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ABSTRACT

Past studies of compensatory education programs in mathematics show that they have not addressed mathematical concerns, have not had clear program goals, have had a narrow view of what constitutes evaluation, and have taught little that would improve students' mathematical abilities. In the future, compensatory programs for mathematics should address the following six issues: (1) the fragmentation of mathematics and the need to show the interconnectedness of ideas; (2) learning as passive absorption; (3) deskilling of teachers; (4) differential opportunity; (5) workbooks/tests as technology; and (6) change as ritual. In developing a contemporary mathematics program, one must consider that the roles and work of student and teacher are complementary and the emphasis should be on creating knowledge rather than just absorbing the history of other people's knowledge. The teacher's goal should be to support, promote, encourage, and in every way facilitate the creation of knowledge by students. The report also discusses how conflicting ideas about individual differences affect the development of compensatory mathematics programs. A list of references is included. (PS)

MATHEMATICS FOR COMPENSATORY SCHOOL PROGRAMS

by

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> Mention of the word school, particularly elementary school, often generates a set of images including red brick buildings; matronly teachers; freshly scrubbed, smiling children; well worn books and some disarray in desks; dusty blackboards; and boisterous recesses. The images we all have could go on and on. They are a product of our upbringing. Schools in other sections of this country and even in other countries seem familiar because most of the same images are present. The physical surroundings may differ, but children and teachers and books remain and there is a facility called a school. (Romberg, 1985, p. 3)

Schools for all children are historically recent and were created in large part to transmit some pre-established knowledge and skills to the young and to inculturate them more quickly and systematically into the prevailing social system. As schools in this culture developed one of the assumptions upon which they were built was that students at a particular age are more similar to each other than they are different (Romberg, 1985). Although there is much rhetoric about attending to the individual needs and desires of each child, the actual groupings of children have rarely reflected those concerns. For example, in a typical elementary school all children within age are subdivided into sets containing 20-30 members and assigned to a teacher for a full school year--a self-contained age-graded classroom. Furthermore, the organization, content and development of the set of lessons that are to be worked on by all students have been developed from a White middle-class perspective.

As America continues to grow in size and diversity the groups of children in classrooms are widely heterogeneous in their abilities, their personalities and their backgrounds. Thus, this similarity assumption so fundamental to how schools are organized is false. During the last quarter of a century educators have begun to face up to the problem that we can no longer assume that the teaching of one curriculum via one set of lessons will best serve all students in our classrooms. Procedures have been developed, or are beginning to be developed, to accommodate the variety of backgrounds of individual children. It has been argued that children from low-income and minority families are less well-prepared than their middle-class counterparts to profit from typical school instruction, particularly in reading and mathematics. As a result programs have been developed since the 1960s to help children who did not have appropriate preparation for the existing curriculum. Given that all social legislation related to compensatory programs to help underprivileged students has been enacted with laudable intentions, it is not my intent to criticize their intents. Rather, what is addressed is the reality of mathematical compensatory programs and how they meet or fail to meet those intentions.

Review of Past Practices

In this section I have not reviewed the detailed characteristics of the compensatory programs for school mathematics during the past quarter century. I believe we need careful and well done analysis of these programs which would include the evaluations of their impact. However, that task is certainly beyond the scope of this paper. What is clear is that lots of compensatory programs have been developed by local school districts. Also, if Federal or state funds were used in their development they were "evaluated". By that I mean some data were gathered with respect to what happened in schools when the particular program was followed. From an ERIC search, over 221 reports on compensatory mathematics programs were located. In addition, five summaries of program characteristics of "programs that worked" were found and examined (Fairley, 1978; Grant & Hoeber, 1978; Lyons & Whitebear, 1978; Park, 1980; and Mullin & Summer, 1983). Finally, a few key scholarly studies were identified (e.g., Kaplan, 1966; Neil, 1978; Alderman, Swinton, & Braswell, 1979; Cooley & Leinhardt, 1980; Kenoyer, Cooper, Saxton, & Hoepfner, 1981; Ragosta, 1983; and Carter, 1984). I am not confident that the most important studies have been found, but I am confident that the comments and issues I am raising are relevant to the approach taken in most compensatory programs as a whole.

The perspective I have used in examining the variety of studies and reports comes from the sociological notion that schools can be described as: a place where work for both teachers and students is organized and defined, and where school work is related to a conception of knowledge (in this case mathematics) which is being distributed by teachers to students. My approach was first to examine the conception of mathematics exhibited viewed in these studies. Second, I reviewed how that conception of mathematics had been translated into activities for teachers and students.

To summarize my findings, first I have made four comments about the studies I have reviewed. Following that I raise six issues which I think must be addressed by those interested in the mathematics in compensatory education programs in the future.

Comments

The following comments have been made to focus attention on what I think are some of the interesting and even disturbing aspects I found from my selective review of past work in the field.

1) Mathematical concerns were not addressed. Mathematics appears in the title of many reports and certainly is mentioned in the overview or introductions. However, there is no real discussion of mathematical concerns in any of the papers. Occasionally, there was a listing of arithmetic skills that are to be taught (often to be mastered one by one). But, there is no analysis of the mathematical deficiencies of low-income children or what constitutes the important ideas from mathematics they (or all children) should know. It is as if mathematics was a commonly understood and agreed upon domain important for all. While I agree that some mathematics should be learned by all students, I found it disturbing that no one challenged this assumption or even suggested there may be a debate about what constitutes basic knowledge in mathematics. Each paper deals with procedures of how to improve students without stating what the mathematics was they were to improve upon. In fact, I had to make inferences about the mathematical topics covered and the approach to mathematics in the studies from their procedural descriptions.

2) Goals were not clear. Since education is goal directed, educators can never be free from questions or problems related to the aims of education. However, the approach to compensatory education in these studies contained no statements of goals or even a description of a desirable end product. In fact, the implied goal in many papers was only "improved test scores" even though no one argued for the validity of the test used. Part of the reason for this deficiency is the nature of American society that resists consensus on what the goals should be. It would simplify the problem if one could say these are the goals of compensatory education and then we could design programs accordingly. While this might be conceivable in some societies, this approach is undoubtedly out of the question in our own. Nevertheless, for the variety of programs that exists, it would have been helpful in trying to pick my way through the lengthy descriptions of procedures to have had a better understanding of the specific goals a particular program was actually designed to meet. However, the papers mainly were descriptions of procedures to be followed in the classrooms and test scores.

3) The meaning of evaluation. The term "to evaluate" means "to judge the value or worth." One problem with these studies is that most have presented a very narrow view of what

constitutes evaluation. This is probably due to a requirement that they must gather some information about the effects of programs. However, the evidence gathered is extremely limited. In fact, this limited perspective raises serious questions about the validity of the arguments being made to support any findings. For example, in many studies the only evidence presented is change in mean score data from pretest to posttest. This is a sparse source of evidence for two reasons. First, the tests used to gather the data are of questionable validity. A typical study used a standardized test. No one built a case for the validity of such instruments. Second, given that such tests were used more than change in mean score should have been presented. For example, in individualized-independent learning programs there ought to be variability in rate of learning. If so, then there should have been both an increase in mean scores and an increase in variability. This could easily be shown in a scatter plot relating the pre- and posttest data. The slope of the regression line should be significantly greater than one (see Figure IV-1). On the other hand, mastery learning programs should reduce variability. Thus, the scatter plot between pre- and posttest scores should indicate a regression line with a slope considerably less than one (see Figure IV-2).

Some studies attempted to rule out alternatives. They used quasi-experimental designs with nonequivalent control groups. Then analysis of covariance was done to adjust for initial differences. Usually the covariate was some ability test. The difficulties of using analysis or covariance with nonequivalent groups are well known. These studies do not control enough variables so that one can rule out alternative explanations for change or growth.

In a few studies, for example, the Instructional Dimension Study (Cooley & Leinhardt, 1980) a lot of data were gathered and regression analysis was used to find predictors of change. This procedure is inadequate if there are prior guesses about effects (e.g., those assumptions underlying mastery learning or individualized-independent learning). A much better approach would be to build causal models and test these models.

Of note was the study reported by Alderman, Swinton and Braswell (1979). They gathered different types of evidence to build a case. Not only did they gather test scores but they also examined those test scores in depth and interviewed students to gather information which would not necessarily have been apparent from test scores.

Overall, the evaluation methods used in this set of studies were naive and inadequate. I have no question that lots of interesting things were done for these students with good intentions. Some activities undoubtedly had a positive

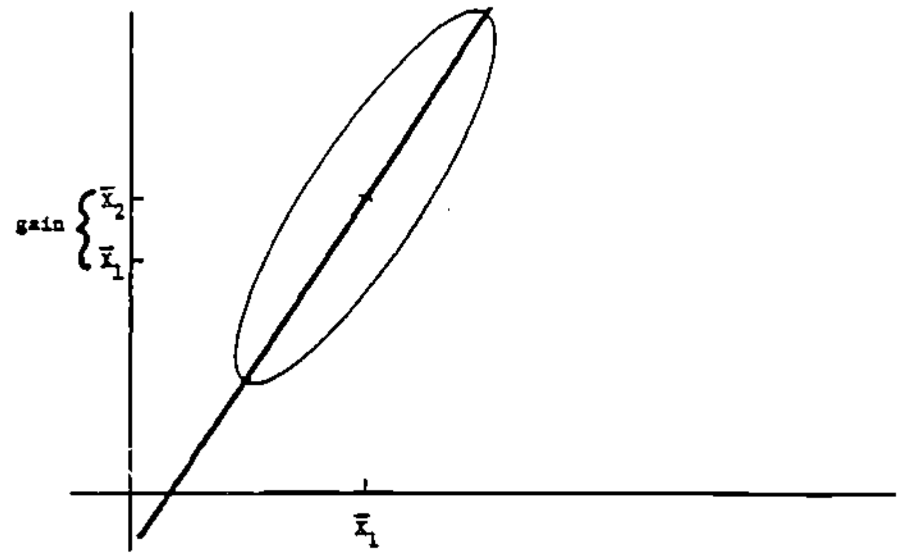

Figure IV.1. Pre-post Test Scatter Plot for an Individualized-independent Program

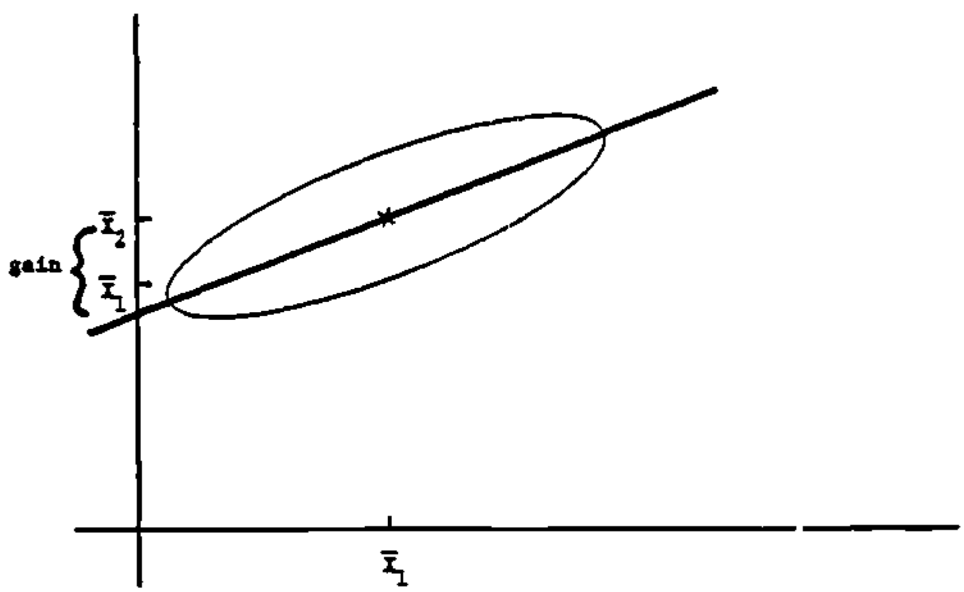

Figure IV-2. Pre-post Test Scatter Plot for a Mastery Learning Program effect. However, the evidence to support any position about activities with these students is weak. Some reported positive findings undoubtedly are illusory. Others probably had real effects but the invalidity of the tests led them to fail to conclude that the procedures were positive.

4) Categories of programs. Based on my readings I organized the programs into three broad categories: enrichment programs, differential programs, and developmentally based programs.

A number of general enrichment programs were built following the argument that low-income children lacked a variety of experiences and needed those experiences and intellectual challenges in order to make them similar to the middle-class students. In fact, many of the early Head Start programs (e.g., Kaplan, 1966) were based on this approach. Children were given toys and games that stimulated their senses and encouraged their reasoning skills. Much of their effort had to do with language skills and what little was done in mathematics seemed to deal mostly with counting and simple calculation. This undoubtedly was the approach followed in middle-class nursery schools which was transferred to compensatory programs. Programs of this type have no longer continued (or have had less emphasis since the 1960s) because the approach is probably futile. A few hours in school are not going to change the cultural and experiential background that children gain outside of school. Furthermore, this approach is probably too indirect to meet the needs of children from these poor families.

Differential programs, begin with the assumption that if children differ, then they need to be treated differently. Operationally this means changing the organization and procedures in classrooms. The most difficult task schools have is how a small number of adults can organize and can manage a large number of children. If children are similar, then they can be grouped for whole-class instruction. If they are different, what can be done? The two procedures that evolved were "independent-paced" instruction and "highly-structured" instruction. Also, in either case there were two instructional strategies which were followed. In most classes, Title I students were pulled out of class for separate instruction. In a few instances students were kept in their classes but worked independently in a separate group.

In independent programs the only difference between children taken into account was rate of learning. In these programs a subset of arithmetic was organized via behavior objectives. The list of objectives was operationally defined as the mathematics to be studied. I assume the objectives were developed around higher hierarchical schemes, some incorporate notions of mastery learning. Most used standardized tests to judge outcomes, although some used objective-referenced tests, and most did not describe the specific mathematical objectives or their sequences in any detail. The paper by Ragosta (1983) is an exception. An outline of a set of mathematical objectives covered in a computer based learning program is given. The computer is only a management tool for helping students through this independent method of instruction. In this regard the computer only provides the teacher with a better way to distribute worksheets. It does not provide students with different tasks to perform.

It is not clear in these programs why rate of learning is the appropriate variable which differentiates children from low-income families from others. Nor is it clear why the specific procedural skills of arithmetic constitute the body of mathematics to be taught to these children. Nevertheless many of the programs were able to demonstrate increased scores on tests related to the specific arithmetic objectives being taught. However, it is my guess that these increased scores are illusory. They only indicate that children have improved on a very small part of mathematics, but have not really gained in knowledge. For example, Alderman, Swinton, and Braswell (1979) reported that students in computer based compensatory mathematics group attained higher posttest scores on a curriculum specific test than a control group. However, they made no fewer total errors on the test. Instead exposure to the computer only improved the student's proficiency in taking tests in that they omitted fewer items. This result conforms with the emphasis in the computer curriculum on drill and practice. By design the treatment neither gave the teachers new concepts in mathematics to teach nor a method to diagnose children's misconceptions about mathematical processes.

The "highly-structured" programs were often modeled after the principles of Engleman and Bereiter (1966). Here arithmetic skills were taught to groups of children using direct drill methods. Other programs were based on the group mastery learning model (Bloom, 1968). Unfortunately, in both types of programs simple drill does not remedy student weaknesses in understanding mathematics. For example, when interviewed individually, the students in Alderman et al. (1979) seemed to view numbers in operations as abstract entities and to have access to few meaningful representations. There was considerable emphasis on right answers rather than on appropriate processes.

Perhaps a more famous study is Erlwanger's (1978) on Benny's conceptions of rules and answers in Individually Prescribed Instruction (IPI) Mathematics. In that study he interviewed several students, Benny being one, on various notions of mathematics. Overall Benny's conception of mathematics was that mathematics was a large collection of skills to be mastered with no connections between skills.

My third category is developmentally based programs. These deal with those based on developmental psychological theories (particularly those of Piaget). For example, in the program developed by Kamii and De Vries (1978) a variety of tasks similar to those developed by Piaget was used both to ascertain the child's level of cognitive development and as inherently important cognitive accomplishments. The distinguishing feature between this approach and other programs is its emphasis in determining the child's thought processes. Teachers are then supposed to act in accordance with the child's level of logical conceptual thought. Unfortunately, I found no examples of studies that were psychologically up-to-date. More recent psychological ideas based on cognitive science which might be useful are not mentioned. For example, there are several modern descriptions of cognitive processing (e.g., Wagner & Steinberg, 1984).

In summary, my cursory review of compensatory mathematics programs was disturbing. In fact, if one views mathematics as things human beings do such as abstracting, inventing, proving or applying (Romberg, 1983) there is nothing in the programs I have reviewed that would give low-income students an opportunity to do any important mathematics.

Issues to be Addressed

If compensatory programs are to be developed in the future which respond to my concerns, the following five issues need to be addressed. If considered, debated and resolved, then I believe a mathematically sound program can be developed which would provide all students an opportunity to learn mathematics.

1) The fragmentation of mathematics. Mathematics to most students is a static collection of concepts and skills to be mastered one by one. Furthermore, the student's task is to get correct answers to well-defined problems or exercises. Compensatory mathematics programs seem to have done little to change most teachers' or students' perception of the subject for several reasons. First, mathematics has been over fragmented. To develop a curriculum, one needs to segment and sequence the mathematical ideas for instruction. However, in many recent efforts, this has been taken to an extreme. The use of behavioral objectives and learning hierarchies, such as advocated by Gagne (1965), and operationalized in many individualized programs, such as IPI (Lindvall & Bolvin, 1976) and in turn reflected in many compensatory programs, has separated mathematics into literally thousands of pieces, each

taught independently of the others. The difficulty with this approach is that while an individual objective might be reasonable, it is only part of a larger network. It is the network (the connections between objectives) that is important. The view of mathematics that students get is of isolated pieces rather than relationships.

Second, this fragmentation (and emphasis on low-level objectives) is reinforced by the testing procedures often associated with such curricula. Multiple-choice questions on concepts and skills emphasize the independence rather than the interdependence of ideas and getting right answers rather than using reasonable procedures.

Third, most teachers have not been exposed to a broader view of mathematics. In the United States, few of our teachers are familiar with the history or philosophy of mathematics or have ever worked as mathematicians. Their knowledge of mathematics is what is done in schools. Therefore, it is not surprising that they see little reason either to view mathematics in a different way or to teach differently. They have little sense of mathematics as a craft, or as a language, or as a set of procedures to get answers. It involves such activities as assigning numbers (measurement), building mathematical models to represent situations, and examining patterns.

Fourth, the segmenting and sequencing of mathematics has led to an assumption that there is a strict, partial ordering to mathematics. In American schools, this has been translated in "you can't study geometry unless you can do arithmetic; you can't study algebra unless you can do decimals; you can't study calculus unless you have had trigonometry; etc., etc." A student who is having difficulty adding fractions with unlike denominators should not be denied the opportunity to study geometric relationships.

In summary, the most serious problem faced by curriculum developers is to realize that while daily lessons (pieces of mathematics) must be taught, somehow the interconnectedness of ideas must be the focus of instruction.

2) <u>Learning as absorption</u>. Most current mathematics programs, including compensatory programs, have conceived of the learner as being a passive absorber of information, storing it in memory in little pieces which are easily retrievable. Note that this view of learning is consistent with the fragmentation of mathematical content.

This conception of learning is based on the tenets of "behaviorism," a theory which evolved during the early part of this century. Actually the theory focuses on the outcomes of learning (behaviors) rather than how learning occurs. It assumes learning occurs by passively, but rationally, reflecting on stimuli from the environment. And, it has been used by scholars to study how desired responses to stimuli (outcomes) become fixed by practice and praise (reinforcement). Learning is viewed as change in behavior (or performance) and change scores (pre-posttest differences) on some measure of performance are often used as evidence for learning. This theory, in its many forms, has strongly influenced all education in the United States and in particular school mathematics. Its strength lies in what Schrag (1981) has called its "generative" characteristics. By this he means that the theory has generated a number of practical procedures which can be used in schools.

Probably the most dramatic research findings of the past quarter century center on the fact that learning does not occur via passive reflection (Wagner & Sternberg, 1984). Instead, individuals approach each new task with prior knowledge. They assimilate new information and construct their own meanings. For example, before young children are taught addition and subtraction, they can already solve most addition and subtraction problems using routines such as counting on and counting back (Romberg & Carpenter, 1985). As instruction proceeds, they continue to use these routines to solve problems in spite of being taught more formal procedures. They will only accept new ideas when it is no longer feasible for them to use prior routines.

Furthermore, ideas are not isolated in memory, but organized in collections in what Anderson (1984) has called "loosely-structured schemas." Such schemas are associated with the natural language that one uses and the situations that one has encountered in the past. This constructive notion of learning is not reflected in current instructional materials or compensatory programs.

The implications from cognitive science as yet have not been drawn to mathematics instruction. However, it is clear that teachers should take into account misconceptions (inappropriate schema) some students have in relationship to new information being presented. For example, many algebra students, when they see an expression $a + b = \_\_$, assume that the equal sign always means "find an answer." This misconception, undoubtedly reinforced by the hand-held calculator, is something that mathematics teachers must deal with when trying to teach students to write equivalent expressions (e.g., $a + b = b + a$).

Recently several authors have described generative features based on notions from cognitive science (e.g., "story shell" units--Romberg, 1983; Romberg & Tufte, 1986; metacognition--Jones, 1986; "structure of learned outcomes"--

Biggs & Collis, 1982). However, to date no examples of research based on these ideas have been done with compensatory programs.

3) Deskilling of teachers. Because of concerns about trying to get teachers to adopt and use new programs, there has been a tendency to overspecify instructions for teachers. Either a detailed individualized program or a highly structured program takes important teaching skills away from the teacher. Often there are no longer decisions to make about what activities to use. Taken to an extreme, the teacher becomes only a conduit in a system, covering the pages of a program without thinking or consideration. The emphasis of teaching is shifted from curricular content and learning to management of individual progress. The teacher becomes a manager of resources and personnel (Berliner, 1982). Teachers are not encouraged to adapt or change to meet local needs or conditions. They are not encouraged to relate ideas of one lesson to another. For students, mathematics becomes completing pages or doing sets of exercises with little relationship between ideas, and teachers reinforce this perspective.

4) Differential opportunity. The most disturbing fact about compensatory programs was the realization that by compensating for an assumed lack in these children's background, educators have created differential opportunity for learning for these low-income students. Most programs probably widen the gap of knowledge about mathematics between those who are affluent in our society and those who are not. This paradox has resulted because we have created a system which has magnified or widened the differences. For example, children in compensatory programs seem to have little access to the computer as anything other than a drill and practice machine (Reisner, 1983). They do not see it as a creative tool. Children in affluent schools, particularly with parents having computers at home, have a different access to this technology. Furthermore, the subset of mathematics that is covered in these programs emphasizes almost exclusively procedural skills, many of which can be done more efficiently and more effectively with a calculator. There is little emphasis on mathematical concepts, understanding relationships, using mathematics to solve problems, proving assertions, etc. For example, Anyon (1981) saw diversity of classroom practices being defined in terms of social class differences. She depicted the teaching of mathematics in a working-class school as spending a great deal of time carrying out procedures (similar to most compensatory programs). "The purposes of which were often unexplained and which were seemingly unconnected to thought processes or decision making" (p. 8). In a middle-class school she discerned more flexibility in regard to procedures which children were expected to follow. There the teachers tended to set out several alternative methods of solving problems and

made efforts to insure that children understood what they were doing. Next, in a professional school the teacher placed a great deal of emphasis on children's building up mathematical knowledge through discovery techniques or through direct experience. And finally, in an executive school these patterns of teaching were extended even further to include explicit problem solving, testing hypothesis about mathematical variables and encouraging pupils to justify the reasonableness of their answers. I am convinced that most compensatory mathematics programs are programs which create increased differential opportunity to learn mathematics for low-income students.

5) Workbooks/tests as technology. Most compensatory programs developed workbooks and associated tests. The result has been that the curriculum has been defined by the workbooks and judged by the tests. The resulting technology includes the text, which is a repository of problem lists, a mass of paper-and-pencil worksheets, and a set of performance tests. Children are to work independently of each other with little opportunity to discuss, argue, build models, or try out ideas collaboratively. In recent years the workbooks and tests have been computerized. This provides for more efficient data collection and feedback. However, the work for students remains the same. Although a few of the books include things to read, there is very little that is interesting to read. Thus, workbook mathematics gives students little reason to connect ideas in "today's" lesson with those of past lessons. The tests currently used measure products, not processes. Answers are judged right or wrong but strategies or reasoning used to derive an answer is not. Also, many of the tests have marginal validity.

6) Change as ritual. The final issue that I want to discuss is the way in which compensatory programs should be viewed as examples of attempts to change American schools. Changes are most often viewed as ameliorative, not radical (Romberg & Price, 1983). Thus, new programs designed to challenge existing traditions are not seen that way within schools. From experience, we know that adopting a curriculum change is not necessarily using it. Moreover, if a curricular innovation is used by an adopting school, it is rarely assimilated into the school in the manner intended by the developer.

Goodlad (1976), in reviewing major educational reform efforts, maintained that the work of teachers and students has hardly changed since the turn of the century. Bellack (1978) argued convincingly that the most interesting phenomenon of major reform is the schools' remarkable resistance to change. Stability, not change, seems to be the dominant characteristic. Romberg (1985), from an analysis of one reform effort, found that most change, however well intended, ended up being nominal with changes in labels, but not practices. Gross (1969), from a case study, demonstrated how enthusiasm and dedication are eroded in a very short time after which practitioners revert to old habits. In a review of the modern mathematics movement, the Conference Board of Mathematical Sciences (1975) was forced to conclude that modern mathematics was not a major component of contemporary education in the United States, and that there was no evidence it had even been given a fair trial.

Nominal change is the most prevalent type of response to innovations. It involves adopting nothing but labels. Educators are good at this. When team-teaching is in fashion this year, groups of teachers are labeled "Team Red" and "Team Blue." When individualism is in vogue, the new term gets prominence in the school reports. But the routines are not changed. As institutions, schools are under considerable political and social pressure to do things they were never designed to do; nor do they have personnel trained to do them. To maintain political viability or to keep pressure groups at bay, nominal change is often reasonable. I suspect that most compensatory mathematics programs have suffered the same fate.

These six difficulties, I believe, stem from a narrow mechanical concept of education. This is true for all education, but it is especially true for mathematics. Too often the acquisition of a prescribed amount of knowledge under competitive conditions and time pressures constitutes mathematics instruction. If we are going to do anything different, now is the time to consider a new approach.

A New Perspective for Compensatory Mathematics Programs

In this section, my intent is to describe the bases which I think should be considered in developing a contemporary mathematics program. To do so the entrenched beliefs, values and traditions of most educators must be addressed. To begin let me again examine--knowledge and the work of students and teachers. The paper then concludes reviewing the claim for differentiated instruction.

Knowledge. The distinction between knowledge and the record of knowledge, knowing and knowing about (Romberg, 1983), is at the root of several of the dilemmas of mathematical education. As a record of knowledge, mathematics has a vast content. Furthermore, the accepted content of mathematics changes. Davis and Hersh (1981), observing that the world is in a golden age of mathematical production, raise the possibility of internal saturation and exhaustion and the notion that there is a limit to the amount of mathematics that humanity can sustain at any one time. Hence, some parts must inevitably be abandoned as new parts are added.

Since the content of school mathematics is of necessity restricted, controversy between mathematics as a science and mathematics as a school subject, arises particularly if the emphasis is on the record of knowledge rather than on knowing. Thus, it becomes essential to carefully reconsider the purposes of mathematical education of children in order to eliminate the redundant while ensuring the crucial.

The intent for students to acquire a structured knowledge of mathematics is enlightening. Scientific management of the record of knowledge resulted in hierarchical classification and taxonomies of knowledge. This approach meant that mathematics to most students was, and still is, the sequential mastery of one concept and skill after another.

Unfortunately, the connectivity of mathematical concepts and the concept of structure so essential to expert thinking remains missing. Stress on isolated parts essentially trains students in a series of routines without educating them to a grasp of the overall picture which will ensure their selection of appropriate tools for a given purpose.

Mathematics as a discipline has not only internal structure but also integral and reciprocal relationships with other disciplines, especially science, and increasingly with the social sciences and humanities. The complexities of these relationships are likely to challenge the traditional hierarchical taxonomies of content. Theories are needed to provide mental models of the relationships between concepts and topics. Students must see and experience the role of mathematics as a language and a science which orders the universe, a tool for representing situations, defining relationships, solving problems, and thinking. They need to experience the powers of its language and notational system in the solution of problems in a wide variety of domains. The connectedness of ideas is critical, and so is the connectedness of process and concept. Students must experience mathematics as part of both larger content and larger process. They need to see it as a process of abstracting quantitative relations and spatial forms from the real world of practical problems and inventing through the process of conjecture and demonstration of logical validity. The emphasis in instruction must now be on experiences which help them to <u>know</u> mathematics (Romberg, 1983).

When mathematical knowledge means knowing and doing mathematics rather than knowing about mathematics, other things follow. Knowledge is both personal and communal in the sense that, while it may originate in an individual, it is validated by the community. Thus, the process of adding to mathematical knowledge through communicating is an integral part of knowing mathematics. Furthermore, the criterion for knowledge is not necessarily that it be true but that it be incorporated into the general system of knowledge (Rescher, 1979). In a sense, adding to the structure of mathematical knowledge is mathematics. This view means that mathematics is, by definition, dynamic and constantly changing and not, as has been the case in schools, a static, bound cumulation. The implications of these views for the whole culture of schools are extensive, suggesting radical change in the work of students and teachers, and in the professionalism of all educators.

The work of students. The work roles of students and teachers are complementary (Skemp, 1979); one teaches, the others learn. However, since schools are ostensibly places where students gather to learn, the role of the teacher should complement that of the student, rather than vice versa. Unfortunately, when knowledge is regarded as knowing about rather than knowing, the vocabulary reflects a reversal of emphasis. The work of the teacher is then to "transmit" knowledge. Logically, this means that the job of the student is to receive it, regurgitating on demand. In fact, the real work of the student is often a matter of negative goals, meeting expectations sufficiently to pass through the system (Skemp, 1979). Clarke's (1985) description of a student's work in a mathematics classroom is:

> ...she tells us what we're gonna do. And she'll probably write up a few examples and notes on the board. Then we'll either get sheets handed out or she'll write up questions on the board. Not very often. We mainly get a textbook. We'll get pages. She'll write up what work to do, page number and exercise. And that's about what happens. (p. 22)

The traditional situation described is organized, routine, controlled and predictable, and an unlikely environment for the creation of knowledge.

Briefly then, the work of students is to constantly extend the structure of the mathematics that they know by making, testing and validating conjectures, which may originate as postulates of conscious thought or be derived intuitively. As long as it is the student making the conjecture, their mathematical knowledge will always be structured, consciously or unconsciously, because conjecture cannot be created from nothing. This amounts to the process of reflective intelligence in which the structure of knowledge is constantly revised by reflecting on events, seeking ways to fit them into the existing structure, and testing its predictive powers (Skemp, 1979).

Verbal and written communication is a crucial part of the process for several reasons. First, communication in the form of logical argument is central to mathematical proof. Second, communication of that proof is the means whereby personal knowledge is submitted for systematizing into the domain and thus accepted as new knowledge (Rescher, 1979). Third, developing competence in the categories and structures of the language system both structures the child's understanding and advances it towards a public mode of consciousness (Russell, 1978). Clearly, the work of students should no longer be a matter of acting within somebody else's structures, answering somebody else's questions, and waiting for the teacher to check the response. Nor is it a matter of evaluating knowledge according to right or wrong answers. In the creation of knowledge, there is only that which fits the structure of mathematical knowledge already created by the student and that which does not, and therefore should prompt conjecture.

The work of teachers. The primary work of teachers is to maintain order and control (Romberg & Carpenter, 1985). There is an inexorably logical sequence when the acknowledged work of teachers is to transmit the record of knowledge. The most cost-effective way to transmit the record of knowledge is through exposition to a captive audience. Theoretically, the child could read and cover the same ground, but that would require a voluntary act which is unlikely as long as children are not setting their own goals. Consequently, that exposition cannot happen unless there is control, which is easier if children talk as little as possible and stay in one place. It is essentially a system for "delivering" knowledge to the group by controlling the individual. This simple sequence has dictated work, furniture arrangement, architecture, etc., for the last hundred years and is the tradition challenged by any attempt at change. The result is that:

> The traditional classroom focuses on competition, management, and group aptitudes, the mathematics taught, is assumed to be a fixed body of knowledge, and it is taught under the assumption that learners absorb what has been covered. (Romberg & Carpenter, 1985, p. 868)

At its simplest, if one regards the roles and work of student and teacher as complementary, when the emphasis is on creating knowledge rather than absorbing the history of other people's knowledge, the work of the teacher is to support, promote, encourage and in every way facilitate the creation of knowledge by students.

In summary, the essential work of teachers should include:

1. Ensuring successful experience for children.

2. Providing for extended and cooperative project work, whose final product is a report.

3. Providing an informal and interdisciplinary approach to mathematics.

4. Encouraging verbal and written eloquence in arguing intuitions.

5. Encouraging self-evaluation and providing for group evaluation of new knowledge and reference to the formal domain.

6. Demonstrably exercising intuition and adding to their own personal knowledge.

7. Providing an emotional and physical environment which supports student work. This includes, for example, recognition of the need for cessation of conscious effort, a change of activity, or an urgency of immediately capturing a thought on paper. It also includes providing for student experience with both physical and intellectual modeling.

8. Changing from structural authority based on negative or extrinsic goals of students to sapiental authority (Skemp, 1979) founded on intrinsic goals. This is the answer to the uniformist/discipline: creativity/ individualistic dilemma.

9. Monitoring the structure of knowledge being created by the child.

10. Using technology appropriately in the processes of: intuition; play; acquisition and manipulation of information; logical argument and communication; evaluating new knowledge against the domain; tracing the development of the student's network of knowledge.

11. In short: to provide the environment; act as a mentor; and get out of the way.

<u>Should Individual Differences be Considered</u>?

Even if a common course of study for mathematics could be developed, the task is not complete, for as Kliebard (1977) argues, while the scope and sequence of a curriculum theory must first address the question of what should be taught, the second question is "who gets taught?" For, although students bring life to mathematics, they add to the instructional complexity, for they also bring to the activities the full range of their differences. To consider those differences implies some sort of criterion that bears on the choice involved about who gets taught what and how they get taught.

Kliebard (1982) has pointed out that the criteria for making such choices are based on claims about schooling from different interest groups. The basic position of any interest group is that schools, teachers, and, in particular, curricular p grams should take into account current knowledge about individuals and their differences. Thus, even though the mathematics as outlined in this paper is for all students, interest groups claim their knowledge about individuals should be considered in making instructional decisions. The interest groups are many and varied. Several have information about differences between individuals based on information from differential psychology, developmental psychology, and sociology.

The first and most prevalent set of claims is based on the extensive work of a number of educational psychologists in the Thurstone tradition of distinct mental abilities (Anastasi, 1953). From test score and psychometric analyses these psychologists have been able to identify differential abilities, traits, aptitudes, styles, and so forth. For example, such characteristics as intelligence, rate of learning, field independence/dependence, or spatial ability have been identified and samples of students ordered from high to low on those traits. Furthermore, it is assumed that these characteristics are fixed, stable characteristics which describe intellectual differences between individuals in the same way as height, weight, stature, and so forth describe physical characteristics. Finally, it has been assumed that instruction would be more socially efficient if some of these differences were taken into account.

The second set of claims is based on information that individuals adaptively interact with the environment and gradually evolve intellectually through discontinuous stages (Langer, 1969). Rather than being fixed, differences between individuals are viewed as a function of growth. Primary age children, for example, usually are at a "concrete-operations" stage, think in terms of themselves (are egocentric), and think of concrete referents near at hand. Hence, they should not be expected to reason about hypothetical, external situations. Instruction then should be tailored to their stage of development.

From vast and various sources, sociological data indicate that children come to school having different social, cultural, and experiential backgrounds. These are differences between individuals in parental background, race, home locale, sex, and so forth. It is assumed that with these differences come differing social expectations; hence it is argued that schools should plan and carry out instruction in light of these differences. It has long been assumed (at least by mathematicians) that mathematics is culturally independent. However, recently this assumption has been challenged because of work in at least four areas. One has been from studies of arithmetical understandings of children from non-technical culture (e.g., G. B. Saxe's work with Oksapmin children of Papua New Guinea, 1982). A second challenge has come from the "ethnomathematics" work of the Brazilian mathematician, Ubiratan D'Amberosio (1985). He stresses the importance of the cultural experiences of students in relationship to instruction. A third challenge comes from the "story skill" curriculum unit notion from cognitive science (Romberg, 1983). The stories in this notion should have cultural relevance. The last challenge comes from research on student's strategies for solving mathematical problems set in contextual frameworks (e.g., Scribner, 1984; Reusser, 1986). The social context of problems has a strong effect on the way they are attacked. Thus, while there is a growing awareness that the teaching and learning of mathematics is culturally dependent it is not yet clear how this awareness can or should influence instruction. In particular there is no research on the relationship of the cultures of the economically disadvantaged children in the U.S. and the learning of mathematics. We do not know whether children from impoverished Black or Hispanic communities come to school with conceptions which make their learning of mathematics different from that of middle-class children.

In addition to information about differences between individuals, there are at least two sources of information about intra-individual differences based on data from social psychology and political science.

In contrast to the "between individuals" arguments about fixed traits, stages of development, or cultural determinants, the argument from social psychology is that individuals differ in interests, likes, motivation, persistence, attitudes, attributions, and so forth. These social characteristics are transient and may change because of curricular unit, environment, teacher, membership in a group, and so on. Instruction should try to capitalize on these transient differences.

Information from political science is based on the notion of "individualism" as an ideological construct in American history. In political thought, this involves the liberal belief in the autonomy of the individual. There are three distinct components of this belief: (1) self-determination—the individual is in control of his own destiny; (2) self-actualization—the good life is attained through acting on one's personal needs and desires; and (3) self-direction—the desire to be free from social constraints. Thus, schooling should offer the student the possibility of studying different (or optional) units. It should be noted that "individualism" assumes the existence of "individual differences" but does not consider identification of those differences particularly relevant. Note that in the first four arguments it was assumed that wise adults can plan, organize, and make decisions about instruction based on information about differences. In this case, the argument is that the learner should make the choices.

Not only do the interest groups who have their claims on one of the five perspectives about individual differences base their arguments on different information; they also reach different conclusions about how instruction should proceed based on that information. One argument is that instruction should be adapted to "complement" differences. For example, if some students learn at a faster rate, they should be allowed (encouraged) to proceed through a program at a faster pace, or if students differ in spatial ability, activities should be adapted so that students with that ability can utilize it in learning and, at the same time, other adaptation should be made so that those low in spatial ability are not handicapped. This is the "aptitude by treatment" interaction argument put forward by Cronbach (1957). It is argued that this approach teaches the same mathematics to all students but in different ways. This is naive because "different ways" imply different processes; hence different mathematics is being learned even if the same concepts or procedural skills are included. Thus, the content of a two-year algebra course, as received by the student, is not the same as a one-year course even though the syllabus is the same.

A second argument is that instruction on the same mathematical units should be adapted to "compensate" for differences. This is often put forward in terms of social equity. Social, cultural, and even intellectual inequities exist, but the school should not exacerbate the inequities. For example, ability grouping is seen as social-class grouping. Thus, differential instruction based on "ability" would only further differentiate social classes.

A third argument is that different students should be taught different mathematics. In particular, the curriculum should not be considered common for gifted or handicapped students. This again assumes that adults (teachers or counselors) are wise enough to decide who gets what mathematics. A part of this argument is that since mathematics is hierarchical, success at one level is a necessary prerequisite for further mathematical study. Thus, half the ninth graders in most secondary schools are counseled to take "general mathematics,"one cannot enroll in Euclidean geometry without passing algebra, and so forth.

The final argument is that different students should have the option of being taught different mathematics. Mathematics, like other subjects (literature, history, science, and so forth), is seen as diverse and interconnected, but not strictly hierarchical. The diversity includes a rich array of activities or topics which all students should have the opportunity to consider and select.

Given that these perspectives and arguments (and others) exist, that they are based in part on valid information, and that some aspects of dealing with individual differences have been incorporated into the traditions of some schools, the question still remains: How should a school react to these interest groups? This is a serious, social-political question. It is a topic upon which considerable open discussion and serious debate needs to be carried out. Without such debate, schools will undoubtedly ignore the additional pressures and maintain existing haphazard traditions.

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