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ABSTRACT

This paper uses a book, "Proofs and Refutations: The Logic of Mathematical Discove ," as an example of Lakatos's use of dialogue. The book was originally adapted from his dissertation and influenced by Polya and Popper. His discussion of the Euler conjecture is summarized. Three purposes for choosing the dialogue form for the book were that it helped to achieve the polemical intent, that it accurately reflected the role of criticism in the growth of knowledge, and that it modeled how mathematics education might be conducted. One disadvantage of the writing style is that the book evokes an emotional reaction from readers, such as delight or disgust. Another disadvantage is that the dialogue itself often veils Lakatos's own position. (YP)

Imre Lakatos's Use of Dialogue

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Background:

Imre Lakatos, during his brief life and tenure at the London School of Economics, was widely known as a ph osopher of science. In particular, his notion of scientific research progra ...es has been very influential. However, his earliest work written in English, his 1961 Cambridge Ph.D. dissertation, was a dialogue in philosophy of mathematics.[1] It was structured as a discussion between a teacher and a group of students of the history of a particular mathematical theory, known as the Descartes-Euler conjecture. Lakatos carefully researched the long discussion and development of the conjecture which occurred in mathematical circles. In the dissertation and its subsequently adapted and published versions,[2] in particular the book entitled Proofs and Refutations: The Logic of Mathematical Discovery,[3] the historical research is restricted to the footnotes, however, while the body of the text is a dialogue in which a

[1] Imre Lakatos, "Essays in the Logic of Mathematical Discovery," unpublished Ph.D. dissertation, Cambridge, 1961.
[2] First as Imre Lakatos, "Proofs and Refutations," British Journal for the Philosophy of Science, 14 (1963-4): pp. 1-25, 120-139, 221-243, 296-342.
[3] Imre Lakatos, Proofs and Refutations: The Logic of Mathematical Discovery, ed. John Worrall and Elie Zahar (Cambridge: Cambridge University Press, 1976).

precocious group of students and their teacher reenact the history in their classsroom.

Lakatos himself attributes this work as arising from two sources, the heuristic of George Polya and the logic of scientific discovery of Karl Popper. Since understanding Lakatos's purposes for using the dialogue form will presumably involve understanding his reasons and motivation for writing at all, some knowledge of these two sources and Lakatos's use of them is appropriate.

Polya was concerned to help students develop their powers of mathematical reasoning, particularly in practical circumstances. His heuristics can be thought of as helpful, though not foolproof, rules of thumb for going about solving mathematical problems. Although Lakatos significantly alters the use of the term "heuristic"[4] in the course of his writing, he takes from Polya the idea that there is something important, something to be studied and learned about the way mathematicians go about solving mathèmatical problems, about the methodology of mathematics.[5] Another important idea which Lakatos gleans from Polya is that understanding a problem is the primary step in being able to solve it.

The work of Karl Popper is the basis of Lakatos's ideas about the nature of knowledge, that is, his epistemology. First both are fallibilists, that is, they hold that human knowledge can never be established with certainty;

[4]For development of this point see Judith Maxwell Greig, "The Epistemological and Educational Arguments of Imre Lakatos's Proofs and Refutations," unpublished Ph.D. dissertation, Stanford, 1987, chap. 3, pp. 67-104.

[5]There seem to be conflation and confusion in Lakatos's writing of different meanings of the term "methodology." Three different contexts, a mathematical, an epistemological, and an educational, all seem to inform his work and his use of the term. For further discussion of this point, see Greig, "Epistemological and Educational Arguments," p. 103.

it is always open to the challenge of criticism and possibility of being shown to be false. Popper claims that this possibility of being shown to be false, or openness to falsification, plays an important role in the growth of scientific knowledge. He claims that scientific knowledge grows through the process of conjectures and refutations. Conjectures are bold suggestions, similar to hypotheses, given in explanation of some phenomenon; Popper considers such suggestions scientific only if they are put in a form which is testable, that is, which allows the making of predictions that can, in principle, be found false.

> The way in which knowledge progresses, and especially our scientific knowledge, is by unjustified (and unjustifiable) anticipations, by guesses, by tentative solutions to our problems, by *conjectures*. These conjectures are controlled by criticism; that is, by attempted *refutations*, which include severely critical tests. . . . Criticism of our conjectures is of decisive importance: by bringing out our mistakes it makes us understand the difficulties of the problem which we are trying to solve. This is how we become better acquainted with our problem, and able to propose more mature solutions: the very refutation of a theory -- that is, of any serious tentative solution to our problem -- is always a step forward. . . .[6]

Just as Popper is primarily concerned with the growth of scientific knowledge, Lakatos is primarily concerned with the growth of mathematical knowledge. In mathematics, the simple traditional description of how growth in knowledge occurs is that claims are put forward and then they are proved. In the dialogue of Proofs and Refutations Lakatos has various characters suggest proofs of the Euler conjecture, very similar to the proofs that were historically suggested; the dialogue continues, however, to show how these supposed proofs were eventually rejected. For Lakatos, as for Popper, it is these rejections or falsifications which are important for the growth of knowledge. Often mathematical proofs have been questioned with

[6]Karl R. Popper, Conjectures and Refutations: The Growth of Scientific Knowledge (New York: Harper Torchbooks, 1965), p. vii.

counterexamples, which Lakatos emphasizes. These counterexamples often su‿‿est key problems which lead to the improvement of the proof, the deepening of understanding, and the growth of mathematical knowledge. Criticism is at the heart of knowledge for Lakatos.

A quick introduction to Lakatos's discussion of the Euler conjecture may be nelpful. This conjecture concerns the relationship between the number of vertices (V), edges (E), and faces (F) of a three-dimensional polyhedron; in simple algebraic form the conjecture is that $V - E + F = 2$. The book opens with the teacher in the ideal classroom offering a proof of the conjecture. The proof involves imagining the poyhedron made of thin rubber and having one face removed; thus, for this new creation $V - E + F = 1$, if and only if $V - E + F = 2$ for the original (because one face was removed). It is then flattened, as against a blackboard. Next the polygons are triangulated, that is, diagonals are drawn until all remaining polygons are triangles. This adds a face and an edge in each case so the relationship remains constant. Then the trangles are removed one by one; two cases are considered and in each the relationship still remains constant. Finally since the equation $V - E + F = 1$ is confirmed for the final triangle, the proof is complete. The students of Lakatos's ideal class immediately attack the proof. One questions whether all polyhedra can be flattened. Another questions whether triangulation always adds both one face and one edge. A third questions whether the two types of triangles removed represent all the possibilities. The teacher then admits that perhaps thought experiment rather than proof might be a better term. The teacher goes on to say, "I propose to etain the time-honoured technical term 'proof' for a *thought-experiment -- or 'quasi-experiment' -- which suggests a decomposition of the original conjecture into subconjectures or lemmas*, thus *embedding it* in a possibly quite distant body of knowledge."[7] Lakatos considers proof and criticism a way of probing what is often called background knowledge, to bring it to light, to demonstrate gaps in our supposed understanding, and to show connections between one bit of information and another. With this example in mind, consideration will now be made of Lakatos's purposes in using the dialogue form.

Purposes:

Proofs and Refutations was originally a dissertation; one might thus expect it to be clothed as a traditional academic work, that is, one in which the research stands as the body of the work. Instead, however, the book is written in dialogue form, with the research relegated to the footnotes. Presumably Lakatos has a purpose in breaking with traditional academic practice and adopting the dialogue form. Three possible purposes will be explored.

First Lakatos seems to have a polemic intent and the dialogue style allows him to develop his case persuasively. It is a case against what Lakatos calls the mathematical philosophy of formalism.[8] Lakatos claims that in trying to explain what it is that mathematicians actually do, the formalist philosophy describes either the algorithmic behavior of a machine or blind guessing. Lakatos denies that this "bleak alternative"[9] is an appropriate description of working mathematicians. He says, "The core of

[7]Lakatos, Proofs and Refutations, p. 9.

[8]Four different "enemies" which Lakatos attacks can be delineated; he refers to them all with the word "formalism." For more detail, see Greig, "Epistemological and Educational Arguments," chap. 2, pp. 40-66.

[9]Lakatos, Proofs and Refutations, p. 4.

this case-study will challenge mathematical formalism. . . ."[10] From this it is clear that his intent is persuasive, that is, to get the reader to agree that formalism as Lakatos portrays it is a false description of mathematics as it is practiced. The dialogue form helps him achieve this purpose by strongly emphasizing the historical or reconstructed events which support Lakatos's case and undermine that of formalism. The most convincing characters in the dialogue are the critical ones; in contrast, those who end up retreating from their original positions are those who have taken what might be described as a formalist line. The dialogue form also allows Lakatos to sidestep some important issues[11] with which a complete philosophy of mathematics would have to come to grips; a conversation simply cannot follow all such issues but rather is focused around one main topic. Thus the dialogue form helps Lakatos to achieve his polemical intent.

Lakatos develops his case against formalism a bit more in saying, "[The book's] modest aim is to elaborate the point that informal, quasi-empirical, mathematics does not grow through a monotonous increase of the number of indubitably established theorems but through the incessant improvement of guesses by speculation and criticism."[12] One of Lakatos's arguments against formalism opposes its "deductivist style."

> In deductivist style, all propositions are true and all inferences valid. Mathematics is presented as an ever-increasing set of eternal, immutable truths. Counterexamples refutations, criticism cannot possibly enter. . . . Deductivist style hides the struggle, hides the adventure. The whole story vanishes, the successive tentative formulations of the theorem in the course of the proof-procedure are doomed to oblivion while the end result is exalted into sacred infallibility.[13]

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[10]Ibid., p. 5.

[11]E.g., mathematical objects, intuition, and the certainty of mathematical truths.

[12]Lakatos, Proofs and Refutations, p. 5.

[13]Ibid., p. 142.

Struggle and criticism, although not of necessity social events, are most often effected in the interplay of scholars. Thus a second purpose for which Lakatos may have employed the dialogue form is to reflect his epistemological thinking with its emphasis on criticism. In the introduction to Proofs and Refutations, he says, in one of his only direct references to the use of the dialogue form, "The dialogue form should reflect the dialectic of the story; it is meant to contain a sort of *rationally reconstructed or 'distilled' history*."[14] For Lakatos, the making of a "rational reconstruction" seems to be a theoretical project which uses the historical data to construct an explanatory account of the growth of knowledge in a particular area. His epistemology "is concerned with the autonomous dialectic of mathematics and not with its history, though it can study its subject only through the study of history and through the rational reconstruction of history."[15] The dialogue, then, is an ideal form through which to convey his epistemological ideas; mathematical history, through the use of the dialogue, becomes interesting and relevant, even for non-mathematicians.[16] In other words, Lakatos used the complementary relationship with the dialogue form to express the importance of criticism in the dialectic of knowledge.

A third purpose for which Lakatos may have chosen the dialogue form is that the classroom setting allows him to model in ideal form how he thinks mathematics classrooms could be conducted. Because Lakatos sees criticism as playing such an important role in the growth of knowledge, he thinks that it should have an important role also in the education of students.[17] In a

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[14]Ibid., p. 5.

[15]Ibid., p. 146.

[16]Thanks to Denis Phillips for underscoring this point in his criticism of an earlier draft of this paper.

[17]See Lakatos, Proofs and Refutations, p. 4.

footnote in Proofs and Refutations, he says, "It has not yet been sufficiently realised that present mathematical and scientific education is a hotbed of authoritarianism and is the worst enemy of independent and critical thought."[18] Several people have developed approaches to education which mirror a Lakatosian philosophy: Dawson suggests that students need to do their own conjecturing and refuting.[19] Robert Davis's torpedoing is a method in which the teacher acts as a critic of student hypotheses in order to inspire more careful or delimited hypotheses.[20] Jere Confrey suggests a "conceptual change" theory for mathematics education in which style and content are more adequately connected; in this approach a rough history of a concept, including important questions, would be helpful to students.[21] Lakatos's point here is to model, through dialogue, how an ideal mathematics education might look.

Evaluation:

The three purposes for which Lakatos may have employed the dialogue form -- that it helps to achieve his polemical intent, that it accurately reflects the role of criticism in the growth of knowledge, and that it models how mathematics education might be conducted -- are not mutually exclusive. In fact, they seem to be interrelated, with the second on the role of criticism being the most important, and the other two being

[18] Ibid., pp. 142-143.
[19] A. J. Dawson, "A Fallibilistic Model for Instruction," Journal of Structural Learning 3 (1971): pp. 1-19.
[20] Robert B. Davis, "Discovery in the Teaching of Mathematics," in Learning by Discovery: A Critical Appraisal, ed. Lee S. Shulman and Evan R. Keislar (Chicago: Rand McNally and Company, 1966), pp. 114-128.
[21] Jere Confrey, "Conceptual Change Analysis: Implications for Mathematics and Curriculum," Curriculum Inquiry 11 (1981): pp. 243-257.

largely dependent on it. Thus in evaluating his use of the dialogue form, all three purposes will be considered together.

Lakatos's use of the dialogue form has both advantages and disadvantages. The primary advantage has already been mentioned: the congruence of content and form between Lakatos's ideas and the dialogue. This may be saying that the book has literary quality in addition to the merits of its ideas. This quality gives it a power and an ability to influence which it might not otherwise have. However, because of this power to influence, many people, scholars included, have had a primarily emotional reaction to the book. The book generally prompts delight or disgust in the reader; few have forged past this initial reaction.[22] Much of the difficulty in so doing is that the dialogue itself often veils Lakatos's own position. So the reader who wants to summarize Lakatos's argument in order to evaluate it is stymied from the first. The thrust of the book is more or less clear from the tone and direction of the dialogue, but the argument which Lakatos would endorse is lost in the maze of conversation.[23] Whether or not Lakatos would accept a statement made by a particular character is often unclear; no character can be said to be consistently carrying Lakatos's viewpoint for him. This makes it difficult to argue with him. Ironically, then, criticism of Lakatos's own ideas on the philosophy of mathematics and mathematics education is impeded.

[22]Peggy Marchi, "Intellectual Maps," Philosophy of the Social Sciences 10 (1980): p. 445.

[23]It is important to note that Lakatos had some deep misgivings of his own regarding Proofs and Refutations. He had a contract for its publication in book form for many years, which he never completed. He regarded it as unfinished and showed hesitation to return to it. See Marchi, "Intellectual Maps," p. 450. The book was finally published posthumously under the editorship of two former colleagues.

Lakatos, then, seems to have used the dialogue form primarily because it reflects his ideas about the importance of criticism in the growth of knowledge. Ironically, however, the dialogue form itself helps to deflect criticism of Lakatos's own ideas.