This paper reviews the literature on reading and mathematics and calls for a new synthesis which views reading as a mode of learning. This synthesis focuses not on the acquisition of techniques but on the process of learning mathematics and the more humanistic aspects of the discipline. Four alternative frameworks for the problem of "reading mathematics" are identified: (1) extracting information from technically oriented mathematics texts; (2) reading to learn so as to achieve the goal of a technically oriented mathematics curriculum; (3) introducing rich mathematical texts to provide students with information on important aspects of mathematics typically excluded from the mathematics curriculum; and (4) shaping a new synthesis in which reading to learn mathematics so as to understand mathematics becomes a "way of knowing." The literature for each of the frameworks is reviewed; most of the existing literature belongs to the first framework. (A figure provides a theoretical grid of the four frameworks; 46 references are attached.) (RS)
A CRITICAL REVIEW OF READING IN MATHEMATICS INSTRUCTION: THE NEED FOR A NEW SYNTHESIS

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A CRITICAL REVIEW OF READING IN MATHEMATICS INSTRUCTION: THE NEED FOR A NEW SYNTHESIS

Introduction

In recent years, researchers and practitioners alike have shown renewed interest in the problem of "reading mathematics." This trend is part of a more general movement toward teaching reading as an integral part of content area instruction. Responding to the difficulties students experience when learning from text, educators have proposed that reading strategies appropriate to each subject matter be included in the curriculum. Reading mathematics, in particular, presents a challenge due to the unique qualities of mathematics textbooks.

Existing reviews of the literature on "reading mathematics" (Nolan, 1984; O'Mara, 1981; Pinne, 1983) indicate that researchers have concentrated on two dimensions of reading mathematics: (1) the language of mathematics, with particular emphasis on the technical vocabulary employed in the teaching and learning of mathematics; and (2) comprehension of word problems, with attention paid to the way in which the syntactic and semantic organization of word problems affects their solution.

Our own critical reading of the work considered in these reviews, as well as more recent contributions (e.g., Cox and Wiebe, 1984; Cordeiro, 1988; Ferguson and Fairburn, 1985; Moyer, Moyer, Sowder, and Threadgill-Sowder, 1984; Moyer, Sowder, Threadgill-Sowder, and Moyer, 1984; Van Den Brink, 1987), confirms the fact that these topics constitute the bulk of the research on reading mathematics, to the exclusion of almost any others.

This preliminary look at the literature suggests that thus far the problem of "reading mathematics" has been interpreted as follows: "Reading can be an obstacle to the learning of mathematics if students lack the reading skills necessary to correctly decode mathematical texts; researchers and teachers should attempt to eliminate this obstacle by
developing instructional strategies that teach the skills of reading mathematics." This statement of the problem leaves any theoretical perspectives implicit, though, on closer examination, it is clear that a very specific view of both reading and mathematics is operating. Once these embedded theories are made explicit, alternative conceptualizations of both reading and mathematics – and, consequently, of “reading mathematics” – become possible.

The scope of this review of the literature or reading mathematics, therefore, will be to interpret the results of existing research in light of a new synthesis of mathematics and reading. More specifically, we present two alternative ways that reading and mathematics, respectively, have been conceptualized, and show how they can lead to at least four different definitions of the problem of “reading mathematics.” We then review current research in terms of this theoretical grid and discuss the research questions and instructional practices implied by the four problem statements. Each differs considerably with respect to (a) what kinds of mathematical texts are read, (b) how they are read, and, most importantly, (c) for what purposes they are read.

Alternative Perspectives

In what follows, we describe, in some detail, a shift in the way the goals of mathematics instruction have been conceptualized before briefly summarizing a similar (and more familiar) shift in thinking that has taken place in the reading profession.

Perspectives on mathematics

Most people equate mathematics learning, especially at the elementary level, with the acquisition of some technical skills – beginning with the ability to perform various kinds of computations with a certain degree of accuracy. Students then move on to the skills needed to successfully apply these techniques, for example, solving assigned word problems or deducing formal geometrical proofs according to some established logical rules and formats. This view is reinforced in the current pre-college
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mathematics curriculum and in most standardized measures of mathematical achievement.

Most mathematicians, on the other hand, would argue that these skills represent a very small part of what mathematics is all about; indeed, many would say that these are the least interesting kinds of mathematical thinking, where machines can best substitute for human performance. In contrast to the more popular view, mathematicians regard mathematics as a "way of knowing" (Bishop, 1988), a particular mind-set for approaching the generation, framing, representation, and solution of problems. Hence, "learning mathematics" cannot be reduced to the passive acquisition of facts, rules, and techniques but should include an appreciation for the process through which new mathematical knowledge is constructed and evaluated (Kline, 1980; Lakatos, 1976), the value-laden nature of problem formation and resolution (Brown and Walter, 1983), and the affective dimensions of mathematical work (Buerk, 1982; Leder, 1982; Oaks, 1987; Resek and Rupley, 1980), among others.

Further, some scholars (see, for example, Borasi, 1986; Brown, 1982; Buerk, 1985) argue that mathematics students become aware of the humanistic nature of mathematics. These authors point out that mathematics is not a black-and-white, cut-and-dried subject, as most people think, but that even in this discipline ambiguity, controversy, and doubt play an important role in both the creation of new knowledge and the learning of established results. Awareness of the humanistic nature of the discipline will help students appreciate that thinking, creativity, and personal judgment are necessary if mathematics is to be successfully learned and used. And, a humanistic concept of mathematics as a discipline may help combat the phenomenon of "math avoidance," which is widespread even among talented people.

Increasingly, the mathematics education profession is coming to appreciate this view of mathematics and realize that it will require a rethinking of the goals — and, consequently, the content and instructional practices — of the pre-college mathematics curriculum. For example,
the National Council of Teachers of Mathematics has re-ently set out the following as key goals for mathematics instruction in the 1990's: "(1) becoming a mathematical problem solver; (2) learning to communicate mathematically; (3) learning to reason mathematically; (4) valuing mathematics; (5) becoming confident in one's ability to do mathematics" (NCTM, 1987, p.12). These goals represent a radical departure from the technique curriculum which is so pervasive in American classrooms and open the door to a reconsideration of the role reading might play in the learning of mathematics. New goals for mathematics education would have an impact on the kinds of reading material considered appropriate and valuable for mathematics instruction, as well as the kind of knowledge, strategies, and attitudes one would hope to foster as a result of these reading experiences.

In a technically oriented mathematics curriculum, reading instruction is limited to teaching the skills necessary to comprehend the information contained in textbooks, such as descriptions of algorithms, theorems and proofs, and word problems. The goal of such reading instruction is the appropriate use of such information in solving assigned problems. A curriculum grounded in a view of mathematics as a "way of knowing" would welcome other kinds of reading materials in addition to textbooks. Historical essays providing insights on the way mathematical knowledge is achieved, philosophical arguments on the nature of mathematics and its applications, and stories and poems which illustrate some of the aesthetic, affective, and value-laden aspects of mathematics are examples of what we call "rich mathematical texts." These materials have the potential to help students gain new insights into the process and nature of mathematics. This potential may be lost, however, if reading is not clearly understood by mathematics teachers.

Perspectives on reading

The shift from a skill-based to a comprehension-based model of reading is well documented in the reading education literature (see, for example, Anderson, Hiebert, Scott, and Wilkinson, 1985; Harste, 1985).
In general, the reading profession has rejected the notion that reading is a set of skills that can be mechanically applied to text so as to extract the information contained in the material. This perspective portrays the reader as little more than a passive information processor; the process is dominated, instead, by the text. Instruction focuses on ways to "decode" the text (e.g., word recognition, vocabulary instruction) and successful comprehension is measured in terms of the reader's ability to recover the author's meaning and duplicate the text.

The idea that reading is a mode of learning suggests that the reader plays an active role in the process, bringing linguistic and domain-specific knowledge, as well as beliefs, attitudes, and strategies to the situation (Anderson and Pearson, 1984; Carey and Harste, 1985; see, also, Gollasch [1982] for the collected works of K. S. Goodman). Reading is thus seen as a fluid, dynamic process in which new meanings are generated from the negotiation of reader, text, and context (Carey, Harste, and Smith, 1981; Rosciblatt, 1978; Siegel, 1984). From this perspective, the text serves as a springboard for constructing meanings; duplicating the text is no longer the criterion for successful comprehension. In brief, reading can be thought of as a mode of learning – an authoring process (Rowe and Harste, 1986) – in which readers transform texts into meanings mediated by their experiences and knowledge and the context of the reading event.

Reading instruction which fosters the understanding that reading is a mode of learning would introduce strategies that encourage readers to be active meaning makers by making connections to other texts and contexts. Of particular interest are strategies which encourage readers to take a new perspective by transforming the text through writing, drawing, drama, or other communication systems (Grumet, 1986; Rowe and Harste, 1986; Siegel, 1984). It is possible that the use of generative reading strategies might help students learn from the "rich" mathematical texts described earlier.

Perspectives on reading mathematics
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When combined, the perspectives on reading and mathematics outlined in the previous section yield four distinct approaches to the integration of reading and mathematics, illustrated schematically in figure 1 below.

**FIGURE 1**
A grid for framing the problem of "reading mathematics"

<table>
<thead>
<tr>
<th>Mathematics as a body of facts and techniques</th>
<th>Mathematics as a way of knowing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reading as a set of skills for extracting information from text</td>
<td>I</td>
</tr>
<tr>
<td>Reading as a mode of learning</td>
<td>III</td>
</tr>
</tbody>
</table>

This theoretical grid offers a way to review the existing literature on reading mathematics. Determining the approximate location of specific studies in this scheme will allow us to discuss their contributions and limitations as well as their potential to promote an understanding of mathematics as a way of knowing. A summary of the kinds of reading materials, strategies, and instructional goals associated with each approach to reading mathematics will be incorporated into this discussion.

**Four Alternative Frameworks for "Reading Mathematics"**

**Box 1: Extracting information from technically oriented mathematics texts.**

As noted, most of the existing literature on "reading mathematics" falls into this category. Researchers working within this framework have identified what they believe to be major obstacles presented by mathematics texts and studied the effects of various instructional strategies designed to overcome these obstacles. Text features such as the specialized vocabulary of mathematics and the syntactic organization of word problems were the most frequently studied obstacles.

The existing reviews of the research (Nolan, 1984; O'Mara, 1981; Pinne, 1983) all indicate that direct teaching of the specialized vocabulary of mathematics contributes to improved success with problem solving. For example, Skrypa (1979) found that teaching mathematical vo-
cabulary can improve students' ability to solve mathematical problems. The emphasis on the meaning as well as the decoding of words in this study is critical. Earp and Tanner (1980) found that sixth graders could decode but not give the meanings of mathematical terms commonly found in mathematics texts; however, students were able to give definitions when the terms were placed in the context of a sentence, leading the researchers to conclude that mathematical texts do not provide much support for comprehension.

Researchers have also given a good deal of attention to the syntactic organization or format of word problems, which are often identified as a major stumbling block for students. Moyer and his colleagues (Moyer, Moyer, Sowder, and Threadgill-Sowder, 1984; Moyer, Sowder, Threadgill-Sowder, and Moyer, 1984) have examined the effects of three different formats of story problems on students' ability to choose the correct operation needed to solve the problem. In the first study, students in grades 3-7 were tested on their ability to extract information from problems written in a telegraphic format and in a verbal format. Results did not support the hypothesis that the telegraphic format would be easier for students; there was some indication that the verbal format was easier for students. In the second study, they compared three formats: telegraphic, verbal, and drawn; the drawn format included a drawing of the objects, with labels, included in the problem. Here again they found no difference between the effects of telegraphic and verbal formats on ability to select the correct operation; however, a significant reading level by format interaction was found. The means for the drawn format were significantly higher at each reading level than the means for the other two formats and particularly so for students identified as low readers. In contrast, a study by Blohm and Wiebe (1980) showed that the inclusion of diagrams had no effect on high school students' success in solving mathematics problems involving the computation of percentages. The authors note, however, that the diagrams may not have served the function they were intended to serve, namely, as an aid to learning. This
study also explored the effects of including extraneous information in the problem. Results indicate that the inclusion of extraneous information greatly interfered with students’ problem solving.

On the whole, these studies are grounded in an implicit theory of reading which assumes that readers are passive and, hence, text factors (vocabulary and the syntactic organization of word problems) are the key variables in reading mathematics. Though comprehension is the intended result of such text manipulations, it is interpreted in a very limited manner as extracting the information necessary to solving the problem.

In summary, this approach to reading mathematics, though it might contribute instructional strategies which increase students’ ability to solve well-defined problems, fails to exploit the potential which might come from an integration of reading and mathematics. In fact, the belief that the “reading” component creates an obstacle to learning mathematics may lend support to the existing trend toward minimizing the use of reading in mathematics instruction. If mathematics educators continue to see the characteristics of reading mathematics texts (such as their density and the need to read them with “paper and pencil” [Nolan, 1984]) as a problem, they will miss a valuable opportunity to help students understand that any kind of text requires the active construction of meaning.

Box 3: Reading to learn so as to achieve the goals of a “technically oriented” mathematics curriculum.

Contributions within this framework are much rarer; however, a few studies can be noted if we include those which clearly give the reader an active role in the process. These studies tend toward a narrow interpretation of comprehension due to the nature of the reading task. In other words, the notion that comprehension involves the construction of new meanings is not considered appropriate when the task of the reader is to correctly solve the word problem.

McCabe (1981) applied a language experience approach to the com-
prehension of mathematics textbooks. First, he assessed ninth graders' understanding of a textbook passage using a cloze procedure. He then asked the students to explain a section of the textbook in their own words. Using the syntactic patterns identified in the students' explanations, the researcher repatterned the passage and reassessed students' comprehension, again with a cloze technique. McCabe found that students' comprehension of mathematics materials improved when the passage was patterned after their own syntactic structures. Other reading educators have pointed to the promise that a language experience approach holds for mathematics instruction (Ferguson and Fairburn, 1985) and this topic deserves further study.

A strategy which invited students to be active meaning-makers was used as part of a study examining the effect of the format of word problems in problem solving ability (Cohen and Stover, 1981). In the first part of their project, gifted sixth graders were asked to rewrite word problems so as to make them easier for their peers; content analysis of their efforts led to the identification of format variables which might affect comprehension of word problems. This rewriting activity is an example of a reading strategy which calls for comprehension in the spirit of "reading as a mode of learning." To rewrite word problems, students had to decide what needed to be changed, thus engaging in problem formation. In addition, the activity required that students shift from reading to writing, a move which helped them take a different perspective on the text and transform it in a personally meaningful way.

However, the mathematical goal implied by the activity was still limited to accurate information processing, and strictly technical mathematical texts (i.e., traditional word problems) were employed. Notice that the students were not encouraged (or even allowed) to change the content of the problem, and explore the mathematical consequences of such modifications—a valuable mathematical activity which would promote problem generation. Indeed, follow-up interviews with the students indicated that they defined the rewriting task as a reading activity, not
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one related to learning mathematics.

An opening to the consideration of non-traditional mathematics texts, such as the newspaper (Becker and Kendall, 1982; Haggerty, 1986), and material written by the students themselves (Van Den Brink, 1987), was found in the practitioners' literature. These contributions suggest new ways that reading and writing can be used in mathematics instruction, and involve students in activities that promote the construction of meaning. Yet, they belong to box 3 because their definition of relevant mathematical content and activities is still very much limited to computations and solutions of traditional word problems—however more interesting when encountered in meaningful contexts.

Box 2: "Rich mathematical texts" are introduced to provide students with information on important aspects of mathematics typically excluded from the mathematics curriculum.

Not surprisingly, contributions to this framework represent the efforts of mathematicians and mathematics educators to introduce aspects of mathematics typically excluded from the traditional mathematics curriculum. The mathematics education community has long recognized the need for texts that communicate the nature of mathematical results, as well as information about their origin and applications, to students. In response, various collections of novel as well as classic, yet non-traditional, mathematics texts have been edited (see, for example, Newman, 1956; Aleksandrov et al., 1963; Campbell and Higgins, 1984; Borasi and Brown, 1985), and proposed as a complement to traditional mathematics textbooks. These efforts reveal the existence of a number of very interesting essays, dialogues, stories, and even poems, which address mathematical topics and issues often neglected by the current pre-college curriculum. Examples include the classic dialogues by Galileo Galilei (1914), in which various mathematical problems and even paradoxes are discussed in a highly understandable and thought-provoking way thus illustrating the thinking process a mathematician would use to approach such situations; or, novels such as "Flatland" (Abbott, 1952), where the readers' experience and imagination is called upon to intuitively under-
stand and explore something as abstract as a four-dimensional space.

All these authors, however, seem to have felt that their task was accomplished once appropriate mathematical texts were produced and made available, evidence, we would argue, of their implicit theory of reading as a straightforward matter of extracting information conveyed in their books. Since mathematics teachers do not see themselves as reading teachers (Singer and Donlan, 1980), and are usually unaware of reading research and instructional strategies, it is very unlikely that they would take the initiative to develop reading experiences that would support students' active comprehension and learning from these texts. Hence, the potential power of these "rich" mathematical texts to add new dimensions to the mathematics curriculum remains largely unexploited.

Box 4: Reading to learn mathematics so as to understand mathematics as a "way of knowing."

Can reading be viewed as an opportunity to learn mathematics rather than an obstacle that stands in the way of such learning? Our review of the literature on reading mathematics suggests that educators have not yet named the problem of reading mathematics in this way. Hence, there is a need for a new synthesis of reading and mathematics, one which views reading as a mode of learning so as to contribute to the goals of a mathematics curriculum which focuses not so much on the acquisition of techniques, but on the process of doing mathematics and the more humanistic aspects of this discipline.

One example of "reading to learn mathematics" was located in the literature. In this case, a teacher described how her sixth graders "played" with the concept of infinity through the use of reading, writing, and art (Cordeiro, 1988). The concept of infinity is not usually part of the elementary school mathematics curriculum and the teacher's emphasis on open-ended, collaborative inquiry into the concept is equally unique. Though the reading strategies used in the course of the unit were not discussed in detail, the inclusion of such writing experiences as freewriting suggests that students were encouraged to be active meaning-makers.
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One can easily envision the way an “authoring curriculum” (Rowe and Ha:stc, 1986), which encourages students to construct their own interpretations by transforming texts into other genres and communication systems, could incorporate “rich” mathematical texts. By emphasizing problem formation, risk taking, collaboration, and meaning making, this new synthesis of reading and mathematics might help students achieve the goals (noted earlier) identified by the National Council of Teachers of Mathematics.

Conclusion

In this paper, we have reviewed the research on reading mathematics in light of four alternative perspectives on the problem. Though most of the existing literature on this topic falls into box 1, other interpretations of “reading mathematics” are possible. Understanding that the problem of reading mathematics can be framed differently, depending on the way reading and mathematics are conceptualized, opens up the possibility of a new synthesis which we call “reading to learn mathematics.”

“Reading to learn mathematics” calls for a new relationship between reading and mathematics educators as well as a new agenda for research and practice. Note that the work reviewed in boxes 2 and 3 were products of either reading or mathematics educators, alone. This work shows the promise that new directions in each field have for the problem of reading mathematics; and yet, these new directions cannot be fully explored due to the limited views these researchers have of the other field being investigated. If, on the other hand, specialists from both field join together in genuine collaborations, a new powerful synthesis of reading and mathematics instruction can be forged.
References


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