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ABSTRACT

One of the strengths of the Pennsylvania Council of Teachers of Mathematics (PCTM) is that it gives mathematicians and mathematics educators the opportunity to exchange and contribute to each other's professional growth. The topic for each yearbook is chosen to coincide with the annual PCTM meeting. This 1987 yearbook contains 14 articles which focus on research, results, perspectives, ideas, and strategies related to mathematics instruction. Topics include: teaching writing in mathematics; teaching reading in mathematics; computer literacy; business education; using computer based materials; relating microcomputers to the mathematics curriculum; textbooks; calculus; discrete mathematics; Pascal; coordinating mathematics programs; classroom methods; using textbooks to teach higher order thinking skills; and mathematics education in West Germany. (CW)

THE MATHEMATICS CURRICULUM

ISSUES AND PERSPECTIVES

1987 Yearbook

PENNSYLVANIA COUNCIL OF TEACHERS OF MATHEMATICS

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ISSUES AND PERSPECTIVES

1987 Yearbook

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PENNSYLVANIA COUNCIL OF TEACHERS OF MATHEMATICS

PENNSYLVANIA COUNCIL OF TEACHERS OF MATHEMATICS
An Affiliate of NCTM

PREFACE

The 1987 PCTM Yearbook — THE MATHEMATICS CURRICULUM: ISSUES AND PERSPECTIVES — is the third yearbook to be developed and distributed to the membership of the Pennsylvania Council of Teachers of Mathematics. The theme was chosen to be congruent with that chosen for the 36th annual meeting of the organization.

The articles in THE MATHEMATICS CURRICULUM: ISSUES AND PERSPECTIVES focus on research results, perspectives, ideas and strategies that should be of interest to elementary teachers, secondary mathematics teachers, college mathematics teachers, teacher educators, mathematics supervisors, and curriculum coordinators as they strive to review and improve existing mathematics programs. The articles were written by teachers, researchers and supervisors from basic and higher education who responded to a call for manuscripts which was sent to all PCTM members in Spring, 1986.

Considerable thanks go to a number of people for their important contributions to the 1987 Yearbook. Genevieve Battisto, Carl Guerriero, Velma Yoder and Lucy Young served as members of the Editorial Board. They shared their insights about the manuscripts that were submitted for consideration and offered many suggestions which we were able to use in the editing process. Suzanne Harpster at The Pennsylvania State University also provided valuable editorial assistance. The authors of the manuscripts deserve considerable credit for taking the initiative and the time to place their ideas in front of their peers. The commercial and institutional advertisers also deserve our thanks for their willingness to invest their money by buying space in the yearbook. Last but not least, the PCTM Executive Board deserves credit for its continuing support of the efforts of the Publications Committee.

We are glad to have had the opportunity to serve as editors for the 1987 PCTM Yearbook. We hope that the readers will carefully read the articles in THE MATHEMATICS CURRICULUM: ISSUES AND PERSPECTIVES and implement the ideas that are relevant for them. We invite response from the readers, authors, and advertisers.

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1987 PCTM YEARBOOK EDITORIAL BOARD

The 1987 PCTM Yearbook is an official publication of the Pennsylvania Council of Teachers of Mathematics Membership in the Council includes a subscription to the Yearbook and Newsletter Other individuals and institutions may subscribe to these publications Inquiries should be sent to Mary Moran, R D #1, Box 172, Canadensis, PA 18325 Opinions expressed in the articles are those of the authors and are not necessarily those of the Council

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WRITING IN MATHEMATICS — LONG OVERDUE

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Several years ago the catch-phrase "Writing Across the Curriculum" became a standard battle-cry on many campuses, with extensive research pointing to the deplorable state of the writing communication skills demonstrated by students nationwide. Now that the dust has settled, it becomes evident that little change along these lines has occurred in mathematics classes. Yet we, as mathematics educators, need to recognize the real value in such exercises and assignments. While term papers come to mind as the most standard type of writing assignment (and are most likely to be avoided by mathematics instructors), sufficient room exists to expect shorter papers, explanations of procedures, and insightful comments regarding the matter at hand.

I must admit that my first attempt at assigning a major research paper resulted in less than satisfactory results — primarily because I never managed to clearly define what I was looking for. But later, research papers submitted by my Mathematics for Finance students were remarkably satisfying and quite enlightening. In fact, I even had the rare pleasure of one student actually thanking me for forcing this assignment! This individual focused his attention on investments he had previously made and recognized that better options were available to him as a result of completing his paper.

In the January 22, 1986, issue of *The Chronicle of Higher Education*, Liz McMillen presents her views of such assignments in an article entitled 'Science and Math Professors Are Assigning Writing Drills to Focus Students' Thinking." Such drills can lead a student to clarify fuzzy thoughts, sharpen his or her perceptions or interpretations, and think through a setting in a much broader way.

McMillen provides numerous examples of cases where productive exercises have been enacted. The benefits are numerous, and she quotes Paul Connolly of Bard's Institute of Writing and Thinking who points out that "It (writing) helps students become autonomous learners, rather than waiting dependently on the teacher to give them the rule that applies," especially when the writing assignment is geared to expressing how one goes about finding the answer. The stress is not necessarily on lengthy research papers, but rather on shorter essays — sometimes to be completed in class.

But what kind of research and essays can be required? Richard Roth, a teacher at Parkland School District's Troxell Junior High near Allentown, routinely expects his students to research suggested mathematics topics and/or various mathematicians. For a student to discover the story of Galois's duelling death or Newton's development of The Calculus while waiting out the Bubonic Plague — this *can* be worthwhile! Or imagine my delight several years ago when I "forced" my Calculus III class to research various curves — the witch of Agnesi, the cissoid, the Folium of Descartes, the cardioid — and obtained results which were insightful, extensively explored, and utterly fascinating. In many instances the students unearthed new aspects of these curves (and even some applications!) of which I was not previously aware, and these facts have become meaningful additions to my in-class repertoire.

Why not assign a first semester calculus class a two-page paper describing what a derivative is? Why not have second year algebra students explore and report on applications of algebra in science, technology, medicine, and finance? Why not have your trigonometry students explain what radians are? Of course many students will initially react negatively to such writing assignments. After all, we *do* have the habit of compartmentalizing our courses, and writing is about as far removed from mathematics as you can get. But the students who can accept such an unexpected development in their mathematics classes often show significant progress, both in their mathematics and science work as well as in their communication skills.

Naturally, extra time is involved in reading such assignments, and most of us do not feel qualified to instruct students in the type of skills necessary for producing coherent, mathematically oriented essays; but the benefits to both the student and the instructor overshadow such qualms. Moreover, alternatives exist to these major writing assignments. I have often required explanations for selected questions on tests — explanations which often require minimal computation. Such responses can be real eye-openers when one realizes the misconceptions the students are clinging to.

For example, consider the following true/false question on a statistics test: "The mean, median, and mode can be found for any type of statistical data." Merely looking for a "true" or "false" response reduces this to a mundane 50-50 chance; expecting a defense *in writing* to support the answer chosen requires a synthesis of knowledge of some rudimentary statistical concepts. Why not ask probability students how to *efficiently* use a table of random numbers to select five out of thirty given data values — with an emphasis on the explanation? Why not ask algebra students to explain why the expression $Ax^2 + Ay^2 + Dx + Ey + F = 0$ will *not* always produce a graph of a circle? Why not ask business math students to explain why a 15% markon will take on different values, depending on whether the markon is based on the cost or the selling price?

Such expectations can help lead our students beyond simply parroting standard procedures; the results can provide unusual insight for both the student and the instructor. A rather unexpected development can accompany the series of writing assignments where students describe procedural steps. McMillen cites Larry D. Kirkpatrick, physics professor at Montana State University, who points out this development, saying that "when students take exams, you notice that they're much more organized about how they go about answering questions."

Marilyn Frankenstein's *Ideas for Teaching a Non-Rote College Arithmetic Course* offers a number of strategies she employs in a remedial setting. Frankenstein suggests having each student maintain a math journal, requiring 15 to 40 minutes of writing weekly on such topics as their feelings in class, approaches taken in attempting the homework, and reactions to the progress being made. Through these journal entries, she gains valuable insights which enable her to offer encouragement, alternate solution techniques, and explanations of how their remarks interrelate with learning in general.

But more importantly, Frankenstein expresses broad opinions which strongly impact not only in remedial classes but also extend to more advanced courses. ". . . Mathematical literacy is not the ability to calculate; it is the ability to reason quantitatively. No matter how many computation algorithms students know, they become mathematically literate only when they can use numbers to solve their own problems, clarify issues, and support or refute opinions."

Moreover, Frankenstein stresses that both content and method need to be stressed. Emphasis on technique alone is stifling and non-creative; critical thinking can be developed only when those of us teaching mathematics also turn our attention to communicating what mathematics is all about and having our students communicate in return. "If content is the only consideration, the curriculum may look good on paper without ever engaging the students. On the other hand, if methods are the major focus, students may develop the self-confidence to master some rote, mechanical skills, without learning to use math in thinking critically about issues that concern them." Getting the students to discuss and write about such concerns seems paramount.

Problem solving must extend beyond the common misconception that we need only assign more word problems, and requiring writing for mathematics courses might be part of the solution. By encouraging — demanding — written responses, we go beyond mere computation. The conceptualization needed to fully synthesize and integrate prior knowledge into coherent, fully-developed, well supported written responses can be a powerful mechanism to foster critical thinking skills. Learning needs to transcend disciplinary compartmentalization, and mathematics educators are in a position to encourage greater in-depth consideration of what otherwise has traditionally been relegated to uninspired, rote procedures.

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TEACHING READING IN MATHEMATICS

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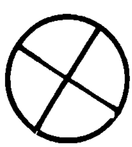
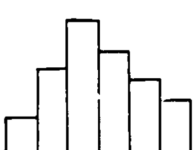
Why should a mathematics teacher be concerned with reading abilities or reading instruction? There are several reasons. Research has shown that the mathematical development of students correlates highly with their ability to read. Secondly, it has been estimated that approximately 35% of the errors on mathematics achievement tests may actually be due to problems in reading (O'Mara, 1981). Thirdly, there is general agreement between reading specialists and mathematics teachers that there are several factors which make the reading of mathematics materials inherently difficult (Nolan, 1981). Listed among these factors, we find that:

1. mathematics is written in a terse, unimaginative style.
2. mathematics is highly compact and requires very slow, deliberate reading.
3. mathematical symbols must simply be memorized.
4. a variety of eye movements are needed, in addition to the normal left to right movements. For example:

a. 113,472 — Right to Left

b.  — Circular

c. $\begin{array}{r} 12 \\ -\ 4 \\ \hline 8 \end{array}$ — Top to Bottom

d.  — Bottom to Top

e. $\frac{2}{3} = \frac{3}{4}$ — Diagonal

5. there are different vocabularies. For example:
   a. General — all walks of life (common, equality, unequality)
   b. Technical — polygon, quotient, hypotenuse, isosceles
   c. Special — one meaning outside of mathematics but a different meaning in mathematics (product, point, property, closed).

Any one or combination of these factors is sufficient reason for poor performance in mathematics as well as cause for difficulties in teaching mathematics. Yet being aware of these factors doesn't ensure that they will be addressed during instruction. Too many teachers assume that such obstacles are just "givens" on the periphery of mathematics education and will somehow resolve themselves. This is not the case and we must take deliberate steps to incorporate each of them in our teaching.

But where does one begin? According to Earle (1976), we must begin with the content. The content of a mathematics lesson will dictate the needed processes. The concepts or solutions to be presented will reveal which reading skills are necessary so that students will be able to grasp the concept successfully or arrive at the correct solution. Using content analysis we can identify the most important concepts or solutions along with a list of words and/or other symbols essential to each concept or solution. Figure 1 is an example. Occasionally, it may be necessary to prioritize this list.

Major Concept:	To reduce fractions to lowest terms	
Vocabulary:	terms	factor
	greatest common factor	greatest common division

Figure 1
SAMPLE CONTENT ANALYSIS

Now we are ready to look at the reading process in mathematics. Earle (1976) has listed the following levels of reading:
1. perceiving symbols.
2. attaching literal meaning.
3. analyzing relationships.
4. solving word problems.

At this point, we must define or describe each of these levels and give examples. *Teaching Reading and Mathematics* (Earle, 1976) is one of the richest resources available for implementing these practices into your daily mathematics instruction. (In his introduction, Earle stated that if the classroom teacher did not find at least one instructional suggestion per page, he will have failed in his goal, which was to write a practical and usable teaching guide.)

Perceiving symbols is recognizing and pronouncing the words and/or symbols identified through the content analysis. It is clear that for a student to reach even a minimal level of success in mathematics he/she must first master this level. How many times do we see mathematics students stumble over the most basic directions or explanations because the words and symbols are foreign to them? There are a variety of activities that will enhance this level of reading such as identifying key words/symbols among other similar words/symbols. There are individual or small group activities that can be done using the overhead projector, chalkboard, labeling flash cards, word lists, etc. The implication for the teacher is that you must make every effort to expose your students to the sight and sound of these key words/symbols. You must at all times use proper vocabulary and hold your students to the same level of communication.

After the students are able to recognize and pronounce key words/symbols, they are ready to attach literal meaning. Attaching literal meaning comes in two parts — vocabulary and explicit ideas.

There are several ways to attach literal meaning using a vocabulary approach. One can use glossing, crossword puzzles, multiple characteristics, multiple meanings and multiple terms, using a single definition which fits several important terms, and context analysis. Glossing and crossword puzzles are familiar; examples of the other strategies are listed in Figure 2.

Strategy	Example
Multiple characteristics	A polygon ______is a circle. ______is a simple closed figure. ______has three or more sides. ______is a prime number. ______has more than two factors.
Multiple terms	Dividing a numerator and a denominator by the same factor. ______multiple ______canceling ______reducing ______simplifying
Context analysis	The students had to find the measure of the *base* of the triangle. The runner stole second *base*.

Figure 2
EXAMPLES OF STRATEGIES TO HELP STUDENTS
ATTACH LITERAL MEANING

Other suggestions include the use of nonsense words in place of key words, use of straight definitions, scrambled words in a sentence, and completion type activities.

Looking at the second part of attaching literal meaning we have explicit ideas. Here we are helping the student to grasp ideas that are stated explicitly in the text material. This can be promoted by giving the students "purpose" questions. These questions guide the student's reading and target important information. Another effective technique is the use of study guides. Study guides are similar to purpose questions but are much more comprehensive.

After the students are able to recognize and pronounce key words/symbols and are able to attach literal meaning to these words/symbols they are ready to identify and explore relationships among the words and ideas.

Earle identifies several strategies for teaching at this level of the reading process. Following a set of six directions, you can construct and use a structured overview. Figure 3 is an example.

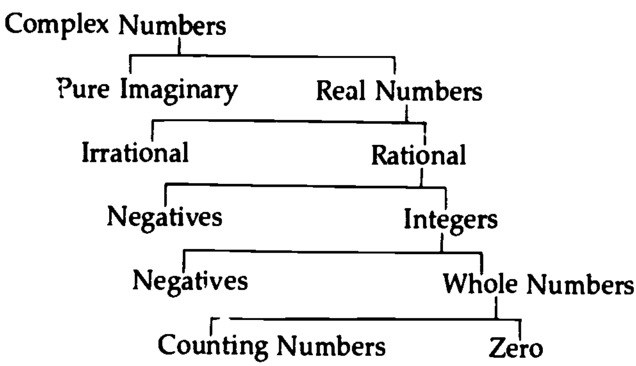


Figure 3
EXAMPLE OF A STRUCTURED OVERVIEW

With a structured overview you are able to keep your students oriented as to where they've been, where they are presently, and where they may be headed. There are several ways to employ structured overviews in your instruction. For example, have students develop their own and compare; you make one, then cut it apart and have students reassemble it; etc.

Another strategy is manipulating symbol relationships. For example, select the idea in each group which includes all the other words in the group. Or, select the word that does not belong in each group.

One other strategy for analyzing relationships is to identify relationships in context. Earle notes two parts to this strategy. For example:

Part I (literal)

Check the statements below which tell what the author *said* in the text (p. 105, section B).

1. ______________________________
2. ______________________________
3. ______________________________

Part II (relationships)

Check the statements below which tell what the author *means*.

1. ______________________________
2. ______________________________
3. ______________________________

Finally we reach the most difficult and sophisticated level of the reading process — solving word problems. In solving word problems, experts in both reading and mathematics agree to the following:

Step 1: Read through the problem quickly.
Step 2: Examine the problem again. Exactly what are you to find.
Step 3: Read the problem again to notice what information is given.
Step 4: Analyze the problem carefully to note the relationship of information given to what you are asked to find.
Step 5: Translate the relationship to mathematical terms.
Step 6: Perform the necessary computation.
Step 7: Examine the solution carefully. Label appropriately. Check for reasonableness.

Note that all steps require the accurate perception of symbols. Steps 1, 2, and 3 require the reader to attach literal meaning. Steps 4 and 5 require the reader to analyze relationships among explicitly stated details. Steps 6 and 7 demand that the reader apply his/her computational ability to the set of relationships and judge the result critically in light of the original purpose.

Being aware of the connection between reading and mathematics is a good first step. But to really address this connection and have your students enjoy a more meaningful study of mathematics, you must pursue the concepts and suggestions that have been presented. Once you begin using the reading levels you will be limited only by your own creativity. One example to address a particular level will often trigger another, then another, and so on.

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COMPUTER LITERACY: MICROCOMPUTER ISSUES IN EDUCATION

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Unlike the development of mainframe computers which emerged gradually through industry, higher education and government, the rapid development of the microcomputer has enabled these machines to become an integral part of the American lifestyle. Microcomputers are everywhere, and they have come on the scene in such a short time and at such reasonable prices that they are now a major force in the educational world. As a result of the proliferation of computers, there is a perceived need that members of our society, especially children, should be educated to cope with this massive influx of technology. Not only do students need to know what these machines can do, but also the influence they will have on social and political institutions.

This article identifies four issues that elementary and secondary school students need to examine as they move through the curriculum. In many school districts, the mathematics department has inherited major responsibility for computers in the curriculum, so these issues are raised for their consideration and action.

Historical Perspective

When computers were initially being developed, the applications were tied to rapid arithmetic calculations that needed to be done in order to increase the speed and productivity of the particular project involved. This branch of the development process led to the hand-held calculator which has had its own influence on the curriculum. Other areas began to emerge for the use of computers as the technological developments began to catch up with the conceptual designs. It was at this point that concern for technological determinism (Jones, 1982) began to be more seriously debated. Was technology beginning to be more than a helping hand by actually shaping the destiny that it was created to enhance? The advances in technology had equal chances for enhancing or degrading the quality of life; important choices needed to be made.

When a student begins to learn about the development of computing and what technological advances have been made, there also needs to be taught an awareness of the limitations of these tools as well as their power (Weizenbaum, 1976). These machines have evolved at such a speed, and with such increasing capabilities that we can become blinded by what they really can accomplish, or should accomplish. Being a beneficiary of the technology is much different from being a victim, and the student needs to be able to distinguish the difference.

Sociological Implications

The effect of computers and related technology on social and political institutions is an important topic for students to examine. As computers become integrated into the economy for increased efficiency in many job areas, there will be effects upon employment (Laver, 1980). The effect will not only be on the numbers of employees, but also on the way in which they are used in the marketplace. In certain job markets such as the auto industry, the effects can be seen already. These changes in employment can result in a general distrust of high technology. Technology can be seen as an agent of social change, affecting the character of the economic sector from one of manufacturing to one of a service industry (Bell, 1979).

What effect does this technological change have upon society? One is a feeling of alienation, when technological inevitability seems all too apparent (Weizenbaum, 1979). Not only can there be a sense of alienation among the general public for all of this "high tech," but as the systems themselves become more and more complex, the accountability is lost. Who is really responsible? This moral issue is one that should be discussed in the classroom. In an information society, dependent upon large networks of electronic systems, where do responsibilities lie? With the managers? With the technicians? With the users? The relationship between humanity and its machines needs to be constantly discussed and evaluated (Kemeny, 1972).

Influences on Education

Much of the seminal work in computing began at the university level and only in this last decade has pre-college education been significantly affected. The accessibility of computing through the development of the microcomputer has the potential for integrating computing into almost all levels of the curriculum in elementary and secondary schools. If the computer is to become an agent of social change (Laver, 1980) then the educational system is already in the process.

It is at this level, especially with younger students, that introducing computers can have a significant effect on how they will view these machines in the future. There are still debates as to the "best" way to approach computing with young children. There are advocates of using programming languages, such as LOGO in order to allow a student to explore and develop thinking skills (Papert, 1980). There are advocates of software technology that propose teaching students to use computers as merely tools to assist their learning, just as a calculator has become a tool. And, there are those who stress the importance and effectiveness of computer-assisted instruction (CAI) in strengthening the educational experience of students (Chambers, 1984). While the effects of each of these various methods continue to be studied, it is apparent that students do need to be instructed not only in how to use these machines, but also in the effects that they will/can have on their own learning. Teaching students to think is a necessary part of the educational process. While it may not be possible in the early childhood years to explore the epistomological effects of computing, it most certainly needs to be part of later secondary education. The successful use of microcomputers in education depends upon two important factors (Malone, 1984): 1) the use of cognitive and motivational processes in learning, and 2) the social structure of the educational setting. Without the incentives to use the computers, there can be little progress made. The attitudes of the persons who are involved in instructing children (and adults!) in the use of computing technology is most important. The content of the instruction is of equal importance and needs to be appropriate to the level being taught.

The Future

If technological development expands geometrically (Bell, 1979), then how do we proceed? Where do we start, or as important, how do we keep abreast? As previously stated, awareness and involvement are the keys to developing any form of computer literacy in students or adults.

As programming capabilities become more and more sophisticated, an understanding of the implications of artificial intelligence research becomes a necessity. After discussing the development of natural language programming, expert systems and robotics, students should examine and discuss how computers can enhance the quality of life (Minsky, 1979) as well as the arguments which place limitations on the extent to which artificial intelligence can develop (Rose, 1985). Computing needs to be put into perspective. There needs to be awareness of what computers can actually do, and what choices we should make about what they should be doing (Winograd, 1979). These choices will change as the technology changes; history has verified this trend.

In the educational world, the use of CAI is still undergoing evaluation with few well-designed studies available. In the interim, the use of the interactive video disk has added a new dimension to the relatively passive designs of a few years past (Chambers & Sprecher, 1984).

The issue of privacy is an important part of any educational attempt to become computer literate. Too often the computer is viewed as the villain. The real problem rests, not with the existence of the machine and its capabilities, but with the design of a system to safeguard the information.

In Conclusion

With over one million microcomputers in the nation's schools now and with the potential for an even greater number to be available, not only in schools, but in homes as well, the challenge to the educational system is at hand. The National Science Foundation will be spending several million dollars over the next five years to encourage projects in curriculum development that will take into consideration the need to integrate computing into the curriculum. This integration process will have significant effects on areas such as mathematics, in ways that no other development in the past decade has seen.

To be truly computer literate at this time is to have an awareness of the historical development of computers, to perceive sociological implications of computer technology on social and political institutions, to explore the diverse approaches to computer use in education, and to be constantly aware of the rapid changes taking place in technological development. The moral and ethical issues need to be debated. Students should not passively accept the changes that are occurring. Computers are a part of American society and they must be dealt with on the educational scene. The choice is whether to let the technology determine the quality of life or to let the quality of life determine the uses for the technology.

In addition to these philosophical issues, students need to have first-hand involvement with computers. Talking about learning to ride a bicycle is one thing; riding it is another. There is no substitute for actually being involved with the machine, whether it be by using software to see its capabilities or by learning to program in several languages to understand how the software works. Acquiring a sense of what is actually happening within the machine helps to dispel some of the fear and awe of the electronic wizardry that can affect the uninitiated. As one becomes a computer user, it is easier to be more objective when discussing the philosophical issues. The choices become more meaningful.

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PREPARING STUDENTS FOR A PLACE IN TODAY'S BUSINESS WORLD

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Many students who enroll in college preparatory mathematics courses in high school change their plans about attending an institution of higher education within one year after graduating from high school. The change in plans can mean a delay in attending college or a decision not to pursue a college degree at all. Many other high school students who are enrolled in mathematics courses, college preparatory or otherwise, have no plans to go on to college after high school. Both groups often suffer from the same problem. They have few or no marketable skills that will help them gain an entry-level job in many businesses and industries. Even many of the college-bound students who will complete their college plans have no "practical" skills that could help them in their collegiate studies.

Is your high school helping students acquire marketable computer skills? Is your school offering a course in computer applications? Are you teaching computer courses other than BASIC? If the answers to these questions are "no," your curriculum is probably not meeting the needs of your students.

Most mathematics departments in high schools teach programming in BASIC, Pascal, or some other language. This is fine for the student going on to computer science or some other career requiring programming, and you do need to offer it. However, many students will have little or no need for programming skills. Unless the nature of a small business involves programming, most small businesses do not require programming skills of their employees. The new software packages now available can do much of what programmers used to do. When specialized programming is needed, the small businesses hire a consultant knowledgeable in the area of their need. Whether a student is going on to further education or directly into the job market, he needs more specialized training to enhance his chances for success. (Since most high schools do not have access to mini or mainframe computers, skills required for them must necessarily be handled by post-secondary institutions. The comments in the rest of this article are focused only on microcomputer concepts and applications.)

Before a school can teach the necessary computer application skills, the correct hardware is needed. Generally, in today's business atmosphere, hardware is IBM or IBM-compatible, so you should acquire some of that type of hardware. Some of your systems should be double floppy and others, hard disk. Several should have graphics and/or color monitors. In addition you should have at least one system equipped with a 1200 baud Hayes compatible modem, a laser printer, and a daisy wheel printer. Several different types of dot matrix printers should be provided, one of which should be IBM graphics compatible. This configuration of hardware would give the students a solid background in equipment now found in modern small business offices.

Before enrolling in an indepth applications course, students must have some basic computer knowledge. This basic literacy knowledge must include:

1. knowing the parts of the computer
2. why and how to format disks
3. how to load programs
4. basic DOS commands
5. understanding the non-standard keyboard keys
6. config.sys and autoexec.bat files
7. why and how to back up disks
8. how to use software and hardware manuals
9. knowing the various classifications of software (i.e. word processors, spreadsheets, etc.) and their purposes.

Whether a student is planning to go directly into the business world or on to college/business school first, proficiency in several areas of computer applications is likely to be quite helpful in obtaining a good job.

Word processing is almost a required skill for the modern office worker. Students should have experience with several different word processors. These should not be limited to *Appleworks* or *Scripsit*, but should include one of the multi-feature business word processors such as *Word Perfect*, *Multimate*, or *WordStar 2000*. Experience with several different word processors is an added plus for several reasons. The chances of finding a job with skill in more than one word processor is increased. Knowing several makes it easier to learn additional ones. Word processing skills should include not only the basic input-edit concepts, but also mail merge, macros, standard form documents, and interfacing with dot matrix, daisy wheel, and laser printers.

Electronic spreadsheets are fast becoming another area where skills are required. If you can only select one package, *Lotus 1-2-3* must be the choice. Students need to learn to set up simple spreadsheets, use labels, formulas, and functions, modify existing spreadsheets by changing the contents of existing cells or adding additional cells, set up printer parameters, and save and retrieve spreadsheets. Additional skills in creating and using macros, creating and printing graphs, and combining several spreadsheets into one are highly desirable.

More and more small businesses are using databases for a variety of applications. Therefore, basic database terminology and uses should be taught. Experiences should include file manager (flat file) programs and relational database programs. Students should learn to set up the database structure, then edit, sort, modify, and print the contents. Additional skills in the relational database area are programming custom applications, relating various files together, and exporting the contents to other programs. There are so many good database programs that there is no one outstanding package. The *PFS* series, *Dbase III*, *R:base System V*, and *Nutshell* are good examples.

Many small businesses are using microcomputers for their accounting. (This is perhaps the hardest area to teach as well as to implement.) If a high-quality, inexpensive accounting package is selected and introduced, the necessary terminology and procedures would be presented. Since almost every package is different, and since there is so much to teach in this area, the computer applications should be integrated into an existing basic accounting course that would be offered by the high school business department. Programs such as *DAC Easy Accounting* or *Peachtree Complete Business Accounting System* would accomplish the task for under $200. (Both these packages also include tutorials and telephone support.)

There are numerous additional computer applications used by many small businesses. Business graphics are used to produce graphs, charts, newsletters, other publications, and to combine clip art, diagrams, etc. into word processing documents. Telecommunications to access a commercial database or electronic mail is opening new concepts and cost savings. Use of software utilities makes using the computer easier and expands its capabilities further.

Instead of teaching a second-level language or a second language (or better yet, in addition to it), high schools should now be offering a course in computer applications incorporating the aforementioned topics. Recognizing that few math teachers or business teachers would have the necessary skills to teach all of the topics, a team teaching approach could be used. Some schools have created a Computer Science Department to bring together mathematics, business, and possibly art, science, English, and history teachers who have interest in computers — each specializing in the area he or she knows best.

This is the age of computers. Secondary schools must take steps to develop and implement an "applications" course so that their graduates will be able to function and compete in the modern business world.

NOTE: The names of the software packages are trademarks of the various companies which produce the software.

ENRICHING AND REDIRECTING THE SECONDARY MATHEMATICS CURRICULUM WITH COMPUTER-BASED MATERIALS

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The publication of the *Agenda for Action* by the National Council of Teachers of Mathematics (NCTM, 1980) and the subsequent increase in availability of microcomputers in schools provided a focus for increased attention to the use of the microcomputer to enhance mathematics instruction. During the 1980s a variety of computer-based materials for secondary mathematics has become available. Many of these materials enable mathematics educators to enrich the mathematics curriculum while others provide an opportunity for pursuing new curricular directions. The examples that follow will examine software that has promise for both enrichment and redirection of the secondary mathematics curriculum.

Roles of Computer-based Materials

There are two key roles that computer-based materials can play in enriching and redirecting the secondary mathematics curriculum. First, computer-based materials, whether they consist of teacher-developed programs for classroom demonstration or commercially-available software, can provide teachers with an opportunity to organize portions of the curriculum around the computer as a source of data and as a tool to interactively manipulate that data. Hatfield (1984) contends that the computer provides a constructive context for learning mathematics, one in which students' primary activities are to search, observe, experiment, conjecture, generalize, and abstract. Programs that allow teachers and/or students to generate data can redirect the current secondary mathematics curriculum toward such a problem-solving approach.

Example 1. Suppose that an Algebra I class that has studied factoring and the solution of quadratic equations is asked to test whether addition distributes over multiplication, namely, whether

$$a + (bc) = (a + b)(a + c) \qquad (1)$$

is true for all real a, b, and c. By testing various values, students will soon see that addition does not distribute over multiplication. When the students are then asked to determine any special values for which (1) is true, they may soon see that (1) holds whenever $a = 0$, and they may find other values such as $a = 3$, $b = 2$, $c = -4$. However, with the teacher-developed program in Figure 1, students can generate data with a, b, and c taking on values in selected ranges from which students can conjecture that $a = 0$ and $a + b + c = 1$ are the only special values for which (1) is true.

```
10 REM   PROGRAM TO TEST DISTRIBUTIVITY OF
ADDITION OVER MULTIPLICATION
100  FOR A = -5 TO 5
120    FOR B = -5 TO 5
130      FOR C = -5 TO 5
140        IF A + (B * C) <  > (A + B) * (A + C) THEN 170
150        PRINT A,B,C
170      NEXT C
180    NEXT B
190  NEXT A
900  END
```

Figure 1
PROGRAM TO TEST DISTRIBUTIVITY

The data and the ensuing conjecture(s) can motivate students to prove that (1) is true only for the special cases in which $a = 0$ and $a + b + c = 1$. In this example the computer facilitates the process of data gathering and conjecturing by generating data requested by the student or teacher and offers motivation for completing an algebraic proof based on the conjecture derived from that data. More extensive opportunities exist for using the computer as a tool for generating data, making conjectures and generalizing results in the computer activities software that accompanies some secondary mathematics texts (e.g., Hopfensperger, 1983; Howsare, Blume, & Graham, 1982; Snover & Spikell, 1981).

Some commercially-available software packages provide teachers with excellent opportunities for enriching the mathematics curriculum by providing an inquiry-oriented focus on problem solving. One such example is *Royal Rules* (O'Brien, 1986).

Example 2. In the *Royal Rules* program, students are challenged to test hypotheses to find a rule that relates various triples of numbers. The triples (3, 5, 15), (16, 3, 48) and (6, 7, 42) might lead one to conjecture that the "rule" for triples (x, y, z) is $z = xy$. However, if (3, 11, 66) also is given as fitting the "rule," students must formulate alternative conjectures (e.g., z is divisible by both x and y) to take into account the fourth data triple. Students and teachers also can create their own rules that are then the subject of conjectures made by other users of the software. When used individually, in small groups, or in a whole-class setting, software of this nature can focus students on important mathematical processes such as looking for patterns, formulating conjectures, and selecting data to test hypotheses.

Example 3. Software in the *Geometric Supposer* series provides an example of computer-based materials that have the potential to redirect the secondary geometry curriculum. Yerushalmy & Houde (1986) contend that this software provides students and teachers with a tool to "create a geometry curriculum based on conjecturing and problem posing." (p. 418). This software allows students and teachers to construct, label, and measure a variety of geometric entities in order to test conjectures based on the geometric figures and data generated. The opportunity to construct numerous examples and to repeat constructions on subsequent figures provides a powerful tool that makes feasible an inquiry-based approach to geometry.

A second role that computer-based materials can play in enriching and redirecting the curriculum is that of providing the intermediate step in the three-step process that often occurs when one learns mathematics (van Deusen, 1985-86). Quite often a mathematical idea is first encountered as a result of manipulating physical objects. Following this initial, concrete encounter with the concept, pictorial representations of the objects often are used as part of the second step in acquiring the concept. This semi-concrete experience often forms the basis for subsequent manipulation of abstract mathematical symbols, the third step in the three-step process.

Example 4. Spatial concepts are encountered in semi-concrete form in software such as *The Right Turn* (Bretl, 1985) and *The Super Factory* (Fish & Kosel, 1985). In these programs students are given commands to manipulate pictures of objects (3x3 grids and cubes, respectively) to achieve desired final states and to predict the results of certain pictorial manipulations.

Computers can provide a vehicle for the transition from the concrete to the abstract by quickly and accurately generating pictorial representations in response to requests from the student. In many cases, however, the pictorial experience must be supplemented initially with teacher- or student-constructed concrete representations. *The Right Turn* and *The Super Factory* provide examples of software that can help to encourage the orderly progression from concrete to abstract representations suggested by Corbitt (1985).

New Directions for the Curriculum

New curricular directions can be expected as the availability of computer-based materials increases. Symbol manipulator programs such as *muMath* that can perform symbolic manipulations of algebraic expressions hold the potential for substantially redirecting the secondary curriculum (Heid, 1983). Promising curriculum projects involving computing and algebra and computing in the remedial mathematics curriculum suggest that it is possible to embed many secondary mathematics topics in a problem-solving and applications context to achieve what

Corbitt (1985) refers to as "a new classroom dynamic in which teachers and students are natural partners in the search for understanding of mathematical ideas and a solution of mathematical problems." (p. 248).

Recommendations for Teachers

1. Review and evaluate software to identify promising materials that provide a means to use the computer as a tool to encourage inquiry, experimentation, and problem solving.

2. Note the objectives that can be addressed by using such software and the topics in the curriculum with which the software might be used. Keep in mind that whole-class use of computer-based materials is often advantageous for providing students with a model for using the computer as a tool to promote problem solving and an inquiry approach. Also keep in mind that manipulatives might be needed to accompany certain software, since students may not be able to deal immediately with the semi-concrete representation provided on the computer screen.

3. Identify additional areas of the secondary mathematics curriculum in which computer-based materials might be used as a context for problem solving. Note areas in which the computer may be used as a tool to facilitate students' examination of important mathematical questions, to stimulate conjectures, and to provide students with a means of verifying and extending their generalizations.

The computer is a powerful tool that can provide teachers and students with a context in which problem solving can be emphasized. The examples above give but a few instances of the many ways in which computer-based materials can contribute to enriching and redirecting the secondary mathematics curriculum.

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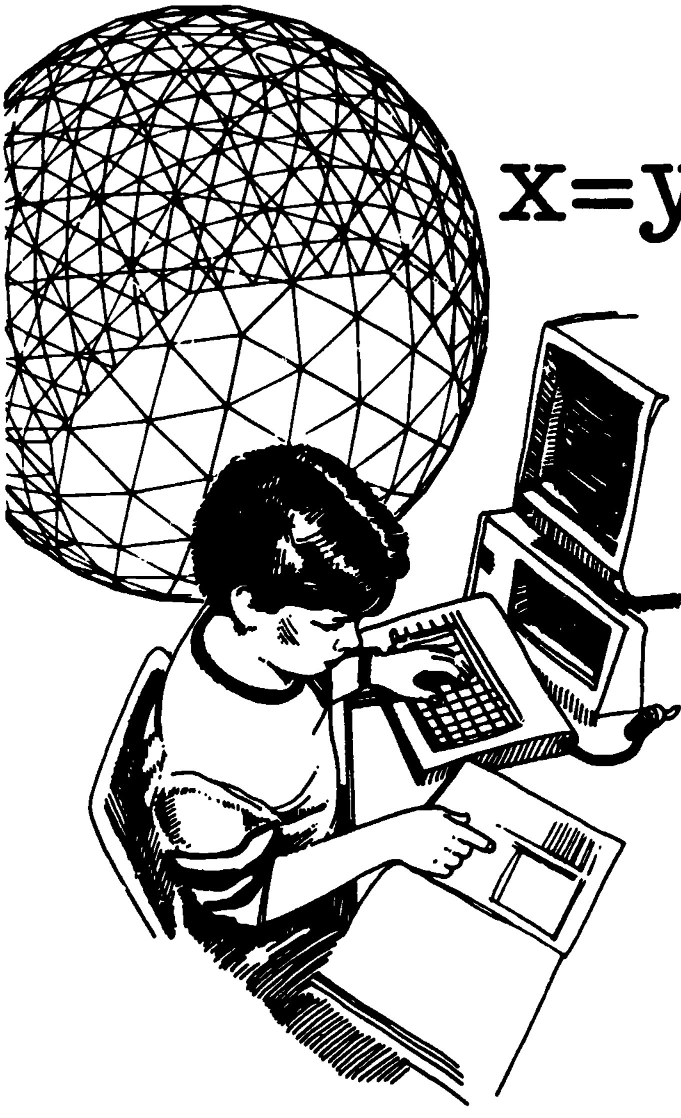

MICROCOMPUTERS AND THE MATHEMATICS CURRICULUM

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Almost fifty years ago the Englishman Harold Benjamin, in an amusing paper entitled *The Saber-Tooth Curriculum* (1939), suggested that the curriculum had become so outdated that it made a virtue out of educating the young in obsolete skills which had been essential to the economy of their ancestors — "fish-grabbing-with-the-bare-hands," "woolly-horse-clubbing" and "saber-tooth-tiger-scaring-with-fire." Vic Kelly in *Microcomputers and the Curriculum — Uses and Abuses* states that it is not difficult to see the parallel with the secondary curriculum of many schools in England at the time the paper was written (Kelly, 1984).

The secondary mathematics curriculum in the United States will be vulnerable to such gibes as "Saber-Tooth Curriculum" if we fail to follow the leadership of our profession. The theme for the 1986 NCTM Annual Meeting was "Better Teaching, Better Mathematics: The Perfect Mix for '86." F. Joe Crosswhite, in his President's Report (Crosswhite, 1986) attributes the theme to Ed Begle, who characterized the period of the late 1950s to the early 1970s as one in which we learned much about teaching *better* mathematics, but little about teaching mathematics *better*. Crosswhite warns that school mathematics suffers when we allow ourselves to be forced to choose between false dichotomies: "the old and the new in mathematics, skills and concepts, the concrete and the abstract, intuition and formalism, structure and problem solving, induction and deduction." Reasonable balances between the extremes should be sought in each case, rather than polarizations of positions which have characterized cycles in school mathematics in the past. Crosswhite alludes to the next cycle in school mathematics: "School programs must take full advantage of calculators and computers — not the other way around" (Crosswhite, 1986).

Vic Kelly warns that as the pendulum swings from the "saber-tooth" curriculum to the "2001" curriculum, microchip technology might be embraced "as avidly for its 'futuristic' attraction as (teachers and schools) once embraced the study of the ancient world for the traditional values enshrined there." His caveat is well taken: "In both cases, there is a failure to evaluate the curriculum in terms of its educational merits and value here and now" (Kelly, 1984).

It is the object of the following examples to illustrate ways in which computers can be used to prevent a "saber-tooth" attitude of mind by teaching *better* mathematics — but more importantly, while teaching mathematics *better*.

Algebra, Structure and Method, Book 1, by Dolciani, Brown, Ebos, and Cole has an accompanying book of *Computer Activities* (Snover and Spikell, 1981) which are ditto-master worksheets corresponding to sections of the text. The following program from Activity 2, *Generating Sequences,* can be keyed in by the student or called up from a disk.

PROGRAM —
```
10 PRINT "HOW MANY NUMBERS IN THE
   SEQUENCE";
20 INPUT N
30 PRINT
40 FOR X=1 TO N
50 LET T=2*X
60 PRINT
70 PRINT "TERM NUMBER"; X;
80 PRINT "HAS THE VALUE"; T
90 NEXT X
100 END
```

PROGRAM CHECK — After typing in the program, the student is asked to make a program check by running the program. If the number 4 is entered after the question, the computer should print

```
TERM NUMBER 1 HAS THE VALUE 2
TERM NUMBER 2 HAS THE VALUE 4
TERM NUMBER 3 HAS THE VALUE 6
TERM NUMBER 4 HAS THE VALUE 8.
```

ANALYSIS — The worksheet then presents the definitions of "arithmetic sequence" and "common difference."

USING THE PROGRAM — The student is asked to change line 50 to read 50 LET T=3*X and to observe the output. After several prescribed changes are made in line 50 and results are observed, the student is requested to put sequence generators such as "5x+6" into acceptable computer format for line 50 (viz., 50 LET T=5*X+6).

EXTENSION — The student is given a set of sequences, such as 4, 7, 10, 13, ..., and asked to find a formula, key in a corresponding line 50, and check the formula by running the program.

The format of the worksheets represents "discovery" learning in computer programming, traditional mathematics content, and enrichment mathematics. Many "what if's" are self-generated or can be posed by the student or teacher when keying in the program (e.g., "What if the student omits the ';' at the end of line 10?"). "Order of operation" questions arise, patterns can be observed, and translations from numerals to algebraic expressions are required as sequences such as 2, 5/2, 8/3, 11/4, ... are suggested. The concept of "limit of a sequence" arises naturally, as does a number crunching answer to the question, "Does the sequence 2, 5/2, 8/3, 11/4, ... have a limit? The spiral approach to teaching limits of sequences is self-motivated and students can have the satisfying experience of identifying limits of sequences they find in calculus books. Such a hands-on approach to limits of sequences can be appreciated by calculus teachers (both high school and college) whose sometime premature jumps to the traditional "epsilon" definition of limit elicit confused stares from students.

The use of computer activity sheets such as those which accompany *Algebra, Structure and Method* have the intrinsic ability to accommodate a wide spectrum of student achievement and motivation in the same setting. The slowest student is ensured a solid grasp of the idea of sequence, for example, while the most precocious student experiences an enrichment lesson in programming, order of operations, algebraic expressions, problem solving, and limits of sequences; yet little previous computer knowledge by the student or teacher is necessary.

The computer activity *Sum of Odd Numbers* (Snover and Spikell, 1981) poses the problem "The first six odd numbers are 1, 3, 5, 7, 9, and 11. Their sum is $1 + 3 + 5 + 7 + 9 + 11 = 36$. Find the sum of the first n odd numbers for any particular n you choose." After keying in the given simple program and finding the sum of the first 12 odd numbers (144), first 5 odd numbers (25), first 83 odd numbers (6889), etc., conjectures are made and responses are checked using the computer. A simple formula for the sum, S, of the first n odd numbers is requested. The *EXTENSION* is to modify two lines in the program to find the sum of the first n counting numbers, rather than the odd numbers. After the sum of the first 10 counting numbers (55) and the sum of the first 100 counting numbers (5050) are printed out, the student can guess the sum of the first 1000 counting numbers without running the program. The traditional idea of mathematics as the recognition of patterns is evident. A teacher who uses the history of mathematics as part of his/her motivational "bag of tricks" can allude to the (real or fictitious) calculation of the sum of the first n counting numbers by Gauss at an early age:

$$\begin{array}{ccccccccc}
1 & + & 2 & + & 3 & + \ldots + & n \\
\underline{n} & + & \underline{(n-1)} & + & \underline{(n-2)} & \underline{+ \ldots +} & \underline{1}
\end{array}$$

$$(n+1) + (n+1) + (n+1) + \ldots + (n+1) = 2(1 + 2 + 3 + \ldots + n),$$

$$\text{thus } 1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}.$$

And the ultimate extension to the definitely non-computerized proof by mathematical induction (included in many traditional texts as an aside or footnote) can be studied by the unusually motivated student.

Additional uses of the microcomputer to teach *better* mathematics and to teach mathematics *better* are legion. "Computer games" such as *Green Globs* and *Tracker* (Dugdale, 1982) are highly motivational, while reinforcing previously studied content. Fresh air and a sense of mathematical exploration, conjecture, discovery, problem solving and experimentation can enliven a traditional geometry class through the use of such programs as Judah Schwartz's *The Geometric Supposer*. Many good graphic utility programs which are available commercially (e.g., *Graphing Equations* and *ARBPLOT* from CONDUIT) and in the public domain are improvements and simplifications of the traditional colored-chalk-and-blackboard approach to studying graphs. More advanced topics in secondary mathematics can be taught better and more easily with the aid of commercial software which gives geometric interpretations of such topics as lower, upper, and Riemann sums for integration and epsilon definitions of limits (e.g., *Mathematics Software* from D.C. Heath).

The leadership role taken by the NCTM in making the taunt "sabertooth" inapplicable to the mathematics curriculum of the 1980s is exemplary. It is up to each of us to keep as our theme (not only for 1986) the goal of "Better Teaching, Better Mathematics."

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NEEDED — USEFUL AND EFFECTIVE TEXTBOOKS

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Is there a mathematics curriculum guide in your classroom closet or file cabinet? When was the last time you looked at it? If you are like most mathematics teachers, the answers to these questions are "yes" and "I'm not sure." Although we do not always like to admit it, the textbook is often the curriculum that we teach.

Mathematics textbooks play an important part in the mathematics curriculum. For most teachers, a textbook represents the heart of their instructional program. A good textbook series provides a sequence, problems, worksheets, review and a testing program.

Heavy reliance on a textbook is not necessarily bad, particularly if the textbook was well-designed. My concern is that some of the textbooks on the market today do not make use of well-established methods to improve mathematics teaching and learning. Even worse, most current textbooks do not challenge talented mathematics students.

The launching of the Russian satellite Sputnik I in 1957 sent shock waves around the country. There was an immediate call to improve mathematics curriculum and instruction in the United States. The result was the so called "new mathematics," designed and developed by mathematicians and mathematics educators. The mathematics, of course, was not new but many of the topics and methods of teaching were new to our elementary and secondary schools. Curriculum and textbook materials of that period emphasized the structure and meaning of mathematics rather than rote memorization of facts.

The new mathematics failed for two reasons. First, the retraining of teachers, particularly at the elementary school level, was inadequate, and second, too many students could not do simple computational exercises. There were some positive results, however. Elementary and secondary textbooks of the late 1960's to early 1970's were the most mathematically rigorous of the last thirty years. I frequently refer to this period as the "Golden Age" of mathematics education.

Dissatisfaction with the results of the new mathematics gradually forced a change in the philosophy of textbook construction. The textbooks of the late 1970's and early 1980's became less rigorous, less structured and provided fewer challenges for our talented mathematics students. On the positive side, they were well organized, reinforced computational skills and provided a variety of useful ancillary materials. They gave the illusion that students were more successful at mathematics because they (the books) were less demanding. A typical sixth-grade textbook of that period would include a section on the addition of fractions. The most difficult common denominator that a student would be asked to find was eighteen, or possibly twenty-four. For the above-average student common denominators of eighteen or twenty-four can easily be found by inspection. Talented students are rarely challenged by such an exercise.

Many teachers supplemented their basal mathematics series to make up for the deficiencies. Often, the supplementary materials came from textbook materials published during the Golden Age. It is not unusual to find an honors or gifted program today using textbooks with copyright dates from that time period. Recent national reports on the quality of education in the United States, including *A Nation at Risk,* have helped to reverse the trend. Progress toward improved mathematics textbooks, however, has been too slow.

We now have the advantage of knowing what worked and what did not work over the past thirty years. I am not suggesting that we return to the methods of Golden Age, but rather that we take the best of that period and the period which followed. We must demand from publishers more useful and more effective textbooks — textbooks that make use of all that we have learned in the last thirty years. We need textbooks that provide a variety of differentiated exercises and ancillary materials, clearly defined objectives, mental computation and estimation exercises, and built-in review and testing. Every set of exercises and every test should include review problems. If our students do not maintain their skills, they will lose them. As teachers, we should be spending our time improving the delivery of mathematics instruction, not supplementing poorly designed textbooks.

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THE MATHEMATICS CURRICULUM: TEXTBOOKS AND HIGHER-ORDER THINKING SKILLS

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During the first third of the twentieth century, the textbook was a major factor in determining what mathematics was taught (Edmonson, 1931). That this reliance on the textbook has continued throughout the remainder of the century has been affirmed by mathematics educators (Johnson & Rising, 1967) and the National Science Foundation (Brandt, 1978). As recently as 1984, Steven Willoughby, then President of the National Council of Teachers of Mathematics, asserted that the "most important factor in determining what mathematics is taught is the textbook used."

Many of the recent national reports have suggested that one way we can improve our nation's schools is to place an emphasis on students' intellectual development. The acquisition of problem solving and higher-order thinking skills have long been goals of our schools in general and of mathematics educators specifically. The National Council of Teachers of Mathematics, in its 1980 "Agenda for Action" recommended that "problem solving be the focus of school mathematics in the 1980's." Students must, therefore, have opportunities to learn how to gather, organize, and interpret information, draw and test inferences from data, analyze and conceptualize problems, experiment, and apply mathematical skills and knowledge.

Since the mathematics textbook has such a powerful influence on what is taught and learned, and acknowledging that the acquisition of higher-order thinking skills by students is (and will continue to be) an important instructional goal, mathematics teachers, supervisors and curriculum committees need to know what opportunities mathematics textbooks provide for students to be actively involved in the development, practice and acquisition of higher-order thinking skills. This article summarizes a strand of research encompassing almost twenty years of analysis of selected content from secondary and elementary school mathematics textbooks.

Analytical Tool and Procedures

The analytical tool used in the research consists of four lists which, when accompanied by decision rules, are designed to enable analysts to classify printed instructional materials according to type of content, level of cognitive activity, stage of mastery, and mode of response. Only the "cognitive level of student tasks" portion of the analysis system is illustrated in detail here because of the aforementioned interest in higher-order thinking skills. Complete descriptions of the other three components – content, stage of mastery, and mode of response — can be found elsewhere (See Nicely, 1970).

The "cognitive level" list consists of twenty-seven verbs which were identified to describe covert tasks. Each verb was given a specific definition so that analysts could accurately classify the printed material based on what it actually was and not necessarily what the textbook author might call it. (For example, "solve" might be used by an author to describe a variety of behaviors at different levels of complexity.) These verbs were grouped into nine categories and arranged in an ordinal scale. Each of the nine categories (plus one additional category entitled "No Task; Observe; Read") was assigned a unique one-digit code. Figure 1 lists the cognitive verbs and their associated levels.

Levels	Verbs
Level 0	No Task; Observe; Read
Level 1	Recall; Recognize; Repeat; Copy (Imitate, Reproduce)
Level 2	Iterate
Level 3	Compare; Substitute
Level 4	Categorize (Classify, Group); Illustrate (Exemplify)
Level 5	Apply; Relate; Convert (Translate); Symbolize; Summarize (Abstract); Describe
Level 6	Justify (Support); Explain (Interpret); Analyze
Level 7	Hypothesize (Theorize); Synthesize (Organize, Structure); Generalize (Induce); Deduce
Level 8	Prove; Solve; Test (Experiment); Design
Level 9	Evaluate

Figure 1
COGNITIVE VERBS AND LEVELS

The cognitive level list was applied to the "complex numbers" portion of secondary school mathematics textbooks that were printed between 1961 and 1984, and to the "decimals" portion of elementary (grades 3 through 6) textbooks that were printed in the mid-1980's. These comprehensive mathematical topics were chosen because they afforded an opportunity to examine how the textbook authors would actively involve students at a variety of cognitive levels throughout the stages of readiness, development, practice, demonstration, overlearning and enrichment as a mathematical concept was developed in its entirety.

The textbooks selected for analysis were in wide use across the United States, and adequately represented those which were popular at the time they were printed. Because the National Advisory Committee on Mathematics Education (1975) asserted that students read very little of the textual material in a mathematics textbook and that the books were used primarily as a source of problems, the analysis focused on only those situations in the books where students could be actively involved in their learning and make an overt response.

Results of Textbook Analysis

The analysis of the complex numbers portion of the textbooks that were printed in the 1960's revealed that a large majority (62% or more) of the problems/tasks were iterative (level 2). The average amount of iterative behaviors per book was slightly more than 75 percent. Relatively few behaviors, other than iterate, appeared more than ten percent of the time. One book had 18 percent at the application level, while another had 11 percent at that level. All the others were significantly lower in terms of application and similarly complex behaviors. Only one book had more than three percent hypothesizing and generalizing and several books had no behaviors at that level. Several 1960's textbooks had at least seven cognitive levels present, and two had 12 percent to 15 percent of their total behaviors in the areas of analysis and justification. synthesis and generalization, and proof. In contrast, one book has 98 percent of student behaviors in just one level — iterate. None of the books had student tasks which require evaluative (level 9) behaviors (Nicely, 1970 and 1981).

The analysis of the complex numbers portion of textbooks that were printed in the 1970's revealed that at least 66 percent of the problems/tasks were iterative (level 2). The average amount of iterative behavior per book was 81 percent. In only two books did behaviors other than iterate appear more than 10 percent of the time. These two books devoted 15 percent to 17 percent of the student tasks to behaviors at the categorizing level. Only two books had many (5% to 8%) behaviors at the analysis and justification, synthesis and generalization, and proof levels. One book had 100 percent of student behaviors in just one level — iterate. Again, no books contained student tasks which required evaluative behaviors (Nicely, 1985a).

The analysis of the complex numbers portion of textbooks that were printed in the mid-1980's revealed that at least 77 percent of the problems/tasks were iterative with the average amount of iterative activity rising to more than 86 percent per book Again, in only two books did behaviors other than iterate appear more than 10 percent of the time. These two books devoted 10 percent and 12 percent of the overt student tasks to behaviors at the prove level. No books contain opportunities for students to operate at the highest level — evaluate (Nicely, Bobango & Fiber, 1984).

Comparing these textbooks across decades, some trends are apparent. The relative emphasis on lower order (levels 1, 2, 3, and 4) cognitive behaviors increased from about 84 percent in the 1960's to more than 93 percent in the 1970's, and then decreased to 80 percent in the 1980's. There was a concomitant decrease in the emphasis on higher-order (levels, 5, 6, 7 and 8) cognitive behaviors — from 16 percent in the 1960's to 7 percent in the 1970's and then to 10 percent in the 1980's. The recent texts show a slight increase in higher-order behaviors, although not a return to the 1960's level. (Nicely, 1985b).

A study conducted by Nicely, Fiber and Bobango (1986) has indicated that recently published elementary school mathematics textbooks also tend to emphasize lower-order cognitive behaviors when teaching decimals. Most of the textbook problems designed for students at the grade levels analyzed were at the iterate level. With one exception — in the third grade book in one series — less than 14 percent of the problems were at the application level. All four series offered enrichment activities in grades four, five, and six, but most of these enrichment activities would only require iterative behavior on the part of the students.

Implications

If we are serious about helping students acquire higher-order thinking skills, mathematics teachers and supervisors will have to carefully select and use textbooks that provide opportunities for students to develop and practice those behaviors. An analysis scheme such as the one described in this article can be useful in determining the extent to which the mathematics books in use (or under consideration) are congruent with the desired student behaviors. In the event that no book can be found which will be likely to help students acquire these behaviors, the teachers will have to make or purchase other instructional materials that do f[illegible] such thinking. Teachers may also have to develop expertise in fr[illegible] ques-tions and guiding discussions that will enable students to operate at the desired intellectual levels.

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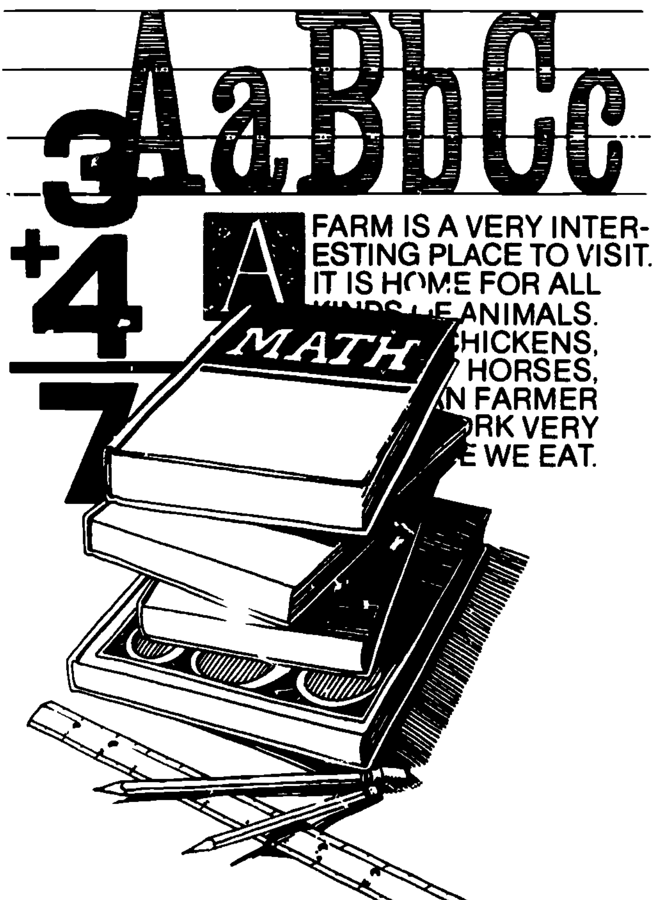

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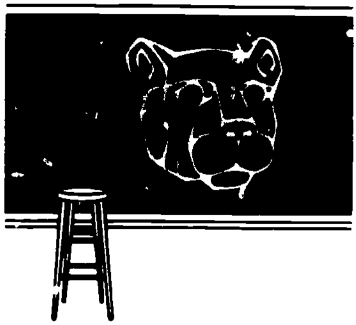

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A CRITICAL LOOK AT CALCULUS

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It is likely that anyone who has taught Calculus at the college level has observed the following sequence of events. On day one a room full of eager students clings to every pearly word which comes forth from your lips; by the first test a few terror-stricken individuals have withdrawn; by the final examination roughly half of the class has disappeared and many of those who remain are praying for a minor miracle; at the start of the next calculus course even some of the finer students have crawled into the woodwork, never to be heard from again.

While the above may be a bit of an over-exaggeration, the picture painted indeed is close to reality. Part of the problem is an outgrowth of syllabi which are overcrowded and/or too rigid. Secondly, the gradual rise of discrete mathematics calls upon mathematicians to extend the traditional applications beyond traditional closed-form methods. Many of you have probably seen calculators which approximate definite integrals and which can provide numerical solutions to equations. With that in mind, it becomes evident that we need to stress more strongly such topics as Simpson's Rule and the Newton-Raphson Method and spend less time presenting a multitude of somewhat extraneous topics, such as many of the various integration techniques. (I grit my teeth as I type this, for these methods are among my favorite topics to teach in calculus!)

Moreover, the influx of such computer algebra systems as *muMath*, MACSYMA, and Maple will impact on calculus the same way calculators have influenced arithmetic. Many routine algebraic and trigonometric processes are readily handled by such software, and even derivatives and indefinite integrals can be determined in closed form! We are in the throes of a major upheaval in mathematics education in which the emphasis needs to be on problem-solving and applicability rather than on the more traditional emphasis of procedure. Unfortunately, the conservative approach of gradual change will likely thwart the urgent need to institute new practices which are more in touch with the available technology and our societal needs.

But precisely what changes are in order? Paul Zorn's *Calculus Redux'* in the March-April 1986 issue of *FOCUS*, the newsletter of the Mathematical Association of America, is a strong presentation of many of the conclusions reached at a four-day Sloan Foundation-sponsored conference. To summarize their findings, change is needed in the existing calculus sequence, both to update and enliven the coursework and to focus attention on the role of calculus as the language of science.

One of the conference subcommittees "drew up a two-semester syllabus that aims to build intuition and conceptual understanding, stressing numerical and geometrical ideas as well as algebraic techniques. To support this viewpoint, access to calculators with definite integral and equation-solving keys will be assumed."

In turn, three variations of the first-year calculus sequence were proposed, ". . . one covering single variable calculus through Taylor's Series and the beginnings of differential equations; another that cuts short the single variable material to cover the basics of several variable calculus in the second semester; and a third that makes full use of the computer algebra systems soon to be widely available."

In fact, Sherman Stein of the University of California-Davis went so far as to suggest that a new streamlined syllabus and a heavier emphasis on calculators will allow time to deal with open-ended questions to be explored outside of class. Stein encourages the freest attack on such problems, by all methods: guessing, estimation, calculation, and even, when all else fails, the full algebraic and analytic arsenal.

Enacting such change is imperative but not easily undertaken. Four-year colleges and universities need to react quickly and relatively uniformly to take advantage of the technology now available. Two-year colleges need to closely monitor the approaches being used by the universities to which their students are transferring to help ensure compatibility of background. Secondary level calculus courses will need to reflect these leads and in fact can even act innovatively by instituting their own emphasis on non-manipulative problem-solving.

Notice again that this stress leads toward conceptualization and away from what one of my former professors referred to as "how-to-do-it-methods." This revitalization of the calculus sequence has the capability of injecting new life into what is often the terminal mathematics course for many students. But more importantly, these changes should help foster thinking skills which are more flexible than those which are needed to merely reproduce technique and procedure.

Regardless of the specific changes in the sequence which have been suggested, attention must be given to the situation as it now stands. Lynn Steen, MAA president, offered these thoughts in an editorial found in the same issue of *FOCUS*: "... Calculus should not and need not be an experience of failure for the majority of college freshmen. Good placement practices combined with good teaching, backed up by timely, sensitive academic support structures, should insure that most students who enroll in calculus will succeed at it. Success breeds success, and students who succeed in calculus will support mathematics as professionals in whatever career they select."

"Second, the current curriculum is overcrowded and outdated. Not even the engineers insist any more on centroid calculations and related rate problems." He continues to reinforce the need for improved calculator utilization, and with this improvement comes a rethinking of the scope of the calculus sequence.

"Third, calculus as presently taught is failing to achieve its most general goal — to help students to develop acumen in analysis and precision in expression. Mimicry mathematics does not develop adequately an appropriate conceptual understanding of the nature of change, which is after all, what calculus is all about."

Short-term results will undoubtedly be a hodge-podge. Some colleges will update their approach to the calculus almost instantaneously. Their students will be expected to be knowledgeable in calculator-related, numerical techniques. Discrete methods and approximation will expand in importance, while more traditional closed-form, procedural techniques will be streamlined and realigned to include only the more flexible, general-purpose procedures. Meanwhile, other institutions will likely maintain course content as it has existed for decades, to the detriment of their own students who are enrolled in other curricula which will call upon non-traditional calculus procedures to solve their specialized problems.

Clearly we need to vigorously expand our knowledge of these processes; let go of cherished, but out-dated, content; and embrace those techniques whose importance are ever-growing.

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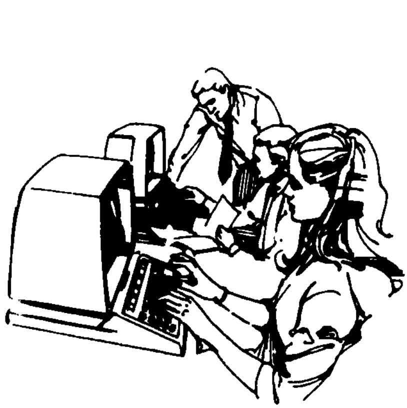

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DISCRETE MATHEMATICS AND ITS ROLE IN THE SECONDARY MATHEMATICS CURRICULUM

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While discrete mathematics has not (and should not) become a revolution in mathematics education (see Hart, 1985), it has, in just a few years, become a significant issue at both the college and secondary levels. For example, the Mathematical Association of America (Siegel, 1986) has formally recommended that, "Discrete mathematics should be a part of the first two years of the standard mathematics curriculum at all colleges and universities," and "Secondary schools should introduce many ideas of discrete mathematics into the curriculum to help students improve their problem-solving skills and prepare them for college mathematics" (p. iii). And, indeed, some high schools are already implementing discrete mathematics courses. Yet the discrete mathematics issue is still not universally understood, especially in relation to the secondary curriculum. It is the purpose of this article to shed some light on this issue by considering what discrete mathematics is, what it is not, and how it fits into the high school curriculum.

What Discrete Mathematics Is

Discrete mathematics deals with discrete objects rather than, for example, with continuous functions. It is mathematics popularized by computers, since computers operate digitally and discretely. This is particularly important since the number of students majoring in computer science in college is growing rapidly, especially in comparison to the decline in numbers of students majoring in mathematics. And, in fact, more college-bound high school students will take discrete mathematics in college than will take (non-discrete) calculus. So there is definitely a need for high schools to be very much aware of discrete mathematics. But still, what exactly is discrete mathematics?

The first and most important point to be made is that discrete mathematics is a way of thinking as well as a collection of mathematical topics. A list of characteristic content might be: graph theory, difference equations, combinatorics, induction and recursion, algorithmics, proof, discrete probability, matrices and linear algebra, abstract algebra, sequences and series, logic and sets, functions and relations, and numerical analysis. The content can and does, vary greatly among discrete mathematics courses. But the unifying themes always present are algorithmic problem solving and recursion.

What Discrete Mathematics Is Not

Discrete mathematics has similarities to other types of mathematics, but it is also distinctly different. It is important to note the differences so that we maintain the true spirit of discrete mathematics and do not fall into teaching other, similar mathematics.

Discrete mathematics is most clearly not classical continuous mathematics. One might say that classical continuous mathematics has been nurtured by natural science while discrete mathematics has been nurtured by computer science. Another dichotomy, to be taken not too precisely, is that discrete mathematics deals with countable sets while continuous mathematics deals with uncountable sets.

The closest relative of discrete mathematics is probably the old "finite mathematics" (see, for example, Kemeny, 1974). But discrete mathematics is very definitely different. First of all, it does tend to have some different topics, for example, graph theory, difference equations, and algorithmics. Secondly, it is not a "terminal" mathematics course as finite mathematics courses usually are. Lastly, and most importantly, discrete mathematics places more emphasis on rigor and algorithmic thinking than does finite mathematics.

With the emphasis on algorithms one might think that we are just returning to old "back-to-basics" mathematics. Not so. The back-to-basics movement emphasized *performing* algorithms, while in discrete mathematics the emphasis is on: (1) using algorithms to solve problems, (2) using algorithms to develop theory, and (3) designing and analyzing algorithms.

So, discrete mathematics is not "back-to-basics." But what about the revolution that spawned "back-to-basics," that is, new math? We're seeing sets, logic, and rigor again, just when we thought we'd gotten rid of them! Well, discrete mathematics is very different from new math. In particular, the algorithmic point of view of discrete mathematics was absent in the new math, where "existential" thinking was dominant (see Hart, 1985, for a more thorough discussion of this point). It is true that some of the content of the new math is reappearing in discrete mathematics, but the treatment is quite different. For example, one finds different number bases in both "types" of mathematics, but they appear for *theoretical* reasons in the new math — to more firmly ground and generalize the meaning of place value, while they appear in discrete mathematics for *practical* reasons — bases like 2 and 16 are dealt with because those are the bases that computers use.

The relevance of computers to discrete mathematics keeps coming up. So maybe discrete mathematics is just what college computer science departments have been teaching for years in a course often called Discrete Structures (see, for example, Stanat, 1977). No again! Discrete mathematics is more than just a service course for computer science. Although it will serve that purpose, it is first and foremost a mathematics course, useful for all the sciences and for further study in mathematics. Some particular differences between discrete mathematics and Discrete Structures are that difference equations are often found in the former but rarely in the latter, and while computer programming is a must in a Discrete Structures course it is strictly optional in a discrete mathematics course.

So far we have had a glimpse of what discrete mathematics is and what it isn't. But now, how does it fit into the secondary mathematics curriculum?

Discrete Mathematics in the High School Curriculum

In thinking about how to fit discrete mathematics into the secondary curriculum it is useful to consider three broad categories of the curriculum: (1) "mainstream" topics, (2) "end-of-the-book" topics, and (3) new topics. Fitting discrete mathematics into each of these categories will now be discussed.

"Mainstream" topics are those topics that are always covered in the secondary mathematics curriculum. Since discrete mathematics is a point of view as well as a collection of topics, it is possible to fit discrete mathematics into the curriculum — without removing anything else — just by teaching some mainstream topics from the "discrete/algorithmic/recursive" point of view.

Two examples of mainstream topics where this could be done are: systems of linear equations and relations. For example, matrix methods, such as Gaussian Elimination or the Gauss-Jordan method, could be used for solving systems of linear equations instead of the (albeit equivalent) equation-manipulation method which is usually taught. Certainly for systems of two equations the equation-manipulation method is fine, but for larger systems matrix methods seem the only way to go.

When teaching relations one could take a graph-theoretic approach using graphs (meaning "vertices and edges") and the matrix representation of graphs. (Graph theory is something that almost everyone has seen in some context or another — for example, networks or the Konigsberg Bridge Problem. For a quick, readable treatment of graph theory, see Ore, 1963.) The idea is that the elements of the set on which the relation is defined become the vertices of a graph, and any time two elements are "related" you join the two corresponding vertices with an edge (actually "directed" edges are used -- edges with arrows to indicate a direction — since x related to y does not necessarily mean y is related to x). One can then represent the graph as a square matrix with entries of T and F (True and False), where a T in the i-j entry means that the ith element of the set is related to the jth element — i.e., the ordered pair (ith element, jth element) is in the relation. For details of this see, for example, Skvarcius, 1986; but in any case, the result is that relations and operations with relations (e.g., composition) can be developed using graphs and matrices.

How about "end-of-book" topics? Such topics are different for different teachers and different books, but they can include such things as combinations and permutations, the Binomial Theorem, sequences and series, and induction — all of which are discrete mathematics topics. Simply bringing these topics "up front" would be a way of fitting discrete mathematics into the curriculum.

As far as new topics go, graph theory and difference equations are two topics from discrete mathematics that deserve a place in the secondary curriculum. (For a quick look at difference equations, see Sandefur, 1985.) But is there room? Well, no. Something else would have to go. But Usiskin (1980) and others have suggested a number of current topics in the curriculum that could be removed, so there is hope for putting in new topics.

Summary

This article has presented a brief overview of the discrete mathematics issue by looking at what discrete mathematics is, comparing it to other mathematics curricula, and suggesting some ways to fit it into the secondary mathematics curriculum. Of course there are many questions and details that have not been discussed. But it is hoped that the picture which has been painted indicates that discrete mathematics is valuable mathematics which is (or can be made) accessible to high school students, and as such is deserving of inclusion in the high school mathematics curriculum.

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EXPONENTIATION IN PASCAL

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Most people will agree that Pascal is fast and elegant and is a good language to use when teaching programming. But while they're saying things like that, they also want the language to do all sorts of things that may or may not be academic in nature. One good example of this confusion is the way that Pascal deals with some arithmetic functions. If I am strictly a programmer, I want the language environment that I'm using to be a completely transparent vehicle for me to communicate my algorithms to the machine . . . and I want the thing to work with as little effort from me as possible. But if I'm a teacher, I want a language environment that causes my students to actually think through what they want the machine to do. In fact, the reason that Pascal has so many adherents in the schools is precisely because Wirth designed it to make people learn rather than to make the life of programmers easier.

Wirth chose to include a number of arithmetic functions into common Pascal, but he also chose to not include several . . . and the philosophical decision to disallow certain functions can cause programmers to gnash their teeth a little bit while giving teachers a glorious opportunity to push their students into doing some mathematical footwork.

The standard arithmetic functions of common Pascal include:

abs(x)	which returns the absolute value of the argument x
sqr(x)	which returns the square of x
sqrt(x)	which returns the square root of x
sin(x)	which returns the sine of x
cos(x)	which returns the cosine of x
arctan(x)	which returns the arctangent of x
ln(x)	which return the natural logarithm of x
exp(x)	which returns base e raised to the power of x
round(x)	which rounds x
trunc(x)	which truncates the real x to an integer.

Now you may be wondering wh[illegible] tangent, arcsine, and arccosine functions aren't on that list. The an[illegible]s that Wirth wanted the language to be as parsimonious as possible. Being an academician, he just assumes that everyone will have a calculus text around with the necessary tables in the back to obtain all of the other trigonometric relationships from the sin, cos, and arctan functions. From Wirth's philosophical position, predefining extra functions in Pascal would be redundant, and make the language less elegant and compact . . . and it would make it easier for students to breeze through trigonometric computations without actually learning how the functions are related.

Another arithmetic function that's not on that list is an exponentiation function, and this simple fact is what often separates the hackers from the real mathematical programmers. Unless you're specifically writing a program that calls for some trigonometric calculations, you can go a long time without having to make a call to arctan(x) . . . but it's a fairly common occurrence to come across exponentiated numbers. In fact, in school environments it seems as if our programs are dealing with exponents constantly: biology students are writing programs to assist them in calculating growth curves of bacteria cultures; physics students are writing programs to calculate the rate of absorption of radiant energy through different mediums; geometry students are writing programs to take the drudgery out of conics; business students are writing programs to calculate loan projections. We seem to be using the word mantissa a lot, but there it is . . . there isn't an exponential function in Pascal and I suspect that Dr. Wirth did it on purpose. Why? Because Pascal is first and last a teaching language. But where there's a will. . . .

Let's go back and look at that list of functions again. We do have a ln(x) function which returns the natural logarithm of x, and we have the exp(x) function which represents e (the base of the natural log system), raised to the real or integer power of x (in other words, e^x). These functions exist primarily because logarithms are so common in formulas and Wirth wanted to simplify expressions like $e^u - e^{-u}/2$ and ln(pi/2) so that they could be written out as (exp(u) - exp(- u))/2 and ln(3.141592654/2).

Now I know from high school algebra that I can use a property of exponents to derive a property of logarithms. For example, $125 = 5^3$; and $125^2 = (5^3)^2 = 5^6 = 5^{2 \cdot 3}$ so $\log_5 125^2 = 2 \cdot 3 = 2 \log_5 125$. Now in most of the textbooks we find that so long as m and b are positive real numbers where b is not equal to 1, and if p is any real number then the property of logarithms is expressed as. . . .

$$\log_b m^p = p \log_b m$$

Since p is any real number, this property can be used to raise a number to a power or to extract a root. Suppose we want to find $(7.27)^5$. Well, $\log(7.27)^5 = 5 \log 7.27 = 5\ (0.8615) = 4.3075$. Therefore $(7.27)^5$ has the approximate antilog of 4.3075 or approximately 20,300. It's these properties of logarithms which make them so popular in physics and biological calculations where large numbers need to be manipulated in small spaces.

Given all of this, we can come up with a simple formula which can be used to carry out exponential in Pascal:

$$a^n = \exp\ (n * \ln\ (a))$$

subject to the following restriction. The mantissa, a, must be a positive real or integer value. So . . . a mathematical expression like 7.27^5 would be written out in Pascal as exp(5*ln(7.27)). Get it? A mathematical expression like $9.87^{-3.51}$ would be written out in Pascal as exp(−3.51*ln(9.87)). The only thing that isn't going to fly is something like $(-5)^{0.15}$ because it has a negative mantissa.

So you see it is possible to do exponentation in Pascal. In fact, you don't really need to understand the properties of logarithms so long as you are able to juggle the formula given and follow the rules. But it is nice to be in a position where you can teach logarithmic functions to your students and have a ready-made excuse to make a practical application of the instruction. Thanks, Dr. Wirth.

The following program which calculates compound interest demonstrates the use of this formula as a function. Notice that Function Power must be written before Function Interest because Interest calls Power, so when we call Interest from the main program it calls Power to itself.

```
Program Investment (input, output);
Var
  start,annualrate,earned   : real;
  years,days                : integer

Function Power (x,y : real) : real;
  [ computes the value of x to the power of y ]
Begin
  Power := exp (y * ln(x));
End;

Function Interest (start,annualrate : real; years,days . integer) : real;
  [ calculates interest earned using compound interest ]
Begin
  Interest := start
              * power (1 + annualrate/100, years + days/365)
              - start;

End;

Procedure Getdata;
Begin
  Write ('Input beginning amount - ');
  Readln (start);
  Writeln;
  Write ('Input annual rate of interest - ');
  Readln (annualrate);
```

```
    Writeln;
    Write ('Input years - ');
    Readln (years);
    Writeln;
    Write ('Input days - ');
    Readln (days);
End;

Begin { Main Program }
    Getdata;
    Earned := Interest (start,annualrate,years,days);
    Writeln;
    Writeln ('Initial investment : $', start:10:2);
    Writeln ('Annual rate : ', annualrate:5:3);
    Writeln ('Duration of note : ', years,' years and ', days,' days.');
    Writeln;
    Writeln (' Interest earned : $',earned:10:2);
End. [of Investment]
```

DO YOUR STUDENTS A FAVOR — COORDINATE YOUR MATHEMATICS PROGRAM, K-12

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Many school systems do not have a coordinated K-12 mathematics program. The primary reason usually given, particularly in smaller school systems, is the cost of hiring a mathematics coordinator/supervisor. However, to avoid hurting the students, someone needs to be given the responsibility for ensuring that all teachers of mathematics in a given school system are aware of when and how mathematics concepts are treated at each level of instruction so that the system will have a coordinated and articulated mathematics program. The person given this responsibility should be someone with teaching experience in all mathematics subjects at the junior and senior high school levels. He/she might enlist the aid of a middle school teacher of mathematics and an elementary school teacher of mathematics. The following examples illustrate the kinds of issues that need to be discussed across grade levels.

Consider the operation of subtraction. Young students are usually given the "take-away" model for subtraction. (Seven objects on a desk, take away three, and four are left). While this concrete model may be fine at one developmental stage, other models for subtraction need to be introduced. For example, students should not encounter subtraction of integers for the first time by having only the "take-away" model for subtraction. While those who are skilled with proper manipulatives can readily demonstrate via "take-away" that $-8 - 3$ is -11, many who are not so skilled may say that $-8 - 3$ is -5, or something worse. Introducing and reinforcing subtraction with the directed distance model on the numberline throughout the elementary grades would make subtraction of integers more realistic to the learner. If the subtraction $7 - 2$ represented the distance from a point with coordinate 2 to a point with coordinate 7 on the numberline, then $6 - (-3)$ would represent the distance from -3 to 6 on the numberline and these subtractions could be readily illustrated.

Consider the fractions $a/0$, $0/a$ and $0/0$ where $a \neq 0$. Teachers of mathematics from kindergarten through grade 12 ought to understand why $a/0$ is undefined, $0/a = 0$, and $0/0$ is indeterminate. This understanding should be shared with students. Many of the problems that older students encounter in operations with zero come from an earlier "nothing" concept of zero. This "nothing" concept of zero is difficult to change because of its reinforcement in everyday life. (The Mudhens defeated the Doves 2-0 — "two to nothing"). Students who maintain this "nothing" concept of zero often have difficulty discerning the difference between zero-slope and no-slope for lines. Another source of confusion for some students is the overgeneralization that "any number divided by itself is one." One property of zero is that $^0/_0$ is not necessarily one. This fraction turns out to be one of the most interesting fractions in all of mathematics.

The tale of the student who reduced the fraction

$$\frac{a + b}{a - b}$$

to -1, by cancelling the a's and then cancelling the b's, leads to another area of curriculum where coordination is important. Teachers at all levels need to understand the problems encountered by students who have misconceptions regarding the use of cancellation to reduce a fraction. It is too easy to acquire the habit of cancelling like factors under the wrong circumstances. The fraction

$$\frac{6x + 12}{3}$$

could be reduced to $2x + 12$, or other expressions, if like factors are the only criterion. The definition

$$\frac{ac}{bc} = \frac{a}{b}$$

needs to be discussed by all mathematics teachers within the school system.

Many teachers of mathematics are unaware, probably not by their own choice, of how the concepts they teach are treated at other levels of instruction. They often teach in isolation from their peers and rarely see the "big picture" — an articulated mathematics curriculum — of which they are a part. A good use of inservice time would be to enable teachers to meet and discuss the entire mathematics curriculum. Perhaps discussions of the scope and sequence of the curriculum would be a place to start. The absence of a mathematics supervisor/coordinator is an inadequate excuse for failing to coordinate the mathematics curriculum. Do your students a favor — coordinate your K-12 mathematics program.

METHODS FOR ENHANCING LEARNING IN THE MATHEMATICS CLASSROOM

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Teaching is imparting knowlec˛ ., it is presenting materials in such a way that students come in contact with the material and make it their own. Some students have difficulty making personal s e out of mathematics. For some reason, the mathem tics curriculum is . ot communicated well enough to these stud nts. This article identifies and explores selected methods that can overcome communication difficulties and enhance student learning.

Simplify or Divide

Let us put ourselves in the shoes of a student taking a final examination in Algebra I. (Throughout the year, the student has passed quizzes but has had difficulty on tests covering more than one chapter.) The test scenario then goes something like this.

The student has just encountered two problems appearing consecutively on the final examination.

1. Simplify: $\dfrac{6x^2-x-2}{2x+1}$ 2. Divide: $\dfrac{6x^2-x-2}{x+1}$

Because instruction in the class has been on "doing problems," the test-taker focuses on the problems and sees two very similar problems. How one "does" the problems mystifies the student.

The nonchalant student may leave both problems blank or may write down anything. The conscientious student tries. The tops (not numerators for the student) look like "things" that could be factored. So he tries to factor the "top." If successful, he finds the factors of $(3x-2)$ and $(2x-1)$ and cancels $(2x+1)$ in the numerator and in the denominator. Then he looks at the second problem and sees that $(x+1)$ is not a factor and is stymied because he cannot cancel.

If the student is unable to factor the numerator in problem 1, he may take another approach. The conscientious student rereads the directions. He sees that the first direction says "simplify" and the second says "divide." Does "simplify" mean "divide?" He may not be certain of the answer to that question, but he may remember how to convert fractions to decimals by dividing. If he transfers his knowledge of division of integers to division of polynomials, he may use long division to divide the trinomials by the binomials By taking the division-concept approach, the student may do both problems correctly.

Individual Differences

Why do some students seem to automatically know what to "do" with problems and others seem not to know where to start? We each have our own particular cognitive style, our own methods of processing information. That cognitive style influences the way we perceive, remember, think, and solve problems. The classes that mathematics teachers address are comprised of students with different cognitive styles. An effective method of communicating to one student may not be an effective way to communicate to a student with a different learning style. To convey mathematical material well to all students, mathematics teachers need to attend to different cognitive styles.

Students, as well as teachers, are visual, auditory, and tactile learners. Many mathematics teachers are more visually oriented than verbally oriented. Some students, especially those who have difficulty in algebra, may respond better to oral presentations and discussions than to visual aids, worksheets, blackboard notes or in-class seat work. For these students it is essential that mathematical materials be verbally communicated clearly, precisely, and in detail. Teachers should supplement visual aids with verbal explanations and discuss board diagrams.

Mathematics and English

Teachers can connect mathematics curricula from elementary, secondary, and college for students. They can even interconnect mathematics with English. The English sentence, "It is not unlikely that residents in Chernobyl will develop cancer" involves two negatives. "Not unlikely" is a double negative that results in a positive "likely." The analogy is that the product of two negative numbers is a positive number.

Compare and Contrast

Teachers need to organize and present materials in such a way that students who come in contact with the material can make it their own. Similarities and differences in problems and in the directions for solving these problems need to be made explicit. The use of colored chalk often helps students structure material presented in class. If blue chalk is consistently used for theorems, principles, or rules; white for working an example; and red for incorrect computations to avoid, the student makes color associations with the concepts. These associations help students retain information.

Expressions and Equations

Distinctions between an expression and an equation are often hard for some students to perceive. Examine the following items.

A. $\dfrac{2x}{x+1} - \dfrac{1}{x}$ B. $\dfrac{2x}{x+1} - \dfrac{1}{x} = 0$

To work both problems, students must find the least common denominator (LCD). What perplexes many students is deciding when to multiply each term by the LCD and when to multiply each term by (LCD)/(LCD). Some students do not know where to stop or when to continue to find a value for x. Perhaps, if teachers stress that the directions for the first are to "perform the indicated operations and simplify" while the direction for the second is to "solve," students will learn to associate direction and problem.

Summary

In order to enhance student learning in basic mathematics, teachers need to effectively communicate with their students. Strategies include paying attention to student differences in learning style, providing clear, unambiguous directions for students, and helping students look at problem situations from a variety of perspectives.

MATHEMATICS EDUCATION IN WEST GERMANY

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Why do American students perform poorly in mathematics compared to students in other industrialized nations? What can we do about our chronic shortage of mathematics teachers? These two questions motivated a sabbatical project to see how mathematics education and teacher preparation are handled in West Germany. There, a surplus of mathematics teachers exists. And German students are among those who have outperformed American students on international standardized tests.

Surely there is much that we can learn from the German experience. In order to see what can be learned, we, as a two-person team (half mathematician and half native German), spent six months in Germany in the spring and summer of 1985. We were guests at the Mathematisches Intitut of the Ludwig-Maximilian University in Munich where we were very warmly received and were able to examine educational materials and discuss problems of mutual interest with members of the mathematics faculty. We exchanged ideas with educators and observed classes at various levels in three of the eleven West German states: Bavaria, Baden-Württemberg, and Hessen. We also spoke with teachers, administrators and parents in four other states. Throughout this report, "German" will mean "West German." We visited no East German schools.

First Impressions and Initial Conclusions

In the first few schools visited, we observed classes in the seventh through twelfth grades. We quickly were able to confirm by personal observation that the results of the international standardized tests had led us to expect: German children are performing higher-level mathematics at lower grade levels, and doing it better, than their American counterparts. For example, in the tenth grade of the Realschule (a school for average students, *not* the elite), students learn not only considerable algebra and trigonometry, but vector operations as well. In a Gymnasium (a school for the university bound) all students begin calculus in the eleventh grade.

How do German students get ahead so fast? Do they sit in austere classrooms where teachers lecture according to the old Prussian authoritarian tradition? Do these stern taskmasters pile on extraordinary amounts of homework? No. This stereotype of German education is totally inaccurate. Most classrooms have a warm, relaxed and cheerful atmosphere which is conducive to learning. Teachers are generally very caring individuals who lead, not lecture, their students to new insights. And the amount of homework assigned at the various grades is comparable to what is expected at those same levels in this country.

What then are the differences? Higher expectations at the Grundschule (elementary school) are an important part of the answer.

Terminology and Organizational Structure

Before proceeding, it is advantageous to get better acquainted with the terminology and structure of German schools. The chart in Figure 1 is typical, although as in America, variations do exist.

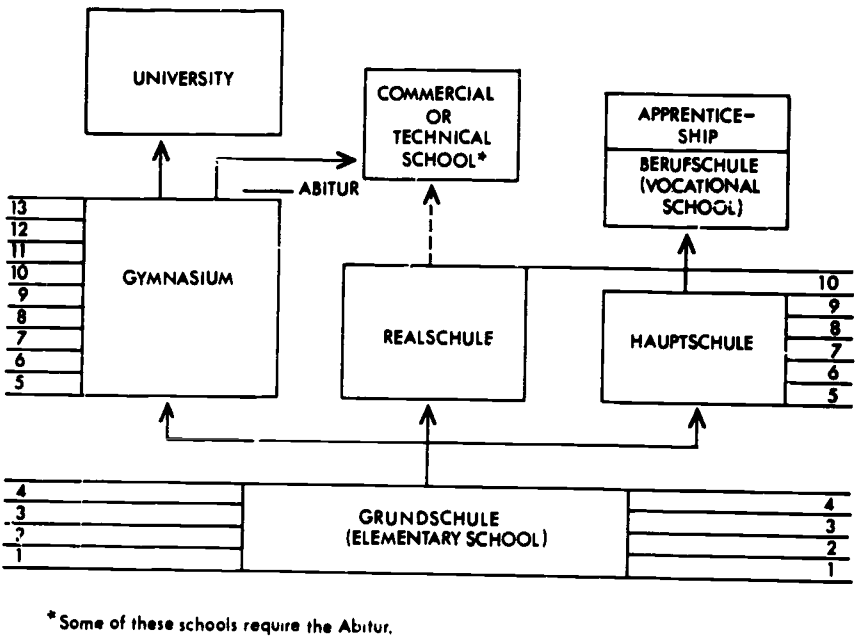

Figure 1
ORGANIZATIONAL STRUCTURE OF GERMAN SCHOOLS

There is no pre-school year corresponding to our kindergarten. A German kindergarten is a nursery school.

The separation into Gymnasium, Realschule, and Hauptschule corresponds roughly to our three tracks: college prep, commercial, and vocational, respectively. Yet there are two major differences. The first is that children in different tracks attend different schools, rather than being housed in one comprehensive school. Secondly, the separation occurs at the fifth grade. This early separation strikes many Americans as premature. However, the decision made after the fourth grade is not final. There is ample opportunity for crossover for those children who are late bloomers, are otherwise improperly placed, or who simply wish to pursue a different course of study.

A Gymnasium is sometimes said to be equivalent to an American prep school plus two years of college. In the description below of the Gymnasium curriculum, one can see that this comparison is quite valid.

Instead of SATs and Achievement Tests, a German student, after finishing the Gymnasium, takes a major examination (covering four subjects) called the Abitur. A composite score (⅓ on this examination, ⅔ on course grades from the last two years) is then calculated. Attaining a certain minimum score enables one to attend any university in the Federal Republic. Universities there have uniform standards, in sharp contrast to the variable caliber of American colleges and universities. A Diplom or Magister from a German university is comparable to an American master's degree.

Curriculum and Expectations

The curriculum in grades one through eight is similar to that in America. The most obvious difference is the earlier introduction of foreign languages. All children begin their first foreign language in grade five. Most start with English but some begin with Latin or French. Those children attending the Gymnasium and many of those in the Realschule start learning a second foreign language in grade seven.

Some curricular differences which are not immediately noticed by the casual observer appear in mathematics in the lower grades: German children are taught more than their American counterparts in the first grade, and then the gap widens with each successive school year. Multiplication, for example, is a second grade topic in Germany, whereas American children normally learn their multiplication tables in the third grade. What factors account for the more rapid progress of German first and second graders? Probably the most important is the setting of higher expectations on the first day of class, and the maintenance of these expectations throughout the school year. German teachers, through their attitudes and actions, make it clear to their charges that school is for learning, and every class period is utilized for its intended purpose, This does not mean that children never get a break nor have any fun in school. Quite the opposite is true as we see below in our discussion of teaching methods.

Another difference in the elementary curriculum comes from the difference in systems of measurement. American children learn the old

English units of measure. They see no logic in them because there is none. Furthermore, the necessity for multiplying or dividing by the various conversion factors (5280 feet per mile, 16 ounces per pound, 7.48 gallons per cubic foot, etc.) imposes a frustrating and unnecessary obstacle to learning. German children, on the other hand, learn the metric system from the beginning, hand-in-hand with the number system. Since both are base ten systems, the children grow up with something orderly. It makes sense to them. Much has been written about "math anxiety" in the United States. We were struck by the lack of it in Germany. We submit that a major factor in this difference is the use in the United States of an illogical measurement system together with its accompanying unnecessary computational burden versus the orderly simplicity of the metric system in Germany.

In the upper grades, one finds additional curricular differences. While American high school students typically have five subjects daily, their German counterparts have about thirteen courses, each of which meets two or three times a week. (A typical schedule of a twelfth grader in a Gymnasium might consist of Mathematics, English, French, German, History, Biology, Chemistry, Physics, Art, Geography, Music, Ethics and Physical Education.) But since German schools have no study halls or lunch period, the time spent in school is normally less than in the U.S.

Algebra and geometry are introduced two to three years earlier than in America, but in smaller doses. Rather than having separate algebra and geometry courses, these two subjects are integrated in a natural manner, each reinforcing and advancing the other. Concepts of logic and proofs are gradually interwoven into this composite development. There is not the sudden shock of having to do abstract reasoning after years of computational mathematics.

By the end of the tenth grade, trigonometry has been thoroughly covered in the Realschule as well as in the Gymnasium and then, for Gymnasium students, Calculus begins in the eleventh grade and Linear Algebra in the twelfth. The calculus texts at this level are not encyclopedic. The goal at this point is a solid intuitive grasp of the main concepts (limit, continuity, derivative, definite integral, etc.) and skillful application of basic theorems (chain rule, Fundamental Theorem of Calculus, etc.). The rigorous development of Calculus is handled at the university.

Funding and Control of Education

Both funding and control of education in the Federal Republic is primarily the responsibility of the eleven states, rather than the local community, as in the United States. This is a crucial difference, as it affects educational quality as well as funding levels. The state has primary responsibility but the federal government plays a role in such matters as helping to set uniform standards and pay scales. The cities and town are involved, of course, in determining local needs.

Because of this funding structure, teachers are paid by the state, according to a step system which is uniform throughout the Federal Republic. Furthermore, this salary scale is applicable to other professionals who are paid by the state. As a result, teacher salaries are comparable to those of state-employed doctors, engineers and lawyers. Education is a sought-after profession, and hence school administrators are able to be very selective in filling vacancies. There are no uncertified teachers employed in the German public school system.

For students, education is free, not only at the lower levels, but also at the universities. For those unable to meet living costs while studying, financial aid is provided by the government and a number of private institutions.

State responsibility for education provides not only higher funding levels than in the U.S., but better quality control as well. This quality control is evident in the state issued curriculum guidelines. The question of curriculum guidelines may not seem like a burning issue, but we have found that the German guidelines are far more meaningful and useful than those put out by most of our states. They provide real guidance for teachers and administrators by clearly spelling out what topics should be covered at which grade levels, and they are followed! Teachers have sufficient flexibility, yet they and their administrators are responsible for assuring that the required topics are covered by the end of the school year. Careful attention to these guidelines helps to translate the higher expectations into higher achievement.

Teaching Conditions

We have already noted the most obvious difference between the German and American conditions for the classroom teacher. In Germany, a teacher's worth to society is recognized in the most tangible of ways, salary, as well as general respect. Clearly, this has much to do with the fact that there is an ample supply of mathematics (and science) teachers, while in the U.S., we have a shortage of very serious proportions.

The actual teaching time is about the same: roughly five hours a day. German teachers, however, have neither study halls to monitor, nor lunchroom duty, as do their American counterparts. Discipline problems exist but rarely do they have a debilitating effect upon a teacher's performance and morale, as is often the case in this country.

Total annual vacation time is fourteen weeks, about the same as in the U.S. However, the breakdown into the several vacation periods is notably different. The summer vacation is six weeks and there are four two-week breaks. This schedule provides the time needed for professional improvement, as well as rest and travel. As a condition of employment, teachers may not accept summer jobs. This restriction is somewhat unnecessary, since a teacher's salary is adequate.

Teacher Preparation

The training of teachers is much more thorough than in the U S. This applies to both pedagogy and subject-matter content. The path to a teaching position in a Grundschule, Hauptschule, Realschule, or Gymnasium follows the following time frame:

1. Gymnasium through 13th grade
2. Abitur (The student is normally 19 years old at this point.)
3. Three to five years at a university
4. First Staatse. en (a comprehensive examination covering content and methodology)
5. Two years of "Referandarzeit" (This includes one year of teaching under the supervision of a master teacher, one half year of teaching somewhat independently, and seminars.)
6. Second S.aatsexamen (more advanced exam on content and methods)

Only after successful completion of the second Staatsexamen is the candidate certified to teach.

The period at the university is, at the very minimum, six semesters or three years for those preparing to teach at the Grundschule, Hauptschule, or Realschule. In practice, it normally takes eight semesters or four years. For potential Gymnasium teachers, the time is four years at the minimum, but normally five years are needed. This means that newly certified teachers are at least 24 years old (25 for Gymnasium), but normally no less than 25 (26 for Gymnasium). For able-bodied men, one must add an additional year because of required military service. These ages for beginning teachers highlight one of the most significant differences between the German and U.S. educational systems. A German teacher starts with a higher level of general maturity, and with considerably more training.

Furthermore, the requirement for the Abitur before entering a university guarantees that all potential teachers start with a solid general education. Recall from our description of the Gymnasium curriculum that this general education includes analytic geometry, linear algebra, and the rudiments of differential and integral calculus. This is important not only from the mathematics standpoint, but as an indication of the depth of instruction across a broad range of subjects taught in the Gymnasium.

A program of studies at a university for a student preparing to teach at the Grundschule level includes at least one semester course on the methods of teaching elementary mathematics. More pure mathematics is not required at this point since the student has already had analytic geometry, calculus, and linear algebra at the Gymnasium. Also, the elementary teacher candidate must take at least twenty courses in one content area. This requirement serves not only to give the student in-depth knowledge in a subject, but also to assure a solid understanding of the scholarly methods of mastering an academic discipline.

For Gymnasium mathematics teacher candidates, the program would include

- 3 semesters of calculus
- 2 semesters of linear algebra
- 1 semester of each of the following:
  real analysis
  complex analysis
  abstract algebra
  number theory
- either geometry (differential, projective and foundations) or probability theory.
- If geometry and probability are not both chosen, then an additional course from the following is required:
  topology
  numerical mathematics
  computer science
  logic

Gymnasium teachers are fully prepared to teach (and do teach) in not just one, but two disciplines. This provides opportunities for the shifting of personnel resources to correct imbalances in the supply and demand for teachers of the various content areas.

Mathematics Teaching Methods

The thoroughness in teacher preparation and the clarity of the state curriculum guidelines are reflected in teaching techniques. This is especially significant at the elementary level where, we believe, is found a crucial part of the answer as to why German students perform better in mathematics. Permeating all of the specific techniques is an attitude of high expectations. One frequently hears American parents or elementary teachers express the following thought: "I was never good at math." The accompanying lowered expectations are very damaging to young children. We did not see that in Germany.

The discovery method is consistently utilized as in the U.S., but German teachers provide more guidance and reinforcement to assure that each new concept is well understood and mastered. Additional drill is provided by a delightful variety of group games which make the drill fun, not boring. For example, one second grade we visited was playing a game to reinforce the multiplication tables. The children were standing in a circle around the room and throwing a large (about foot on each side) foam rubber die. The children with the die would call out a number and then throw the die to another child. The receiving child would then have to give the product of the number called out and the number showing on the die. We were astonished at the speed and accuracy of the responses, and saw enjoyment and a sense of accomplishment on the faces of the children.

Teachers also provide more guidance in the form of direct note-taking. Under the teacher's supervision, each child develops a clean set of well-organized notes. These notebooks are so well done that they are in reality mini-texts; and the child is actively engaged in creating a finished product to be saved and used. A conspicuous feature of these notebooks is the tidiness. Neatness in written work is stressed in German schoo much more than here. Teachers are very careful to set an orderly example in their own work at the blackboard. Some of the specific practices include the use of graph paper for notebooks and worksheets so that each character has a place, the use of a straight-edge for geometric figures or underlining, and the use of two or three colors. In writing out definitions, rules, and theorems, results are stated completely, grammatically, and with mathematical correctness (proper use of equal signs, etc.). This orderliness and accuracy in writing serve to foster both logical thinking and precise thinking. The extra time taken to be neat, complete, and precise pays off. By developing good work habits in the elementary school, German children and their teachers are spared the woes of spending years trying to break ingrained bad habits and sloppiness.

Some Modest Recommendations

We have seen that Germans expect more learning from their children, and get more, beginning in first grade. Which of their ways might we realistically consider adopting?

By starting with an attitude of higher expectations that costs nothing, individual teachers, parents, and school principals can have an impact. Unfortunately, that impact can be of limited duration and extent after the initial bursts of energy have waned. This is where curriculum guidelines should play a role. Higher expectations and achievement can be made lasting by more meaningful and demanding guidelines to which teachers are required to adhere.

We recommend that elementary teachers *not* avoid drill on basics (multiplication tables, etc.) for fear of boring their pupils. Instead, make the drill fun, as was done in the class we described in the section on teaching methods. Involving a certain amount of social interaction and physical activity, as in that class, not only makes the activity fun, but gives the children a break from sitting at their desks.

Another classroom technique that could be easily implemented by the individual teacher is the practice of directed note-taking. To be effective, however, this should be accompanied by stress on the importance of writing that is complete, orderly, and precise.

We have identified what we believe is a link between the problem of math anxiety and the frustrations caused by our clinging to an outdated, illogical system of measurement. At ti very least, a child's progress in tackling problems involving units of measure is impeded by the use of our cumbersome system. Renewed calls for a speedy conversion to the metric system are in order. Administrators, mathemati al organizations, and more general associations within the educational community should all be involved in this effort. In recent years, the arguments in support of going metric have concentrated on international trade considerations. Business executives have long recognized that our products should be metric if they are to be competitive in world markets. Now is the time for leaders in education to join forces with those in industry to stress the twin goals of educational reform and economic prosperity.

Finally, we return to the chronic shortage of mathematics (and science) teachers. That school superintendents across the country, year after year, are forced to fill vacancies with unqualified personnel, is simply unacceptable. It does not have to be that way. Germany experiences no such shortage, but rather has a surplus. The biggest difference in conditions for teachers in the two countries is monetary remuneration. If we in this country really want to attract and retain sufficient numbers of qualified teachers, salaries must be raised substantially. No radical restructuring of our system is required. All we need is the commitment to providing (and this implies the willingness to pay for) a good education for our children.

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