This pamphlet reports on a project in Tasmania exploring whether the "natural learning conditions" approach to language learning could be adapted for mathematics. The connections between language and mathematics, as well as the natural learning processes of language learning are described in the pamphlet. The project itself is described—small groups of teachers alternating workshops with classroom trials—and three examples of classroom work resulting from the project are provided ("Thomas' Banana--Math Conferencing", "Matthew's Square Root," and "Now You See It--Now You Don't."). A summary outlines major findings of the project: (1) many of the processes are already in place; (2) other processes can be easily adapted; (3) changes have to be made to the mathematics; and (4) children must be allowed to record their work in their own way. The pamphlet contains a discussion and sample listing of "challenges" (open-ended problems used to present math to children). It concludes with a brief description of future project plans. (SR)
The project reported here resulted from a deceptively simple idea. Many Tasmanian schools are successfully implementing an approach to language learning based on natural learning conditions. Is it possible to develop an analogous approach to learning mathematics?

THE CONNECTIONS BETWEEN LANGUAGE AND MATHEMATICS

In recent years teachers have become more aware of the importance of the use of language, in all its forms, in learning mathematics. Children need to talk and write about mathematics to help develop their mathematical understanding, to challenge their thinking, to refine their concepts and to precisate mathematical vocabulary and 'language structures'. A dilemma arises for teachers in that the language for mathematics has usually been considered to be the highly symbolised and specialised mathematical formalism. But it is obviously impossible for novice learners to communicate their mathematical ideas in formal ways. Children will learn mathematical language by talking about mathematical situations using their own vocabulary and language. Thus they should be encouraged to use whatever language skills they have available to express or document their mathematics (Pengelly 1986). Initially this will be based on their speaking and writing skills and the limited grasp of mathematical language they have picked up in their lives. The teacher will support and encourage their attempts and, when appropriate, model a more sophisticated mathematical language or documentation to them.

If natural language is so important for learning mathematics, then it is reasonable to consider the possibility that the processes used for language learning might be adapted for mathematics. In the latter half of 1986 the author and a small group of early childhood teachers set out to investigate this proposition. This PEN aims to make the results of our work available to a wider group of teachers. A more detailed report has already been published (Edmunds and Stoessiger 1987).

THE DEVELOPMENTAL APPROACH TO LANGUAGE LEARNING

The developmental approach to language teaching and learning being implemented in many schools throughout Australia is based on natural learning processes such as young children use when learning to talk. Holdaway (1979) and Cambourne (1986) have argued that the same learning processes which facilitate oral language learning should be allowed to operate for all language learning. The conditions which allow these processes to operate have been summarised by Cambourne (p. 9) as follows:

- Immersion
- Demonstration
- Engagement
- Responsibility
- Approximations
- Employment

Mutual exchanges between expert and novice

Young children are immersed in a rich oral environment. From the moment they are born, they are expected to learn to speak and are engaged in that process. They decide for themselves what they will learn and the sequence of learning. When learning to talk, children are not expected to produce the 'correct' adult model. Immature approximations are warmly accepted. The learners are provided with ample opportunities to use their skills in exchanges with adults. These exchanges are non-threatening and there are no penalties for not using the conventional 'correct' form.

These conditions do not seem to characterise conventional mathematics teaching in schools. Children are not engaged in learning from a maths-rich environment, with mathematics being used for a variety of purposes. They are not expected to be responsible for their own maths learning. Approximation is not readily accepted in classroom mathematics, where the answers to most questions are judged to be either right or wrong. As a consequence, there are threatening conditions attached to many school maths activities. What then would teachers do if they used natural learning conditions for mathematics?
THE PROJECT

Groups of Tasmanian classroom teachers have been exploring this new approach since 1986. Initially small groups of teachers known to be successfully implementing a natural approach to language learning were recruited. We reasoned that they would quickly tell us if the project was unnecessary or inappropriate. Their success with the approach encouraged us to work with the whole early childhood staff in a number of schools, and more recently to extend the approach into the primary area.

In working with teachers we attempted to model the processes that teachers might use with children. We asked teachers to do maths activities similar to those for children and hence be maths learners in their own right. It proved important to alternate workshops with classroom trials so that theory and practice were built up together. The next three sections give examples of the classroom work resulting from the project.

Thomas's Banana — Maths Conferencing

One group of children in a Year 2/3 class were working on activity cards introducing the idea of fractions. They were asked to divide some pictured objects into halves, thirds, and so on. Next they were asked to draw some objects of their own, similarly divided into equal parts.

Thomas drew a banana divided into four like this.

Looking over his shoulder, the teacher asked Thomas if he thought the banana had been divided into four equal bits. Thomas could see that the left piece would be smaller than the others and so his teacher challenged him to find a way to divide the banana into four equal pieces (quarters).

Some time later, when Thomas had made no progress, the teacher was asked what she would do if a child came to a stop with some writing. She explained that the child would be asked to conference with one or two other children. The child would explain what had been written so far and the others would make suggestions.

Hence Thomas came to conference on the banana challenge. Because the process of conferencing was already in place in the classroom, Thomas knew immediately what to do without any additional instructions. He sought out another child, explained the nature of the problem and together they came up with a solution — slicing the banana lengthwise.

This simple example illustrates a number of features of a process approach. Firstly the teacher kept the challenge open, refusing to give a 'correct' answer when the child came to a dead stop. The responsibility for learning remained with the child; he took the action to get the help he needed. While an appropriate learning process was available in the classroom (conferencing), it would not usually have been used for maths work. Finally, by conferencing, the child had to explain the problem to a friend, thus increasing the amount of language used in maths work, giving him a better understanding of the problem and increasing his engagement in the activity.

Matthew's Square Root

One of the features of the project was the use of open-ended challenges to help reveal children's thinking. All the teachers in the project challenged the children with this question:

What is the most interesting number sentence you can think of?

Matthew, in Year 3, was determined to get a square root into his sentence. He successfully multiplied 16 by 16 on the side of his page to give 256 as the number to aim for so that the square root could be the last operation. He then proceeded to write

\[ \sqrt{3 + 9 - 0 + 0.5} \]

to start his sentence. He was asked what the result was so far and answered 24, revealing that he understood how to divide by 0.5.

Next he began dividing 24 into 256 to find the next number to multiply by. He explained to a friend that he would do this by a complicated process resembling long subtraction rather than long division. He could see that his result was not likely to be correct and so he was asked to estimate the answer.

A little later he had written

\[ \sqrt{3 + 9 - 0 + 0.5 \times 10.25} = 16 \]
as his number sentence. When asked about the 10.25, he explained he was not sure if it should be that or 10.75. Further discussion revealed that he was trying to express the remainder as a decimal but was not quite able to do so without some help.

Later, Matthew's teacher asked him if he would like to take the rough drafts of his interesting number sentence and produce a version for publishing. The refining process was already in place for language work and so Matthew knew exactly what to do. He worked on his sentence on several occasions over the next few days, making it more elaborate and indicating both new knowledge and some deficiencies in his mathematics.

This one activity showed an amazing amount about Matthew's mathematical work. For example, he could divide successfully by 0.5. Teaching point — could he divide by other decimals and fractions? He understood square roots as the reverse operation to squaring.

Teaching point — could he produce a number sentence with a cube or higher root? He needed to do more work on division and was ready to be introduced to long division.

Matthew's teacher was clearly a little overpowered by the maths he produced. However, she commented:

I was not sure how to set challenging enough work for Matthew before, but now I can see that if the questions are open-ended, he will challenge himself.
It should be mentioned that at the same time as Matthew produced his number sentence, other children produced such sentences as:

(6 + 2) + 1 = 3  
(8 + 2) + 1 = 4  
(10 + 2) + 1 = 5  
(12 + 2) + 1 = 6.

The important point is that these children achieved just as much success as Matthew did from the very same activity.

Now You See It — Now You Don’t

One teacher reported that her Kindergarten class included three children who were very mathematical. She had talked to them about subtraction but they had not needed to formally record their answers. She posed this challenge:

Pretend that I don’t know anything about take-aways and you have to tell me what they are.

The children presented some examples such as:

If you had three cans and one rolled away you would only have two left.

The teacher next asked them to try and produce something without using number to show what they meant. They returned some time later with the device illustrated below. The hand could move through a slit and cover up the plums.

The teacher commented:

While the three children were showing me their teaching aid all the other children came from nowhere. They were taken with the device and started making their own.

The challenge was sufficiently open-ended to lead the three children to construct their own teaching aid. This proved to be appropriate and interesting to other members of the class — so much so that the maths lessons took over from other classroom activities.

WAT TEACHERS FOUND — A SUMMARY

In a very short time the teachers and project organisers found that a natural learning approach to mathematics could be developed. It is based on the language approach with input from other areas, particularly the expressive arts. Our major findings are outlined below.

Many of the processes are already in place.

It should perhaps have come as no surprise that many of the language processes are readily available for use in mathematics. For example, when children have difficulty with their maths work, they can be asked to conference on it with some friends. They know immediately what to do. Importantly, this step leaves the responsibility for learning with the child rather than the teacher. Similarly, the language process of editing for publication can be used immediately for mathematics so long as teachers are willing to value approximations as the starting points of the process.

Other processes can easily be adapted.

Teachers have found that they can successfully adapt other language conditions for mathematics. They can immerse children in a maths-rich environment, often by simply drawing attention to the mathematical nature of their world. They can organise mutual exchanges with the children rather than operating as the expert with ‘the answers’. Expectation and engagement are just as important for maths as they are for language.

Changes have to be made to the mathematics.

Very early on we realised we would have to make changes to the way the maths was presented to children if all the language conditions were to be brought into play. One consideration was the lack of an equivalent for quality literature, which is so vital to language learning. Another was the difficulty we had in accepting approximation and in initiating refining and editing processes when traditional maths questions were used. The question, ‘Four and three more, how many?’ does not invite approximation.

We found inspiration in a Tasmanian arts-in-education project (Felton and Coman 1986), in which teachers were using open-ended problems, or challenges, to initiate natural learning processes. Problem solving is of fundamental importance in mathematics and making the problems open-ended allows the refinement processes to be readily engaged. Similarly, the use of challenges encourages mutual exchanges between teacher and child without considerations of right or wrong dominating. Because challenges are so important in initiating and supporting natural learning processes, they will be discussed in detail in the next section.

Children must be allowed to record their work in their own way.

One of the features of the writing process is that teachers can see from children’s work what they are capable of and what is likely to help them progress further. The teaching points are revealed by the work. If we insist on maths
being recorded only in formal ways, we get access to a very limited amount of information. Teachers are often amazed by what they find out about children's knowledge of mathematics when they allow them to record and present their work in their own way. Of course teachers must also provide models of ‘expert’ forms and encourage children to move towards them, just as they do in language.

For reasons outlined above we found a need to present maths to children in the form of open-ended problems, though we prefer to call them ‘challenges’ because of the negative connotations of ‘problems’. We have found no reason to change the maths content that is taught; it is only the way it is presented to children that needs to be altered. Because open-ended problems are not normally posed in mathematics, it proved difficult at first for teachers to develop their own challenges. As they came to value the increased enthusiasm expressed for this approach, by children as well as themselves, they grew more adept at developing and using challenges. Within twelve months some of the teachers were teaching mathematics entirely through this approach.

The following are examples of challenges used by teachers in the project.

- **What is the most interesting number story you can write? Post it to a friend.**
- **Make something to show Alf what take-away means.**
- **Design a clock for your room.**
- **What is special about zero? Write about this.**
- **What weighs two nuts less than the blackboard duster?**
- **Plan the route for Little Red Riding Hood.**
- **What can you make to measure length? What are the smallest and longest things you can measure with it?**

We shall be publishing an extensive list of challenges in the near future.

Ideally, children investigate challenges in small groups since this fosters much important language use. Because they are open-ended, children with different abilities can always respond at their own level. Children are encouraged to reflect on their work, select the most suitable parts to share with others and refine (edit) it for an audience. The same challenge, or a variation, can be repeated at intervals with particular children until it loses its edge. They have then learned all they can from it.

Challenges may be highly structured. They may be designed by the teacher to focus on a very small part of mathematics. Other challenges may be developed to meet a specific need of a particular child. But as long as the challenges remain open-ended, children are able to respond in ways that make sense to them and retain responsibility for their own learning.

We have used challenges very successfully with teachers to engage them in thinking mathematically and to re-kindle their enthusiasm for the subject. Many challenges are so open that they are suitable for both adults and children. For example:

- **Use counters to make triangles like these.**
  
  ![Triangle Patterns](image)

- **What patterns can you find? Record them.**

Both adults and children find this a valuable challenge.

### The Future

The project is continuing in 1988 as a Commonwealth Schools Commission Project of National Significance. We hope to further define the use of natural learning conditions in mathematics. We would like teachers to observe children closely to determine the processes they go through when tackling challenges. We believe that the new approach to maths may have special benefits for girls, given their strengths in language. We plan to investigate this further.

In any new approach to mathematics teachers must be particularly aware of the need to include parents. Challenges seem to provide an ideal opportunity for parents and children to work together on maths in the same way as they read together.

### References and Further Reading


Felton, H. and Coman, S. (1986), Links between Related Arts and Basic Learning, Education Department of Tasmania, Hobart.


Further information is available from the author, Education Department of Tasmania, PO Box 256, North Hobart, 7002.

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