. If 1128 soldiers are being bussed to their training site, how many buses are needed?" Although a majority of the children were able to correctly perform the arithmetic algorithms to obtain an answer of 31 remainder 12, less than one third realized that 32 buses are needed in order to transport all of the soldiers. Resnick (1987) and Schoenfield (in press) provide further examples of how commonsense is lost in the formalized context of school mathematics.

The difficulties that children have in applying commonsense reasoning to school tasks is a consequence of both the structure of the school curriculum and the students' interpretation of the school culture. These difficulties may be particularly acute for minority children. Heath (1983) studied children in North Carolina who had differing experiences with literacy practices before entering school. One group of children from the neighborhood that Heath referred to as Trackton entered school with language skills such as telling stories, making metaphors and discerning patterns. But they quickly experienced failure in school because they lacked what the school curriculum considered more basic literacy skills. By the time these children's skills were valued in the classroom, the children were no longer able to overcome their failures in the lower grades by recovering earlier strengths. Heath worked with the teachers to find "bridges" between the children's homes and school experiences, thereby enabling the children to successfully participate in classroom activities from
the beginning while developing basic school skills, and hopefully maintaining their early strengths.

Students may also have difficulty using commonsense reasoning in schools because of an undue emphasis on formalisms in lieu of deeper knowledge of academic subjects. Just as expert reasoning is based on qualitative understanding of scientific and mathematical principles, students need to learn qualitative reasoning skills about school subjects that will enable them to use knowledge productively across contexts (diSessa 1982, 1983; Linn and Songer 1988; Resnick and Omansson 1987). In science education, the formalisms are typically equations or quantitative definitions such as "heat is the number of calories in a substance." In math the formalisms are things like the algorithms for long division or subtraction with regrouping, and formal proofs in high school geometry classes (Schoenfeld ip). Such formalisms inhibit the productive use of general reasoning skills because the students become engrossed in learning the rules of what appears to be a closed symbolic system.

Despite the apparent importance of everyday experiences in other academic domains such as physics and early literacy, to date a limited number of nonschooled mathematical skills have been studied, primarily counting and basic arithmetic operations. Gelman and Gallistel (1978) explored what children as young as two know about counting. Through a series of ingenious experiments they found that young children understand the principles of counting (e.g. one-to-one correspondence between number names and objects, invariate order of number names, cardinality, etc.) even though they are not always able to apply these principles with larger set sizes. The cross-cultural studies of unschooled persons cited earlier (Lave 1977; Carraher, Carraher and Schliemann 1985; Petitto 1982) have shown that people learn to do simple computation using procedures not typically taught in school. These include decomposing numbers into smaller, easier to handle numbers; using physical
aids to computation such as counters and money denominations; transferring results from known problems to new problems, and using elaborate counting strategies. Before they begin school, many American children also learn to do the simpler versions of these nonschooled computations (Ginsburg and Russell 1981; Starkey and Gelman 1982). Some children continue to develop their mathematical skills ahead of their school instruction (Resnick 1986) and invent the same procedures as unschooled adults in other cultures.

Much less work has been done on how children come to understand the written system of numerals despite its obvious importance in school mathematics. It is known that in the United States and other literate societies many children are able to read numbers before they begin school (Ferreiro and Teberosky 1982; Ginsburg and Russell 1981). They also understand how numbers differ from letters and what some of the functions of numbers are in different contexts (Sinclair and Sinclair 1984). Sinclair and Sinclair found that between the ages of 4 and 6, Swiss children develop their understanding of number from simple recognition (what they term description of numeral) to specific function, i.e. recognition that a number has a very specific purpose which can differ in different contexts.

My study seeks to extend the work on nonschooled mathematical knowledge by considering how young children come to understand the meaning of written numerals and how they use this knowledge to interpret nonschool situations. Children's understanding of written numerals merits further study for a number of reasons. For many children the first major stumbling block in mathematics classes is the transition from what Ginsburg (1977) calls informal math to symbolic math as embodied in the written number system. Written numbers continue to be problematic as children struggle throughout elementary school to understand place value as represented by the numerals. Written numbers may also constitute a natural bridge between everyday quantitative reasoning and school mathematics. In American society the physical
environment is full of written numbers: on billboards and signs, in stores in the form of prices and quantities, throughout our houses on appliances and written materials, and ubiquitously in board games and videogames. These contextualized instantiations of the number system offer children many opportunities for seeing the relation between the written symbols and quantitative ideas such as ordinality, ordered sequences, relative quantities, decimal notation, ratios, etc. Real world uses of numbers may also develop children's problem solving skills such as recognizing reasonable results, estimating outcomes, etc., skills which are notably missing if many children's classroom problem solving repertoires.

In addition to examining children's understandings of written number, I seek to examine how they develop and use this knowledge. There are almost no studies with American children that explore how children understand and learn about math outside of school. Although some psychologists have gone so far as to hypothesize that there is an innate ability to learn about number, just as there is an innate ability to learn about language (Starkey and Gelman 1982), studies in other cultures and with adults in America have found that the development of mathematical skills is closely tied to the contexts in which they are used (Lave 1988; Saxe; Scribner 1984). Most research to date showing that American children learn math outside of school has depended upon test-like skill inventories or experiments in lab settings which are carefully designed to uncover very specific aspects of mathematical knowledge. Little is known about the contexts in which children use number and even less about how the children themselves conceptualize the quantitative aspects of everyday settings as embodied in the numerals.

The study to be reported here has several components. Most of the data are from interviews done with the same children in the middle of kindergarten and again at the end of second grade. This is supplemented with test data obtained from these children when they began kindergarten. This study is one of a series exploring
children's use of math in nonschool settings. Extensive observations have been done in nonschool settings including homes and stores. In addition interviews were conducted with the families of 4 year old preschool children. The data from observations and home interviews will only be referred to tangentially in this report.

In this paper I will answer the following questions:

1) Do children begin kindergarten with skill at reading numerals?
2) How well can children interpret the meaning of numerals that are commonly found in everyday contexts? How much does this skill develop between kindergarten and the end of second grade?
3) How do children reason about numbers in specific contexts? How much is their reasoning structured by the specifics of the context versus mathematical principles embodied in the number system?

METHODS

The subjects in this study were twenty-eight native Hawaiian and part-Hawaiian children in the laboratory school of the Kamehameha Early Education Project. The children at the lab school are selected from communities throughout the island of Oahu, with a disproportionate number coming from a lower socioeconomic background so as to represent the educationally-at-risk population. Native Hawaiian children fall below the norms for school achievement as compared to both national samples and other ethnic groups in Hawaii (Brenner et al. 1985, Native Hawaiian Educational Assessment Project 1983). However, they are expected to have many practical skills at an early age and have an impressive range of household responsibilities at an earlier age than many Mainland samples (Jordan 1981). All of the children in the study were native speakers of English although many of them spoke a dialect known locally as "pidgin", more formally as Hawaiian Creole English.

The children were given a series of individually administered tests at the beginning of kindergarten to assess what math skills they brought with them to school.
The children were also interviewed in what I'll refer to as the Picture Task in the middle of kindergarten (February) and two years later at the end of second grade (May). There were twenty-six children in the kindergarten class and twenty-four in the second grade class. Twenty-two of the children were interviewed in both kindergarten and second grade. These 22 children were in the same class for all three academic years covered by this study so they have had generally the same exposure to the school curriculum. The other two children in the second grade class had entered at the beginning of second grade and so shared only one year of the curriculum with the other children.

The Picture Task is based upon a technique reported in Sinclair and Sinclair (1984) in which the children are shown pictures of everyday settings and asked to explain what the numbers in the pictures mean. Nine pictures representing most of the same settings in the Sinclair and Sinclair study were selected from children's books. The children were also shown a real measuring cup and a real receipt from a grocery store. As summarized in Table 1, the nine pictures include: a store with a customer and cashier standing by a cash register, a street of houses, a picture of people running a race, a picture of a car at a gas station with its license plate prominently showing, a girl speaking on a telephone in kitchen, a fruit and vegetable stand, a birthday cake, and an elevator with people entering it. Each picture/item included written numerals, sometimes several numerals.

Sinclair and Sinclair report their findings primarily in terms of how quantitatively specific the children's interpretations of the numerals are. They coded children's responses as a description, as specifying a global function or as specifying a specific function. I was more interested in how children actually reasoned about the meanings of the numbers—what kinds of information they used, what role quantitative considerations played in their reasonings and how large their knowledge base was
about each setting. So for each picture the following sequence of topics was covered with each child:

1) The child was asked to identify the setting in the picture and tell us an experience in that setting. If the child had difficulty, he/she was told what the setting in the picture was intended to represent.

2) The child was asked to identify whatever numbers were in the picture and what they mean or how they are used in the setting.

3) Target questions were asked about each picture that were particular to the setting in the picture. For instance, after identifying a picture of a speed limit sign and defining speed limit, children were asked about speed limits of 100 miles per hour and ten miles per hour and whether these were real speed limits and why or why not. After identifying the function of buttons in an elevator, each child was asked where someone would go after pushing the button with a five on it. The target questions were asked of every child except in a few instances when a child professed total ignorance about the setting in a particular picture. These questions were designed to uncover children's perception of numerical properties that were present in the real life settings shown in the pictures.

4) Extension questions were asked based upon a child's previous answers in order to explore in more depth a child's particular understanding of a situation. For instance, children were asked if they know what words come after the number in a speed limit. If a child said 'miles per hour' or some other label, he/she was asked more about what 'miles per hour' means.

In the same session as the Picture Task, each child was interviewed about knowledge and experiences with money. These data are not reported here and will be combined with observations of children's buying behaviors in natural settings in another paper.
The interviewing style used was very informal and the children were repeatedly reassured that this was not a test, and that guesses and other speculations about the pictures were strongly encouraged. Since the children had nearly daily contact with the interviewer for the three school years covered by the study, the conversations were in general lively and related. However a few of the boys in second grade were not totally comfortable and gave relatively brief responses to the questions. Each interview with the children was audiotaped and notes were made to back up the recordings and to keep a record of students' behaviors during the interview. The tape equipment did fail at times, so some of the more detailed data is incomplete due to mechanical problems. All of the second grade interviews were transcribed while only sections of the kindergarten interviews were transcribed.

RESULTS

Reading Numerals

In order to interpret children's understanding of numerals in the environment, it is necessary to ascertain if the children can actually read numerals and understand their quantitative referents (although early literacy studies have shown that children do not need to accurately read words in the environment to attribute meaning to them). At the beginning of kindergarten before formal instruction had begun, the children in this sample were given a series of tests to determine their level of math knowledge. Results from two sections will be reported here. One test was a paper and pencil test in which the children were asked to count a group of objects and circle the corresponding numeral from a choice of three numerals. There were twelve questions covering numerals between zero and twelve. Overall the children answered 79% of these questions correctly. They were most accurate with numbers between zero and four (88%), nearly as accurate for numbers between 5 and 8 (85%) and least accurate for numbers from nine to twelve (65%). Of the mistakes that were made, just as many
were due to counting errors as inability to read the numerals. Only one child did not show solid understanding of the correspondence between a set of objects and the numerals--she only answered 2 questions correctly. Only one other child answered less than 67% of the questions correctly. Thus it can be safely assumed that the children were able both to read the numerals and understand their quantitative significance.

These test circumstances provided the children with very strong cues about the quantitative significance of the written numerals. So another test was included to assess children's skill with numbers in a less academic task. In this test, the interviewer and child used puppets to act out a visit to a simulated store which sold stickers. The stickers in this store were divided into three prices groups--3¢, 5¢ and 10¢. These numerals had no correspondence to a quantity of items in the visible environment and thus there were fewer cues to the meanings of these symbols. In this situation the children were a bit less accurate in reading the numerals, having accuracy rates of 88%, 84% and 62% respectively. But they still had fairly strong control over the quantitative meanings of the numerals with over 80% of the children being able to state which numerals represented more and a similar number being able to count out a corresponding number of pennies.

The children in this study began school with enough numeral reading skills to enable them to quantitatively read their environment and begin to interpret it (if indeed this skill is needed to do this!). Although interviews were not conducted with the parents of these children, another study with the parents of Hawaiian children beginning preschool in the lab school facility (Levin, Brenner and Kotubetey ms.) found that most parents don't attempt to teach their children to read numbers. Rather, they concentrate their efforts on teaching children to count and were often surprised during the course of the joint parent/child interviews to find out that indeed their children knew how to read some numbers. While there are many routes besides
parental instruction through which children can learn such a skill (e.g. SESAME Street, daycare, siblings), our impression has been that, as with early literacy skills, children actively seek to interpret the symbolic representations that are so ubiquitous in American society. They learn to attribute meanings in terms of the activities that they see people doing in the context of numerals.

During the picture task itself, the children never had any trouble detecting the numerals in the pictures and they never confused numerals with words. In kindergarten some of the children were unable to read the numbers with more than two digits except as strings of single digits, although in many of the pictures this didn't affect the meanings attributed to the numbers (e.g. 3 digit house numbers).

**Numeral Understanding**

We found that the children in our sample had extensive knowledge about the uses of numerals in the pictured contexts, apparently more than has been reported for some other samples. We also found substantial growth in knowledge over the two years covered by the study. There were also some significant differences in the answers given by the younger and older children. These points are each covered in turn in this section of the paper.

**Identifying functions of numbers**

Table 2 summarizes what percentage of the samples of children were able to specifically state the function of the numbers in each picture. Unlike the Sinclair and Sinclair (1984) coding strategy, an answer was judged correct only if it was correct in both specificity and content. A child was considered to have understood a specific function if the response conveyed a sense of the information being represented by the numeral itself. At times it was difficult to distinguish a child's general knowledge of the setting from knowledge of the numeral. During the interviews an attempt was made to ask questions to clarify what a child meant. Some examples are of how these
distinctions were made are as follows. (Further examples can be found in Sinclair and Sinclair (1984) on page 179.)

Some typical initial responses for the numbered buttons in an elevator are "So you can go up and down" and "you push the buttons to go somewhere". If a child gave such an answer the interviewer followed with a question such as "If you push the button with the two, where do you go?" A child had to specify the second floor to be credited with a correct response. Responses such as "up", "to the middle" or "to my aunt's house" were not considered to indicate an understanding of the numeral's function in the context.

For the store setting, children were shown a picture featuring numerals on a cash register and an actual receipt. Answers about the cash register such as "for paying", "they press the button and the number come out" and "for the money" were judged as an insufficient answers for indicating knowledge of the meaning conveyed by the numerals. Correct answers were "that tells how much money to pay", "it says how much it costs" and "so you can give that much money" were judged sufficient. The responses from the kindergarten children about the receipt were not tallied because none of the children were able to differentiate the several different kinds of numbers (date, item prices, tax, total, change, etc.) from each other.

For the birthday cake, the child had to connect the numeral with a person's age, not just with the number of candles on the cake since some children discovered the latter relationship by counting items in the picture. It was discovered in the course of the interviews that there was a local custom in which one more candle than the person's actual age is added to the cake for good luck. Since the cake had 8 candles and a numeral 8 on it, the child could say the person was either seven or eight years old. In this instance there is a double symbolism mapped both by the numeral and the number of candles.
Overall, the children had extensive understanding of the meaning of common numerals in their environment. Every kindergarten child was able to identify at least half of the numerals. As a group the kindergarten children achieved 59% accuracy and the second grade children achieved 86% accuracy. This exceeds the knowledge shown by the children in the Sinclair and Sinclair (1984) sample. Their five year old sample, which is comparable in age to this kindergarten sample, gave specific function answers for only 45% of their responses. This is particularly surprising since their criteria for an accurate answer was more lenient, requiring simply that a child name a specific function, not necessarily a correct specific function.

**Comparison of kindergarten and second grade responses**

There was substantial growth in knowledge between kindergarten and second grade, particularly on the more difficult items. This was evident in a number of ways. As Table 2 indicates, the children in second grade were more accurate in identifying the function of the numbers in every picture. The older children also knew more about the settings in the pictures. They were able to tell more details about both the setting and the numbers, and they related more personal experiences with the settings. The following excerpt is from an interview with a second grade boy who knew just about everything about license plates except what the number signified. [Note: Interview excerpts are edited to remove redundancies. These are indicated by dots.]

**Int:** What does the number mean?
**A.:** The number for the license plate means, I dunno what...I dunno.
**Int:** ...Have you ever noticed the license plates on the car?
**A.:** Uh huh (yes), the back is like this, the front is same thing.....My dad said M, A, T is "My Automatic Transition". (laughs).
**Int:** Oh, yeah? Well, is that what his number has on it?
**A.:** M, A, T, it has M, A, T and some numbers are on...I think it was 1, 6 9, I dunno.
**Int:** ...yeah, some people[...] pay extra and get one that says a special word...
**A.:** Like LORI, 'cause ...when I ride my bike I go down the hill and I see this white car that has LORI. That's the owner's name.
A. knew that the front and back plates are the same on a car, he knew that in Hawaii license plates have letters as well as numbers (unlike the picture) and he knew what was written on both his father's plates and a neighbor's plates. Even though he did not know what information a license plate conveys (he finally guessed pounds) he would probably only need to be told once and would remember with little difficulty. In contrast, the kindergarten children never offered any extra information about the license plates. Many of those children who did not know what a license plate was for indicated that they hadn't noticed that cars had such things on them. (After being interviewed, a few children told us that they had gone home and seen the plates on the cars for themselves.) Although part of this response difference might be attributed to a difference in the verbal skills between the two time points and perhaps a different understanding of how to talk with the interviewer, it is probably a real difference in what children know about different settings. The birthday cake was a more familiar picture for the children than the car in the gas station. As noted earlier, many of the kindergarten children pointed out that on their cakes there was an extra candle for good luck and that therefore there was something strange about the picture. They had no qualms about telling this to the interviewer and were quite clear in their explanations.

In general, the items had the same relative difficulty in both kindergarten and second grade. One interesting exception is the picture with a speed limit sign in it. In kindergarten very few children even attempted to guess what the sign might mean. A few children indicated awareness of signs in general, guessing that it might name a street or indicate a bus stop. Most children professed ignorance of all signs and a few maintained that in fact there were not such signs in Hawaii. A few of the children were asked about speed limits and could not say much of anything about them. The second graders, in contrast, were quite conversant with speed limit signs. In describing the picture, one girl said "It's a picture of a bus going fast and uh, numbers in this one is
speed limit! And there's always speed limit around!" Part of the difference between this picture and the others is in the interrelation of alphabetic text and numerical text. Unlike the kindergarten students, most second graders were able to read the words 'speed limit' and those who could not made a point of asking about the words. In contrast, not one kindergarten student asked to be read the words on the sign. In fact, the kindergarten children generally ignored the text in the other pictures as well, including the labels on the receipt, the name on the street sign and the letters on the shirts of the runners in the race picture.

As would be expected, there were many qualitative differences in the ways in which the children in the two age groups expressed their answers. The older children had a more developed and standard vocabulary for describing some functions of numerals including labels such as "address", "license plate", "receipt" and "cash register" as well as quantitative phrases such "how much it costs", "how old someone is" and so on. However for many of the pictures there was no particular relationship between a child's skill at using a label or quantitative phrase and their understanding of the quantitative concept embodied in the picture.

Less expected was that the second grade children were less succinct and precise in the ways they defined certain numerical functions. For the pictures of the house numbers, the license plate, the race, and the speed limit, the kindergarten children who knew the function of the numerals said direct and simple phrases such as "so you know where you live", "so you know whose car is that", "so they know who did the best", "because, so you can go to the right house". In contrast most of the second grade children gave 'situational' definitions for these same pictures. Three typical answers for the license plate by second graders are:

1) M.: It's the...what the people knows if someone takes, if someone is trying to rob you and you see their car ...and if you don't know ...what their car looks like then you maybe can take down their number.
they need that for, like if you're in a movie house and then you block something, then you get, then they call whatever the thing (number) is, and then you have to move it.

...like if the bus stop an' something was wrong with the bus, like didn' have enough oil. An' you wanted to catch your mom, an' your mom was coming down, then you...can hitchhike your mom. ...If the people didn't have licen' plates, you wouldn't know which one was your mom car.

Only two of the second graders gave more succinct answers for this picture (14% of the correct answers) while nine of the kindergarten children gave succinct answers (90% of the correct answers). The situational answers seem to have their origin in both the child's developing idea of what constitutes evidence for a position and in their efforts to construct meaning out of specific incidents. When children were asked how they knew something, they often responded with such situational explanations similar to the ones given here. But another example, that of the measuring cup, exemplifies the constructivist nature of children's explanations.

More kindergarten children knew the name of the measuring cup and they gave much more succinct, 'adult' sounding definitions of the function of the cup and its numbers. So typically the kindergarten children said of the numbers on the measuring cup, "maybe for water, so it can tell you how much water you put inside the cup", "it tells you how much you want", "so they know how much it is." The second grade answers were more along the lines of "like if you wanna measure water you can like, they, you wanted to put eight, an' you put in 11, then you have to pour some and then measure, an' then measure, and then pour some until it's 8!" And few children in second grade knew the name for the measuring cup although some gave descriptive answers such as "ruler cup" and "cup measuring holder".

The answer to this discrepancy in kindergarten and second grade performance can be found in the classroom experience of the kindergarten children. The first few kindergarten children interviewed could not identify the measuring cup nor describe
the function of the numerals. Then suddenly all of the children were giving this information. When we asked them where they had learned this, they told us that they had just had a lesson on measuring cups in class and were using them at a science project center featuring a water table. Very few of the kindergarten children remembered ever seeing a measuring cup at home and thus most could not give any examples of its use outside of school (although in fact the teacher did tell them this.) In contrast, the second grade children were usually able to recount specific uses of the measuring cup and most of them tried to describe the units of measurement, although they were often wrong and commonly said centimeters, teaspoons or inches. Thus the kindergarten children were telling us a recently acquired piece of information that they had been given in school while the second grade children were bringing together a variety of pieces of information and constructing their own definitions at our request. Since these were the same children at two different points in time, we can see that children may give awkward or situationally specific definitions not because of a lack of being told the correct definition, but because they need to make sense of the concepts in their own terms.

Reasoning About Numerals

As the discussion above indicates, children not only learn about numerals by being told what they mean, they also learn about numerals through their own observations and interpretations of numeral usage in various contexts in their own lives. This section of the paper describes some of the principles which shape the constructive reasoning of the children.

Functional Transfer of Meaning

Cross-cultural psychology has been stymied by the increasingly clear fact that it is invalid to infer about people's cognitive processes when using materials that are meaningless to the subjects of study.
In educational practice it is also increasingly accepted that learning proceeds best when children are familiar with the materials. The question arises from this, what constitutes familiarity? In educational practice, there is also a question of what enables a child to transfer a piece of knowledge from one problem to another. In other words, how does a person learn to perceive that two problems are similar? How does familiarity help children to perceive similarity?

In this study, the settings shown in the Picture Task were of varying familiarity to the children. And it has already been noted above that children were unable to respond to pictures which showed totally unfamiliar stimuli such as the speed limit signs were to the kindergarten children. In that example, both road signs and speed limits were unfamiliar to the children. But once a basic level of familiarity is achieved with a setting, the primary information that children use to reason about numerals is the function of those numerals in the setting. The interaction of functional reasoning with familiarity will be shown by two examples: identification numerals and money numerals.

In four pictures (house numbers, telephone numbers, race, license plate), the primary function of the numbers was to identify someone or something, but the settings differed greatly in terms of how much the children knew about them. By second grade all children knew their phone numbers but few knew their full address. In discussing their addresses, many children used what they knew about phone numbers to think about their addresses. For instance, one child made the hypothesis that perhaps the house number is the same as the phone number but was unable to reconcile the fact that phone numbers have 7 digits but his own house number did not.

In the case of the phone number and house number, the phone number is more familiar as judged by both the number of children who can state what it's for and the number of children who claim to know their phone number. But children also made connections across less familiar settings. One boy called a house number "the license
plate for the house. There were other grounds for similarity between pictures besides the function of the number, but the children almost never made reference to these similarities.

Money is an example of a functional use of a number which overrides many other ways of conceptualizing familiarity. As Table 2 shows, the pictures with money (cash register, fruit stand) were among the easiest for the children to identify. Money was also a very salient feature of many environments even when it was not explicitly indicated in a picture. For several of the pictures, the second grade children were asked what other numbers they might see in a real place like the picture. A comparison of the children’s responses to the gas station setting in the license plate picture and the kitchen setting in the telephone number picture highlights how a functional number is more salient than other numbers for children. Although a kitchen is a setting in which children have daily experience, the children in the sample had a very difficult time thinking of other numbers that might be present in a kitchen. Twenty-nine percent of the second grade children mentioned the temperature setting on the oven as a number found in a kitchen. Another twenty-five percent of the children mentioned other numbers (price tags, calendar, recipe) while the other forty-six percent couldn’t think of anything. What was particularly surprising was that not one child mentioned the numbers on a measuring cup even though a cup was sitting in front of the child during the interview, the picture of the kitchen showed a girl preparing food and many children indicated that they’d seen measuring cups used for food preparation.

The gas station, on the other hand, is a setting more rarely visited by children and one in which children are rarely involved in the major transaction of pumping and buying gas. And yet almost all children were able to think of other numbers they would see in a gas station and most of these numbers involved money. While most children
mentioned that the gas pump indicates how much money one pays for the gas, a few
also mentioned the prices on the soda vending machines.

For children, familiar settings with numerals serve as a way of making sense of
less familiar settings with similar numeral usage. In addition, a familiar kind of number
can help a child to make sense of a relatively unfamiliar setting while numerals in a
familiar setting can remain relatively unnoticed when the numbers are less
meaningful.

Reasonability

As mentioned in the introduction, it has often been noted that children seem to
lose sense of what constitutes a reasonable answer while working a math problem.
But this study found that children have strong ideas about what constitutes a
reasonable numerical value for quantities in everyday situations. It was also found
that the children would use seemingly unreasonable numbers to reinterpret a picture
so that the numbers were then reasonable.

After identifying the speed limit sign which had a speed of 30 mph, children
were asked about several different values for speed limits and if a car could go these
speeds. All but one child indicated that 100 would be too much because it would
cause the car to blow up or would cause an accident. Three children indicated that
100 would be ok under special circumstances such as a race or on the freeway (of
which there are very few in Hawaii!). When asked about a speed of ten miles per hour,
only one child said this was not a good speed. Many indicated that it was
acceptable speed because the speed limit indicates the maximum but it's alright to go
slower than the limit. A couple of children mentioned that under some circumstances
10 miles per hour was preferable because it was dangerous to go the speed limit
because of heavy traffic. Two children showed an even more finely honed sense of
practicality—they said that it was alright to go 5 mph over the limit but no more because
then you might get a ticket!
The values of the numerals in another picture caused many children to redefine what the picture was about because the numbers didn't make sense in the pictured context. In the representation of a produce stand, the various items for sale range in price from 3 to 12 cents. One price that struck many children as too low was that of 3 cents for a cup of apple juice. While several indicated that it would really cost somewhere between forty cents and a dollar, other children reinterpreted the picture to make more sense as shown in these two examples.

1)
Int: And, do they seem like good prices or are they too high, or too low, or?
C.: It seems good.....This is a good price, for 3 cents.
Int: 3 cents for the apple juice.
C.: No, no, no! Just the cups. One cup.
Int: Oh yeah, this is for the cup. That would be...a normal price for a cup. What about a cup of juice? Would 3 cents be a good price for that?
C.: No-ho.
Int: What would be a better price for that?
C.: Dollar, dollar something.

2)
Int. So are you telling me that these prices are, are too ......compared to our supermarket prices?
J.: No, too less. Cause um, apple juice should be about 5 or at least one dollar. But I know why they make it 3 cents, cause they only have little cups.
Int: Oh, I see.
J.: Like, about that big. [Indicates small size with his hands] Just try to taste it.

C. interprets that the cups alone are for sale, while J. assumes that the little cups are samples for people to taste. Another child noted that the people selling the things in the picture were children and therefore the prices were good so that other "children like me" could buy the things. The two examples also show how the children transform the picture prices into 'real' prices, by putting the price in dollars instead of cents. As another girl said, "almost all prices these days are dollar something".

Some other children reinterpreted the prices within the price range given in the picture. Several felt that carrots should not be the most expensive product (a matter of
taste or reality?) while apples (an expensive imported product in Hawaii) should cost more than the other produce.

School Math

It can not be said from this study that children totally forget what they've learned in school when confronted with everyday situations, such as those represented by the pictures in this study. In kindergarten the children were obviously using information they had just learned in school to talk about the measuring cup. The second grade children used their reading skills to combine text with their interpretations of the numerals. They also were glad to see instances of numerical notation that they’d learned in school used in some other context. When looking at the date on the receipt, one child exclaimed "Oh, that's how Mrs. B. always writes the date on the board!"

But there were many aspects of school knowledge that did not transfer over to the Picture Task context. Lots of school vocabulary was not used when looking at the pictures. One instance of this was the lack of ordinal number n...mes in the context of the elevator picture. This is particularly surprising because in standard English ordinal numbers are typically used to refer to elevator destinations such as the second or third fl... Jr, (although this might not be true in Hawaiian Creole English which was spoken by some but not all of the children.) Although both the kindergarten and second grade classes had had lessons on ordinal numbers and many children in both classes were able to answer questions about ordinal numbers on a standardized test, many fewer used them when questioned in the Picture Task. Of the children who correctly identified the function of the elevator buttons, only 6% of the kindergarten children and 55% of the second grade children used an ordinal number when asked “Where do you go when you push the '2' (or '5')?"

It was also very difficult to elicit any recognition from the children that the numerals in some pictures had ordered attributes such as they have learned about in school. And it was almost impossible during the interview to lead them to use these
attributes to solve a practical problem in the picture context. The picture with house numbers exemplifies this. All of the second grade children knew that house numbers serve to identify a house. Each child was asked (with actions and clear reference to specific positions in the picture) if knowing the house numbers would help a person to find house 317 when that person is standing in front of 304.

Int: Okay, if I'm walking up the street and I wanna come visit you, and you live in house 317, and I come walking like this, I walk past a few houses and I get here to 304. Do I have any help to figure out, what can I do to figure out where you live. I mean, how to find your house? If you live in 317?

P: Go knock on the door and ask them?

Int: Uh-hm (yes). Is there anything else that would help me?

P: Look at the address!

Int: Yeah. Okay, ...when I'm standing here I can see 304. Does that give me any clues about how to find 317?

P: I dunno. Yeah, that gives me some clues!

Int: What are the clues?

P: You're almost there.

Int: That's true 'cause--how do you know that's almost there?

P: Cause. Oh, Never mind. Forgot what I was gonna say anyway.

Int: Oh you did! Okay.

P: It's going like this. Like that....Starts with three hundred, then it goes across the street, then back to the other street, it was on. Then it keeps going across the street and back to the other street.

Int: That's right. Did you ever notice that before?

P: No.

Int: ...Does that give you any clue?

P: Yeah.

Int: What was the clue?

P: Probably when you go across the street come back, go across again, come back, then you might come over here, and then you might see it across the street.

Int: Okay. Is there anything else I know? If I'm standing here and I've walked this way do I know whether to go this way or this way?

P: Yeah.

Int: How do I know which way to go?

P: Cause then, this is not close to 317.

Int: Yeah, so, which way would I go? Left or right?

P: Left or right. It's left.

Int: This way is right. This way is right, and that way's left.

P: Geez, you going right then.

Int: How would I know that right was the right way to go?

P: Cause, if a lower number is lower than a big number then it's probably going up the street.
This was the only instance in which a child was induced to use the pattern of numbers to find the house. Like most children she began with a social solution to the problem (ask someone) and she also knew the fallback position of keep looking until you see it. But her utilization of the numerical pattern was clearly a result of the interviewer's persistent prodding and not a solution from her own experience. Four other children noticed that the houses went in numerical order but they didn't use this information as a clue for solving the problem. None of the children noticed that the houses were also organized with odd and even numbers on opposite sides of the street even when explicitly asked, although one child mentioned this attribute of the elevator buttons. As in the case of ordinal numbers, the children had displayed competency in ordering numbers and identifying odd and even numbers in classroom exercises and on standardized tests.

SUMMARY AND CONCLUSION

The children in this study began school with a substantial base of knowledge about numerals and their usage in everyday life. All of the children could read numbers when they started kindergarten and all but one had clear quantitative referents for those numerals. They were also able to describe how numerals are used in several everyday contexts, particularly when the numerals refer to money or have an identification purpose. By second grade these same children knew more functions of numerals including measurement and a wider variety of identification purposes. They also knew substantively more about the settings in which numbers are used and they are actively engaged in a process of trying to make sense of all the numerals they notice.

There was evidence that children have the skills to be good problem solvers in everyday situations. Descriptions were given of some of the processes that children
use to reason about number. Children make analogies across situations in which the numbers seem to have the same function such as identification and measurement. These functional meanings of numerals also help children to reason about less familiar settings. Children also struggle to make reasonable sense about the numbers they see around them. They have an idea of what the normal range of a numerical quantity in such diverse areas as money and speed limits. Many of the children will struggle to figure out why numbers in a certain context don't make sense by redefining the context or by changing the standard interpretation of the numbers. Less encouragingly, the children didn't use all the reasoning tools that seemed to be available to them for interpreting the everyday settings represented by the pictures. In particular, they didn't utilize some of the skills they have learned in school such as vocabulary and knowledge about properties of the number system. This suggests that there is some kind of barrier between everyday and school reasoning about quantitative situations.

Some observations about classrooms shed some light on this problem and suggest further areas for study. Some of these observations relate to the schools in Hawaii while others are probably true of schools in general. There is a general lack of attempts to bridge between home and school. The children in this study come from an educationally-at-risk group characterized by low socio-economic status and use of a nonstandard dialect of English. To some degree this means that the children may not have as much apparent expertise with some of the topics in which math classes seem to overlap most with everyday life such as measurement (measuring cup) and vocabulary (ordinals, measurement terms, names for things like license plates). From the teachers' point of view, this makes it very hard to build from the children's knowledge base as standard educational practice. As the teachers often said "they don't know anything about math". But this study has shown that children have substantive knowledge about written numerals and in selected domains such as
money (Brenner 1989) when they begin school. This knowledge continues to develop over the early years of elementary school.

The teachers are not helped by the standard educational materials available at the time of this study. The picture of the produce stand with the ridiculously low prices was taken from a first grade math book. This type of picture eventually persuades children to detach what they do in school from similar situations outside of school. Where else but first grade could you buy enough groceries for a family of six for 29 cents? Teaching materials for ordinal number also fall into this trap. For conceptual clarity, lessons on ordinal number typically use examples such as arrays of objects which have no intrinsic order in ordinary life. While this teaching strategy can help children to extract the quantitative essence of the terms, the failure to reinterpret everyday usage of the terms prevents children from developing a full functional use of the vocabulary, and probably the concept. Written numerals also tend to be excluded from children's picture books. An examination of many dozens of children's books revealed that numbers are absent even in stories about settings such as stores where quantification is an intrinsic part of the major activity in the setting.

There is much to be done to better understand how children learn to use math and to solve problems in nonschool settings. While the studies in more exotic settings provide dramatic evidence for the possibility of mathematical conceptual development without the support of formal schooling, American children (daily life in Honolulu is amazingly similar to life in the Mainland United States) are trying to combine their school and everyday lives. Children such as the Native Hawaiian children in this study do not always receive direct instruction on school skills at home or experience school relevant activities such as storybook reading with parents (Brenner and Levin 1986; Levin, Brenner and Kotubetey ms). Consequently, they may be even more likely than children from white, middleclass families to have problems integrating school and everyday reasoning. Even when everyday reasoning could greatly benefit from
the precision and deliberate evaluation of school based reasoning (Reeve, Palinscar and Brown 1987), as in the house number example, the children have little experience at using school based skills outside of school. At the same time these children have more responsibilities (Jordan 1981) and some experiences such as independent shopping (Brenner 1989) at an earlier age than children in other parts of the United States. Through their active and curious engagement in these parts of everyday life they have developed reasoning skills and confidence in their own abilities that could greatly enhance their school performance. While this study has used interviewing as its major data collection method, observational studies and methods involving more direct involvement with children at home are needed to reveal the competencies that children bring to school.

Note: I would like to thank Karen Nose for helping to do some of the second grade interviews and for the transcriptions of all the second grade interviews. This research was supported by the Center for Development of Early Education, Kamehameha Schools, Honolulu, Hawaii.
References


<table>
<thead>
<tr>
<th>Setting</th>
<th>Short label</th>
<th>Numerals</th>
</tr>
</thead>
<tbody>
<tr>
<td>A girl talking on the telephone in a kitchen while preparing food</td>
<td>telephone number</td>
<td>numerals 1 to 0 on a telephone dial</td>
</tr>
<tr>
<td>People entering an open elevator</td>
<td>elevator</td>
<td>numerals from 1 to 10 on buttons;</td>
</tr>
<tr>
<td></td>
<td></td>
<td>same numerals over door</td>
</tr>
<tr>
<td>A woman standing by a cashier and cash register paying for groceries</td>
<td>grocery store</td>
<td>amount of purchase showing on</td>
</tr>
<tr>
<td></td>
<td></td>
<td>cash register--$5.75. Same amount on</td>
</tr>
<tr>
<td></td>
<td></td>
<td>receipt</td>
</tr>
<tr>
<td>A real printed receipt from a supermarket</td>
<td>receipt</td>
<td>date, time, individual item</td>
</tr>
<tr>
<td></td>
<td></td>
<td>prices, total, tax, change returned</td>
</tr>
<tr>
<td>A street with houses on both sides</td>
<td>house number</td>
<td>numbers on houses from 300 to 318</td>
</tr>
<tr>
<td>A birthday cake with 8 candles</td>
<td>birthday cake</td>
<td>numeral 8</td>
</tr>
<tr>
<td>A real measuring cup</td>
<td>measuring cup</td>
<td>ounces 1-16, 1, 2 cups, fractions</td>
</tr>
<tr>
<td>Photo of people running a race with numbers on their shirts</td>
<td>race</td>
<td>various 3 digit numbers</td>
</tr>
<tr>
<td>A car getting gas in a gas station</td>
<td>license plate</td>
<td>4 digit number on rear plate</td>
</tr>
<tr>
<td>A street with a school bus and speed limit sign</td>
<td>speed limit</td>
<td>speed limit of 30</td>
</tr>
<tr>
<td>2 children selling produce at an outdoor stand</td>
<td>produce stand</td>
<td>prices from 3 to 12 cents</td>
</tr>
</tbody>
</table>
TABLE 2
PERCENTAGE CORRECT IDENTIFICATION OF NUMERALS

<table>
<thead>
<tr>
<th>Item</th>
<th>kindergarten (26)</th>
<th>second grade (24)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Telephone number</td>
<td>96%</td>
<td>100%</td>
</tr>
<tr>
<td>Grocery Store</td>
<td>65%</td>
<td>100%</td>
</tr>
<tr>
<td>House Number</td>
<td>90%</td>
<td>100%</td>
</tr>
<tr>
<td>Birthday Cake</td>
<td>65%</td>
<td>100%</td>
</tr>
<tr>
<td>Elevator</td>
<td>65%</td>
<td>88%</td>
</tr>
<tr>
<td>Measuring Cup</td>
<td>65%</td>
<td>95%*</td>
</tr>
<tr>
<td>Race</td>
<td>46%</td>
<td>63%</td>
</tr>
<tr>
<td>License Plate</td>
<td>42%</td>
<td>58%</td>
</tr>
<tr>
<td>Speed Limit</td>
<td>0%</td>
<td>88%</td>
</tr>
<tr>
<td>Average</td>
<td>59%</td>
<td>88%</td>
</tr>
</tbody>
</table>

Note: The following two items are missing data from some of the kindergarten sample.

Receipt: NA 100%
Produce Stand: NA 100%

*N=18