Intelligent tutoring systems (ITS) aim to improve upon previous computer-aided instruction programs by embedding much of the expert knowledge that good human teachers possess. While ITS research has a practical goal of developing programs that assist students in learning, there is little data on their instructional effectiveness. This paper presents preliminary student outcomes from the initial introduction of a prototype intelligent computer-based algebra tutor in a California high school. The focus is on student characteristics, including their background and attitudes about computers and computer-based instruction, and their evaluation of the tutor's usefulness for learning. Pre- and post-semester algebra achievement tests and final course grades provide data on student learning for 80 participants. It was concluded that: (1) positive attitudes toward learning with computers, experiences with this algebra tutor and previous computer experience appear to be unrelated to actual achievement in algebra; (2) there is a significant negative relationship between programming and overall algebra achievement; (3) gender differences existed; and (4) the preliminary data tell little about the relationships between student characteristics, overall achievement, and the nuances of learning algebra with an intelligent tutoring system. (Appended is the student questionnaire and achievement test.) (PK)
AN INTELLIGENT TUTOR FOR BASIC ALGEBRA:
PRELIMINARY DATA ON STUDENT OUTCOMES

Cathleen Stasz

April 1988
AN INTELLIGENT TUTOR FOR BASIC ALGEBRA:
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The RAND Corporation

OBJECTIVES

Intelligent tutoring systems (ITS) aim to improve upon previous computer-aided instruction programs by embedding much of the expert knowledge that good human teachers possess. While ITS research has a practical goal of developing programs that assist students in learning, there is little data on their instructional effectiveness (Ohlsson, 1986). This paper presents preliminary student outcomes from our initial introduction of a prototype intelligent computer-based algebra tutor in a local high school.² We focus on student characteristics, including their background and attitudes about computers and computer-based instruction, and their evaluation of the tutor's usefulness for learning. Pre- and post-semester algebra achievement tests and final course grades provide data on student learning. Although these results represent preliminary findings from the first year of a multi-year study, the lack of reported data on effects of intelligent tutoring systems prompts us to report them at this time.

²This project is being funded by the National Science Foundation's Applications of Advanced Technologies Program. The opinions expressed herein are the author's own, not necessarily shared by the sponsoring agency. The author thanks her colleagues on the RAND Algebra Tutor Project for their help in conducting this research and reviewing the paper: David McArthur, Abby Robyn, Tor Ormseth, Don Voreck, Mary Zmuidzinas, Orli Peter, Chris Burdorf, and John Hotta.
METHODOLOGY

Overview of the RAND Algebra Tutor

We are designing the algebra tutor to include several types of knowledge, including knowledge of the subject, the student, and of teaching. The current version is an expert problem solver in basic algebra. It includes an inspectable expert algebra system, capable not only of solving all the algebra problems we expect the students to solve, but also able to produce intermediate reasoning steps similar to those of an "ideal" student. This version does not have sophisticated knowledge of the student, nor does it possess intelligent tutorial policies that determine how to present concepts and administer feedback. Nevertheless, the tutor is able to respond to specific student requests and thus can assist the student in reasoning in various ways.

The student sees the tutor as a collection of windows and menus, as shown in Figure 1. The large window in the upper right is the display window. The student is presented with a problem to solve and the student's problem solving is represented as a reasoning tree. Each branch in the tree represents an alternate solution or line of attack of the problem, and can be compared with different solutions, both the student's and the tutor's. The boxed equation represents the student's current focus of attention in problem solving. The student can either input algebraic expressions using a mouse and menu selections, or write them by hand, using a tablet and pen. This input appears initially in the workspace window (below, left of the display window). The student taps a carriage return when ready and the tutor responds by placing the new expression in the display window. The student uses the menu selections (left of the display window) to manage his or her reasoning or to ask the tutor for assistance. The tutor gives feedback or makes other comments to the student in the comment window (lower, right of screen).

The menu selections are fairly self-explanatory and easy to execute. For example, if the student wants to check an answer he or she simply selects the Answer OK? option and the tutor responds by indicating whether or not the answer is correct. The Step OK? option
Figure 1: The Algebra Tutor interface
permits the student to check each new problem-solving step. An option called Elaborate Step allows the student to see more detailed intermediate steps which illuminate the tutor's reasoning; the tutor automatically inserts these steps appropriately in the reasoning tree. Hint Next Step provides both high level hints (e.g., try isolating the variable) and low level hints (e.g., isolate $7x = 4$ by dividing both sides of the equation by 7) that correspond to different reasoning goals. Explain Step provides a textual description of how and why the tutor reasoned to take a particular step. Hints and explanations appear when requested in special pop-up windows. These and other features of the tutor constitute a set of tools that support debugging and goal-directed reasoning skills (see McArthur, Stasz, and Hotta, 1987, for a more complete description).

Procedures and Instrumentation

The tutor runs on Sun Microsystems workstations, six of which were installed in two classrooms in a local high school. Five first-year algebra classes (about 150 students of average or below-average algebra skill) had access to the tutor for most of the second semester. About 14 percent were taking the course for the second time. Students were individually trained and project staff members were available to provide assistance whenever the computers were in use. Students used the tutor during regular class times and data about each session (e.g., problem tried, correctness of answer, selection of menu options, time) was automatically recorded. We administered pre- and post-measures consisting of a questionnaire (background information, attitudes about and experience with computers) and an achievement test (see Appendix). Here we present data from 80 students, across the five classes, who completed pre- and post-measures. Since this pilot study did not include a comparison group of students who did not use the tutor, this design does not provide direct evidence on the tutor's teaching effectiveness. However, it does permit us to assess the statistical properties of our achievement test and to examine the relationship between achievement and various student characteristics.
RESULTS

Student Characteristics

At pretest students ranged from 14 to 18 years of age (average=15.8) and were in the tenth to twelfth grades (51.25, 40, and 8.75 percent, respectively). Fifty-four percent were male. About half received a grade of B or higher in their last mathematics course and about three-quarters expected these grades in the present course. Although students significantly reduced expectations at posttest (only 55 percent expected an A or B), in actuality over three-fourths of the students obtained a grade of C or less. If nothing else, this suggests that these students' own estimates of their algebra skills are highly inflated and that such estimates may be poor indicators of actual performance.

Perhaps in correspondence with these course grades, students' interest in algebra and feelings about algebra (as rated on four-point scales) significantly decreased over the course of the semester.

Overall, these students had extensive and varied previous experience with computers. Fifty-two percent had taken a course about computers or computer programming. Half had used computers in other classes and about two-thirds (65.8 percent) had used them outside of school. Computer uses included doing homework (25 percent), programming (43 percent) and playing games (95 percent). Eighty-five percent thought that they would like learning algebra on a computer.

Student Attitudes

Students were asked a series of six attitude questions about computers and learning with computers, that were derived from our previous work in evaluating telecourses (Shavelson, et al., 1986). They rated on a 6-point scale (1 = strongly disagree, 6 = strongly agree) statements like: "I believe many algebra courses could be improved by the use of computers"; or "Computers are poor substitutes for algebra teachers." While students, on average, held positive attitudes toward learning with computers, these positive attitudes consistently declined slightly from pretest to posttest for each item (over all items, F =
7.67, p < .0001). We observed the same decline in attitudes in our telecourse evaluation. These changes may reflect similar declines in liking algebra, having interest in algebra, and in course grades.

Student Evaluation of the Tutor

At posttest, about two-thirds of the students reported enjoying the tutor "a good bit" or "a great deal." Eighty-nine percent thought it fairly or very easy to learn to use. Students were less enthusiastic, however, in their assessment of whether it helped them learn to solve algebra problems or graph equations: about forty percent thought it helped "a good bit" or "a great deal." Only 21 percent thought the tutor useful for "learning about computers." Despite the somewhat disappointing assessment of the tutor's usefulness for learning, about a third of the students said they would use computer tutors again if given the opportunity. Given the global nature of this assessment, it's difficult to determine the precise source of the students' disappointment with the tutor's teaching capabilities. It might be attributed to specific aspects of the tutor or to the students' overall class experiences during the semester.

Looking at average student ratings of specific tutor options in Table 1, we see that students found some options more useful than others. Options that provided precise feedback on the correctness of a step or answer were judged the most helpful. Judged least helpful was the option that allowed the student to obtain the tutor's solution for any step in the reasoning tree. A student might choose this option, for example, if he or she could not figure out an error in a step and wanted to see the tutor's solution. Elaborate Step also received a lower rating; this option shows the tutor's more detailed reasoning from one step to another. These results suggest that students want immediate, specific feedback. This is understandable, since students rarely get such feedback on their problem solving in the classroom, or when doing their homework. Students commented that they liked the Step OK? option in particular because it allowed them to check their work as they went along. Often, students experience frustration when they work through a long problem in class, only to discover that their answer is wrong. The
Table 1

STUDENT EVALUATIONS OF TUTOR HELP OPTIONS

<table>
<thead>
<tr>
<th>OPTION</th>
<th>Percent Reporting Helped &quot;a good bit&quot; or &quot;a great deal&quot;</th>
<th>Average Rating*</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>N</td>
<td></td>
</tr>
<tr>
<td>ANSWER OK?</td>
<td>80</td>
<td>78.75</td>
</tr>
<tr>
<td>STEP OK?</td>
<td>79</td>
<td>75.95</td>
</tr>
<tr>
<td>HINT NEXT STEP</td>
<td>70</td>
<td>61.43</td>
</tr>
<tr>
<td>ELABORATE STEP</td>
<td>60</td>
<td>50.00</td>
</tr>
<tr>
<td>EXPLAIN STEP</td>
<td>69</td>
<td>66.66</td>
</tr>
<tr>
<td>DO NEXT STEP</td>
<td>65</td>
<td>67.69</td>
</tr>
<tr>
<td>DO SOME STEP</td>
<td>49</td>
<td>48.98</td>
</tr>
</tbody>
</table>

*Rated on a scale from 1 = "no help" to 4 = "helped a great deal"

Step OK? option permits them to detect an error right away and try to recover.

Our classroom observations suggest that students may find less usefulness in tutor-generated elaborations or explanations because they don't know how to use that information to aid their problem solving. Collins and Brown (in press) posit that ITS can become a powerful tool for learning reflection. In our tutor, for example, students can compare the details and structure of their own performance with that of the expert system, thereby discovering elements that need improving. Our data suggest, however, that expert help may be at a student's disposal, but that doesn't guarantee that he or she can or will make use of it. Students may require explicit training to use the computer as a tool for learning through reflection.

Student Learning

In consultation with the two algebra teachers who were using the tutor in their classes, we constructed a 19-item achievement test. The test aimed at evaluating non-traditional skills, like debugging, reversibility and flexibility (cf., Rachlin, Matsumoto, and Wasa, 1985).
as well as more traditional problem-solving skills. Across skill types, the items also represented linear equations, inequalities, and simultaneous equations.

The test was very difficult. Although the difference was statistically significant, the average score increased from only 4.71 to 5.71, out of a possible score of 19 (F = 5.13. p < .0001). Item difficulty, the proportion of examinees who get that item correct, ranged from 0 (no one got the item right) to .9 (see Table 3). Test results support our notion that items are arrayed along a difficulty continuum—from linear equations to inequalities to simultaneous equations. An examination of the set of debugging (1-3) and flexibility (10-12) items, shows student performance following this progression. We

Table 2

EXAMPLE PROBLEMS FROM ALGEBRA ACHIEVEMENT TEST

<table>
<thead>
<tr>
<th>TYPE</th>
<th>PROBLEM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Debugging</td>
<td>Hector made an error when he solved this problem for x. See if you can find the error and put a circle around it. Then try to solve the problem correctly.</td>
</tr>
<tr>
<td></td>
<td>x + 4 = 10</td>
</tr>
<tr>
<td></td>
<td>x + 4 + 4 = 10 - 4</td>
</tr>
<tr>
<td></td>
<td>x + 8 = 6</td>
</tr>
<tr>
<td></td>
<td>x = 2</td>
</tr>
<tr>
<td>Reversibility</td>
<td>We will give you the answer to a problem and ask you to write an equation. Write an equation in which the answer is x = 2. The equation should have at least one set of parentheses.</td>
</tr>
<tr>
<td>Flexibility</td>
<td>Try to solve each problem below. Then, if you can, try to solve the problem in a different way and mark the method that you like the best.</td>
</tr>
<tr>
<td></td>
<td>Solve for x: 7x - 2 = 5x - 8</td>
</tr>
</tbody>
</table>
Table 3
SELECTED ITEM STATISTICS FOR ACHIEVEMENT TEST

<table>
<thead>
<tr>
<th>ITEM</th>
<th>PROBLEM TYPE</th>
<th>DIFFICULTY [a]</th>
<th>DISCRIMINABILITY [b]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Pre</td>
<td>Post</td>
</tr>
<tr>
<td>1</td>
<td>debugging, linear</td>
<td>.80</td>
<td>.92</td>
</tr>
<tr>
<td>2</td>
<td>debugging, inequality</td>
<td>.39</td>
<td>.44</td>
</tr>
<tr>
<td>3</td>
<td>debugging, simultaneous</td>
<td>.01</td>
<td>.16</td>
</tr>
<tr>
<td>4</td>
<td>linear, single parentheses</td>
<td>.36</td>
<td>.51</td>
</tr>
<tr>
<td>5</td>
<td>simple inequality</td>
<td>.26</td>
<td>.32</td>
</tr>
<tr>
<td>6</td>
<td>linear, embedded parentheses</td>
<td>.25</td>
<td>.31</td>
</tr>
<tr>
<td>7</td>
<td>inequality, parentheses</td>
<td>.24</td>
<td>.24</td>
</tr>
<tr>
<td>8</td>
<td>simultaneous, easier</td>
<td>.02</td>
<td>.10</td>
</tr>
<tr>
<td>9</td>
<td>simultaneous, harder</td>
<td>.01</td>
<td>.02</td>
</tr>
<tr>
<td>10</td>
<td>flexibility, linear</td>
<td>.60</td>
<td>.65</td>
</tr>
<tr>
<td>11</td>
<td>flex., linear w/paren.</td>
<td>.42</td>
<td>.50</td>
</tr>
<tr>
<td>12</td>
<td>flex., inequality</td>
<td>.25</td>
<td>.19</td>
</tr>
<tr>
<td>13</td>
<td>flex., simultaneous</td>
<td>.04</td>
<td>.25</td>
</tr>
</tbody>
</table>

[a] Proportion of students who got the item correct
[b] Point-biserial correlation between item and total test score.

also see a progression in the two sets of items: 4 and 6, and 5 and 7. Within the linear equations, the item with single parentheses (item 4) was easier than that with embedded parentheses (item 6). The simple inequality (item 5) was a little easier than the inequality with parentheses (item 7), particularly at posttest. Also interesting is the sharp increase in performance from pretest to posttest in items 3 and 13. Although these items were still difficult for most students at posttest, it's clear that some proportion of students had mastered simultaneous equations, which were taught during the semester.

Although the test exhibited less than ideal psychometric properties, it correlates well with other achievement measures (see below). In addition, the item analyses indicate, as expected, that linear equations were the easiest, followed by inequalities and simultaneous equations. Student performance increased significantly from pretest to posttest for simultaneous equations, which were covered
in class during the semester. Overall, these results increase our confidence in the test as a valid achievement measure.

Correlations Between Outcome Measures

Correlations between selected student characteristics (ability, computer experience) and the algebra achievement posttest are presented in Table 4. Measures of "ability," including a student's pretest score, final grade in his or her previous math course, final grade in the current course, and estimates of algebra difficulty were all significantly correlated with posttest performance. Prior computer experience was consistently correlated with lower posttest performance.

Student attitudes toward computers and learning algebra with computers (summarized as a single "opinion" score) were significantly correlated with their experiences in using the tutor. For example, they found the tutor more enjoyable to use \( (r = .60) \); felt the tutor helped them to solve algebra problems and to graph equations \( (r = .56 \) and \( .40, \)

<table>
<thead>
<tr>
<th>CHARACTERISTICS</th>
<th>N</th>
<th>CORRELATION</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pretest</td>
<td>80</td>
<td>.42</td>
<td>&lt; .01</td>
</tr>
<tr>
<td>Grade Last Math Course</td>
<td>80</td>
<td>.26</td>
<td>&lt; .05</td>
</tr>
<tr>
<td>Final Grade</td>
<td>80</td>
<td>.62</td>
<td>&lt; .01</td>
</tr>
<tr>
<td>Estimate: Difficulty of Algebra</td>
<td>80</td>
<td>.28</td>
<td>&lt; .05</td>
</tr>
<tr>
<td>Experience with computers:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Taken course</td>
<td>80</td>
<td>-.16</td>
<td></td>
</tr>
<tr>
<td>Used at home</td>
<td>80</td>
<td>-.20</td>
<td></td>
</tr>
<tr>
<td>Used for Homework</td>
<td>80</td>
<td>-.20</td>
<td></td>
</tr>
<tr>
<td>Played games</td>
<td>80</td>
<td>-.17</td>
<td></td>
</tr>
<tr>
<td>Programming</td>
<td>80</td>
<td>-.26</td>
<td>&lt; .05</td>
</tr>
<tr>
<td>Literacy*</td>
<td>80</td>
<td>-.27</td>
<td>&lt; .05</td>
</tr>
</tbody>
</table>

*Literacy is a composite score created by summing over the experience items.
respectively); found the tutor easy to learn to use \((r = .38)\); and would use computer tutors again \((r = .45)\). This suggests that students who were positively disposed toward learning with computers tended to enjoy their experience with the algebra tutor. However, students' positive attitudes and experiences with the tutor were negatively, although not significantly, related to test performance. A similar pattern appeared with regard to final course grade: students with more previous computer experience, more favorable attitudes, and more positive evaluations of their experience with the tutor tended to receive lower grades in the course.

A final interesting pattern appeared with regard to feelings about algebra and performance. At pretest, girls had significantly lower expectations about their grade \((r = -.24)\) and disliked algebra \((r = -.25)\) more than boys. At posttest, girls still judged algebra to be more difficult than did boys \((r = -.28)\). However, gender was basically unrelated to scores on the pretest and posttest or to final grades \((r = .07, -.03, \text{ and } -.05, \text{ respectively})\).

CONCLUSIONS

Positive attitudes toward learning with computers, experiences with this algebra tutor and previous computer experience appear to be unrelated to actual achievement in algebra. Perhaps this "non-finding" is not a general cause for concern. If students with less computer experience or negative attitudes did poorly, then we would need to worry about how to make the tutor more motivationally appealing or how to help students overcome negative feelings about the technology. On the other hand, student comfort with the technology, while perhaps necessary for learning, does not ensure that the technology will be effective as an instructional tool.

The significant negative relationship between programming and overall algebra achievement perhaps questions the assertion that programming may lead to the acquisition of general planning and problem-solving skills (e.g., Papert, 1980; Soloway, Lockhead, and Clement, 1982). A closer examination of our test items with programming
experience indicates that programming was negatively, although not significantly, correlated with all but one item—a word problem on simultaneous equations ($r = .14$).

The gender differences found in our study suggest that girls have less confidence in their ability to learn mathematics (McLeod, 1985). Although girls' feelings and expectations were unrelated to their performance, they are disturbing nonetheless. Research suggests that such feelings make girls disinclined to pursue coursework or careers in mathematics and science (Goleman, 1987). If improving the quality of mathematics education is a national goal (NSB, 1983), then further research should continue to pursue the origin of these differences. In addition, our results run counter to studies which find a positive relationship between mathematics self-concept and achievement, regardless of gender (Marsh, 1986).

The preliminary data reported here tell us little about the relationship between student characteristics, overall achievement, and the nuances of learning algebra with an intelligent tutoring system. For example, we have not yet determined how these variables relate to time using the computer, time to solve different types of problems, frequency of use of the different tutor options, and the student's success or failure on individual component skills that comprise each problem the student attempted. Since the tutor automatically gathers these and other data, further finer-grained analyses may shed some light on these and other student outcomes. In addition, these preliminary conclusions await verification in a study (currently under way), which includes the requisite control groups.
REFERENCES


CONGRATULATIONS! Your class has been selected as one of five SAMOHI Algebra I classes to participate in a special project. A research team from The RAND Corporation wants you to work with them in the development of an intelligent computer tutor for algebra.

Very few students have the opportunity to use intelligent computers to help them learn algebra. SAMOHI will be one of the first schools in the country to have such sophisticated computers in their classrooms. The RAND research team is very excited about this project, but to be successful, we need your help. We hope that you will cooperate with us, and that you will be interested and enthusiastic about the project.

We’d like you to take this Diagnostic Test today. The first part is a questionnaire that asks about your background and your opinions about algebra and learning with computers. The second part is a test that will give us an idea about how much algebra you know already. If you have any questions while working on the test, please raise your hand.

THANKS FOR YOUR HELP!
DIRECTIONS: Please fill in the parentheses next to each question with the appropriate number. Select only ONE answer to each question; please answer all questions.

1. What is your sex? ( )
   (1) Male
   (2) Female

2. How old were you on your last birthday? ( )

3. What grade are you in? ( )
   (1) 10th grade
   (2) 11th grade
   (3) 12th grade

4. What was your final grade in your last MATH course? ( )
   (1) I withdrew from my last math course.
   (2) F
   (3) D
   (4) C
   (5) B
   (6) A

5. What grade do you think you'll get in this course? ( )
   (1) F
   (2) D
   (3) C
   (4) B
   (5) A
6. Have you completed Algebra 1 before? ( )
   (1) no
   (2) yes

7. How interested are you in learning Algebra 1? ( )
   (1) No interest
   (2) Little interest
   (3) Some interest
   (4) Great interest

8. How do you feel about Algebra? ( )
   (1) I dislike it
   (2) I mildly dislike it
   (3) I mildly like it
   (4) I like it

9. How difficult is Algebra for you? ( )
   (1) Easy
   (2) Hard
   (3) Uncertain
   (4) I have not worked on Algebra

10. Have you ever taken a course about computers or computer programming? ( )
    (1) no
    (2) yes

11. Have you ever used computers in other classes? ( )
    (for example, in English, Math, or history class?)
    (1) no
    (2) yes
12. Have you ever used computers outside of school? (for example, at home, at a friend's house, at camp?)
   (1) no
   (2) yes

13. Have you ever used a computer to do your homework?
   (1) no
   (2) yes

14. Have you ever played computer games?
   (1) no
   (2) yes

15. Have you ever programmed a computer?
   (1) no
   (2) yes

16. Do you think you'll like learning Algebra on a computer?
   (1) no
   (2) yes

Please indicate the degree to which you agree with the following statements about computers. There are no wrong or right answers, we just want your opinion.

17. I believe many algebra courses could be improved by the use of computers.
   (1) Strongly disagree
   (2) Disagree
   (3) Slightly disagree
   (4) Slightly agree
   (5) Agree
   (6) Strongly agree
18. Using computers to teach algebra is a bad idea.

(1) Strongly disagree
(2) Disagree
(3) Slightly disagree
(4) Slightly agree
(5) Agree
(6) Strongly agree

19. Computers can do a lot more teaching than most people realize.

(1) Strongly disagree
(2) Disagree
(3) Slightly disagree
(4) Slightly agree
(5) Agree
(6) Strongly agree

20. It is a waste of time to try to learn using computers.

(1) Strongly disagree
(2) Disagree
(3) Slightly disagree
(4) Slightly agree
(5) Agree
(6) Strongly agree

21. Computers can show students what's important to be learned.

(1) Strongly disagree
(2) Disagree
(3) Slightly disagree
(4) Slightly agree
(5) Agree
(6) Strongly agree

22. Computers are poor substitutes for algebra teachers.

(1) Strongly disagree
(2) Disagree
(3) Slightly disagree
(4) Slightly agree
(5) Agree
(6) Strongly agree
THANKS FOR YOUR HELP! PLEASE CHECK TO MAKE SURE THAT YOUR STUDENT I.D. NUMBER IS ON THE FIRST PAGE.
Achievement Test

DIRECTIONS: The purpose of this test is to find out how much you know about algebra. Some of these problems are difficult. No one expects you to know all the answers. If you cannot solve a problem, don’t be discouraged, just go to the next problem. Please try to solve all the problems and show all your work.

1. Hector made an error when he solved this problem for x. See if you can find the error and put a circle around it. Then try to solve the problem correctly.

   \[ x + 4 = 10 \]
   \[ x + 4 + 4 = 10 - 4 \]
   \[ x + 8 = 6 \]
   \[ x = -2 \]

2. Jane did not solve this problem correctly. Try to find the error and put a circle around it. Then try to solve the problem correctly.

   \[ 5 < -3x - 7 \]
   \[ 5 + 7 < -3x - 7 + 7 \]
   \[ 12 < -3x \]
   \[ 12/-3 < -3x/-3 \]
   \[ -4 < x \]
IF YOU CANNOT SOLVE A PROBLEM, JUST GO TO THE NEXT QUESTION.

3. Hyon and Jim worked on this problem together, but they made an error. See if you can find and circle their error. Then solve the problem correctly.

Solve the following equations for \( x \):

\[
\begin{align*}
2x + y &= 4 \\
2x + y &= 4 \\
x + y &= 2 \\
y &= x + 2 \\
2x + (x + 2) &= 4 \\
3x + 2 &= 4 \\
3x + 2 - 2 &= 4 - 2 \\
3x &= 2 \\
x &= \frac{2}{3}
\end{align*}
\]
IF YOU CANNOT SOLVE A PROBLEM, JUST GO TO THE NEXT QUESTION.

4 - 5 Directions: Solve each problem for $x$.

4. Solve for $x$:
   \[ 7(x + 9x + 4) = 12 \]

5. Solve for $x$:
   \[ -7x < 70 \]
IF YOU CANNOT SOLVE A PROBLEM, JUST GO TO THE NEXT QUESTION.

6 - 7 Directions: Solve each problem for x.

6. Solve for x:

\[ 3[x + 4(2x + 2)] = 10 \]

7. Solve for x:

\[ -4(x + 1) < 4(3 + 1) \]
IF YOU CANNOT SOLVE A PROBLEM, JUST GO TO THE NEXT QUESTION.

8 - 9 Directions: Solve each problem for $x$ and $y$.

8. Solve for $x$ and $y$:

   
   
   \begin{align*}
   -7x + y &= 3 \\
   5x - 4y &= -1
   \end{align*}

9. Solve for $x$ and $y$:

   
   
   \begin{align*}
   -7x + 3y &= 3 \\
   5x - 4y &= -1
   \end{align*}
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10 - 11 Directions: Try to solve each problem below. Then, if you can, try to solve the problem in a different way and mark the method that you like the best.

For example, you could solve $x + 4 = 2x + 2$ one way:

<table>
<thead>
<tr>
<th>Example</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x + 4 = 2x + 2$</td>
<td>You add $-x$ to both sides of the equation</td>
</tr>
<tr>
<td>$4 = x + 2$</td>
<td>Then you add $-2$ to both sides of the equation</td>
</tr>
<tr>
<td>$2 = x$</td>
<td></td>
</tr>
</tbody>
</table>

or another way:

<table>
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<td>$x + 4 = 2x + 2$</td>
<td>You add $-2$ to both sides of the equation</td>
</tr>
<tr>
<td>$x + 2 = 2x$</td>
<td>Then you add $-x$ to both sides of the equation</td>
</tr>
<tr>
<td>$2 = x$</td>
<td></td>
</tr>
</tbody>
</table>

I like the second way best.

10. Solve for $x$:

$$7x - 2 = 5x - 8$$

11. Solve for $x$:

$$4(x + 1) - 2(x + 1) = 6$$
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12 - 13 Directions: Try to solve each problem below. Then, if you can, try to solve the problem in a different way and mark the method that you like the best.

For example, you could solve $x+4 = 2x+2$ one way:

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</table>

or another way:

$-3x > -9(3 + 5)$

13. Solve for $x$ and $y$: $x + y = 2$

$x - y = 6$
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14 - 16 Directions: Here are some word problems. Write each problem as an equation and then solve the equation.

14. If the width of a rectangle is 3 inches less than its length, and the perimeter of the rectangle is 94 inches, what is the length of the rectangle in inches?

15. You are taking a history course. There will be 4 tests. You have scores of 89, 92, and 95 on the first three. You must make a total of 360 to get an A. What scores on the last test will give you an A?

16. A telephone coin box contains 15 coins. The box contains only nickels and dimes, and the total value of the coins is 95 cents. Find out how many coins are nickels and how many are dimes.
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17 - 19 Directions: We will give you the answer to a problem and ask you to write an equation.

For example, write an equation in which the answer is \( x = 3 \). The equation should have at least five appearances of \( x \):

<table>
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<td>( x = 3 )</td>
<td></td>
</tr>
<tr>
<td>( x + 2x = 3 + 2x )</td>
<td>Add 2x to both sides</td>
</tr>
<tr>
<td>( x + 2x - 9x = 3 + 2x - 9x )</td>
<td>Add -9x to both sides</td>
</tr>
</tbody>
</table>

17. Write an equation in which the answer is \( x = 2 \). The equation should have at least one set of parentheses.

18. Write an inequality in which the answer is \( x < 0 \). The inequality should have at least two negative coefficients.

19. Write a system of equations in which the answers are \( x = 4 \) and \( y = 6 \).