DOCUMENT RESUME

ED 302 515 SP 030 756

AUTHOR Owens, John E.

TITLE Curriculum and Evaluation Standards for School Mathematics: Report of the National Council of Teachers of Mathematics' Commission on Standards for School Mathematics.

PUB DATE 9 Aug 88

NOTE 17p.; Paper prepared for the Summer Workshop of The Association of Teacher Educators (Starkville, MS, August 9, 1988).

PUB TYPE Speeches/Conference Papers (150) -- Reports - Descriptive (141)

EDRS PRICE MF01/PC01 Plus Postage.

DESCRIPTORS *Academic Standards; *Course Objectives; *Curriculum Development; Educational Resources; Elementary Secondary Education; Evaluation Criteria; *Mathematics Instruction; *Teaching Methods; Textbook Preparation

ABSTRACT

The National Council of Teachers of Mathematics' Commission on Standards for School Mathematics was created in 1986 as a vehicle for coalescing current thinking of the profession on content, instructional methodology, and program and student evaluation of school mathematics for K-12. This report is intended as a guideline for development of curricula, textbooks, resource materials, and evaluation criteria. Each of the four sections provides from 11 to 14 standards, including a statement of mathematical content to be learned, expected student outcomes, and a discussion of purpose, emphasis, and appropriate instructional strategies. A list of recommended reading is included. Appended is a list of underlying assumptions about mathematics instruction for different grade levels. A summary of changes in content and emphasis in the study of algebra, geometry, trigonometry, and functions for grades 9 through 12 mathematics is included. (JD)

Curriculum and Evaluation Standards for School Mathematics:

Report of the National Council of Teachers of Mathematics'

Commission on Standards for School Mathematics

John E. Owens

The University of Alabama

Prepared for the Summer Workshop of the

Association of Teacher Educators

Starkville, MS

August 9, 1988

Curriculum and Evaluation Standards for Sch 1 Mathematics: Report of the National Council of Teachers of Mathematics' Commission on Standards for School Mathematics

The NCTM Commission on Standards for School Mathematics was created in 1986 as a vehicle for coalescing current thinking in the profession on content, instructional methodology, and program and student evaluation of school mathematics. During the summer of 1987 four working groups (K-4, 5-8, 9-12, and Evaluation), each representing a cross section of classroom teachers, supervisors, teacher educators, mathematicians, and researchers, were assigned the task of developing a draft set of criteria for their respective areas.

Since the release of the Working Draft (NCTM, 1987) in October, 1987 the Council has actively sought input from various interested parties through hearings at regional and national conferences. The final report, implementing these suggestions, is scheduled for completion during the summer of 1988. The NCTM anticipates dissemination of the final version at its annual meeting in Orlando, Florida in March of 1989.

The Standards report is intended as a guideline for development of curricula, textbooks, resource materials, and evaluation criteria. Each of the four sections provides from eleven to fourteen standards, including a statement of mathematical content to be learned, expected student outcomes, and a discussion of purpose, emphasis, and appropriate instructional strategies.

Shifts in Curricular Emphases

Few, if any, of the recommendations in the Standards report are new. Rather, the document represents a compilation of exemplary and innovative programs developed in mathematics classrooms over the last several years. Its major impact is not in the development of new ideas but in presenting for the first time a coherent program of reform in mathematics education.

Taken as a whole, however, the recommendations present a vision of mathematics education vastly different from that now experienced by most students. These changes and their implications for the classroom teacher and teacher education are briefly considered here. The complete effect of the Standards will only become evident as they are discussed and adopted by teachers, state and local curriculum writers and textbook publishers.

Equity of mathematical opportunity

Qualitative and quantitative differences continue to exist in the mathematical education of our children. Women and minorities have traditionally been under-represented in mathematics classes and subsequently in technology-intensive careers; resources, including calculators and computers, that are readily available in more affluent communities are scarce or non-existent in poorer schools. These are societal problems and, although recognized by the report, are unlikely to be solved through curriculum reform.

There is a subtler form of discrimination in mathematics, however, that can be approached through restructuring of the curriculum. For many students the study of mathematics begins and ends with computational skill; mastery of pencil-and-paper procedures is believed by many to be prerequisite to the investigation of applied problems, algebra, geometry, or other mathematics. Those students not demonstrating an early ability at fast and accurate computation are often relegated to remedial classes dominated by repetitive

drill. At the secondary level these students typically end their mathematical studies by the ninth or tenth grade, denied the opportunity for many careers.

In order to correct imbalances in the mathematical content studied by students, the report proposes that schools: 1) require all students take 12 years of mathematics (K-11; 13 years is suggested, particularly for college-intending students); 2) implement a core curriculum that allows all students the opportunity to study the important ideas and methods of mathematics; and, 3) remove pencil-and-paper computational ability as a necessary prerequisite to the study of other mathematics.

The twelve-year requirement is becoming more common in schools and is likely to be the norm in the not-too-distant future. The core curriculum, however, is radically different from the way mathematics curricula is now structured in most schools. The de-emphasis on computational skills, a necessary complement to the core curriculum, is likely the most controversial aspect of the report.

Core curriculum

The implementation of a core curriculum is most evident in grades 9-12 where mathematics has traditionally consisted of a series of "tracks" comprising significantly different mathematical content. The "college prep" track, for example, typically consists of Algebra I, Geometry, Algebra II, and so on; a "general math" track may include courses in consumer techniques with introductory algebra but is unlikely to include topics such as trigonometry or analytic geometry; a "basic math" track may merely serve to enhance arithmetical skills.

The standards report states bluntly that no basis exist for the belief that pencil-and-paper skills are prerequisite to the study of other mathematical ideas, recommending that all students be exposed to essentially the same topics--a much broader set than is currently included in any of the present tracks. Obviously, not all students need to, or will be able to, study these topics at the same level of mathematical rigor. And, in adding a wide variety of new topics to an already overcrowded curriculum, something must go.

The proposed solution to differing abilities in the classroom is to vary not the content, itself, but the "depth and breadth" in which content is covered. For example, all tenth graders would study some geometry--some would study geometry at an informal level emphasizing spatial visualization and simple problem situations, others would experience a more formal, axiomatic approach including a stronger emphasis on formal proof and introduction to non-Euclidean geometries.

The 9-12 standards present goals for all students followed by additional goals for college-intending students; in the lower grades this difference is not made explicit, students are assumed to work at different levels while considering the same subject matter. The report is somewhat ambivalent on the nature of classroom organization supporting such differentiation, generally criticizing tracking models (particularly for "gifted" students) but leaving the option of separate classes for different levels of rigor open to school interpretation.

The major effects of the core curriculum are seen as changing middle school practice of repeating arithmetic operations (only 30% of the material presented in a popular 7/8 grade text series was found in a recent study to constitute "new" material), particularly for lower achievers, and sharply revising secondary general and basic level mathematics tracks.

Additions to the curriculum include a renewed emphasis on generally neglected parts of the current curriculum (geometry, probability and

statistics, for example) and the inclusion of new topics from discrete mathematics (see below: Technology, Content additions). Room is made for these inclusions through a reduction in time spent practicing pencil-and-paper computational skills.

Computational skills

Central to the development of a core curriculum is the assumption that students need not necessarily be fast and accurate calculators in order to study geometry, probability, or other areas typically relegated to advanced classes. Coupled with the increased availability of calculators and computers that perform these tasks, the report asserts that de-emphasis of these skills are not only possible but past due.

The report is clear that computational skills remain an essential component of mathematical learning. The shift in emphasis is from an instruction system seemingly obsessed with computational skills to one in which these abilities are considered in the context of a broader interpretation of mathematical knowledge. In particular, the report recommends: 1) delaying skill building exercises until a firm conceptual base can be laid; 2) using calculators and computers for more involved problems; 3) embodying skills instruction in problem solving contexts; and, 4) stressing estimation skills.

Computational skills are defined as more than mere arithmetical competencies. They include, at the secondary level, the extensive manipulation of expressions and equations that are central to current study in algebra--a considerable portion of the current Algebra I curriculum, for example, is devoted to manipulation skills associated with solving equations, simplifying complex rational expressions, and factoring. As with arithmetic skills, these are not envisioned as unnecessary but as areas that should be de-emphasized; conceptual development of the ideas underlying such operations should take precedence, calculator and computer solutions are stressed, and applications serve as both motivators of operations and practice opportunities for developing skills.

Conceptual understanding

In addition to de-emphasizing complex pencil-and-paper computational operations, some traditional algorithmic skills are recommended to be developed later in the students coursework. This deferment is in order to support a better fit between children's' developmental readiness for instruction and to allow more time for maturation of conceptual understanding to precede skills acquisition.

Operations with fractions, for example, are one of the few areas where American students do relatively well on international comparisons in the early grades. However, this seems less attributable to better instruction than to the fact that other countries tend to teach these skills latter in the curriculum. The year fraction operations are introduced, their students immediately jump ahead of American students--presumably based on the laying of a sound conceptual base in the earlier years. Conversely, operations involving multi-digit addition and subtraction tend to be introduced later in the U. S. Curriculum.

The recommended conceptual approach includes an emphasis on children's' model-building activities. A rote skill approach to adding single digit numbers, for example, might merely concentrate on drilling addition tables. A model-building approach, on the other hand, stresses strategies--e.g., to add 5 and 7, you might think 5+5=10 and 10+2=12; adding 8 and 9 can be thought of as 10+10=20 and 20-3=17. In this approach the mental models are not left

unsaid for the students to develop (or not) but made an explicit part of classroom discussion.

Problem solving

Problem solving has been a central issue in mathematics education for at least the last decade; An Agenda for Action (NCTM, 1980) identified problem solving as the "focus of school mathematics in the 1980's." In the past several years much has been said and written on the nature of problem solving and instruction designed to develop students' (and teachers') heuristic faculties.

Many arguments remain as to the most effective problem types (non-standard versus application problems), instructional strategies (explicitly teaching strategies versus immersion in problem situations), classroom organizations (small group versus whole class versus individualized), and evaluation criteria for classroom use. However, little question remains that American students, while demonstrating increasing computational skills, are seriously deficient in critical thinking skills (Dossey, et al; 1988).

The Standards report argues for an increased curricular emphasis on problem solving with a decidedly applications-oriented perspective. This is an attempt on the part of the Standards writers to provide students a rationale for the mathematics they study and a better understanding of the relevance and pervasiveness of mathematics in their daily lives and future careers.

Problem solving activities are also viewed as context for skill development. The report suggests that real-world problems serve both as progenitors of procedural topics and as avenues for building expertise. At the middle school and elementary levels problems are to be associated when possible with manipulative materials; for example, the algorithm for adding fractions would not be taught as a "given" but worked out by the students through physical manipulation of fraction bars, candy bars, and other contexts that support the addition of fractions. Similarly, skills practice is to be embedded in problem contexts--allowing students to measure the correctness of their work against actual phenomena.

The problem solving emphasis is evident at all grade levels. Elementary courses are encouraged to use "natural" context for teaching all skills; story and word problems predominate in the middle and upper grades. In grades 9-12 the practice of teaching isolate contrived problems (a section on "age" problems, followed by sections on "percent-mixture" problems, "work" problems, and "coin" problems--often taught from a formulaistic approach) is to give way to realistic situational problems that require the application of a range of mathematical ideas and skills.

Integration

At the elementary level integration suggests differences in sequencing and in teaching strategies. For example, it is not uncommon for instruction in integer operations to proceed stepwise from single digit addition to two digit addition without renaming to two digit addition with renaming and so on, followed by a similar sequence of single digit subtraction to two digit subtraction without borrowing to two digit subtraction with borrowing. Students often know which algorithm to invoke based simply on the fact that all problems taught today (or this week) are of the same type.

An integrated approach goes beyond merely including problems of the "old type" when introducing new topics. Such approaches may include teaching addition and subtraction in parallel or with explicit emphasis on relationships between the operations. Stigler and Perry (1988) described a

first grade math class as follows:

> For example, one first-grade U.S. class started with a segment on measurement, then preceded to a segment on simple addition, then to a segment on telling time, and then to another segment on addition. The whole sequence was called "math class" by the teacher, but it is unclear how this sequence would have been interpreted by a child. (p. 216).

Given this introduction, it is little wonder that children often view mathematics as a disjointed collection of rules and tricks.

By comparison, a Japanese teacher was quoted as asking a first grader, at the beginning of a mathematics class

> Would you explain the difference between what we learned in the previous lesson, and what you came across in preparing for today's lesson? (p. 217)

The question, which would not be asked in the vast majority of American first grade classes, was easily answered by the student--because student construction of classroom coherence is an area that is of conscious, constant concern to the Japanese teacher; the teacher makes connections explicit and asks the students to do likewise.

Beyond this interpretation, integration also means developing relationships between mathematical ideas. Probability at the elementary level, for example, can be employed not as a separate topic isolated from arithmetic but as an integral component of teaching, say, fractions. In this approach the selecting of colored marbles from a jar may serve to develop students' concepts of ratio.

Integration follows in the middle school with a similar emphasis on the melding of topics. Geometry and statistics are to be integrated into the teaching of arithmetical skills--not left as separate chapters to be covered if time allows (and rote skills mastered). Problem solving emphasis continues to be the tie that binds the curriculum.

At the secondary level the report recommends a three-year integrated program as a replacement for the traditional Algebra I/Geometry/Algebra II/Trigonometry sequence (New York state has used such a tactic for several years and several other areas are implementing or considering such an approach).

Communicating mathematically

Mathematics has often been described as a language--one that includes words, symbols, pictures, and graphs. Through this language we access the ideas that are mathematics in terms ranging from concrete to abstract. In the classroom, however, mathematics frequently seems limited to the mere act of writing a number or formula, or drawing a graph in answer to a clearly defined problem. A "complete answer" may mean nothing more than stating the units for the solution.

At each of the three grade levels the report places "Mathematics as Communication" as the second standard (behind problem solving). This serves as a signal of the importance placed on students' communications skills and is central to the teaching methodology inherent in the Standards' philosophy of instruction.

Communications skills begin in the elementary grades with an emphasis on using a variety of representations (pictures, words, physical models, symbols, etc.) and progress through the student taking an active role in communicating his/her developing mathematical constructs. At the upper grades the abilities to read mathematics and to express ideas--orally and in writing--through the

formulation of generalizations and construction of clarifying and extending questions are primary instructional goals.

As an instructional strategy, communications is enhanced through direct teacher action "by stressing active student participation in learning through individual and small-group explorations, discussions, questioning, listening, and summarizing." Students are expected to "clarify, paraphrase, or elaborate" on the topic at hand in a progression from informal assessments and justifications in the elementary grades to formal proofs and symbolic representations in the upper grades.

In more specific terms, communication, combined with applications, conceptual understanding and integration of topics, suggests a different instructional pace--one characterized by more attention to fewer problems. The common practice of stressing quantity (30-40 similar problems for homework, for example) is to be replaced by an emphasis on students' generalizing from examples, comparing and contrasting operations with previous processes, and using newly learned algorithms in a variety of contexts.

Technology

Technology is seen as both integral to the study of mathematics and as progenitor of change in the content and methodology of school mathematics. The report assumes availability of calculators (one per student) and computers (at least one per classroom) for all mathematics classes.

Calculators are recommended at all grade levels. Four-function calculators are considered mandatory through grade six with scientific calculators for subsequent study. Graphing calculators are recommended for students beginning with the third year of secondary mathematics.

The Standards recognize that calculators are more than mere tools to replace pencil-and-paper computation; calculators have the potential to both change the nature of mathematics instruction and redefine emphases in the curriculum. Specific recommendation in the elementary grades include limiting pencil-and-paper integer operations to cases with fewer than three or four digits, using calculators to develop number patterns and in problem solving contexts, and stressing mental computation and estimation skills.

Computers, like calculators, can change the nature of instructions. Also like calculators, computers can effect the content and emphases of the curriculum. As calculators are recommended to alleviate repetitive and time consuming arithmetical tasks, allowing for the exploration of patterns, computers are touted as graphical equivalents--allowing quick and easy generation of even the most complicated graphs in an effort to develop intuition and visual imagery.

Content additions

Changes in content include different emphases on certain topics and the addition of new topics. Probability, statistics, data analysis, and estimation are current topics that are given the strongest push by the report. These topics are supported due to the increased mathematization of the social sciences, business, and other field that were traditionally considered "non-quantitative." Their inclusion is across the curriculum, ranging from simple charts and probability experiments in the elementary grades through more complex statistical analysis in grades 9-12.

Renewed emphasis on estimation and the addition of topics in discrete mathematics are primarily outgrowths of technology. Estimation is encouraged as a strategy for discerning reasonableness of calculator and computer generated results. Computers are inherently discrete machines and, combined with the increased mathematization of many fields, have spawned a great deal

of new, discrete mathematics. These recent additions to the mathematical agenda, however, have been slow to find their way into the curriculum.

The dispute within the mathematics community over what exactly constitutes "discrete mathematics" is certainly not resolved by the report. The report takes a conservative view of the field; curriculum changes recommended include emphasis on linear algebra, finite graphs, matrices, sequences, and series.

Evaluation

The evaluation component of the Standards report considers two facets: student achievement and program evaluation.

Student progress is to be evaluated through an expanded repertoire of formative techniques.

> Assessment is not simply a matter of noting whether or not a student obtained the correct answer; rather, it involves determining the thought processes that produced the answer.... Many assessment techniques are available. They include structured and open interviews, teacher probing, observations of students working individually and collectively in small or large groups, and observations of students communicating mathematics in a variety of circumstances. (p. 166-7).

More important is the need for "fit" between instruction, curriculum, and evaluation. The techniques suggested must be embodied in an instructional mode that encourages student participation in problem solving activities in a student-centered classroom.

Program evaluation is envisioned as a process of continual development and investigation of the curriculum as it is implemented in the classroom. The report recommends formal programs of classroom observation, student interviews and testing, and consultation with outside sources to determine the status of the curriculum. The evaluation program requires a high degree of professionalism from all mathematics teachers in the school, demanding they be current with the literature and reflective on the nature of their subject and their teaching.

Missing Pieces

The Standards report is not intended as a complete curriculum guide. The amount of detail, while impressive, is hardly satisfactory as a syllabus, nor is the structure of the document designed to provide specific course outlines. Some elements are cursorily covered or not considered (for example, the report calls for mathematics to integrate with other classes (science, social studies, etc.)--however, the mechanisms for this melding are unclear). Other topics, such as mathematical history and social implications of the subject are generally overlooked as instructional topics.

But the report does not claim to be complete. Rather, it provides an outline to be completed through a process of change--a process involving classroom teachers, textbook and materials publishers, school districts, and mathematics educators. In a sense the Standards are more vision than substance, a vision rooted in an understanding of what can be done, and needs to be done, and has been done in exemplary programs.

A significant danger lies in the possibility of partial implementation. For example, adopting a problem solving perspective while maintaining current testing criteria seems self-defeating; the core curriculum cannot succeed without the decreased emphasis on computation which requires calculators which

demand estimation skills; building student communication skills demands teacher moves that exhibit and engender classroom cooperation.

Implementation of the complete standards will be contingent on the perceptions of the program by the educational community and the general public, the availability of resources for changing current practice, and the degree to which teachers and students develop mathematical and instructional conceptions conducive to changing instructional patterns.

Perceptions of the program

The research base for such a major curriculum reform is, of necessity, slim (RAC, 1988). Simple pre/post test comparisons of different teaching styles or curricular emphases are not capable of assessing beforehand the magnitude of major reform; research on conceptual understandings and problem solving abilities has not yet found its way into mainstream classroom activities. In a way, this is a strength of the Standards--the reform contains elements not often found in American education: forward and long-term thinking. Critical thinking skills are necessarily life-long skills that develop over time and are not easily measured in terms of daily or weekly behavioral objectives.

A major reform must be judged against its own criteria, not against those of a previous era. If the measure of success of the Standards is, for example, judged in terms of the memorized rote skills stressed in current testing then it will clearly be a failure. Judged against a set of criteria that values using (versus "knowing") mathematics the results should be different. To this end the evaluation criteria may be the most critical aspect of reform. Changes must not only affect students in a positive way, the perception of growth must be felt by teachers, parents, administrators, politicians, and, most importantly, by the students themselves.

The Standards commission's decision to include evaluation as an equal partner (along with the three grade-specific areas) suggests the importance attributed to both student and program evaluation. Another set of evaluation standards, however, is present--the public perception of the goals and successes (or failures) of the implemented reform.

The public (and school administrators) are often chastised by teachers as being more concerned with test scores than with learning. Employers criticize the schools for producing graduates incapable of applying simple mathematical and communications skills. The task of reform is to satisfy both groups. Satisfying employers means developing students' reasoning, communications, and problem solving skills. Satisfying measurement concerns will require a significant effort at redeveloping evaluation instruments--a task somewhat underway with calculators admitted on College Board Achievement Tests and a proliferation of (admittedly first generation) problem solving tests.

Changing resources

Current integrated texts seem to offer little more than re-sequencing content traditionally taught in separate courses (Amsco, Merrill, and Houghton-Mifflin have developed sequences for the three year secondary program). The question of varying levels of rigor over a consistent set of topics is not yet approached by these or other texts, nor does problem solving appear to be integrated into the instruction. This process should evolve as new offering are published, but the prognosis for rapid production of exemplary texts is not good--similar textbook reforms in the past have not been rapidly accommodated, with new ideas (such as "calculator corners," problem solving exercises, and BASIC programs) appearing first as blocked-out

"extras" at the end of units, only slowly, and often inadequately, becoming integrated into the instruction.

Even if new texts can be developed to meet integrated curricula requirements, the question of embodying the suggested reforms as a central themes remains. Can problem solving, communications skills, and conceptual understanding be adequately addressed in a textbook?

Several attempts are now under way to provide applications material for classroom use. COMAP (the Consortium for Mathematics and its Applications) has produced applications pamphlets, primarily for undergraduate mathematics, for some time. The Consortium has recently developed a series of applications modules specifically aimed at the high school (HIMAP--High School Mathematics and Its Applications), along with newsletters for both high school and elementary (The Elementary Mathematician) teachers. COMAP has also produced, with the support of the Annenberg/CPB Foundation, a 26-part television series and accompanying text (For All Practical Purposes: Introduction to Contemporary Mathematics) for use in upper level high school courses or as a first-year liberal arts mathematics course for colleges.

The Mathematical Association of America (MAA) has developed, to date, six application modules for high schools, ranging from business applications (Pricing Auto Insurance) to elementary Physics (Capturing a Satellite). Each module includes student and teacher material, videotapes, and associated computer programs. Other material, including Challenge of the Unknown, and Voyage of the Mimi, is also available for the teacher, but availability of significant amounts of curriculum-specific applications material is at least several years off.

These materials share a common interest in mathematical applications in a problem-solving context with an emphasis on student communication of findings. However, integrating such material is in itself a formidable task for the classroom teacher. Not only is the relative lack of material a problem (particularly at the elementary level), curricular and instructional decisions abound: Where does the material fit in the curriculum--i.e., which concepts and skills are prerequisite, which can be developed using the material, and which can be enhanced or practiced in the given context? How can the material be altered for different levels of mathematical ability? What classroom organizational structures best support implementation of new materials? What resources are available for purchasing materials, and what allowances are made for teacher time and support to develop material? Will the test-driven nature of the curriculum be altered to allow for different goals and strategies? If curriculum reform is to be more than simply "teaching a different text," questions such as these must receive critical attention.

Obviously, these questions can not be immediately answered--in most cases, complete answers are unlikely. Reform, like mathematics, is a process, not a product. Some evidence of change on a resource level is beginning to take place: The three year (9-12) integrated program is in place in some school districts (albeit typically in a college-prep track only); better materials for non-college intending courses (e.g., informal geometry texts, applications resources) are beginning to crop up; California recently refused to adopt a single elementary mathematics text series based on an absence of higher order skills and has developed a K-12 technology program; major school systems (in Illinois, Pennsylvania, and other states) have purchased calculators for each student; Texas' 1990 textbook adoption guidelines reflect significant aspects of the reforms; many states, after years of ignoring educational needs, are renewing monetary support for schools.

The extent to which these beginnings will produce suitable material and an atmosphere in which they may be successfully implemented depends on the

involvement of teachers and school officials in the reform process. After all, it is the teacher who sits on textbook adoption committees, curriculum writing committees, and specifies resource material for the classroom. If there is no support or encouragement from officials, however, there seems little likelihood of movement.

Changing the participants

The experiences of the "new math" movement during the 1960's clearly demonstrates the ineffectiveness of curriculum reform without active participation from the classroom teacher (Cooney, 1988). But is this participation to be forthcoming?

Fey (1981) reports that the predominant mode of instruction in the elementary grades is that of teacher explanation and questions followed by individual seatwork on pencil-and-paper exercises. Surveys found little interest among elementary teachers in inquiry or laboratory experiences; drill in facts and procedures seemed to fit both the instructional style and mathematical perceptions of the teachers.

This pattern follows in the upper grades. A common routine might be as follows:

> First, answers were given for the previous days' assignment. The more difficult problems were worked by the teacher or a student at the chalkboard. A brief explanation, sometimes none at all, was given of the new material, and problems were assigned for the next day. The remainder of the class was devoted to working on the homework while the teacher moved about the room answering questions. (p. 18).

The observer remarked, "the most noticeable thing about math class was the repetition of this routine." Teachers at the secondary level tended to stress algorithmic skills unconnected to applications; teachers and students shared a common expectation of mathematics "as a discipline of well defined procedures and 'right' answers."

The Standards' emphasis on applications suggests a reversal of the "new math" movement of the 1960's. "New math" curricula treated applications as afterthoughts, tacked on as contrived contexts for practice of the "real" mathematics, not as central to the development of mathematical concepts. "Critical thinking" instruction, as defined by the Standards, seems to have taken a turn from an emphasis on abstract mathematical structure towards an emphasis on using mathematics in a variety of contexts.

More importantly, the Standards signal the final death knell of the "back to basics" movement of the 1970's. The "minimal competency" curriculum, with its emphasis on mastery in a limited context, of a limited set of skills, in a "teacher-proof" curriculum is inconsistent with the teaching style and content inherent in the report.

But the majority of practicing mathematics teachers grew up in the "new math" age, where mathematics was dominated by mathematicians' conceptions of the subject, and received their formal training and experience in the standardized-test-oriented "back to basics" days. Now these teachers are asked to alter their conceptions of their subject and of their instruction.

The fact that conceptual change is required is critical--merely changing the texts, or curriculum guides, and requiring group work, or term papers, or projects is not sufficient. A teacher does more than merely transmit mathematical fact; he/she communicates a sense of the value of mathematics and the processes of mathematical thought. The teacher who considers mathematics a mere set of rules to be followed can not teach in a problem solving environment; neither can the teacher who sees instruction as a wholly teacher-

centered activity develop students' skills of independent, creative thinking and communication of mathematical constructs.

How can these conceptions be changed? If student conceptions of mathematics and its instruction are formed in the classroom, then it follows that teachers' conceptions may be re-formed in teacher education programs (given that some unlearning or modifying of previous conceptions may initially be necessary). Breaking the cycle of "teach-what-I-was-taught-like-I-was-taught-it" can only happen through the active involvement of teachers in an educational process that emphasizes and practices the philosophy that permeates the standards (Cooney, 1988). Such a program must involve preservice teachers in the construction of mathematical and pedagogical conceptions in the context of the content they will teach.

The retraining of current teachers and corresponding changes in teacher education are beyond the scope of the Standards report. These concerns must, however, be considered if reform is to be successful.

References and Suggested Readings

Brown, C. A., Carpenter, T. P., Kouba, V. L., Lindquist, M. M., Silver, E. A., & Swafford, J. O. (1988). "Secondary School Results of the Fourth NAEP Mathematics Assessment: Algebra, Geometry, Mathematical Methods, and Attitudes." Mathematics Teacher, 81 (May, 1988b): 337-48.

------. "Secondary School Results of the Fourth NAEP Mathematics Assessment: Discrete Mathematics, Data Organization and Interpretation, Measurement, Number and Operations." Mathematics Teacher, 81 (April, 1988a): 241-48.

Cooney, T. J. (1988). "The Issue of Reform: What Have We Learned From Yesteryear?" Mathematics Teacher, 81 (May, 1988): 352-363.

Dossey, J. A., Mullis, I. V. S, Lindquist, M. M., & Chambers, D. L. (1988). The mathematics report card: Are we measuring up? Princeton, NJ: Educational Testing Service.

Fey. J. T. (1981). Mathematics teaching today: Perspectives from three national surveys. Reston, VA: NCTM.

Hirsch, C. R. & M. J,. Zweng (Eds.). (1985). The secondary school mathematics curriculum: 1985 Yearbook of the National Council of Teachers of Mathematics. Reston, VA: NCTM.

Kouba, V. L., Brown, C. A., Carpenter, T. P., Lindquist, M. M., Silver, E. A., & Swafford, J. O. (1988). "Results of the Fourth NAEP Assessment of Mathematics: Measurement, Geometry, Data Interpretation, Attitudes, and Other Topics." Arithmetic Teacher, 35 (May, 1988b): 10-16.

------. "Results of the Fourth NAEP Assessment of Mathematics: Number, Operations, and Word Problems." Arithmetic Teacher, 35 (April, 1988a): 14-19.

National Council of Teachers of Mathematics, Commission on Standards for School Mathematics. (1987). Curriculum and evaluation standards for school mathematics (working draft). Reston, VA: NCTM.

National Council of Teachers of Mathematics, Commission on the Education of Teachers of Mathematics. (1981). Guidelines for the preparation of teachers of mathematics. Reston, VA: NCTM.

National Council of Teachers of Mathematics. (1980). An agenda for action: Recommendations for school mathematics of the 1980's. Reston, VA: NCTM.

Research Advisory Committee of the National Council of Teachers of Mathematics. (1988). "NCTM Curriculum and Evaluation Standards for School Mathematics: Responses from the Research Community." Journal for Research in Mathematics Education, 19: 338-44.

Stigler, J. W & Perry, M. (1988). "Cross Cultural Studies of Mathematics Teaching and Learning: Recent Findings and New Directions." In D. A. Grouws & T. J. Cooney (Eds.) Effective Mathematics Teaching (pp. 194-223). Reston, VA: NCTM.

The Mathematical Association of America, Committee on the Undergraduate Program in Mathematics Panel on Teacher Training. (1983). Recommendations on the mathematical preparation of teachers. Washington, DC: MAA.

Thompson, C. S. & Rathnell, E. C. (1988). "NCTM's Standards for School Mathematics, K-12." Arithmetic Teacher, 35 (May, 1988): 17-19. (Note: also in Mathematics Teacher, 81 (May, 1988): 348-51.

Assumptions on the Nature of Mathematics and Instruction
NCTM Commission on Standards for School Mathematics

1. All students should be required to study mathematics for at least 12 years (grades K-11).
2. The study of mathematics should revolve around a core curriculum that allows all students an opportunity to learn the important ideas and methods of mathematics.
3. Mathematics should be studied as an integrated whole so that students understand it is a dynamic discipline and an integrated part of our culture.
4. Doing mathematics involves problem solving.
5. Communication is an important of mathematics instruction.
6. Mathematics should help build students' abilities to reason logically.
7. New topics (e.g., data analysis, estimation) must be introduced into the mainstream curriculum.
8. Mathematics should be taught in a natural context.
9. Students should be encouraged to create, invent, and participate.
10. Calculators and computers should be used throughout school mathematics.
11. Success in paper-and-pencil computation need not be a prerequisite to the study of other mathematics.

Underlying Assumptions, Grades K-4 Mathematics

- The K-4 curriculum should be conceptually oriented.
- The K-4 curriculum should be developmentally appropriate.
- The K-4 curriculum should emphasize the development of children's mathematical thinking and reasoning abilities.
- The K-4 curriculum should actively involve children in doing mathematics.
- The K-4 curriculum should include a broad range of content.
- The K-4 curriculum should emphasize applying mathematics.
- The K-4 curriculum should emphasize the interrelationship of mathematical knowledge.
- The K-4 curriculum should make full use of calculators.

Assumptions about Instruction, Grades 5-8

- All students should experience the full range of topics addressed in the standards.
- Topics in the individual standards should be integrated through a selection of rich activities.
- Students should be actively involved in the learning process, investigating and exploring individually and in groups.
- Relevant situational contexts should motivate instruction. Students should experience ideas in context--real world and/or mathematical.
- Instructional activities should take place outside as well as inside the classroom.
- Mathematics activities should help relate mathematics to other school subjects such as science and art.
- Teachers should be facilitators of learning, not merely dispensers of knowledge.

Assumptions about Classroom Conditions, Grades 5-8

- Every classroom will have ample sets of manipulative materials and supplies (e.g., spinners, cubes, tiles, geoboards, pattern blocks, Miras, scales, compasses, scissors, rulers, protractors, graph paper, grid and dot paper, etc.) for student use.
- Appropriate resource material providing problems and ideas for explorations will be available for teacher and student use.
- In grades 5 and 6, a four-function calculator will be available at all times to each student. In grades seven and eight, a scientific calculator will be available at all times to each student.
- Every classroom will have at least one computer available at all times for demonstrations and classroom use. Additional computers should be available for individual, small group, and whole class use.

SUMMARY OF CHANGES IN CONTENT AND EMPHASES IN 9-12 MATHEMATICS

In the study of ALGEBRA

Topics to receive REDUCED ATTENTION:
- word problems by type such as coin, digit, and work
- simplifying radical expressions
- factoring to solve equations and to simplify radical expressions
- operations with rational expressions
- paper-and-pencil graphing of equations by point plotting
- logarithm calculations, especially using tables and interpolation

Topics to receive INCREASED ATTENTION:
- using real-world problems to motivate and apply theory
- computer graphing to develop conceptual understanding
- computer-based methods such as successive approximations and graphing utilities for solving equations and inequalities
- structure of number systems
- matrices and their applications

In the study of GEOMETRY

Topics to receive REDUCED ATTENTION:
- Euclidean geometry as a complete axiomatic system
- initial postulates/theorems for distance, betweenness, and angle measure
- geometry from a synthetic viewpoint
- two-column proofs
- theorems involving circles
- analytic geometry as a separate course

Topics to receive INCREASED ATTENTION:
- integration across topics at all grade levels
- coordinate and transformation approaches
- component skills for deductive proof
- the development of short sequences of theorems
- recording deductive arguments in sentence or paragraph form
- computer-based explorations of 2-D and 3-D figures
- three-dimensional geometry
- real-world applications and modeling

In the study of TRIGONOMETRY

Topics to receive REDUCED ATTENTION:
- verifying identities
- numerical applications of sum, difference, double- and half-angle identities
- table reading skills and interpolation
- contrived triangle applications
- paper-and-pencil solutions of trigonometric equations

Topics to receive INCREASED ATTENTION:
- use of scientific calculators
- realistic applications and modeling
- connections among the right triangle ratios, trigonometric functions, and circular functions
- computer graphing techniques for solving equations and inequalities

In the study of FUNCTIONS

Topics to receive REDUCED ATTENTION:
- treatment as a separate course
- paper-and-pencil evaluation
- graphing by hand using tables of values
- formulas given as models of real-world problems
- transformation of function equations to standardized form in order to graph

Topics to receive INCREASED ATTENTION:
- integration across topics at all grade levels
- the connections among a problem situation, its model as a function in symbolic form, and the graph of that function
- function equations in standardized form as checks on the reasonableness of computer-generated graphs
- functions that are constructed as models of real-world problems

OTHER TOPICS TO RECEIVE INCREASED ATTENTION:

The study of STATISTICS | The study of PROBABILITY | The study of DISCRETE MATHEMATICS