This issue contains abstracts and critical comments for published reports of research in mathematics education. The reports are concerned with: (1) relationships between class size, teaching practices, and student achievement; (2) sex differences related to attitudes; (3) comprehension of mathematical relationships expressed in graphs; (4) effect of semantic structure on strategies for solving addition and subtraction word problems; (5) using computer-based analysis to improve subtraction skills; (6) cultural variations in family beliefs about children's performance in mathematics; (7) a case study of a highly skilled mental calculator; (8) learning of geometrical concepts through Logo; (9) parent attitudes and career interests in junior high school; (10) relationships between attitude and achievement for elementary grade mathematics and reading; (11) adapting mathematical problems to student interests; and (12) writing geometry proofs. Research references from "Current Index to Journals in Education" (CIJE) and "Resources in Education" (RIE) for April–June 1987 are also listed. (RH)
INVESTIGATIONS IN MATHEMATICS EDUCATION

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Abstract and comments prepared for I.M.E. by J. PAUL MCLAUGHLIN, Purdue University Calumet, Hammond, Indiana.

1. Purpose

This article reports the results of research into how teaching practices differ between smaller classes and larger classes. The subjects were 63 Grade 5 classes in 33 schools in metropolitan Melbourne, Australia. A causal model was proposed and tested to explain differences in achievement that were linked to class size. The details of the statistical model are not discussed in this abstract.

2. Rationale

Numerous studies have shown that students in smaller classes achieve more. This study sought to determine some factors which account for this greater achievement. Basically two groups of sequential questions are addressed:

1. Is there a relationship between class size and achievement in the data gathered? If so, what is the nature of the relationships?
2. To what extent is the postulated causal relationship mediated through teaching practices? Can specific teaching practices linking class size and achievement be identified? (p. 559)

3. Research Design and Procedures

The research involved gathering data through observations (8 to 10 observations in each class over a 12-week period) and through
questionnaires. All observations were of mathematics lessons. Achievement was measured using a pre/post-test on the four arithmetical operations with whole numbers.

Since there might be more than one "teacher" in the room during the lesson, class size was the average number of students per teacher for each 5 minutes of lesson time observed. Classes ranged from 12 to 33, with both the mean and median being 25. While over 10% of the classes had fewer than 20 students, no quantitative measure of "larger" and "smaller" was given.

As part of the causal model multiple linear regression analysis was used. This analysis indicated that school size and socio-economic status of the "catchment" area was linked to achievement through class size. Student ability, in addition to being linked directly to achievement, was indirectly linked to achievement through class size. Teacher experience was not linked to class size; however, teacher experience teaching Years 5 and 6 was retained in the analyses of class size and achievement.

The hypothesis was that the use of different teaching practices employed in classes of different sizes caused variations in achievement. Thus, identifying these practices would help us to understand why smaller classes are better, given students of equal ability and schools with similar characteristics.

4. Findings

Fourteen teaching practices were significantly correlated individually to class size. As a group, but not individually, they were significantly correlated with achievement. Table I, reproduced from the article, shows the teaching practices identified. Because of collinearities, the final block of teaching practices was the group of 9 practices above the line. Positive correlations are in the direction of larger classes.
### TABLE I

**Relationships of Teaching Practice Variables with Class Size and Achievement (N = 63, correlations of 0.20 were required for significance)**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Range</th>
<th>Mean</th>
<th>SD</th>
<th>Correlations with Class size</th>
<th>Correlations with Achievement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Use of whole class teaching</td>
<td>0-3.5</td>
<td>2.6</td>
<td>0.8</td>
<td>-0.24</td>
<td>0.16</td>
</tr>
<tr>
<td>No. of groups used within the class</td>
<td>1-4.2</td>
<td>1.6</td>
<td>0.6</td>
<td>0.33</td>
<td>-0.13</td>
</tr>
<tr>
<td>Any interactions between teachers and students</td>
<td>239-384</td>
<td>293</td>
<td>38.5</td>
<td>0.31</td>
<td>0</td>
</tr>
<tr>
<td>Amount of noise tolerated in the class</td>
<td>0-1</td>
<td>0.5</td>
<td>0.5</td>
<td>0.34</td>
<td>-0.14</td>
</tr>
<tr>
<td>Non-academic management</td>
<td>2-26</td>
<td>11.4</td>
<td>5.4</td>
<td>0.29</td>
<td>-0.15</td>
</tr>
<tr>
<td>Teacher probes after a question</td>
<td>0-4</td>
<td>1.3</td>
<td>0.8</td>
<td>-0.25</td>
<td>0.04</td>
</tr>
<tr>
<td>Student questions</td>
<td>0-6</td>
<td>2.4</td>
<td>1.5</td>
<td>0.32</td>
<td>-0.15</td>
</tr>
<tr>
<td>Teacher waits for a response</td>
<td>0-1</td>
<td>0.5</td>
<td>0.5</td>
<td>-0.20</td>
<td>0</td>
</tr>
<tr>
<td>Homework and assignments assessed</td>
<td>0-6</td>
<td>2.0</td>
<td>1.6</td>
<td>-0.28</td>
<td>0.19</td>
</tr>
<tr>
<td>Teacher directly interacting with students</td>
<td>20-86</td>
<td>54.3</td>
<td>14.5</td>
<td>-0.27</td>
<td>-0.04</td>
</tr>
<tr>
<td>Teacher monitoring student work</td>
<td>3-64</td>
<td>32.0</td>
<td>13.1</td>
<td>0.29</td>
<td>0.02</td>
</tr>
<tr>
<td>Positive teacher response to answer from student</td>
<td>1-17</td>
<td>4.7</td>
<td>2.7</td>
<td>-0.22</td>
<td>-0.09</td>
</tr>
<tr>
<td>Teacher lectured or explained</td>
<td>6-34</td>
<td>14.8</td>
<td>6.0</td>
<td>0.39</td>
<td>0.07</td>
</tr>
<tr>
<td>Use of oral tests for assessment</td>
<td>0-4</td>
<td>2.5</td>
<td>0.9</td>
<td>-0.28</td>
<td>-0.17</td>
</tr>
</tbody>
</table>

(p 563)

The direct path correlation from class size to achievement narrowly failed to reach significance. Thus, with student ability controlled, the teaching practices in the smaller classes were examined as a source, collectively, for higher achievement.

1. **Class groupings:** In smaller classes, there was more whole class instruction. In larger classes, teachers tended to form more groups during mathematics lessons.

2. **Teacher–student interaction:** In smaller classes, there were fewer student questions and fewer non-academic procedural arrangements. Larger classes required more management; student questions were often requests for clarification or other help.

3. **Questioning behavior:** Teachers in smaller classes probed more frequently, and waited (more than 5 seconds) more frequently for a response, before making another teacher move.
4. Homework practices: Smaller classes were given more homework and apparently devoted more class time to homework-related activities.

5. Noise levels in the classroom: While noise levels varied considerably across the board, smaller classes were less noisy.

5. Interpretations

Some specific teaching practices linking class size and achievement were identified which account for a significant part of the variation in achievement. These practices by no means account for all of the variation even in the teaching practices block. Other research using the same causal model, which was shown to provide an acceptable interpretation of the data, should look at such things as the classroom context and student dynamics. The quality of certain aspects such as student engagement, and not just the quantity, needs to be studied. A knowledge of teaching practices used in smaller classes and some rationale as to how they promote achievement will be helpful in assisting teachers to employ teaching practices that are most effective with classes of different size.

Abstractor's Comments

This article reported on research on how teaching practices differ in classes of different sizes. While mathematics lessons were observed, the objective and focus of the study was on general teaching practices. In fact, no mention of the subject is made except to emphasize that all observations were made in the same content area and achievement was measured using a test on arithmetical operations with whole numbers.

Even though it is not directly research in mathematics education, it is of interest to, and has implications for, educators in mathematics. Particularly relevant might be its implications for
teaching problem solving. A detailed discussion of or references to statistical analysis is given, and the discussion of the causal model will be of interest to many researchers in mathematics education.

One challenge for mathematics education at all levels is to look at the teaching practices as a block -- a more holistic approach if you will -- and seek to improve teaching by incorporating these or similar practices into one's teaching in a balanced way.

A challenge for researchers is to look for other significant practices, or other factors or conditions which affect achievement in the classroom.

Finally, is the use of those teaching practices found in the smaller classes something that is possible only in smaller classes? Could some of them be successfully implemented in larger classes? If so, how and how can we help teachers to develop the requisite skills for using those practices in larger classes?

This research is a good reminder that we are teachers first and foremost. As teachers, we need to focus on principles of effective classroom practices as well as on the mathematics we teach.
Abstract and comments prepared for I.M.E. by MICHAEL T. BATTISTA, Kent State University.

1. Purpose

The purpose of the study was to explore secondary males' and females' attitudes toward mathematics and computers.

2. Rationale

There are documented sex differences in secondary students' attitudes toward mathematics. Because attitudes are related to participation and achievement, many sex-bias intervention strategies have as a goal the improvement of female students' attitudes toward mathematics. In addition, because the use of computers within mathematics instruction may enhance interest in mathematics, it has been hypothesized that such use might play an important role in these intervention strategies. According to the author, a possible complication in this hypothesis is that there are also sex differences in students' attitudes toward computer studies; female students exhibit relatively negative attitudes toward computers. It was hypothesized that one factor that contributes to these negative attitudes is that females associate computer studies with mathematics.

3. Research Design and Procedures

The subjects were 539 male and 479 female students at grade 8 and 419 male and 381 female students at grade 12. They were each given two sets of Likert-type attitude items: a set of 24 items concerning computer use and computer users, and a set of 4 items concerning personal attitude in mathematics and comfort with regard to doing mathematics.
The mathematics course for the grade 8 students included 20 hours of computer activities that utilized BASIC programming for various mathematical tasks such as finding means, prime factors, and geometric formulas. At the time the data for the study were collected, 41% of the girls and 47% of the boys in grade 8 had completed the activities; the remainder of the students had not yet started them.

4. Findings

The results of a canonical correlation analysis indicated a significant relationship between computer attitudes and mathematics attitudes in each of the four sex-by-grade groups. Two variate pairs (in each pair, one variate for computer attitudes and one for mathematics attitudes) were significant for all but the eighth grade females, with the canonical correlations ranging from .34 to .55. At grade 12, the mathematics variates accounted for approximately 19% of the variance in the original set of computer variables; at grade 12, approximately 12%. The computer variates accounted for approximately 4% of the variance in the mathematics variables.

The author interpreted the canonical variates as follows. For male students at both grade levels, feelings of frustration with mathematics tended to be associated with disinterest in computers, and feelings of self-esteem in mathematics were related to feelings of pleasure with computer use. For females at grade 8, feelings of frustration with mathematics tended to be associated with "You have to be smart to use computers," while the relationship between mathematics self-esteem and computer-related attitudes was not interpretable. For females at grade 12, feelings of frustration with mathematics tended to be associated with "I would be embarrassed to tell by friends that I would like to join a computer club," and feelings of self-esteem in mathematics seemed to be related to feelings of self-esteem in computers.
For the grade 8 students, a multiple regression was completed, with computer use in mathematics class (yes/no) as the criterion and the 24 computer-attitude items as independent variables. The relationship between computer use and computer attitudes was significant for the male, but not the female, students. For the males, the computer-attitude variables were positively correlated with computer use in mathematics; for the females, 8 of 24 of these variables were negatively correlated with computer use. The author also indicated that, for females, computer use in mathematics was negatively correlated with self-confidence in mathematics. As an example of this trend, the author cited the fact that girls who had used computers in the mathematics class were more likely to disagree with the statement "Using a computer in mathematics class would make math more fun" than girls who had not used computers. (I am not sure what this statement has to do with self-confidence in mathematics, however.)

5. Interpretations

According to the author, the results indicate that secondary school students' attitudes toward mathematics are positively related to their attitudes toward computers, and that students' attitudes toward mathematics strongly influence their attitudes toward computers (but not vice versa). She concluded that "female secondary school students are more likely than male students to associate negative attitudes toward mathematics with generally negative opinions about computer use and with stereotypes about computer users" (p. 300). Thus, practices such as using computer programming in mathematics classes, staffing computer studies and computer science courses with mathematics teachers, and focusing computer courses on mathematics-related examples and topics "may produce a negative predisposition toward the computer activities for female students" (p. 400). It was also concluded that "the study did not support the assumption that providing secondary school students with computer experiences in a
mathematics class yields an improvement in the attitudes of the female students toward either computers or mathematics" (p. 401).

Abstractor's Comments

This study provides some useful information about students' attitudes toward mathematics and computers. The data indicated that the students' attitudes toward computers were strongly influenced by their attitudes toward mathematics, that the relationship between the students' attitudes toward mathematics and computers was complex, and that the relationship differed for males and females. Using computers in mathematics class seemed to be a more positive experience with computers for males than for females. Also, apparently, some females who used computers in mathematics thought that it made mathematics less fun.

I would like to discuss several questions and important issues that are raised by the study. First, the attitude construct consists of several theoretical factors—feelings toward content (mathematics/computers), self-concept related to content, and sex-role stereotype related to content. The study's attitude instruments did not deal with these factors in a systematic way. In fact, there were 24 computer items and only 4 mathematics items. I believe that the reliability and validity of the two instruments would have been greatly improved if the two attitude scales had been constructed to be parallel both in number and theoretical factors.

Second, several of the author's conclusions do not seem to follow from the data. For example, it is stated that female "students are more likely than male students to associate negative attitudes toward mathematics with generally negative opinions about computer use..." (p. 400). However, examination of the mean scores on the attitude items reveals that the females had positive attitudes toward computers and felt positive about their ability to do mathematics (although their attitudes were slightly less positive than the males). For
example, in response to the item "I would like to learn how to use a
computer," the males' mean responses were 4.24 (grade 8) and 4.04
(grade 12); the females' were 4.10 and 4.09, respectively (4 = agree,
5 = strongly agree). Thus, though these data do not invalidate the
possibility that the conclusion is technically correct, the tone of
the author's statement is misleading. Also, the canonical correlation
between attitudes toward computers and attitudes toward mathematics
was actually slightly stronger for males in 3 out of 4 comparisons.
Furthermore, there was no indication in the reported data that the
association between the negative mathematics-attitude factor and the
negative computer-attitude factor identified in the canonical analysis
was any stronger for the males than for females. Thus, I was unable
to find any reported data that supported the author's conclusion.

Finally, with the statement "computer activities that emphasize
programming as a major way to integrate computer use into mathematics
are what need to be most critically examined" (p. 401), the author
raises an issue that is extremely important not only for this study
but for the investigation of attitudes toward mathematics and
computers in general. To examine this statement, it will be useful to
delineate three goal domains for computer use. Each domain provides a
different perspective from which to view the statement.

The first domain deals with how to teach students about computers
and develop in them positive attitudes toward computers. The author
suggested that associating computer use with mathematics may encourage
students to transfer any negative attitudes they have toward
mathematics to computers. From this perspective, the study's data
suggest that integrating BASIC programming into a mathematics class
may not be the best way to improve students' attitudes toward
computers.

The second domain deals with how to teach students the role that
computers play and how to apply computers in mathematics. The author
suggests that there are many computer uses in mathematics (e.g., graphing, spreadsheets) that are just as important as programming and probably more beneficial for students' computer-related attitudes. While I agree that there are many applications of computers in mathematics that are more important than those associated with programming, this suggestion actually confounds the first and second domains. Students should become familiar with those mathematical applications of computers that experts in mathematics deem most important, irrespective of attitudes towards computers. We do not avoid word problems in mathematics simply because some students do not like them.

The third domain deals with how to improve students' learning of mathematics with the use of computers. The author suggests that using the computer to increase students' motivation to learn mathematics may not be the answer. However, the study's data suggest only that if the "computer use" is BASIC programming and if the students are female, such use will not be beneficial. Such use did seem beneficial for the males. And using a more appropriate programming language such as Logo, or using good mathematics computer courseware such as the Geometric Supposer may produce far different results. Furthermore, increased motivation is not the only mechanism by which computers can enhance mathematics learning. For instance, there is both theory and empirical data that support the notion that the learning of certain geometric concepts can be enhanced by the proper use of Logo (Clements & Battista, in press). Thus, from the perspective of this last domain, the original statement's phrase "integrate computer use into mathematics" seems not to be relevant. The goal in this domain is not to integrate the computer into mathematics; it is to teach mathematics the best we can.

It is conceivable, therefore, that the three different goal domains may place conflicting demands on computer use in mathematics instruction. For reasons related strictly to attitudes toward
computers, some might advocate avoiding certain uses of computers in mathematics courses. However, to teach students about the role of computers in mathematics or to help them better understand certain mathematical ideas may demand these very same types of uses. Hopefully, mathematics and computer educators will recognize the complexity of this issue and strive to deal with all three domains effectively.

Reference


Abstract and comments prepared for I.M.E. by CHARLES VONDER EMBSE, Central Michigan University.

1. Purpose

The purpose of this study was to apply the principles of schema theory of reading comprehension to graph reading by investigating the effects of prior knowledge on students' ability to understand the mathematical relationships represented in certain types of graphs.

2. Rationale

Schema theory of reading comprehension (Adams & Collins, 1977; Smith-Burke, 1979) hypothesized that the amount of "meaningful exposure" (p. 383) a student had to the topic, content, and form of a reading passage affected how the reader incorporated new information into related schemata. These three aspects of a reading passage were applied to graphs. The topic of a graph was the title or axes labels. The mathematical content of a graph included the number concepts, relationships, and operations embedded in the graph. The form of the graph referred to the four types of graphs used in the study: line graphs, bar graphs, circle graphs, and pictographs. Since reading and mathematics achievement levels were universal predictors of overall school achievement, these factors were also tested to assess the need for prior knowledge beyond general reading and mathematical ability.

The study used fourth- and seventh-grade students. It was assumed that by fourth grade, students had done "elementary" (p. 383) work with graphs and were able to read and do arithmetic at a level sufficient to read graphs. Seventh graders were assumed to have had
"growth and achievement" (p. 383) in their graph-reading abilities. Sex-related differences were assumed to be manifested by the time students reached seventh grade.

3. Research Design and Procedures

The study, conducted in one middle-class New York City school district, consisted of 204 fourth graders (101 boys and 103 girls) and 185 seventh graders (102 boys and 83 girls). Only data from native speakers of English were analyzed in the study. Four instruments were used to measure graph comprehension, prior knowledge, reading achievement, and mathematics achievement. Graph comprehension was assessed by the Graph Test, a researcher-developed instrument. Items on the Graph Test gauged three types of graph comprehension: literal reading of information, reading between the data, and reading beyond the data. Prior knowledge was assessed by the Prior Knowledge Inventory, another researcher-design instrument. This test consisted of three subtests designed to measure topic, mathematical content, and graphical form. Reading and mathematics achievement were assessed by the SRA Achievement Series tests. Data were collected by trained test proctors during four separate testing sessions in the Fall of 1980.

4. Findings

Second order partial correlations (controlled for reading and mathematics achievement) between graph comprehension and the three prior knowledge variables (topic, content and form) were significant (p < .01) for grade 4. Similar comparisons for grade 7 were significant (p < .05) for topic and content, but not for form. Regression analysis confirmed that the largest portion of the variance in graph comprehension at both levels was accounted for by reading and mathematics achievement (in nearly equal amounts). Entering each of the prior knowledge aspects into the regression analysis showed that
even though the amount of variance was small, it was significant
(p < .01) for both grade levels. Calculation of beta weights
indicated that reading and mathematics achievement and previous
knowledge of topic, content, and form were predictors of graph
comprehension for grade 4. Reading and mathematics achievement and
prior knowledge of content were the only significant predictors for
grade 7. Correlations between sex and the other variables were not
significant at grade 4. At grade 7, correlations between sex and
reading and mathematics achievement and graph comprehension were
significant (p < .05); however, they were low (.16, .23, and .15
respectively). Sex was not a significant predictor of graph
comprehension at either grade level.

5. Interpretations

The investigator concluded that no significant sex-related
differences for graph comprehension were found, supporting previous
research (Peterson & Schramm, 1954; Strickland, 1938/1972). Seventh
graders seemed to have more prior knowledge of the topic and form of
graphs than fourth graders, as was assumed. This may have accounted
for the failure of topic and form prior knowledge as predictors of
graph comprehension for seventh graders. Fourth graders may also have
needed more knowledge of the "concrete, visible, explicit aspects of a
graph" (p. 391) which were the salient aspects of the topic and form
of a graph. Prior knowledge of the mathematical content of a graph
was needed by both age groups since mathematical content was implicit
to the graph and not as readily apparent as the topic and form. The
investigator concluded that experience in collecting and graphing
"real world" (p. 391) data might help develop students' mathematical
schemas so they could better understand the implicit mathematical
relationships embedded in graphs.

Abstractor's Comments

This study attempts to explain the relationship of prior
knowledge to understanding the information encoded in graphs. The
study raises questions about procedures and findings as well as future research.

The investigator gave no information about the graph reading experiences of the group of seventh-grade subjects. Was the lack of significant correlations between topic and form and graph comprehension at the seventh-grade level due to specific experiences this group of subjects had, or was it due to other, more general learning factors? Understanding the topic of a graph (title or axes labels) seems to be more dependent on previous knowledge in general rather than recall from some specific mathematical schema.

The issue of sex-related differences was confusing. The investigator claimed there were no significant sex-related differences found, but reported significant (p .05) correlations between sex and reading and mathematics achievement and graph comprehension at the seventh-grade level (.16, .23, and .15 respectively). The investigator reported significant correlations between graph comprehension and topic prior knowledge for seventh graders at the same significance level (p < .05) and the same correlation value (.15) as the sex-related differences.

Graph comprehension in today's technological society is an increasingly important area for research and curriculum development. More than of the references (19 of 33) are from the 1970's and many (10 o. 3) are from the period 1927-1969. Are the findings of this study the most current information about how children understand graphs? Does more recent research shed new or different light on this subject? Is the cited research from the 1940's, 50's and 60's relevant to learning about graph comprehension in 1988?

This study tells mathematics educators that previous knowledge of the mathematical content of graphs is a good predictor of, and significantly correlated to, graph comprehension at fourth and seventh
grade. The investigator suggests that more experience in elementary grades graphing "real world" (p. 391) data "might" (p. 391) help students develop appropriate mathematical schemata. This suggests many future research questions. What kind of experiences are most effective at different grade levels? Is "real world" data better than contrived data when learning to comprehend graphs? Are there specific schemata for graph comprehension which differ from schemata for comprehension of a table of numbers? More research needs to be done before the exact nature of early graphing experiences can be prescribed accurately.

Other research questions could be based on issues raised in this study. Is it more important to teach students to read from the data, between the data, or beyond the data at different grade levels? If topic and form are not significant factors at seventh grade as they were at fourth grade, are there other factors which are significant? The mathematical content of a graph is defined to be the number concepts, relationships, and fundamental operations of the graph. Of these, which are most important, when and how should they be taught, and are there other distinguishable elements of the mathematical content?

Better understanding of how the visual information of a graph is decoded and cognitively processed will require experimental rather than descriptive studies with larger random samples and more comprehensive analysis. This study only points out the tip of a very large "iceberg."

References


Smith-Burke, M. T. (Speaker). (1979, March 5). Comprehension as a constructive process (Sunrise Semester Episode No. R19) [Film]. New York: WCBS-TV.

1. Purpose

The purpose of the study was to determine the strategies first graders use to solve simple addition and subtraction word problems, how those strategies develop, and whether problem structure influences children’s solution strategies.

2. Rationale

This study complements related research, particularly that by Carpenter and Moser (1984), that has examined the difficulty of different types of word problems and the strategies and errors children exhibit when solving them. Riley, Greeno, and Heller's (1983) classification of problems as Change, Combine, and Compare was used in this study. An expanded version of Carpenter and Moser's classification of solution strategies into concrete (direct modeling), counting, and mental (number fact) categories was employed. The present study included additional distinctions among the concrete strategies used for addition problems and among the mental strategies for addition and subtraction.

The study attempted to verify the previously observed development of children's solution strategies, to determine whether semantic structure of addition problems influenced strategy choice as it had been found to in subtraction problems in other studies, and whether the semantic structure of subtraction problems affects mental strategies as well as concrete (material) and counting strategies.
3. Research Design and Procedures

The sample consisted of 30 first graders from three classes in one school in Belgium. The children were individually interviewed in September (prior to instruction in addition and subtraction), in January (after learning to mentally solve addition and subtraction exercises with sums up to 10), and in June (after instruction on basic facts for addition and subtraction exercises with sums up to 20). The children's instruction in word problems occurred in the latter half of the year, emphasized searching for an operation "hidden" in the problem text, gave little attention to material or verbal counting strategies, and focused on a limited number of problem types.

In each interview eight Change, Combine, and Compare word problems, four addition and four subtraction, were read to the children. Manipulatives were available, but their use was not required. For each problem the child was asked to retell the problem, solve it, explain and justify the solution strategy, model the problem with manipulatives, and write a number sentence. When the children encountered difficulty the interviewer's assistance consisted of rereading the problem, suggesting the use of manipulatives, or pointing out an arithmetic error. All of the interviews were videotaped.

4. Findings

Problem difficulties essentially agreed with those of other studies. For example, Change problems involving a decrease from an unknown starting set were among the most difficult. The extent to which children developed internal solution strategies also agreed with the findings of other studies, with mental strategies being used on about two-thirds of the items by the end of the school year.

Three of the material strategies for addition problems were variations of the process of "counting-all-with-models," and a fourth
was "reversed matching," the complement of the matching strategy for solving comparison subtraction problems. These were found to be used differentially on the addition items, supporting the hypothesis that the semantic structure of addition problems influences the methods children use to solve them. Children's counting and mental strategies for addition problems also provided evidence that problem structure was related to children's choice of strategy.

Material and verbal counting strategies for subtraction problems also were related to problem structure much as they had been in previous studies. In this study, however, mental strategies were similarly related to problem structure. For example, all 27 mental strategies (employing known number facts or derived facts) used for the Change problems involving an increase with the change set unknown were indirect additive strategies rather than direct or indirect subtractive strategies.

5. Interpretations

The authors conclude that the influence of problem structure on strategy use is even stronger than that found by Carpenter and Moser (1984), with effects being found on both addition and subtraction items and continuing with mental strategies even when children no longer rely on manipulatives or verbal counting. Also, the authors hypothesize that children's strategies for solving word problems depend not only on semantic structure but on the sequence in which the known quantities are introduced in the text of the problem.

The authors state that their findings suggest that word problems can be used to develop the meaning of formal arithmetic operations, serving to promote concept acquisition rather than simply providing settings for application of the operations once they have been learned.
Abstractor's Comments

This study is to be commended for extending the findings of a well-established chain of inquiry. The authors clearly present a well-conceived study that provides important new information about the approaches first graders use to solve word problems. Although a portion of the present study replicates findings of previous studies, this study also extends them by providing a more detailed look at the relationship of problem structure to various material strategies children use on addition problems and the types of mental strategies children use on addition and subtraction problems. The study also provides further support for the contention that word problems have exceptional potential for being the vehicle for introducing children to the operations of addition and subtraction.

Although it is interesting to study and attempt to relate children's ability to restate, solve, explain and justify a solution, model the problem with objects, and write a number sentence for problems that have been read to them, the extensive nature of such a task calls into question the accuracy of children's retrospective descriptions of their work when so many subtasks are included. It would be interesting to determine whether, when the children in this study were required to restate a problem prior to solving it, they in any way restructured the problem and, thereby, used potentially different strategies from those they would have used had they only been asked to solve the problem.

The interviewer's assistance to children who could not initially solve a problem raises questions as well as suggesting an area for further investigation. It is possible that assistance such as pointing out a counting error or suggesting the use of manipulatives may have influenced the choice of strategy. For example, the suggestion to use manipulatives to represent the problem may have encouraged children to pay additional attention to the structure of
the problem. Although children who required systematic help were excluded from the analysis, no evidence is given that children who solved the problem "more or less independently" (p. 366) were not systematically influenced by the interviewer's intervention. It also would be interesting to determine the extent to which children were able to profit from that assistance, since such data have implications for instruction.

This study provides yet another instance of strong support for the importance of encouraging teachers to determine, analyze, and take advantage of the rich repertoire of solution strategies that young children have available during their initial instruction in mathematics.

References


1. **Purpose**

The purpose of this study was to investigate the efficacy of combining computer-diagnosis of subtraction errors with direct remedial instruction to detect and correct systematic (procedural) errors.

2. **Rationale**

Research has demonstrated that analysis of the errors that children make provides an effective basis for subsequent instructional decisions. Yet observation of daily strategies employed in the classroom, as well as an analysis of current mathematics texts, suggests that error analysis procedures are not practiced consistently. Thus children receive feedback relative to the correctness of their work, but direct instruction to correct specific kinds of errors is rarely provided. Although not stated as such, the implication made is that diagnosis by computer would 1) assist the regular classroom teacher in classifying children according to error types and 2) provide the basis for direct instruction intended to remediate the existing error patterns.

3. **Research Design and Procedures**

From two rural schools in which third and fourth grades were combined, only the third-graders were selected for the sample. Of the original sample, only those completing both pre- and posttests were included in the final sample of 23; 13 received the experimental
treatment, 10 the control. Bases for excluding the fourth-graders, or for the existence of such small class sizes or other sample description is not provided. It is the case that the lead author is a special educator, but whether or not the study involved special learners is unreported.

All students in both schools were administered the SKILL TESTER, a computer analyzed arithmetic test written by one of the investigators. The instrument contains test items ranging in difficulty from first- to sixth-grade equivalence covering addition, subtraction, multiplication, and division. Only the number of subtraction items (18) is reported.

Results of the computer analyses group students upon the basis of a stated common error type (e.g., "subtracts the smallest from the largest digit"). No other description of the test (item format, open vs. forced-choice, paper-and-pencil vs. computer assisted, etc.), nor possible error types (number of different types, frequency of error leading to classification, etc.) is provided in the report.

The experimental teacher initiated an "error correction period" for the investigation. The five-step process was:

1. tell students the specific error
2. show students the correct algorithm
3. guide students through a practice problem
4. students solve two more practice problems
5. students independently complete a sheet of problems

All students with the same error type received their remediation as a group. The sessions were 50 minutes, once a week, over three consecutive months. Although the focus of this investigation was third-grade subtraction, both third and fourth graders received remediation on all error-types regardless of operation. This was so the teacher would not concentrate solely on one grade or operation.
The control group, like the experimental group, was taught using district-selected basal series and workbooks during 50-minute periods. No remedial changes were instituted in the control class.

At the end of the three months, only the third graders were given the SKILL TESTER again, as a posttest.

4. Findings

Mean performance on the pre- and posttests for the control declined from 10.9 to 10.5 (out of a possible 18). The mean experimental group score of 14.0 on the posttest was a 3.5 point increase over the pretest. Adjusting for the pretest as a covariate, posttest performance of the experimental group was found to be significantly greater than that of the control group ($T = 2.69; p < .05; df = 20$).

Performance on other portions of the test, frequencies of remedial sessions per group, or number of different subtraction error groups existing are not reported.

5. Interpretations

Acknowledging that other factors could influence the results, the researchers believe that the combination of computer diagnosis coupled with time for direct instructional remediation offers "a possible direction for teachers." Further, they suggest that instituting this program at the beginning of a school year would enable the teacher to provide new learning while reviewing continually through remedial sessions.

Abstractor's Comments

The results of this investigation reaffirm the position that diagnosis of errors is more important than simply declaring that a
particular frequency of errors has been made. The use of the computer as diagnostician in the remediation process did give groupings and error types to the teacher. Relative to adding to the research literature, however, this investigation raises several questions:

1. Does computer diagnosis force students into error-type classifications which might be false?

We are not told in this report, but it is fairly safe to assume, that the test was multiple-choice. Probably (again, not told) the students are classified upon the basis of selecting the same type of distractor which has been created by applying a systematic error identified by previous research. With only three or four foils available, several systematic errors are not covered by the item. If the entire set of 18 items covers more than just three or four error-types, then how students respond to items not measuring their error type might lead to classification errors. Particularly for the remedial procedure used in this investigation, mis-classifying students could reduce the effect of the remediation.

Since we are told that the test items span first- through sixth-grade levels, a third grader might miss items due to lack of specific instruction. What would be identified by the computer as a systematic error would more apt to be a measure of the lack of transfer of training. For problems at his/her level, the student might very well not have a systematic error.

Other questions related to computer-based diagnosis include what effect does “Christmas treeing” have upon the diagnosis? and, how can we most effectively diagnose blank responses?

2. Is once in three months often enough to apply diagnosis? The report argues that analysis of children’s daily performance is important. Yet the study provided diagnosis only once, at the beginning of the three-month investigation. Related questions include:
a) Did one systematic error change to another error-type over time (e.g., zero-related error to regrouping errors)?

b) Did learning of new content interfere with past learning to create systematic errors (e.g., the multiplication algorithm being reflected in the addition algorithm)?

c) Did the computer diagnosis stimulate the experimental teacher (Hawthorne effect) to provide additional diagnosis?

3. Does telling the child his/her specific error reinforce the error?

There is research which suggests going over tests before returning student test papers in order that students will not focus on their mistakes. It might be the case that telling students the specific, erroneous algorithm tends to reinforce its existence. Although significant differences between experimental and control groups existed, the experimental group still only achieved 78% of the items, on the average. Reinforcement of errors may be one of the contributing factors for continued low performance on subtraction in third grade.

". For the rote-learning remediation provided, could both diagnosis and remediation be computer-based?

A quick perusal of the steps in the remediation process outlined above suggest that none are outside the capabilities of a computer. In fact, if computer-based, the remediation would also provide continuous diagnosis, which may or may not have been offered by the teacher (see #2, above). Granted the teacher could provide more conceptual explanations, using manipulatives, etc., but following the prescription reported for this investigation, conceptual understanding is not important and the pure symbolic approach can be computer-generated. Of course, the prescription itself could serve as the basis of future investigations.
5. To what extent was there an increase in academic-engaged time resulting from the remediation provided?

Studies have shown the significant effect that keeping students on task has on learning. Having small remedial groups under direct instruction would tend to increase time on task for them, but might have reduced it for those doing seatwork. A third group receiving enrichment, not remediation, might have provided measure relative to this question.

Abstract and comments prepared for I.M.E. by GRACE M. BURTON, University of North Carolina at Wilmington.

1. Purpose

The authors of this study wished to examine the beliefs of mothers in three cultural groups concerning the mathematics performance of their sixth-grade children.

2. Rationale

Family beliefs about achievement influence both parental and child behavior. It has been known for almost 20 years that students from the United States do not perform as well as Asian students on tests in mathematics and science. An examination of cultural differences in family beliefs about children's performance in mathematics and mothers' reports of their behavioral responses to student performance might help explain the achievement differences.

3. Research Design and Procedures

Three groups of mothers were included in this study: Caucasian-American, Chinese-American, and PRC-Chinese. Responses from the Caucasian-American parents were obtained as part of an earlier study of 67 native-born, first-born 4-year-olds from a variety of socio-economic backgrounds in San Francisco and a follow-up study of 47 of these children when they were in grade 6. The 51 Chinese-American mothers and sixth-grade children were recruited from a variety of socio-economic backgrounds in the San Francisco Bay Area. The sample was not restricted to first-born children. The 47
PRC-Chinese mothers were also from a wide socio-economic range and lived in the capital city of Beijing. The PRC-Chinese children, like the Chinese-American children, were not necessarily first-borns.

In all three sample groups, interviews of the mothers and the children were conducted separately. The maternal interview consisted of reports of (1) how well the child was doing in mathematics, (2) the strength of each of five attributional phrases (sic) for this behavior (indicated by the distribution of 10 chips on the five cards bearing the statements), and (3) what they would say if their child brought home an unusually good (or unusually bad) grade in mathematics. The child interview was parallel in construction with only appropriate referent changes. Translators were used for the PRC-Chinese participants. All questions were part of the original (and only) interview for the 47 mothers and their sixth-grade children in the PRC-Chinese sample. They were part of a telephone follow-up for the Caucasian-American and Chinese-American samples. Data were obtained from 18 mother-child pairs of the former and 41 of the latter.

4. Findings

The mothers from the three samples differed in both the causal explanations they gave for child performance in mathematics and in the responses they said they would give to high or low performance by their children. All three groups placed more weight on effort than on other causes, but this was especially true of the PRC-Chinese mothers. Blame for poor performance was more evenly distributed among all five sources by the Caucasian-American mothers, somewhat less so by the Chinese-American mothers. Both the PRC-Chinese sample and the American-Chinese sample saw training in the home as more influential than training in school. This pattern was reversed for the Caucasian-American sample. The Caucasian-American mothers tended to place responsibility for poor performance on factors not under their control (ability, luck, school training) more than did the
American-Chinese or the PRC-Chinese mothers. The data from the children followed the same patterns, with somewhat less emphasis on controllable causes.

In accounting for good performance, mothers and children in the PRC-Chinese sample gave the most credit, by far, to the schools. The American-Chinese mothers and children viewed the home as most important but gave a great deal of credit to ability, effort, and school training. The Caucasian-American sample gave about equal credit to all five attributions.

Specific responses to good performance differed greatly. The PRC-Chinese mothers tended to be much less lavish with praise than did either of the other groups, especially the Caucasian-American. The three groups of mothers responded similarly when asked about their responses to an unusually poor grade. However, while in the American groups the typical response was to ask for a reason and to offer help, about one-fifth of the PRC-Chinese mothers said they would express anger or punish the child in some way.

The multivariate analyses of variance performed showed school training to be considered the most important reason for student success or failure to a significant degree \( p < .001 \), with the second most important explanation being effort. Luck was seen to be fifth in importance, following home training and ability.

When maternal attribution of good performance was analyzed, the three groups were found to vary in the weight they placed on school training \( p < .001 \), home training \( p < .001 \), and luck \( p < .01 \). School training was seen as more influential by the PRC-Chinese than by either of the other groups, and home training was of more importance to the American-Chinese than to the other two groups. Boys and girls differed in patterns of attributions between the genders, but these patterns were not consistent across the three cultural groups. Mothers and children from each group also differed from each other.
An analysis of maternal attributions for poor performance revealed that lack of effort was chosen as most important by all respondent groups. Luck was considered the least potent explanation. Caucasian-Americans saw lack of ability as a more important reason than did PRC-Chinese, who placed more weight on effort and less on luck than did either other group. Children tended to differ in their beliefs from mothers across cultural groups, with Caucasian-American children citing luck more often than their mothers or their age-counterparts from other cultural groups.

5. **Interpretations**

For the PRC-Chinese, respect for school and reluctance to claim individual credit lead to a tendency to make individuals responsible for their failures and to share credit for their successes. The American-Chinese, on the other hand, gave more credit to the home and held the parents highly responsible for the success of their children. The PRC-Chinese tendency to treat bad grades more severely may be explained by their belief that the children did not try hard enough.

**Abstractor's Comments**

Cultural differences are of interest to anyone hoping to understand how children learn, and the authors are to be commended for seizing a serendipitous opportunity to study representatives from a country which embraces about one-fourth of the world's people. They admit to several methodological flaws that, due to the difficulties in locating members of a previous sample, they were unable to eliminate. They state that sample groups could not be equated, that similar data-gathering procedures could not be employed, and that caution must be used in using inferential statistics based on such small samples. These reservations did not prevent the authors from reporting and drawing inferences from F-values of even 3- and 4-way interactions, however. Recognizing the unwillingness of some publications to accept articles without statistical fireworks, I quietly suggest that the informative graphs and representative quotations may tell the story.
of this study more convincingly and in a more trustworthy fashion than the multitude of F-values presented.

It was somewhat puzzling that the authors cited neither the small but informative literature base of mathematics curriculum and instruction in China (c.f. Becker and Ware, 1983; Madell and Becker, 1979; Parker, 1977; Sen, 1984; Steen, 1984; Ye, 1973) nor representatives of the many articles relating to causal attribution in elementary students in China (c.f. Burton, 1986; Kessen, 1975; Robinson, 1978; Sidel, 1982; and Wenning, 1983). Discussion of this material might have provided useful background for readers of the present study.

Despite these shortcomings, this article will make interesting reading for researchers in attribution theory, scholars of comparative education, and mathematics educators.

References


Abstract and comments prepared for I.M.E. by THEODORE EISENBERG, Ben-Gurion University, Beer Sheva, Israel.

1. Purpose

The purpose of this study was to identify the processes and the procedures used by a highly skilled mental calculator. Specifically, the study examined a 13-year-old calculating prodigy with respect to: 1) the speed and accuracy with which she did multi-digit mental products, 2) the methods and verification procedures she used, 3) the base functions and numerical equivalencies she drew upon, 4) the capacity of her short-term memory and 5) the environmental factors fostering her exceptional calculating ability.

2. Rationale

Many instructional hours are currently spent in developing in students' competency in working with conventional paper-and-pencil algorithms. There is a need, however, in the everyday world, for having mental estimation skills. As such, mental calculation is becoming an integral activity of school mathematics programs. But few contemporary researchers have studied the cognitive processes involved in proficient mental calculation. This case study was undertaken to identify the processes and the procedures that characterize a highly skilled mental calculator.

3. Research Design and Procedures

Charl, as a 13-year-old eighth grader. She was recommended for the study by an older friend participating in another study on mental calculation. She was interviewed on three separate occasions.
In the first interview she was asked to perform 50 mental multiplication problems. Some of the problems were straightforward, like $8 \times 999$; others were more complex, like $123 \times 456$. The problems were presented orally with the word "times" always being used. Charlene was instructed to perform the calculation and then to explain her method of reasoning. Probing was used when her explanation lacked detail, in order to classify her solution method. The sessions were audiotaped and transcribed. Her response times were calculated from the recordings.

Because Charlene commented that she could recall some squares of two- and three-digit numbers, her memory of them was evaluated. In all, she was asked for the squares of 81 two-digit numbers, from 11 to 99.

Her ability to recall the squares of larger numbers, her short-term memory, and her ability to determine prime and composite numbers were assessed in the second interview. Starting with 101 she attempted to recall the squares of consecutive integers, skipping over multiples of ten. Fatigue set in at 349, but later in the interview she recalled this square and those of a few more three- and four-digit numbers. Her short-term memory was assessed with the digits forwards, digits backwards subtests of the Wechsler Intelligence Scale for Children. Carefully chosen odd numbers which were prime or with large prime factors, like $(899 = 29 \times 81)$, were read to her. Charlene was instructed to determine if the number was prime, and if not, to give at least one factor. Her reasoning was probed, the session was recorded, and her response times were calculated.

Charlene and her parents were questioned in the third interview about the development of her calculation ability.

4. Findings

With a median response time of 1.75 seconds Charlene correctly answered 46 of 50 multiplication items in a single attempt. The
Charlene used three general methods to solve the mental multiplication tasks. These were distributing, which she used in about 33% of the exercises; factoring, which was used in 30% of the exercises; and recalling. For example, she calculated:

i) \( 16 \times 72 \) by reasoning \( 16 \times (70 + 2) = 16 \times 70 + 16 \times 2 = 1120 + 32 = 1152 \).

ii) \( 18 \times 72 \) by reasoning \( 18 \times (18 \times 4) = 18^2 \times 4 = 324 \times 4 = 1296 \).

iii) \( 50 \times 64 \) by reasoning "half of 64 is 32, and double 50 is 100. The answer is 3200".

Charlene seemed to do no calculating whatsoever for many of the products presented; she recalled them by memory, "75 = 5625; it's a fact". She was able to coordinate the different techniques she used very efficiently, as in mentally finding the product of 123 \( \times \) 456, which she did in 50 seconds.

Her memory of squares was extraordinary. She correctly squared 69 of the 81 two-digit numbers presented, each within a second. She correctly recalled 47 squares of three-digit numbers and even a four-digit one. She often used divisibility rules and estimation techniques to verify the reasonableness of her answers.

Her ability to discern whether or not a number was prime or not was equally impressive. Again she used mental calculation, recall of products and quotients, and divisibility rules to determine if a number was prime or composite. It took her one second to find the
factors of 507, one second to state that 599 is prime, and two seconds
to factor 187. Her digit span memory placed her at the 95th percentile.

Charlene seemed to be largely self-taught, with most of her
 techniques being developed as she "played around with numbers". For
 example, she noted that \((a + 1)^2 - a^2 = a + (a + 1)\), and then she
 used this technique to find the squares of other numbers: \(31^2 = 30^2 + 30 + 31 = 900 + 61 = 961\). Her ability to calculate was not
 noticed by her parents until she was about ten years old, and it
 escaped her teachers' attention as well. Both her parents and
 Charlene believed that she was even more adept at calculating when she
 was younger. None of the other six children in the family seemed to
 have this ability, although her mother though that Charlene's
 seven-year-old sister was starting to think as Charlene did when she
 was that age and confronted with a calculation problem.

5. Interpretações

"Charlene serves as a reminder that success in mental
calculation, as in many other everyday cognitive tasks, depends more
upon the ability to select the 'right tool for the job' than upon the
possession of some innately superior mechanism for processing
information."

Interest in number patterns seems to be a prerequisite for
acquiring a high level of proficiency in mental calculation.
Deliberate memorization of techniques seems not to enter into the
development. The author addresses Charlene's perceived loss in
calculating ability and makes a plea for longitudinal studies of
proficient mental calculators.

Abstractor's Comments

This study was carefully done and the report of it is very well
written. But I wonder how much I got out of the article. I'm
introduced to Charlene, a calculating prodigy, at least with respect to multiplication--the only subject tested. The techniques she used and her methods of orchestrating them, which are well documented in the article, are well known and standard school topics. The really interesting thing is how Charlene's ability to "play around with numbers" was awakened. This was not addressed in the article. Perhaps it can't be documented, but it seems like the author missed a golden opportunity to try.

The perceived loss in calculating ability is another area which should have been explored further. The author refers to the analogy of modeling information-processing to the architectural limitations of computers where short-term memory is compared to storage capacity. With respect to Charlene's perceived loss, the model looks promising. But the idea should have been taken further.

Calculating prodigies have had a checkered past in the history of mathematics. The author should have at the very least given us an historical overview.

The author has done a wonderful job in documenting Charlene's multiplication skills. That was the easy part--the hard part lies ahead.

References


Abstract and comments prepared for I.M.E. by GEORGE W. BRIGHT, University of Houston, Houston, Texas.

1. Purpose

This exploratory study investigated the effects of Logo experience on children's understanding of length and angle. Emphasis was given to identifying specific aspects of these concepts that might be affected by the Logo experience, rather than to investigating the entire range of geometric understandings.

2. Rationale

There appears to have been little previous investigation of the changes in geometry knowledge that might be attributable to Logo experiences. Research on understandings of length and angle (apart from a Logo environment) suggests that each concept is really a constellation of subcomponents, many of which seem to be learned by children without explicit teaching. Understanding the relationships between Logo experiences and the learning of these subcomponents seems important.

3. Research Design and Procedures

This study was part of a larger study (118 students from five classrooms) of the use of Logo to create a mathematical environment. Each class had one computer, a printer, and a turtle robot. Logo was accommodated by each teacher within the curricular activities of each classroom; little direction seems to have been given by the experimenter. Emphasis was given to teaching students to program in Logo, without making any explicit ties to mathematics in general or
geometry in particular. Students worked for a median time of 75 minutes per week on projects of their own choosing.

Phase One of the study took about 10 weeks, during which students learned about the turtle robot and began to develop a goal-directed style of programming. Phase Two took the remainder of the year. Various programming concepts were introduced (e.g., procedure, variable), and distinct learning strategies were observed to be identified with various kinds of mathematics activity.

To study the understanding of length and angle, a test was developed and given to students (N = 84) in four classes (one grade 3, one grade 4, two grade 5) out of the original five classes which had studied Logo and to students (N = 92) in four other classes chosen as suitable comparisons. The test was given at the end of one-year's exposure to Logo. The comparison class for each of the three oldest classes was selected from the same school and the same grade as the Logo class and represented at least equal mathematical ability in the eyes of the school staff. The grade 3 Logo class was compared to a grade 4 class in the same school, because of a lack of another class of the same age; this comparison class was deemed to be of higher mathematical ability.

The test was 12 items, six on length and six on angle; four of the items had multiple parts. The items were taken from or modeled on items from the Concepts in Secondary Mathematics and Science (CSMS) study (Hart, 1980). For length there were three subscales: length conservation, length combination, and length measurement. For angle there were also three subscales: right-angle conservation, angle conservation, and angle measurement. Two pilots of the tests were made to refine the items.

4. Findings

Each of the six subscales were considered separately. Data were analyzed by log-linear modeling; treatment, sex, and school were used
as classification factors, though the Logo experiences did not constitute a well-defined "treatment" across classes. Most of the items showed no differences across any of these factors.

For length conservation, boys outscored girls in both Logo and comparison classes; the difference between scores of Logo boys and girls was smaller than the difference between scores of comparison boys and girls. The Logo boys outscored (previously tested) CSMS students (who were one to three years older) on two parts of one item, even though in general the CSMS students scored higher than students in this study.

For length combination, there appeared to be no important differences among groups, even though a dramatic difference between Logo and comparison students in one school created a statistical interaction.

For length measurement, the Logo students scored higher than CSMS students on comparison of measurements using different non-standard units, though the comparison students scored about at the same level. Logo girls outscored Logo boys and comparison boys outscored comparison girls, though the effect was not statistically significant.

For right-angle comparison, statistical effects again seemed to be due to large differences in favor of Logo students in one school. The Logo boys scored a little higher than the Logo girls, but the comparison boys scored about 40% higher than the comparison girls.

For angle conservation, comparison of equal angles produced no significant difference between groups. But for comparison of unequal angles (with rays for the smaller angle being longer than for the larger angle), the Logo students performed significantly better. This is consistent with the notion of angle being a rotation of the turtle's path in Logo. The difference between Logo boys and girls was
small, whereas the comparison boys performed noticeably better than the comparison girls.

For angle measurement, the Logo students outperformed the comparison students on finding the angle with the "least turn." On finding the "greatest turn," the Logo girls outscored the Logo boys slightly, but the comparison boys outscored the comparison girls noticeably.

5. **Interpretations**

For the length subscales, the Logo experiences seemed to have a positive effect for length conservation and the concept of unit measurement, at least as compared to CSMS students. However, the variations due to school suggest that age and amount of Logo time may be critical factors. Lack of differences for other items may be due to a wide variety of reasons; the data of this study do not allow clear interpretations.

For the angle subscales, the Logo experiences seemed to have a positive effect for angle conservation and angle measurement. Lack of differences for angles embedded in figures seem consistent with the findings for length measurement.

The sex differences indicate a trend (though largely non-significant statistically) supporting a greater facilitating effect of Logo for girls than for boys. That is, the Logo experiences seemed to help girls "close the gap" observed in the comparison classes.

The greater apparent effect for angle than length items is consistent with other research that new knowledge may have to "compete" with old knowledge. Logo provides extensive experience with angles that may not be provided in other contexts. Since students'
beginning knowledge about angles is less secure than their knowledge
about length, new understanding of angles may more easily displace
existing (mis)conceptions. This explanation would also help explain
the sex differences, since girls are stereotypically expected to have
had fewer "spatial experiences" than boys.

Logo experiences seem to support the development of geometric
corcepts. The improvements noted here seem to have developed
spontaneously rather than as a result of specific instructional
interventions, so more careful correlation of the experiences to
mathematics instruction might be necessary for more pervasive learning
effects arising from Logo instruction.

Abstractor's Comments

The focus of this study is very important; it is critical that
mathematics educators know what mathematical payoffs there are from
Logo experiences. Many teachers seem to use Logo with the expectation
that there will be improvements in mathematics learning, but little
data seem available to support this expectation. This study provides
hope that effects can be clearly identified.

The gender differences are especially intriguing. Can Logo
provide an environment that will support the development of spatial
skills in girls so that gender differences typically observed on
spatial tests for junior high school students might decrease? If so,
will the improvements be stable, and will improved spatial skills
support girls' taking more mathematics courses in high school?

The lack of a clearly defined treatment is a problem, at least
for this reader. What did the teachers do? If they did different
things, which is almost certainly the case, then how did these
differences show up in the data? Just knowing that there might be
differences in favor of a Logo group prevents me from being too
encouraging of teachers to use Logo to teach geometry; I would need to know what type of instruction to suggest. Too, the observed effects may really represent fairly random events.

The selection of the comparison classes also makes me uneasy; I am not convinced that the groups are really comparable, especially in terms of the kinds of instruction on geometry provided by the teachers. If this study is treated only as exploratory, then the concern about comparability is lessened. But in terms of interpreting the results for other classrooms or for regular classroom teachers in my part of the world, the lack of comparability creates quite real problems.

The use of CSMS items is nice. The built-in comparison to CSMS data provides a way to strengthen the interpretations. Other researchers should be encouraged to follow this lead and to use existing items for future studies.

One overlooked interpretation that might be put on the results is that they represent a practice effect of the dynamic nature of Logo. That is, the items showing the most difference seem to be those most closely related to the dynamic nature of turtle graphics. Perhaps the superiority of the experimental groups simply represents the additional exposure to dynamic graphics displays. If so, then we need to search for ways to demonstrate concepts dynamically, other than through Logo.

Follow up to this work is essential. Developing a clear conceptualization of ways to integrate Logo into mathematics instruction is important; currently those ways are not at all clear. I am encouraged that mathematics learning might really be enhanced by instruction in Logo. But the generation of data to demonstrate this clearly has only just begun.
Pederson, Katherine; Elmore, Patricia; and Bleyer, Dorothy. PARENT ATTITUDES AND STUDENT CAREER INTERESTS IN JUNIOR HIGH SCHOOL. Journal for Research in Mathematics Education 17: 49-59; January 1986.

Abstract and comments prepared for I.M.E. by MARGARET KASTEN, The Ohio State University.

1. Purpose

The major purpose of this study was to investigate parent attitudes and student career interest relative to their contribution to a theoretical model of mathematics achievement in junior high school. A second purpose of the study was to investigate the multivariate relationship between student attitudes and parent attitudes, between student attitudes and student career interests, and between parent attitudes and student career interests.

2. Rationale

Researchers have identified a variety of variables related to mathematical achievement. For high school students, variables which have been found to be correlated to mathematics achievement include: attitudes toward mathematics, spatial visualization ability, sex, parent attitudes, career interests, and participation in mathematics courses. While many of the variables have also been established as correlates with mathematics achievement in junior high school, little research has been done relative to the contribution of parent attitudes and student career interests to mathematics achievement in junior high school. The researchers felt that there was "sufficient evidence to suggest that such research might be fruitful."

3. Research Design and Procedures

The sample consisted of 497 boys and 477 girls in seventh grade and 520 boys and 488 girls in eighth grade from 13 midwestern junior high schools. Variables and the methods of measurement were:
a. Mathematics achievement measured by the national percentile rank on a standardized achievement test

b. Spatial visualization ability measured by the space relations test of the Differential Aptitude Tests

c. Sex, a dichotomous variable

d. Parent attitudes measured by each parent's responses to the Math as a Male Domain Scale and to an adaptation of the Mother Scale or the Father Scale of the Fennema-Sherman Mathematics Attitude Scales

e. Student Career interests measured by the Unisex ACT Interest Inventory

The tests, scales, and inventories were administered by the researchers, data on mathematics achievement and sex were obtained from school records, and students took home and brought back the parent questionnaires.

Multiple regression analysis was used to determine which of the variables -- spatial visualization ability, student attitudes, sex, parent attitudes, and students career interests -- are predictors of mathematics achievement over and above the contribution of the others. Canonical correlation analysis was used to study the relationship between pairs of variables.

4. Findings

The contribution of parent attitudes to the variance of mathematics achievement was significant over and above the contribution of the other variables in the model. It was also determined that student career interests were a significant predictor of mathematics achievement over and above spatial visualization ability, student attitudes, sex, and parent attitudes. Additionally,
it was found that spatial visualization ability and student attitudes each contributed significantly to the variance of mathematics achievement over and above the other variables. Three canonical variates were identified between student attitudes and parent attitudes, three between student attitudes and student career interests, and one between student career interests and parent attitudes.

The interpretation of the canonical variate found between student attitudes and parent attitudes indicated the following relationships:

1. Student attitude measures of confidence, teacher, usefulness, mother, attitude toward success, effectance motivation, anxiety, and father are related to mother's and father's attitude toward their child as a learner of mathematics.

2. A positive correlation was found between student attitude toward mathematics as a male domain and mother's and father's attitude toward mathematics as a male domain.

3. The third relationship was between student attitude measured by the Father Scale and a father's attitude toward mathematics as a male domain and the father's perceptions of his child as a learner of mathematics.

The variates between students attitudes and student career interests are:

1. A positive correlation was found between all student attitude measures except anxiety and career interest in science, services, and business operations.

2. A relationship existed between a high score on mathematics as a male domain and a lack of interest in business operations and technology.
3. The third relationship was found between high anxiety towards mathematics and interest in arts and technology.

The final positive correlation that was suggested was between student interests in science and business operations and the mother's and father's perception of the child as a learner of mathematics.

Abstractor's Comments

The researchers attempted to investigate a variety of things which they conjectured might be correlated to the mathematics achievement of junior high school students. Because the mathematical achievement of junior high school students is deemed below desirable levels on a variety of measures, study of achievement at this level is important.

The authors collected data on many types and ran a variety of statistical analyses via the computer. The fact that they were able to find some significantly positive correlations in this process indicates a need for further study in this area. Often the correlations, though statistically significant, were at levels that supported further study, but could not by themselves be said to indicate a "high" correlation between variables.

Several elements in the study or the analyses seen: subject to question or clarification, such as:

1. The use of standardized achievement tests, which may or may not accurately reflect the curriculum, needs further thought, especially since the test used was not the same across the sample.

2. The student population was drawn exclusively from small rural communities, which would seem to limit generalizability.
3. The return rate on the parent questionnaire was a high 85%, but questions still remain concerning the 15% who did not respond.

4. The computer has made it possible to do a lot of "data fishing." That is, it is now possible to collect large amounts of data, do statistical analyses with a lot of iterative processes, and produce the statistically significant correlates. This procedure is important and useful in educational research, especially when formulating hypotheses. But care must be taken not to substitute computer power for well-thought-out hypotheses, nor statistical significance for educational significance.

5. The authors are careful not to imply causal relationships from the correlational data. Such care must be reinforced whenever results of correlational studies are reported. This is especially true in a study of this type that would be of interest to the lay public.

6. The grouping of student attitude variables under the label of self-confidence does not seem to be fully explained or justified.

Overall, the study seems to have made progress toward the stated goal of the development of a theoretical model of achievement in junior high school mathematics. One would hope that the hypotheses suggested by the results would be followed up with a series of more narrowly focused studies, with careful attention given to defining and measuring achievement, to the identification of a more universal population, and to variable definition and explanation.
1. Purpose

The study was conducted to test the widely held assumption that there is, in both mathematics and reading, a meaningfully strong causal relationship between attitude and achievement.

2. Rationale

Although many researchers have suggested that a positive student attitude toward mathematics or reading contributes meaningfully to achievement, previous research has not provided clear evidence for such a relationship. Past correlational studies looking at mathematics attitude and achievement have generally found 15% or less common variance between the two variables. Similarly studies of reading attitude and achievement have found 10% or less variance in common.

Because most of the correlational studies have provided very little evidence on the existence and direction of a causal relationship between attitude and achievement, it has been suggested that a cross-lagged panel analysis be performed to help clarify the cause-effect situation.

3. Research Design and Procedures

Cross-lagged panel analysis is a quasi-experimental procedure that compares the various correlations that can be calculated when two variables are measured at two or more points in time. The relative sizes of the correlations are supposed to provide a logical type of evidence about the direction of causality.
Such a procedure was used to perform a secondary analysis on data from three previously conducted studies: a) the Beginning Teacher Evaluation Study, which was designed to identify effective instructional characteristics and teacher attributes in grades 2 and 5; b) a study of the effects of teaching phonics to elementary school students (second-grade students were used in the cross-lagged analysis; and c) an evaluation of a compensatory education program called Special Elementary Education for the Disadvantaged (students in grades 3-6 were included in the re-analysis).

Fifteen sets (panels) of correlation coefficients were computed from the three studies. The appropriate pairs of correlation coefficients were tested for statistical significance using the Pearson-Filon test for dependent correlations.

4. Findings

None of the cross-lagged correlations were significantly different; i.e., the analysis did not find any predominant causal relationship between attitude and achievement. The findings also indicated that it is unlikely that there is an equal, reciprocal, causal relationship between attitude and achievement.

5. Interpretations

It was concluded that changes in attitude toward mathematics and reading would not necessarily lead to changes in achievement and vice versa. It is possible, however, that attitude may be causally related to achievement through a third variable. Any causal relationships that might be found would be expected to be weak.

Abstractor's Comments

By providing evidence that attitude and achievement may not be causally related, the authors have given us cause to reassess an assumption that many educators hold. Although I had a few concerns
about the precise logic of their overall approach, they did use methods which are basically state of the art.

Among the concerns I had were: a) Rogosa's (1980) severe criticism of cross-lagged analysis (the authors cited his article but did not specifically address his criticisms); b) the relatively unknown quality of some of the tests (e.g., when developed by the project) used in the prima, rtuaies; and c) the authors' not considering the use in the future of a well-designed experimental study to address the causality question.

Despite my concerns, I found the article enlightening and a refreshing caution backed with evidence against accepting a commonly held assumption.

Reference


Abstract and comments prepared for I.M.E. by JOHN G. HARVEY, University of Wisconsin-Madison.

1. Purpose

The study examined the relationships between college student problem-solving achievement and problem context. In this study the students were individually permitted to choose from one of four contexts for the instructional treatment they received.

2. Rationale

There is a substantial amount of research that links problem-solving performance to the context in which the problems are stated. This link may account, in part, for the difficulty that students are having in solving mathematics "story" problems; that is, that poor performance on story problems may be less a function of computational skills and more one of an inability to read, translate, and comprehend the problems. Thus, it has been hypothesized that performance on story problems might significantly improve if more familiar, personally relevant context themes were used. In an earlier study, Ross (1983) varied the thematic context of problems in a probability unit; one treatment related concepts to education, another to medicine, and a third to abstractions. Results of this study showed that education majors learned best from the education contexts and nursing students, from the medical contexts. Ross, McCormick, and Krisak hypothesized that achievement might be further improved if students were first permitted to choose the context for the problems they receive during their study of probability concepts.
3. Research Design and Procedures

This study was replicated with two groups of college-level subjects. One group of subjects (n = 80) was nursing students enrolled in a required educational statistics course; the sample consisted of students who volunteered to participate for extra course credit. The second group (n = 50) consisted of undergraduate education majors enrolled in a required human development course who, too, were promised extra credit for participating.

There were four instructional treatments; within groups, subjects were randomly assigned to treatments. In the standard-adaptive treatment, all of the subjects were taught using a context appropriate for that group (i.e., medicine or education). In the standard-nonadaptive (i.e., abstract) treatment all of the subjects were taught using an abstract context. The subjects who participated in the learner-control adaptive treatment first rank-ordered the four context options from most to least favored; the context options were: medicine, education, sports, and abstract. During the learner-control adaptive treatment each subject was taught in his or her first choice. The learner-choice nonadaptive treatment taught each subject in the thematic context (i.e., medicine, education, or sports) ranked as least favored by that subject. The instructional treatments and the administration of the test instruments occupied one class period. The content taught by each treatment was the same: four introductory probability rules. Each rule was presented as a separate unit consisting of a definition, a mathematical formula, and five supporting examples.

There was no pretest of student knowledge of the four rules. Three measures were administered. After the first three examples of Rule 4, a 6-item attitude measure and a recall test were administered. The attitude measure is not described. The recall test consisted of asking subjects to write down as much as they could remember about each of the three Rule 4 examples they had studied; the responses
for each example were scored as 0 or 1 in four categories; those categories were: (a) any information given, (b) thematic information recalled, (c) thematic information and all numerical values given, and (d) the example's correct answer was stated. By category the scores were summed across examples.

The achievement posttest consisted of 20 items. On this test there were five items per rule; each item within a rule subset was different. Three of the items were mathematical problems identical in format to the examples presented in the instructional treatments; the items different in the context used: medicine, education, and abstract. The fourth item in each rule subset was a multiple-choice one that asked subjects to identify a correct rule formula. The fifth item was a transfer-of-knowledge problem.

4. Findings

Nursing subjects. For the attitude and recall measures, the data were analyzed using ANOVA. There were no significant differences between treatment groups on the recall measure. There were significant differences (p < .001) between the adaptive group mean and the nonadaptive group mean; the adaptive group mean was higher than the nonadaptive group mean. There were no significant interaction effects between material selection (i.e., learner vs. group choice) and contextual adaptation (i.e., adaptive vs. nonadaptive).

The achievement data were analyzed initially using a 2 x 2 MANCOVA (material selection by contextual adaptation) with the five test items as the dependent variables. Subsequently, these data were further analyzed using a 2 x 2 x 5 mixed ANOVA, with item type as the within-subjects factor. Finally, post hoc analyses were performed using Tukey's HSD test. The initial MANCOVA indicated that there were no significant main effects or two-way interaction. The mixed ANOVA yielded a significant test item main effect (p < .001) and a
significant contextual adaptation \times \text{test item interaction} (p < .01). For the test item effect, Tukey’s HSD test showed overall performances on abstract (M = 82% correct), education (M = 78%), and medicine (M = 75%) exceeded those on formulas (M = 60%) and transfer problems (M = 51%). Formula scores were significantly higher than transfer scores.

The conceptual adaptation scores were analyzed using simple t tests. The adaptive group (M = 58.8%) was superior to the nonadaptive group (M = 42.3%) on transfer items.

Education students. The same statistical tests were performed on the data gathered from these subjects. There were significant treatment differences on the first and second recall measures; there was a significant material selection by context adaptation interaction on the fourth recall measure. There were no significant attitude effects.

The following significant effects resulted from the mixed ANOVA: test item (p < .001), material selection (p < .05), and three-way interaction (p < .05). As for the nursing students, the test item main effect showed significantly higher performances on abstract, education, and medical items than on formula and transfer items. In addition, there was a conceptual adaptation main effect that showed higher overall achievement by the adaptive group (M = 73.2%) than by the nonadaptive group (M = 60.27%).

5. Interpretations

For both the nursing and the education students, familiarity with the content was an influential factor in learning the mathematics content, and when the context was familiar, achievement was improved. Results from the two groups differ, however. Contextual adaptation effects were found only for transfer items in the nursing group but across all of the item types in the education group. The recall of
examples was superior for the adaptive context only in the education students. Attitudes were favorable toward adaptive contexts in only the nursing student group.

The differences in outcomes between the two groups may be attributable only to error variance, but differences between the two groups must also be considered. The nursing students were older and more advanced academically than were the education students, and because they were enrolled in an educational statistics course, they may have been more experienced in solving mathematics story problems. Attitude differences favoring the adaptive treatment were found only for the nursing sample; this may be because they do not ordinarily find materials outside their majors so clearly oriented to their preferences, while education students may not have viewed the adaptive materials as unusual or relevant.

Since the material selection variable was not influential, the effect of the variable upon learning, especially less homogeneous students, is still unknown. The authors urged that future research should test this hypothesis.

Common practice in mathematics (e.g., in textbooks) seems to be one of introducing new material in unfamiliar or technical contexts; to the investigators, the result of this practice seems similar to their learner-choice nonadaptive treatment. In contrast, the conceptual adaptation treatment employed in this study seems directly supportive of processes that learners must be able to use in problem solving.

Abstractor's Comments

The question investigated is an important one; it also seems to be one that has not been extensively studied in mathematics education. Thus, this study seems relevant; it was well designed and carried out. The results from the study affirm that, at the very least, mathematics
teachers should be knowledgeable about their students' interests and prior problem-solving experiences. As the investigators point out, at the school level it may be impracticable to place problems in an appropriate context for every student.

There are some questions that need answers and some concerns that need consideration.

1. If we consistently find that students learn better from word problems whose contexts are the ones chosen by them, how do we incorporate this knowledge into mathematics instruction without further "atomizing" problem solving? At present and all too often students do not understand that the techniques they use to solve one set of problems is exactly the same as the ones they use on another set—even when they are very able at solving both kinds of problems.

2. The time allotted to both treatment and testing in this study was one class period of an unspecified length. The length of an instructional treatment seems to be an important variable, and so we do not know that the differences favoring the learner-adaptive treatment would persist under more "normal" circumstances.

3. Do we really wish to restrict further the contexts in which we present mathematics problems? It seems to me that if problem-solving is context-bound, as this study suggests, then we need to study more effective ways of teaching students how to learn about contexts.

4. Perhaps it was intentional, but the examples given from the instructional materials were often ambiguous and can be improved. This was given as Example 1: Education Context:
A student makes a completely random guess on each of two multiple-choice items containing three alternatives. The probability of randomly guessing the correct answer is thus \( \frac{1}{3} \). What is the probability of randomly guessing it on both items?
1. Purpose

The purpose of this study was to determine how well students who have completed a year-long course in geometry write geometric proofs.

2. Rationale

While writing proofs is a central focus of the traditional geometry curriculum, how well students actually write geometric proofs has remained largely conjectural. None of the three National Assessments of Educational Progress assessed students' proof-writing ability. While some researchers had attempted to measure students' ability to write proofs, their studies have been made with small samples and most are quite dated.

3. Research Design and Procedures

The author constructed three forms of a six-item test on writing geometry proofs. The items were grouped so students were asked to do the following:

(1) fill in missing proof statements,
(2) translate from verbal statements to a figure, and
(3) write complete proofs.

A total of 1,220 students in seventy-four geometry classes in five states were given the test about a month before the end of the school year. Most students were tenth graders (62%) with the remaining students divided approximately equally between grades nine
and eleven. Students were from honor classes (24%), regular classes (46%), and single-track classes (30%). There were 49% females and 51% males.

Tests were randomly assigned in each class so approximately one-third of each class had each of the three forms of the test. Students were given 35 minutes to complete the test. Using a scale of 0, 1, 2, 3 and 4 where 4 was a completely correct response, reliability estimates for the three tests were 0.86, 0.85, and 0.88. Grader consistency was measured by having two graders grade the test papers. Analyzing the three forms of the test separately, the correlation between the two graders was between 0.81 and 0.95 for the six items, with an average of 0.86.

The percentages of students solving the various test items were reported. Sex differences were also investigated.

4. **Findings**

Using a one-way analysis of variance on total test scores, the three test forms were not equivalent ($\gamma^2(2,1517) = 8.73, p < 0.0002$). The proof-writing achievement data were analyzed separately for the three forms of the test.

The percentage of students able successfully to solve an item, i.e., scoring three or four points out of four, ranged from 6% to 77% on the six items on the three forms of the test. The percentage of students not able to solve an item, i.e., scoring zero or one point, ranged from 1% to 57%.

Writing a proof was more difficult than filling in the missing steps in a proof and than translating a verbal statement to an appropriate figure.

"Approximately 30% of the students in full-year geometry courses that teach proof reach a 75% mastery level in proof writing" (p. 453).
About 25% of the students have virtually no competency in writing proofs, i.e., were unable to write any of the proofs; about 25% of the students could write simple proofs; while about 30% of the students showed mastery of proofs.

No consistent sex related differences in proof-writing achievement were found.

5. Interpretations

The baseline data on students' ability to write proofs are discouraging. As only about 50% of high school graduates in the United States complete a year course in geometry, the results suggest that fewer than 15% of the high school graduates from high school in the United States master writing proofs.

The author feels the results imply that there is a need to improve the teaching of the traditional material in high school geometry. The author also feels that there is a need to examine if the traditional content of high school geometry is appropriate with its emphasis on writing proofs.

When teaching proofs the author suggests the need to assist students with (1) starting a chain of deductive reasoning; (2) understand a proof as many students used the theorem to be proved as a step in the proof; and (3) understanding how to use embedded figures and auxiliary lines in diagrams with a proof.

Abstractor's Comments

When reading this article one must realize that the journal, the Mathematics Teacher, is not primarily a research journal. For example, the statistical analysis of the sex differences are not given but are referenced as being in the author's dissertation.
no statistical analyses reported or cited to support the claim that writing geometry proofs is harder than completing the proof statements or translating from words to an appropriate diagram. In fact Table 1, which gives selected percentage scores for selected items on different forms of the test, has no data for the item on translating from words to an appropriate diagram!

While some of these omissions are serious, given the nature of the journal this paper would seem to be an exemplary paper that links current research on an important issue to teaching. The article investigates a fundamental issue to the secondary mathematics curriculum, namely how well do students write geometric proofs and what do the results suggest about teaching? The article is very well written, and the geometry proofs given seem very appropriate for what is traditionally taught in geometry. The article is surprisingly rich in statistical detail. For example, reliability and grader consistency estimates are given. While the study does not directly address the teaching of geometry or how to improve the teaching of geometry, the author provides a nice summary in the Recommendations section of other current research on geometry with possible implications for teaching practice.

In summary, this is an important paper that deserves to be read by those teaching or researching the teaching or learning of geometry. It raises some fundamental questions that need to be researched as part of any reform of the curriculum or teaching of writing proofs in geometry.


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