The realization that the areas of second language and mathematics education share some common features and that educators in these fields may profit by an exposure to research and practice in both fields motivated the production of this paper. From an introduction to the literature, parallels in the history of the two disciplines were recognized: in mathematics, movements emphasizing problem-solving and critical thinking, and in second-language education, the reconceiving of language as a tool for communication and for the making of meaning. This paper focuses on several contributions from theory, research, and practice in second language instruction, showing their relevance to mathematics education. Most of the paper analyzes three methods developed to teach second languages: (1) delayed oral production; (2) the silent way; and (3) counseling-learning/community language learning. The characteristics and principles of each method are discussed along with their implications for mathematics instruction. Also, an illustration of how the same pedagogical principles can be applied to the teaching of a mathematic topic is included. The discussion of this paper is presented in the form of a dialogue with the words of the mathematics educator appearing in Roman type and the second-language educator writing in italics. (RT)
POTENTIAL IMPACT OF RESEARCH IN SECOND-LANGUAGE ACQUISITION
ON MATHEMATICS INSTRUCTION:
THE BEGINNING OF A CONVERSATION

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INTRODUCTION

This paper was inspired by the realization that the areas of second language and mathematics education have much more in common than it is usually perceived, and that educators in these fields may profit by an exposure to research and practice in both fields.

As colleagues responsible for the development of secondary school teachers in mathematics and second languages respectively, the two authors discovered that their concerns regarding the learning and teaching of their specific disciplines were often very close. This, in turn, promoted further and more focused exchanges, and an introduction to the literature in each other’s field.

As a result, we recognized interesting parallels in the history of our disciplines: in mathematics, the recent movements emphasizing problem-solving and critical thinking, and in second-language education, the reconceiving of language as a tool for communication and for the making of meaning. We identified a number of issues of common interest which have been approached independently in the two fields, for example: Why does traditional instruction fail to produce good results, either proficient problem-solvers in mathematics, or fluent speakers in second languages? Given the reconceived view of language/mathematics, what is really involved in its learning? How can we measure students’ progress in the acquisition of complex skills?

Though neither field claims absolute solutions to these complex issues, new learning theories and radical alternatives to traditional instruction have been developed and studied in the last thirty years.
Neither of us had been aware of these results in the other field, and being exposed to them caused each of us to draw valuable parallels for our own field, to look at problems from a different perspective, and in some cases to suggest new resolutions.

In this paper, we will focus on a few recent contributions from theory, research, and practice in second-language instruction, discussing their relevance to mathematics education. Each topic we selected gave rise to many more that we look forward to addressing in the future. This paper represents the beginning of our cross-disciplinary dialogue.

Most of the paper will be devoted to the analysis of three unusual methods recently developed to teach second languages: "Delayed Oral Production" (Postovsky, 1977), the "Silent Way" (Gattegno, 1976), and Counseling-Learning/Community Language Learning" (Curran, 1976; Rardin, 1977; La Forge, 1977; and others). We will briefly describe the characteristics and principles of each method. Then we will discuss the implications of each method for mathematics instruction, presenting an illustration of how the same pedagogical principles could be applied to the teaching of a mathematical topic.

Prior to an analysis of the three methods, we will briefly present our views of language, mathematics, and the learning of each. To capture and express both the exploratory nature of our enterprise and the potential for opening a channel of communication between the two fields, we have presented our thoughts and discussion in the form of a dialogue. The words of the mathematics educator appear in roman type, and the second-language educator writes in italics.
In order to fully appreciate the potential contributions of research and practice in second-language learning and instruction for mathematics education, let us begin by clarifying our conceptions of these two subjects. We are hardly the first to have noted a similarity between learning mathematics and learning a foreign language, but others may have identified commonalities which describe the worst of practice, rather than offering hope for better teaching and learning.

For example, I have heard geometry teachers justify their emphasis on memorization, rather than on problem-solving and open exploration, with arguments along these lines: "Geometry is very much like learning a language. You must memorize the new vocabulary and the grammar rules before you can do anything else." The problem with this argument is that it assumes a conception of mathematics and language as a body of facts/words and rules which students must master in order to become proficient users, a view I suspect we would both like to challenge for each of our respective fields.

For my part, I doubt that the ability to recollect facts and correctly apply learned rules to carefully selected exercises makes for a good mathematician. Along with many other mathematicians and mathematics educators, I believe that mathematics students should be enabled to approach novel problems, evaluate alternative solutions, and appreciate the structure of mathematics. They should be motivated and able to generate questions.
Similarly, I expect that most mathematics educators (including myself) would approach the study of a second language with expectations that you would challenge. As a start, I would like to hear what language educators have to say about what characterizes language and what makes a person proficient in it.

[BJA] I agree with you that how one teaches language is inevitably related to how one conceptualizes language. Your geometry teachers (like many of their colleagues in the second-language department) see their subject as something outside of ourselves, a set of rules to be analyzed and manipulated. Teachers who hold this view of language find it reasonable to present language, structure by structure, in an analytical way. Unfortunately, though knowing grammar structures and vocabulary may lead to a sense of security and to an "intellectual appreciation" of a language, it does not necessarily result in one's being able to use that language. I appreciate football, but that appreciation would not take me far on the playing field.

While language is indeed composed of grammar rules and vocabulary, its primary function is to embody meaning. Language use is a quintessentially human activity. In everyday use, language is almost transparent. It functions as a tool, in Dodson's (1967) words, rather than a target. Even when language is consciously crafted, it still retains its basic function of communication, and fails if communication does not occur at some level. Form is not enough.

Even where formal rules are important, we have tended to define rules too narrowly. Those forces which govern language include more than
grammar rules. There is a much greater emphasis these days on the context in which communication occurs. A sentence can be well formed and still be "wrong" if it violates certain commonly understood conventions of communication: We expect our interchanges to be informative, relatively true, relevant, and reasonably unambiguous (Grice, 1975).

Similarly, an utterance can be "wrong" if it violates certain cultural patterns (Wilkins, 1976). One culture, for example, may value "saving face," making the other person comfortable, more than the absolute honesty or accuracy which another culture considers a virtue.

It is also interesting to think about ways in which language may possibly constrain or facilitate certain kinds of awareness. To address someone in Arabic, you must recognize, through your choice of words and grammatical forms, whether you are taking to one male, two or more males, one female, or two or more females. There is no possibility of being gender-neutral in Arabic! And when a man and woman are alone together, even the grammar of their language acknowledges that fact.

[RB] There is much of what you have said about language that I think most mathematicians would agree can apply to mathematics. In particular, I believe that mathematics, as well as language, is a generative tool that helps shape one's thinking and one's interpretation of reality, assisting in the search, creation, and communication of meaning.

Your observation that "convention" or "culture" plays an important role in language makes me question whether there is also a "mathematical culture" that our students should come to appreciate as an integral part of their learning of mathematics. For example, in order to be able to
follow and produce mathematical "proofs," students may first need to understand the very special requirements mathematicians have set for something to be "convincing," as well as to appreciate the philosophical and historical reasons behind such decisions. But this is a topic for another time.

Now, however, let me come back to our original discussion of what is involved in being able to speak a language and how a person can become fluent in a new language.

[BJA] First, let's review what language teaching is not. It is not the systematic piling up, like building blocks, of elements of language. This "building-block pedagogy" has been alive and well for decades. It has generated wonderful arguments: Should we teach the negative form before the question? Do subject pronouns come before object pronouns? Are the negative and question forms equally difficult?

However, if one sees language as a tool for human interaction, for the conveying of meaning, a structural approach starts getting messy. An example: Our first question to a visitor from another country is, "How do you like the U.S.A.?" And our next question, inevitably, is, "How long have you been here?" We require present perfect tense in our first social interaction, but present perfect is not one of the first tenses we present as teachers of a structural approach. We belabor present, past, irregular past, and future tenses before we gird our loins to attack the "difficult" ones. Our pedagogy does not recognize that people need difficult tenses from the very beginning. These tenses fit later into
the elegant structures in which we have a deep investment and an abiding love.

Not only are the building blocks alone inadequate, but the idea of language as linear is a hindrance to teaching and learning. Language learning (and, I would venture to say, most learning) is cyclical, rather than linear. Aspects of language which were unclear at one point become clarified later, often through a context which makes full understanding and accurate use communicatively significant.

The educational system's legitimate need for assessment partly accounts for our continuing emphasis in language upon testable aspects of language—th. rules, the spelling, the irregular verbs, the culture "facts," etc. It is convenient to evaluate elements which can be counted clearly right or wrong. We justify such evaluation by claiming that, if the "building blocks" are carefully reduced to their smallest components, mastered in isolated, sequential units, language use will result eventually. This is a dangerous view, language educators are not alone in espousing it.

Elsewhere (Agor, 1987), I have called for an "evaluation by approximation," a concept which makes some educators distinctly nervous. Yet if we are to affirm that good language use involves the giving and receiving of meaning, we need to recognize that meaning can be successfully transmitted with something less than perfect linguistic accuracy.

[RB] I find these observations about the non-linearity of language learning extremely intriguing. It is common to think of both mathematics
and second languages as subjects that need to be built sequentially, with students mastering each component to a certain extent before they can proceed to new, more complex ones. A considerable amount of research and energy in my field has been put into finding the most logical and "natural" sequence in which to present mathematical topics, as well as the elements within each topic. Your argument makes me question the soundness of the assumptions behind such views, and encourages me to look for alternatives.

If language learning is not linear, and if it is not learned by piling one building block of language upon another, what is it?

[BJA] Stephen Krashen's theory of second-language learning, the input hypothesis (1982), offers us a new way of understanding the teaching and learning process. According to the input hypothesis, an essential requirement for learning is "comprehensible input." Like Frank Smith, Krashen believes that we learn language by hearing it in a context that is relatively understandable, and by seeing people respond to words in meaningful ways. We can sit forever next to a radio broadcasting in Mandarin, and with no clues about the meaning of the sounds, we will never come to understand it. Unlike an experience with a disembodied radio, Smith (1984) notes that learners find meaning in their observation of common events. The young child hears a parent say, "Please pass the salt." She then watches the salt shaker passed across the table. Even if the child may not understand the niceties of "please" or the nature of the imperative, she has experienced language with meaning.
Krashen also has redefined the word "learn." He reserves this word for conscious, rule-governed study, what we think of as school learning. He proposes another term, "acquisition," for a kind of learning which he considers more powerful. Acquisition is that kind of learning which seeps in around the edges. It's the stuff you "pick up," not knowing quite where it came from. Krashen believes that the acquisition process provides us with that part of language which we use fluently. Learning gives us knowledge about language, but does not facilitate our use, our performance.

Krashen has gone so far as to suggest that those parts of language which are "learned" remain in the conscious part of the mind, needing to be consciously accessed. Learning, seen in this light, can actually get in the way of mastery. "Mastery learning" becomes an oxymoron.

Krashen's input hypothesis also recognizes the existence of an affective "filter" (Dulay and Burt, 1977). If the filter becomes "clogged," it will keep the learner from acquiring knowledge. Factors which affect the filter include motivation, self-confidence, and anxiety. Learning itself can clog the filter. Krashen notes that "monitor over-users" often have difficulty because they pay too much attention to the form and structure of their language use.

[RB] I have never seen a distinction between "learning" and "acquisition" made with respect to mathematics, but such a distinction seems potentially very interesting.

Your definition of "learning" certainly applies well to what most teachers attempt to achieve in the mathematics classroom, yet I think I
can point to instances of mathematics acquisition as well. It has been observed that those who we tend to consider "mathematically talented" seem able to grasp the meaning of what they are doing in math, to somehow access appropriate strategies even when they are facing new problems and situations, to transfer with appropriate modifications what they know, to see connections. On the contrary, less able students in the same course are lost when presented with anything slightly different from what they have previously learned. It looks as if these higher-level abilities are rarely developed as the result of direct instruction and could instead be the product of some sort of "acquisition."

Perhaps mathematics educators need to appreciate what "mathematical acquisition" can mean, and to explore what we could do to foster it in school instruction. The fact that people all over the world have been able to become fluent in languages other than their native one makes me hopeful that these abilities are not "innate" and that it is possible to create learning experiences through which our students can be led to "acquire" mathematical knowledge.

I am also intrigued by the implications of an affective "filter" operating for the learning and "acquisition" of mathematics. In recent years, the mathematics education community has certainly been made aware of the power of emotions for mathematics learning, as revealed by many studies of "math anxiety" (Leder, 1982 and Tobias, 1978).

From all that you have said, I can now better appreciate your dissatisfaction with the traditional method of teaching a second language by stressing grammar structures and vocabulary building. Such a method
may be appropriate for language "learning," but does not seem to help much with language "acquisition."

Once we accept the limitations of teaching the "building blocks" of language in a careful, sequential, analytical way, what are the alternatives that language teachers have adopted to help their students "acquire" a second language?

[BJA] In the next three sections, I will offer three methods which many of us in second-language education have found helpful in freeing us from traditional classroom practice.
ALTERNATIVE METHODS: DELAYED ORAL PRODUCTION

[BJA] The foundation of the first method I will present comes from the sensitive observation of second-language learners in natural settings. Sandra Savignon (1983), Susan Ervin-Tripp (1974) and others have noticed that children almost inevitably go through a silent period when placed in a new language environment. During this quiet time, they appear to be absorbing a great deal, but they resist speaking. Later, when the children do begin to talk, it is often with a fair degree of sophistication, and their subsequent progress is rapid.

The late Valerian Postovsky (1977) based a language teaching approach on the silent period. His carefully researched and reported conclusions of his Russian teaching at the Defense Language Institute are compelling. Postovsky compared students whose initial stages of language learning did not include speaking practice with others who were given massive amounts of practice from the first day. Those who delayed oral production outperformed their colleagues in all four skill areas—reading, writing, and listening, as well as pronunciation.

Postovsky, Ervin-Tripp, and others have speculated on a number of reasons why practice does not—at least initially—make perfect. They note that production is more complex than reception, and suggest that we needlessly complicate early learning by combining the two demands in our insistence upon performance. This emphasis on production causes learners to us (misuse) incompletely developed skills, and may also force them back to the inappropriate use of their first language's structure and phonology.
Postovsky further suggests that this premature demand for oral production makes comprehension more difficult, requiring the student to use "a serial mode of processing a message when the process is likely to be hierarchical in nature." (Postovsky, 1977, pg. 21) This linear processing puts a burden on short-term memory:

Since most of the foreign language material learned by the student needs to be habituated before it becomes automatically retrievable, the student initially holds this material in the short-term memory for immediate recall. But the rate of presentation of new materials is always greater than the rate of habituation; the student's short-term memory, therefore, early... the course reaches a point of saturation, thereby causing considerable inhibition of the learning process. (p. 21)

Removing the demand for early performance may free students to learn.

[RB] You are clearly enthusiastic about this approach, but I am not sure I understand what it really means in the classroom. I do not see much difference between what you are proposing and what is already common practice in school mathematics. The teacher explains things and acts as a model, expecting students to be able to understand and perform similar tasks, a result which rarely happens. Such an approach appears to be quite mechanistic. It ignores what we know about the learning process, and it takes opportunities for discovery away from the students.

[BJA] Though the role of the teacher is important, there is little in Postovky's theory which inevitably leads to a passive, mechanistic classroom. Postovsky briefly described his method, and set three criteria which practitioners should follow as they develop their own classroom translations:

1. The language material presented to the students must convey meaning from the very first hour of instruction;
2. A provision must be made for a student response which will verify comprehension of each utterance or a short passage immediately after delivery; and

3. Students must be challenged to problem-solve and guess at the meaning of unfamiliar elements in a foreign utterance on the basis of context and other cues in the given linguistic environment. (p. 22)

And yet troubles have occurred in the classroom application of delayed oral production. One example is Total Physical Response, commonly known as TPR (Asher, 1977), which can at times look very much like the mechanistic, teacher controlled classrooms you deplore. The teacher gives commands, acts them out, and students (singly or in groups) silently perform the commands. Whether Asher and others originally intended this or not, in classroom use, Total Physical Response (TPR) has become a variation on the old audio-lingual method. This should not be surprising, since teachers, trained in audio-lingualism, consciously or unconsciously bend new techniques to a framework in which they are comfortable. Thus we find classrooms in which the teacher acts as drill sergeant, and students practice by rapidly carrying out orders. While the results can be fun and humorous ("throw the flower pot out the window if you think George is cute"), knowledge and direction tends to flow from teacher to student.

[RB] You see a connection between the translations of delayed oral production and another method, audio-lingualism. Though I am more familiar with the audio-lingual method, since I experienced it as a beginning learner of English and French, I think it would be helpful if you would discuss it further. I would appreciate a clarification of its difference from the new instructional methods you are now presenting. In
addition, I would be interested in other objections to the audio-lingual method, since it is probably the method that most closely resembles traditional instruction in the mathematics class: The teacher presents a concept or rule, works out a few sample exercises at the blackboard, and then asks the students to perform similar exercises, at first with the teacher's guidance and later on their own.

[BJA] My first objection is pragmatic—it didn't work. The method was founded on behavioristic theory, which now is generally considered to be an incomplete view of human psychology at best. It also rested on a limited view of language. Language was considered to be a series of habits. Habits, it was believed, would be learned by practice, which usually took the form of repetitive drills. If our students did not learn, it was because we weren't purely audiolingual. We drilled more.

I see an even more serious problem with audio-lingualism. It relates to its "hidden curriculum." Students learned as much about school learning as they did about language, and their conclusions, while probably accurate, describe an educational pattern which is not empowering. The teacher's role is to command or structure learning. Students respond. While there is meaning, it flows exclusively from teacher to student. The pace is fast, with no time for reflection. A quick stimulus-response is desirable. Errors are seen as sins to be avoided, rather than as occasions for learning (Borasi, 1987, 1988).

[RB] Your criticisms about the audiolingual method should be seriously considered by mathematics teachers, since so much of instruction and learning going on in school mathematics seems based on a
unshakable belief that all students will experience success in mathematics if only they practice harder.

Back to "delayed oral production." How does it then differ, in practice, from audio-lingual methods?

[BJA] There are some differences which I find in several non-TPR applications of Postovsky's theories (Blair, 1983; Winitz, 1981; and others). These are generally built around a problem-solving approach. In fact, some even look a little bit like a mathematics class. They use a series of pictures or diagrams. Students listen to a description of each frame, guess the meaning of the words, and continue to revise or clarify their hypotheses as they move through a series of frames.

Students find the exercises absorbing. Once they truly realize that they do not have to memorize vocabulary, that will not have to say the words or phrases they are hearing, they relax. The "filter" goes down and, with "comprehensible input," they appear able to "acquire" language.

[RB] I can see now how many of the challenges put forth by the delayed oral production approach are relevant for mathematics as well, and could help us uncover some common fallacies in what we have so far perceived as unshakable principles behind much of current practice in mathematics instruction.

Perhaps a concrete mathematical example could help at this point to better illustrate and further explore the possibility of translating some of the principles of delayed oral production to mathematics instruction. I think that the topic of graphing will serve our purpose.
Despite individual variations, students are usually introduced to graphing through the following moves:

1. First they are asked to "plot" on a grid the points correspondent to some assigned pairs of coordinates;

2. Once students have (hopefully) "understood" the principle that relates points with their coordinates, the relationship between straight lines and linear equations will be addressed (since this is the "simplest" function which can be represented graphically);

3. Then, when students are sufficiently proficient with plotting lines corresponding to a given linear equation and with recognizing the equation of a given lines, they can then move to consider other curves such as parabolas or circles (the second next in terms of "difficulty"); and so on with other curves of increasing complexity.

It seems to be a reasonable approach. But your observations in support of delayed oral production may now allow us to recognize some considerable drawbacks. First, students' difficulties with graphing at these beginning stages seem to be caused primarily by their clumsiness in plotting. A few points wrongly plotted may make the curve look different and thus impede the student's ability to recognize its equation or properties.

We may now question whether being able to plot from the very beginning is really valuable for understanding the meaning of graphing and using graphing in mathematics. We can in fact argue that the "traditional" approach described above does not allow students to appreciate the power and "meaning" of graphing until very late, perhaps too late.

Mathematicians use graphs because they are a good way to represent complex relationships and thus predict events, and they provide an efficient way to solve problems connected with the solution of equations.
and inequalities. These benefits, however, are not evident to students of mathematics until they deal with equations which are too complex to be solved algebraically. In the traditional approach, students will not realize these values of graphing perhaps until a year after their introduction to graphing, when they are shown how graphic methods can help to solve quadratic inequalities without getting confused with signs or "forgetting pieces."

But is there really an alternative to the traditional approach? Once we accept the possibility of delaying the production of graphs, some new possibilities indeed emerge. After all, even laymen with little mathematical background know roughly how to interpret the information contained in a graph representing, for instance, the average consumption of gasoline as a function of the speed at which a certain car is driven. Long before introducing equations and plotting, students could discuss a graph like the one described above, answering questions for which they need the information contained in the graph, creating problems which could be answered using the graph, or comparing similar graphs corresponding to different types of cars.

Once students appreciate what graphs are for, they may have less difficulty with understanding the role of the x and y coordinates, and the relationship between algebraic and graphical representations of a function. Their own plotting of equations may wait almost until the very end (especially now that computer graphics technology allow to produce the graph of a given equation with a few commands). But when it finally occurs, we may expect that it will not be so difficult, and that the students themselves will have developed an intuition that will allow them
to make sense even if some "plotting mistakes" or imprecisions mar the product.

Of course, the considerations stated above are just a rough sketch of how one could interpret and apply the principles of delayed oral production with respect to graphing. Careful attention would be needed in the design of an actual instructional experience based on these ideas. Appropriate research also needs to be conducted in order to correctly evaluate the results of this approach, taking into account not only the "learning" but also the "acquisition" of the relevant mathematical content reached by the students. However, I hope that my sketch has helped for the moment to show how a translation of method from languages to mathematics is possible, and research on the topic worth pursuing.

[BJA] I find your example very encouraging. Students are allowed to concentrate on the meaning of graphing. Performance comes later, after understanding. This delay in performance would indeed appear to assist students' initial comprehension.

It will be interesting to see if your results with graphing parallel Postovsky's. If they do, your students should be better performers—in this case, better graphers—even with less specific practice in graphing skills.

Your design also has implications for an integrated curriculum. There is no reason that some of the graphing discussion cannot take place in the social studies class. But that's for another paper.
ALTERNATIVE METHODS: THE SILENT WAY

[BJA] I am struck by the importance that approaches based on "deferred oral production" have given to silence, and the next method I would like to discuss continues that emphasis. In this case, it is the teacher who is silent, or nearly so.

Caleb Gattegno, the father of the Silent Way, has designed wall charts (fidels) of single sounds, sound clusters, and words. These fidels and a box of cuisinaire rods are the tools of the Silent Way.

The teacher may first have students practice a few sounds from the wall charts. She will point to a letter or small group of letters ("s," for example, and "sh"). She makes the sound of the letters once. Then the teacher is silent. Students reproduce her sound(s). The teacher, a silent conductor, points to the student whose pronunciation is most like her own. The teacher's relative silence forces students to listen carefully to each other.

From the reproduction of single sounds, the teacher moves to the combination of sounds. Pointing to successive sounds on the fidel, she encourages the students to form syllables or words. Intonation and stress are conveyed by gestures. This combining of sounds is done entirely without meaning, yet students do not seem to rebel. The exercise is relatively brief, and it has an engaging puzzle-like aspect.

The language lesson moves quickly to add meaning. After a short time with the fidels, the teacher lets out a few rods and describes them in some way. She may simply identify them (for example, short blue rod, long green rod) or she may do something with them and describe the
action. The teacher’s words and the movement of the rods brings meaning to the sounds which the students have already practiced. Skillful use of the rods gives students a sense of confidence: While there is some initial ambiguity about the meaning of the teacher’s language, the concreteness of the rods promises ultimate understanding.

By not burdening students with too many nouns ("rod" becomes the all-purpose noun), it is possible for students to concentrate on using language in a variety of ways, all accompanied by the manipulation of a handful of the colored rods. It is not necessary to master a carefully specified number of nouns (building blocks) before assembling elements of language into meaningful communication. Hence the motto associated with the Silent Way, "Few words, much language."

[RB] I have long been aware of Gattegno’s contributions to mathematics education, but I never knew of his interest in language instruction as well. Our two fields are closer than I thought. I am, however, concerned about the passive role of students in the illustration you have given.

[BJA] Ten minutes of participating in such an exercise will convince you that the listening and hypothesizing required is hardly passive. In fact, one of the problems of introducing the Silent Way in public schools has been the frequency of classroom interruptions which break the students’ intense concentration. You are right, however, to note the teacher’s control. Variations on the method have students practice with each other as soon as they are comfortable with the forms and meaning of the lesson.
[RB] With these clarifications, I can see a parallel for the Silent Way's "few words, much language" in the recent work on "problem-posing," developed by Stephen Brown and Marion Walter. These authors, too, have argued that one does not need a great deal of factual knowledge before being able to engage in the creative activity of generating questions and problems. They have supported this argument with various illustrations within elementary mathematical topics.

Let me briefly report one of their examples, dealing with the well-known Pythagorean theorem. Usually, we present mathematics students with the statement of the theorem ("In a right triangle, the square of the hypotenuse is equal to the sum of the squares of its legs")), sometimes along with some justification for this result, and we expect them to be able to apply it in a variety of examples involving right triangles.

Approaching the teaching of this theorem differently, Brown and Walter suggest that students could gain a better understanding of the theorem and engage in more valuable mathematical activities if they are instead asked to generate questions inspired by this theorem. To help them in this enterprise, the instructor may suggest that students list the attributes of this situation, change some, and then analyze the potential consequences of such changes. For example, by changing the "square" attribute, the students themselves could propose (and perhaps begin to investigate) questions such as: What happens if we construct an equilateral triangle on each side of a right triangle? What if we construct other regular polygons on each side? Would the same
relationship (i.e., the figure built on the hypothenuse equals the sum of the figures built on the legs) hold? Why?

An entirely different line of inquiry could be initiated by focusing on the fact that the Pythagorean theorem deals with right triangles. Given a generic triangle, what would be the relationship between the square built on its larger side and the squares built on the other two sides? Is there any other triangle for which the relationship \(a^2 + b^2 = c^2\) holds?

As a result, the students may not only discover some unexpected mathematical results, but also may understand more deeply the Pythagorean theorem and its possible applications.

[BJA] Your example picks up on the fact that students can do interesting work in geometry with only a beginning knowledge of the subject. Such an approach does, indeed, parallel the Silent Way's "few words, much language."

However, your emphasis on student initiative in questioning improves the traditional teacher-controlled Silent Way model in ways similar to Earl Stevick's variations on the method. Stevick (1980), for example, places a single rod on the table and encourages students to describe it. Students then generate information about that rod, and in no time at all, the single rod becomes a twelve-story building. This approach makes me think of a new motto, "few rods, much imagination." Both your example and Stevick's add power and richness to the Silent Way by borrowing elements from the third method I would like to discuss.
ALTERNATIVE METHODS: COUNSELING-LEARNING/
COMMUNITY LANGUAGE LEARNING

[BJA] The silence which is part of the first two methods gives a
great deal of authority to the teacher. In the third method I would like
to present, a different use of silence shifts that power balance
significantly. Let be begin my presentation of Counseling-Learning/
Community Language Learning (CL/CLL) with an illustration. While CL/CLL
is more than the "circle," many people in language tend to identify it
with the method. A description of the circle may help clarify philosophy
and values behind CL/CLL.

Students sit in a conversational group, a tape recorder in the
middle of the circle. When a student wishes to speak, she picks up the
microphone. This is a signal for the teacher (called the "knower") to
move to a space behind the student. The student says what she wishes to
say in her native language (or a brave combination of the native and
second language). The teacher restates the utterance in the second
language, using natural speed and intonation. The student practices it
as long as she wants, signals for repetitions of all or part of the
utterance by the knower, and when she is satisfied with her production,
she turns on the microphone and records the second-language translation
of her original statement. The conversation continues as another student
indicates her desire to speak by picking up the microphone.

My concern with the "hidden curriculum" of our teaching is satisfied
by the emphasis which CL/CLL places upon the role of the student in the
making of meaning. While in the circle, students are responsible for the
content of their discussion. Later the knower, as teacher, will shape
the students' understanding of the language forms which are embedded in that content. This reversal of initiative is distressing for many teachers and students. Dutiful students ask the teacher to specify a topic. Teachers, accustomed to the power of instructional decision-making, often agree.

Where teachers and students agree that learners select the content of their discussions, CL/CLL also has a period of silence (La Forge, 1977). The CL/CLL silence occurs at the beginning of the circle experience, when students grope for their topic(s), and it may occur again later in the circle experience whenever students need to change topics, express opinions, or otherwise generate their own thoughts. A measure of the knower's success is in that ability to wait, giving students time to think, and showing them in yet another way that she will wait as long as is necessary for them to assume their responsibility.

[RB] I can see that Brown and Walter's use of student-posed questions fits here as well. It requires that students assume much the same kind of responsibility and initiative. But please go on. I am eager to know more about the rationale and implications of this method.

[BJA] Let me first return to the assumption of responsibility embedded in CL/CLL, since that change is not always accomplished easily. CL/CLL pays explicit attention to the affective needs of learners, recognizing all those elements which too rarely are discussed in the teaching and learning of languages (and mathematics as well I suspect)—the fear, anger, sense of inadequacy, feelings of lack of control, need for security, and more.
Students deal with these issues in the "period of reflection" which follows each circle. During this time, students are encouraged to talk about themselves as learners and as members of the learning group. In addition to celebrating successes and supporting each other, students typically mention feelings of anger, frustration, embarrassment, and the seemingly inevitable conflict between the dependency a learner feels toward the teacher (the knower) vs. the desire to be an independent speaker of the language. Students also reflect upon the content of their learning, as well as on the process. As you might expect, this process requires extraordinary skills of a teacher. Though the proponents of CL/CLL emphasize that the period of reflection should never approach therapy, the skills of a very good listener are essential.

The technology of CL/CLL also addresses other affective needs. One of the biggest barriers to second-language proficiency or mathematical skill is the students' sense of themselves as not able. Through the "circle's" selective use of the tape recorder, students create a tape which makes them sound like fluent speakers of the language before they truly are. I find this a fascinating use of technology. In a persuasive way, the circle-produced tape shows students that they can hold meaningful, reasonably accurate conversations in a second language.

[RB] I can certainly appreciate a concern with the affective components of learning. As I mentioned before, "math anxiety" is unfortunately a well-known and widespread disease, especially among women, and mathematics teachers are realizing the importance of dealing with the emotions that do not allow students to engage in mathematical
activities in an efficient way. Reflecting on and talking about the process of learning mathematics or of solving a problem could certainly prove to be useful for mathematics students as well. Furthermore, I can see some cognitive benefits in this activity, too, since a discussion of how we "do" mathematics can help identify new approaches and heuristics, and this in turn could result in an improvement of study skills and problem-solving strategies.

CL/CLL suggests to me the value of students working collaboratively on a meaningful and complex problem, and invites us to consider the possibility that such problems could be addressed even if the students do not have all the necessary technological background to deal with them on their own—provided that the teacher can play the role of the "technology expert and helper" who can furnish the necessary answers as requested by the students, just as computer packages do when we need to apply statistical manipulations to our research data.

To respect the concern for engaging the students in "the making of meaning," it seems important that the problem chosen be something for which neither students nor teacher "know" the answer, so that they may genuinely work together towards a solution, feeling free in the process to redefine the problem itself or what could be considered a suitable solution (as one does in real life).

This collaborative problem-solving and problem-framing also seems to call for the choice of problems which are sufficiently open-ended so that all the participants have an opportunity to shape the direction in which the inquiry is going, to change it when suitable, and to pursue side issues. Though the teacher may need to take a more active role in the
choice of the starting point of the inquiry (so that a suitable situation is chosen), she will have to step back from this point on, except when called in to provide answers and partial solutions which the students have explicitly requested.

The conditions described above do not seem easy to meet in mathematics instruction, but they are not impossible either. Once again, an example may be helpful.

I think that an appropriate context for a "CL/CML" experience, that is, for an open-ended mathematical exploration conducted collaboratively by students with the teacher playing the role of a consultant, could be provided, for example, by "taxicab-geometry" (Krause, 1986; Borasi, 1981). This is the "geometry" relevant for a taxicab driver, who has to move around the city along the streets, and for whom the shortest way is often not defined "as the crow flies," since he cannot go through buildings. If the street pattern of the city is sufficiently regular (as in New York City and many other urban centers), this situation may be represented by an approximately square grid, where "distances" are computed as the length of the shortest path on the grid connecting two points. Because the "distance" is defined differently here than it is in Euclidean geometry, many metric loci (such as "circle," "ellipse," "perpendicular bisector") take on a new shape, and many problems involving distance may lead to unexpected solutions.

I can imagine the teacher providing the starting point of the "circle" by simply presenting the situation, and by observing that the constraints created by the buildings affect the way we compute distances, so results from Euclidean geometry are not very useful here. The
students can then start an inquiry/conversation by generating some questions which they may be interested in pursuing: Where are all the points at a given distance from the City Hall? What is the quickest way to go from one given point to another in the city? What would be the best location for special buildings such as a bus station? Given the location of elementary schools, how should the group decide how the city student population should be partitioned among them, i.e. who goes where depending on his/her home address? They can decide to focus on one problem or to pursue several questions at the same time.

While they are pursuing these questions together, they may call in the teacher to provide needed information on the context—for example, returning to the case in question, students may decide that they need to know the city's population and how many elementary schools would be needed. They may ask whether certain areas of the city are more populated than others, and so on. In some cases, the students may also feel that they do not have all the mathematical tools necessary to complete some steps in their problem solution. Upon request, the teacher could then provide an algorithm or formula which enables the students to complete their task. The teacher may even provide a partial solution if the procedure is too complex.

In any case, the initiative and control over the problem should remain with the students, and the teacher should intervene only when her help is explicitly asked, and then limit her intervention to providing only the help requested.

Whenever the students feel that they have reached a worthwhile resolution, or than an interesting hypothesis has been generated, they
could record this result in a notebook, along with relevant information about the process which produced it.

It is clear how in such a "brainstorming" session various questions can be raised and abandoned, and final solutions perhaps never reached.

The "lesson" I just sketched may seem to have little to do with what is usually perceived as "school mathematics." However, it would certainly engage the students in important mathematical activities, such as generating and refining problems and trying to reach original solutions, and it may make them appreciate the nature and concerns of geometry better than most traditional approaches.

[BJA] Your example captures one of the essential aspects of the CL-CLL method—that need comes before language. First students have to want to say something. Then it is a matter of learning how to say it. You have offered students an important degree of responsibility, as they translate the teacher-posed topic into questions which are, at least, personally meaningful variations on the teacher's theme.

If you added a period of reflection to the notebook, I think you would have an exciting way for doing mathematics in school.

[RB] I agree. It would certainly be helpful to have each brainstorming session followed by a period of reflection, when the notebook is reviewed and reflected upon in order to "extract" some important mathematics results, to generalize valuable procedures, to introspect on the process that led to the generation of questions and solutions, as well as addressing the affective elements involved in the inquiry.
ISSUES FOR IMPLEMENTATION OF ALTERNATIVE APPROACHES

[RB] All three methods you have presented seem extremely interesting and profitable, yet they all challenge some of our well-entrenched beliefs about language and its learning. I suspect teachers' decisions and behavior are shaped by their beliefs, especially those dealing with the nature of the subject matter they teach, and by their history as learners. I would expect then, that this could create a serious obstacle in convincing teachers to abandon the traditional, more intuitively "sound" approaches for the teaching of second languages for any of these new ways.

[BJA] And indeed it has. Krashen has suggested that many of us who become language teachers are strange ducks, that we have somehow acquired second-language proficiency through "learning," where most of the world's bi- or multilingual people have arrived at that state through "acquisition." He attributes the failure of American second-language teaching and learning partly to the mismatch between the way most teachers learned (heavy use of the language "monitor") and the way that most students would profitably learn (acquisition).

The effect of prior exposure or learning on the way one teaches is also implicitly confirmed by relatively easy experiences introducing new methods to one particular kind of second-language teacher, those who teach English to speakers of other languages (ESOL). ESOL teachers in the United States are, in general, native speakers of English. They have never been "learners" of the subject they are teaching—they "acquired" English. As a result, they cannot hark back to a teacher they had to
taught them what they are now learning. They have no school model (good or bad) for their own teaching style.

For teachers who do have a history as learners of their subject, research reporting the success of new instructional methods can certainly contribute to gaining their support. Yet research is rarely enough to fully convince them that the new methods are better than whatever worked for them in their own learning. One useful strategy is to have teachers learn something in their subject area, and think about that learning. Here both ESOL and modern foreign language teachers have an advantage in that they can place themselves in a position similar to their own students, becoming learners of a language they do not know. A weekend as a student in an intensive Japanese language immersion program powerfully demonstrates to an experienced teacher the painful complexity of second-language learning. Thoughtfully analyzed experience as a learner is one of the most effective ways to encourage teachers to change. An essential component of my "methods" course for prospective language teachers includes having them experience as learners lessons using each approach.

[RB] I agree with you that the power of teachers' beliefs should not be underestimated and that an effective practice is having the teachers themselves experience as learners some new methods, and reflecting on the experience. And I think that in mathematics, too, we can create a learning situation comparable to your teachers' becoming beginning learners of a new language.
In my mathematics courses, for example, I have often engaged the participants in the study of a simple, non-standard mathematics situation, such as the "taxicab geometry" discussed before, using methods alternative to the usual teacher-centered ones which they experienced as students.

The consideration of teachers' beliefs and expectations raises another important issue. We can in fact expect that students, as well, may be influenced by the explicit as well as implicit messages communicated in their previous schooling, and may resist in various forms the new "crazy" methods we are trying to introduce.

[BJA] And this is certainly what often happens. Perhaps we, as teachers and educators, should recognize that students, too, need to reflect about and discuss the learning process, goals of teaching, and the rationale behind a method, if we want to secure their collaboration. The CL/CLL method's "period of reflection" provides a possible structure for that inquiry. Similar opportunities could be built into the other approaches as well.

[RB] Since we are discussing the obstacles that existing expectations about the nature of language and its learning can create for classroom implementation, let me bring in another component in this process: the system's expectations. I think they are most evident in the issue of students' evaluation. Obviously evaluation is important to students, teachers, the educational system and the community. I think this is an area in which mathematics educators can benefit from research in second language, since you seem to have the advantage of having clear...
standards for successful performance—the ideal of native proficiency—and you can study how language is acquired spontaneously, since most children in the world learn to speak without any need of formal instruction.

[BJA] That is true, but our task is not as easy as you might expect. What is ideal native proficiency? Is it the language of the taxi-driver, a university professor, or an energetic kindergarten child? And at what point do we measure that proficiency? The kindergartener is native and fluent, but I wouldn’t want to evaluate her use of the past perfect tense.

Evaluation is relatively easier when fluency has been achieved. We encounter more difficulty, however, when we try to assess the intermediate steps toward that goal—the partial results, the meaningful approximations.

[RB] I see a parallel in mathematics instruction as well. Indeed, the problem has never been that of determining whether a student has become a proficient problem-solver, because this can be tested by presenting her with a variety of novel problems to see whether she can approach them with a reasonable rate of success. The real difficulty lies in measuring any movement or intermediate step in the process. I can see how this can seem very troublesome for most teachers, given the emphasis on assessment in schools and the widespread love for standardized tests in mathematics.
Current evaluation procedures, with the process writing as a notable exception, are tied either to the end product (native fluency) or to the measurable building blocks which can be "learned" and demonstrated along the way toward that final proficiency. Before we can significantly change prevailing evaluation procedures, we as educators need to understand the nature of those middle steps.

Our dialogue in this paper has set out the need for measures of meaning and of partial proficiency, but a definition of the problem is only one step toward the solution.

To conclude this section, let's touch upon another problem that I think has to be addressed in order to change school practice. While all the methods you have presented seem to share some common assumptions about language—the importance of meaningful language, the acceptance of partial products—in some respects they also seem almost contradictory. For language teachers interested in applying some of these research results, this may appear confusing and discouraging. It may thus be quite important to clarify whether these methods are mutually exclusive or rather potentially complementary. I also think that such a discussion could be especially relevant for mathematics teachers, since in our area, too, researchers and curriculum developers have proposed quite different approaches to the teaching of mathematics. Think, for example, of the "new math" emphasis on communicating to students the "structure" of mathematics, versus the more recent methods which emphasize "problem-solving" or "critical thinking."
I agree that there are contradictions in practice among the three methods I have presented. These are particularly evident when one plans for the first day of instruction in a new language: Are the students silent, or is the teacher silent? Or do the students begin speaking immediately? Some kind of accommodation of the three methods may be made by thinking of them as variously relevant over the long period of language learning. But this is only a partial answer.

Perhaps some of these contradictions can be reconciled, however, if we look less at the attributes of each method and rather allow for modifications so that a teacher can transform them to fit her own educational goals and teaching style. Rather than "what method, or combination of methods, is best for learning," the questions addressed by researchers should rather become: What are the assumptions behind each method? What kind of outcomes can they produce? For what personalities and contexts would each seem most suited? Each teacher could then use this information to decide what to use—and how—in her work.

A reflective practitioner may be able to reconcile these methods in ways we have not anticipated. And, more importantly, such a thoughtful professional will see problems and benefits which we have not raised in this preliminary dialogue.
CONCLUSIONS

The sociolinguist Robert Shuy (1982) points out that topics recycle through a given conversation until the participants come to some kind of resolution. The conversation between second language and mathematics educators which this dialogue represents certainly have not yet reached such a natural conclusion. Page and time constraints, rather than lack of more topics to discuss, have brought us to a stop at this point.

Nevertheless, this exchange has lead us to challenge some of the most basic "pedagogical truisms" common to both our areas. First, it would seem that both mathematics and language have traditionally assumed that the approach to learning is to break it down into many sequential "chunks," to have students master each one before moving to the next. In fact, the most common response to students' learning difficulties is often limited to breaking the chunks even finer and providing more time and practice to master each. Another of the dogmas of instruction in both areas seems to be that teachers should strive for providing students with material that learners can perfectly understand and reproduce. Our discussion instead suggests that there is a value in students' "partially understanding" a complex discussion for which they have not yet acquired all the necessary background knowledge.

It may be that, in both fields, a learner needs to be able to see the "larger picture"--the meaning, the context--before being able to understand and master its components.

Second-language educators have shown how this challenge to focus on meaning can be translated into practice through radical alternatives to
traditional methods of instruction, backed by a sound foundation in theories of learning and documented success. In turn, our conversation has suggested how some basic principles and ideas of these instructional approaches could be translated in mathematics education.

Most of the issues we have touched upon, however, are far from being resolved. This is certainly true with regard to the problem of finding appropriate ways to evaluate students' "partial" understanding and acquisition. Dealing with students' and teachers' beliefs about subject matter, learning, and schooling is another challenge for which at the moment we can only suggest partial resolutions. Ways in which research can help teachers in their ultimate decisions about how to teach their students are a continuing challenge for the field of education.

Furthermore our dialogue, like any worthwhile conversation, has introduced more questions than it has been able to answer. Among the issues saved for another day are the nature and role of culture in both mathematics and language; the treatment of errors; and the potential of a meaning-based integrated curriculum.

We hope that this beginning conversation has shown the value of an exchange between the fields of second-language and mathematics instruction, and that some of the new questions it has generated will stimulate further exchange between our two fields.
REFERENCES


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