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ABSTRACT

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In Classroom Teaching: An Experimental Study

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Assisting in all phases of the research were Deborah Carey, Janice Gratch and Cheryl Lubinski. Both the experimental treatment and the data collection were facilitated by Glenn Johnson and Peter Christiansen III of the Madison, WI Metropolitan School District and Carolyn Stoner of the Watertown, WI Unified School District.

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Abstract

This study used knowledge derived from classroom-based research on teaching and laboratory-based research on children's learning to improve teachers' classroom instruction and students' achievement. 20 first-grade teachers, assigned randomly to an experimental treatment, participated in a month-long workshop in which they studied findings on children's development of problem solving skills in addition and subtraction. Other first-grade teachers ($\underline{N}$ = 20) were assigned randomly to a control group. Although instructional practices were not prescribed, experimental teachers taught problem solving significantly more and number facts significantly less than control teachers. Experimental teachers encouraged students to use a variety of problem solving strategies, and they listened to processes their students used significantly more than did control teachers. They believed that instruction should build upon students' existing knowledge more than did control teachers, and they knew more about individual students' problem-solving processes. Experimental students' exceeded control students in number fact knowledge, problem solving, reported understanding, and reported confidence in problem solving.

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One of the critical problems facing educators and researchers is how to apply the rapidly expanding body of knowledge on children's learning and problem solving to classroom instruction. Theory and research on cognition and instruction have now reached a point where they can be used profitably to develop principles for instruction to guide curriculum development and the practice of teaching. Implications for instruction do not follow immediately from research on thinking and problem solving, however, and explicit programs of research are needed to establish how the findings of descriptive research on children's thinking can be applied to problems of instruction (Romberg & Carpenter, 1986). Thus, researchers need to investigate how research-based knowledge of children's learning and cognition can be used and applied by real teachers to instruction of real children in actual classrooms with all the complexity that is involved.

To undertake such a task requires building on knowledge derived from classroom-based research on teachers and teaching as well as knowledge derived from laboratory-based research on children's learning and cognition. Traditionally, research on children's learning and research on classroom teaching have been totally separate fields of inquiry governed by different assumptions, asking different questions, employing different research paradigms, requiring different standards of evidence, and conducted by different groups of researchers (Romberg and Carpenter, 1986). The present investigation was unique because we, as researchers from these two distinct paradigms, came together and used knowledge derived from both paradigms to attempt to improve the teaching of actual teachers. In the discussion that follows we describe how the present investigation built on knowledge derived from each of these research paradigms.

Research on Children's Thinking

Research on children's thinking has tended to focus on performance within a specified content area, and the analysis of the task domain represents an important component of the research. This study draws on the extensive research on the development of addition and subtraction, concepts and skills in young children. Researchers have provided a highly structured analysis of the development of addition and subtraction concepts and skills as reflected in children's solutions of different types of word problems. In spite of differences in details and emphasis, researchers in this area have reported remarkably consistent findings in a number of studies, and researchers have drawn similar conclusions about how children solve different problems. This research provides a solid basis for studying how children's thinking might be applied to instruction. (For reviews of this research see Carpenter, 1985; Carpenter and Moser, 1983; or Riley, Greeno, and Heller, 1983).

Analyses of Addition and Subtraction Problems

Research on addition and subtraction word problems has focused on the processes that children use to solve different problems. Recent research has been based on an analysis of verbal problem types that distinguishes between different classes of problems based on their semantic characteristics. While there are minor differences in how problems are categorized and some researchers include additional categories, the central distinctions that are included in almost all categorization schemes are illustrated by the problems in

Table 1. Although all of the problems in Table 1 can be solved by solving the mathematical sentences 5 + 8 = ? or 13 - 5 = ?, each provides a distinct interpretation of addition and subtraction.

Insert Table 1 about here.

The join and separate problems in the first two rows of Table 1 involve two distinct types of action whereas the combine and compare problems in the third and fourth row describe static relationships. The combine problems involve part-whole relationships within a set and the compare problems involve the comparison of two distinct sets. For each type of action or relation, distinct problems can be generated by varying which quantity is unknown, as is illustrated by the distinctions between problems within each row in Table 1. As can be seen from these examples, a number of semantically distinct problems can be generated by varying the structure of the problem, even though most of the same words appear in each problem.

Children's Knowledge and Strategies

These distinctions between problems are reflected in children's solutions. Even before they encounter formal instruction, most young children invent informal modeling and counting strategies for solving addition and subtraction problems that have a clear relationship to the structure of the problems. At the initial level of solving addition and subtraction problems, children are limited to solutions involving direct representations of the problem. They must use fingers or physical objects to represent each quantity in the problem, and they can only represent the specific action or relationship described in the problem. For example, to solve the second problem in Table 1, they construct a set of 5 objects, add more objects until there is a total of 13 objects, and count the number of objects added. To solve the fourth problem, they make a set of 13 objects, remove 5, and count the remaining objects. The ninth problem might be solved by matching two sets and counting the unmatched elements. Children at this level cannot solve problems like the sixth problem in Table 1, because the initial quantity is the unknown and, therefore cannot be represented directl·· with objects.

Children's problem-solving strategies become increasingly abstract as direct modeling gives way to counting strategies like counting on and counting back. For example, to solve the second problem in Table 1, a child using a counting-on strategy would recognize that it is unnecessary to construct the set of 5 objects, and instead would simply count on from 5 to 13, keeping track of the number of counts. The same child may solve the fourth problem by counting back from 13. Virtually all children use counting strategies before they learn number facts at a recall level.

Although children can solve addition and subtraction problems using modeling and counting strategies without formal instruction, the learning of number facts remains a goal of instruction (NCTM, 1987). Number facts are learned over an extended period of time during which some recall of number facts and counting are used concurrently. Children learn certain number combinations earlier than others. Before all the addition facts are completely mastered, many children use a small set of memorized facts to derive solutions for problems involving other number combinations. These solutions usually are based on doubles or numbers whose sum is 10. For example, to find 6 + 8 = ?, a child might recognize that 6 + 6 = 12 and 6 + 8 is just 2 more than 12. Derived facts are not used by a handful of bright students, and it appears that derived facts play an important role for many children in the learning of number facts.

Applying Cognitive Research to Instruction

Until recently, researchers on children's thinking and problem solving have focused on children's performance without considering instruction. They have provided a picture of how children solve problems at different stages in acquiring skill in a particular domain, but they have not addressed the question of how children learn from instruction. Researchers are beginning to turn their attention to this important question (Carpenter & Peterson, in press), but much of the work represents an extension of the carefully controlled clinical approaches employed in cognitive science (c.f. Anderson, 1983; Collins & Brown, in press).

Although we do not deny the importance of research that investigates how instruction may operate under optimal conditions, it is not clear that the findings from this research always can be exported directly to typical classrooms. Some researchers have found that instructional programs that were effective in tutorial settings or with small groups of children were much less effective when they attempted to train teachers to apply them in traditional classroom environments (Fuson & Secada, 1986; Resnick & Omanson, 1987). Thus researchers need to begin to explore directly how to apply research on children's thinking to instruction in real classrooms with all the complexity that is involved.

Several approaches for applying cognitive research to classroom instruction are consistent with the findings of research on children's thinking and problem solving. One possibility would be to design instruction that directly teaches the knowledge and strategies required for competent performance. An alternative would be to specify instructional sequences that lead children through the primary stages in the development of competence (Case, 1983). A third alternative would be to use the detailed knowledge that research provides about the errors that children make and the knowledge deficiencies that cause them (Brown & Van Lehn, 1982) to develop specific diagnostic procedures for teachers to assess children's knowledge and misconceptions so that instructional programs could be matched to outcomes of the assessment to build upon children's existing knowledge or explicitly redress their deficiencies.

Although we concur that instruction should be matched to children's existing knowledge and that instruction should take into account what researchers know about the thinking involved in the acquisition of competent performance, instructional approaches that attempt to specify explicit programs of instruction ignore two critical variables in classroom instruction: 1) the classroom teacher and 2) the classroom setting. To address these two critical variables, we need to draw on knowledge derived from classroom-based research on teachers and teaching.

Classroom-Based Research on Teachers and Teaching

Over the years, researchers on teaching have documented that to understand teaching in an actual classroom setting, one needs to recognize that teaching is an interactive process. In early process-product studies of teaching effectiveness, researchers such as Flanders (1970) and others developed teacher-student interaction observation systems to describe the teaching-learning processes in actual classrooms. These researchers then correlated observed teacher-student interaction patterns and behaviors with students' scores on standardized achievement tests and defined effective teaching practices as those that correlated highest with achievement (Dunkin and Biddle, 1974). Present day researchers on teaching have continued to focus on the interaction of the teacher and students in their studies of classrooms. As Shulman (1986) put it,

> Teaching is seen as an activity involving teachers and students working jointly. The work involves exercise of both thinking and acting on the parts of all participants. Moreover, teachers learn and learners teach (Shulman, 1986a, p. 7).

Findings from classroom-based research on teaching have demonstrated the importance of studying teaching and learning in actual classroom settings. As Good and Biddle (in press) noted, "Early teaching effectiveness research in the 1970's was motivated by a dissatisfaction with previous research which had been conducted in laboratory settings... and a dissatisfaction with solutions that were not based on an understanding of existing classroom practices and constraints" (Good and Biddle, p. 26). Another assumption shared by most researchers who have conducted classroom-based research on teaching is that the teacher has a central role in classroom instruction (Shulman, 1986a). In teaching mathematics in the elementary school the teacher, ultimately, has the responsibility for planning, developing, and carrying out instruction that facilitates students' meaningful learning of mathematics.

A Cognitive View of the Teacher

In keeping with the emerging cognitive view of the learner described above, researchers on teaching have begun to take a cognitive perspective on the teacher (Clark and Peterson, 1986; Peterson, 1988). Like their students, teachers are thinking individuals who approach the complex task of teaching in much the same way that problem solvers deal with other complex tasks. Researchers studying teachers' thinking and decision making have documented that teachers do not mindlessly follow lesson plans in teachers' manuals or prescriptions for effective teaching (Shavelson and Stern, 1981; Clark and Peterson, 1986). Teachers interpret plans in terms of their own constructs and adapt prescriptions to fit the situation as they perceive it. Teachers' knowledge and beliefs affect profoundly the way that teachers teach. Moreover, previous efforts at curriculum reform may have failed because reformers attempted to prescribe programs of instruction without taking into account the knowledge, beliefs, and decision making of the teacher implementing the program (Romberg and Carpenter, 1986; Clark and Peterson, 1986). Thus, teachers' knowledge, beliefs, and decisions have become important foci of study.

Research on Teachers' Knowledge, Beliefs, and Decisions

Previous research on teachers' decision making suggests that teachers do not tend to base instructional decisions on their assessment of children's knowledge or misconceptions (Clark and Peterson, 1986). Putnam (1987) and Putnam and Leinhardt (1986) proposed that assessment of students' knowledge is not a primary goal of most teachers. They argued that keeping track of the knowledge of 25 students would create an overwhelming demand on the cognitive resources of the teacher. Putnam and Leinhardt hypothesized that teachers follow curriculum scripts in which they make only minor adjustments based on student feedback. The evidence is far from conclusive, however, to support the belief that teachers do not or cannot monitor students' knowledge and use that information in instruction. Furthermore, Lampert (1987) has argued that a concern for monitoring students' knowledge should be related to a teacher's goals for instruction. Although teachers may be able to achieve short-term computational goals without attending to students' knowledge, they may need to understand students' thinking to attain higher level goals of facilitating students' meaningful understanding and problem solving.

Researchers have begun to investigate how teachers' knowledge of and beliefs about their students' thinking are related to student achievement. In an earlier study, based on the same group of teachers as the study reported here, we measured 40 first-grade teachers' knowledge of students' knowledge and cognitions through questionnaires and an interview (Carpenter, Fennema, Peterson, and Carey, in press). We found that these first-grade teachers were able to identify many of the critical distinctions between addition and subtraction word problems and the kinds of strategies that children use to solve such problems. However, teachers' knowledge was not organized into a coherent network that related distinctions between types of word problems to children's solution strategies for solving the problems, nor to the difficulty of the problems. In the same study, we found that teachers' knowledge of their own students' abilities to solve different addition and subtraction problems was significantly positively correlated with student achievement on both computation and problem solving tests. Similar results were reported by Fisher, Berliner, Filby, Marliave, Cahn, and Dishaw (1980), who found that teachers' success in predicting students' success in solving specific problems on a standardized test was significantly correlated with their students' performance on the test.

In a related study (Peterson, Fennema, Carpenter, and Loef, in press), we found a significant positive correlation between students' problem solving achievement and teachers' beliefs. Teachers whose students achieved well in addition and subtraction problem solving, tended to agree with a cognitively-based perspective that instruction should build upon children's existing knowledge and that teachers should help students to construct mathematical knowledge rather than to passively absorb it.

In none of the studies cited above did researchers address the critical question of, "How might knowledge of the very explicit, highly principled knowledge of children's cognitions derived from current research influence teachers' instruction and subsequently affect students' achievement?" Research provides detailed knowledge about children's thinking and problem solving that, if it were available to teachers, might affect profoundly teachers' knowledge of their own students and their planning of instruction. Shulman (1986b) called this type of knowledge pedagogical content knowledge, and Peterson (1988) referred to it as content-specific cognitional knowledge.

The purpose of this investigation was to study the effects of a program designed to provide teachers with detailed knowledge about children's thinking. We have termed this approach to applying cognitive research, Cognitively Guided Instruction.

Cognitively Guided Instruction

Cognitively Guided Instruction (CGI) is based on the premise that the teaching-learning process in real classrooms is too complex to be scripted in advance, and as a consequence, teaching essentially is problem solving. Classroom instruction is mediated by teachers' thinking and decisions. Thus, researchers and educators can bring about the most significant changes in classroom practice by helping teachers to make informed decisions rather than by attempting to train them to perform in a specified way.

The guiding tenet of Cognitively Guided Instruction is that teachers' instructional decisions should be based on the goals of instruction which can be achieved through careful analyses of their students' knowledge. The goals of instruction include development of problem solving, understanding of concepts and the acquisition of skills. To accomplish these goals, teachers must have a thorough knowledge of the content domain, and they must be able to assess effectively their students' knowledge in this domain. Relevant knowledge integrates content knowledge with pedagogical knowledge. In the domain of early addition and subtraction it includes an understanding of distinctions between problems that are reflected in students' solutions at different levels of acquiring expertise in addition and subtraction. Teachers must have knowledge of problem difficulty as well as knowledge of distinctions between problems that result in different solution strategies. Teachers' ability to assess their own students' knowledge also requires that teachers have an understanding of the general levels that students pass through in acquiring the concepts and procedures in the content domain, the processes that students use to solve different problems at each stage, and the nature of students' knowledge that underlies these processes.

Two major assumptions underlie CGI. One is that instruction should develop understanding by stressing relationships between skills and problem solving with problem solving serving as the organizing focus of instruction. The second assumption is that instruction should build upon students' existing knowledge.

Several broad principles of instruction may be derived from these assumptions. The first principle that is embedded in all the other principles is that instruction should be appropriate for each student. A second principle is that problems, concepts, or skills being learned should have meaning for each student. The student should be able to relate the new idea to the knowledge that he or she already possesses. Third, instruction should be organized to facilitate students' active construction of their own knowledge with understanding. Because all instruction should be based on what each child knows, the necessity for continual assessment is the fourth principle. Teachers need to assess not only whether a learner can solve a particular problem but also how the learner solves the problem. Teachers need to analyze children's thinking by asking appropriate questions and listening to children's responses. Research on children's thinking provides a framework for this analysis and a model for questioning. Fifth, teachers need to use the knowledge that they derive from assessment of their children's thinking in the planning and implementation of instruction.

Purpose of the Study

The purpose of the study reported here was to investigate whether giving teachers' access to knowledge derived from research on children's thinking about addition and subtraction would influence the teachers' instruction and their students' achievement. Our hypothesis was that knowledge about different problem types, children's strategies for solving different problems, and how children's knowledge and skills evolve, would affect directly how and what teachers did in the classroom. Perhaps more importantly, we hypothesized that such knowledge would affect teachers' ability to assess their own students, which, in turn, would be reflected in teachers' knowledge about their own students. Teachers' knowledge of their own students would affect instruction, allowing teachers to better tailor instruction to students' knowledge and problem solving abilities. Thus, students' meaningful learning and problem solving in mathematics would be facilitated.

A Merging of Paradigms in the Design of the Study

Because we were attempting to build on research-based knowledge derived from both research on children's learning and research on classroom teaching, we drew on both paradigms in designing the study. We measured student achievement with standardized achievement tests in the tradition of classroom-based process-product research on teaching, but we constructed additional tests and scales that were sensitive to the distinctions between different levels of problem solving identified by cognitive research on children's learning. To assess learning, we interviewed children using techniques derived from research on children's problem solving in arithmetic to identify the processes that children used to solve different problems. We developed two classroom observation systems that followed procedures commonly employed in studies of classroom instruction. However, we derived many of the observation categories from a cognitive analysis of the content of instruction, the strategies that children use, and different assessment and grouping practices that we hypothesized that teachers might employ in applying their knowledge about children's thinking. Our observation categories required observers to go beyond students' overt behavior to infer the cognitive strategies that teachers expected or encouraged students to use and that students were actually using to solve problems. Finally, in this study we employed a large sample of classrooms and quantifiable measures to provide the kinds of evidence commonly reported in traditional process-product studies of teaching. In spite of some similarities in design and methodology to traditional process-product experimental studies, such as that of Good, Grouws and Ebmeier (1983), the present study differed in several important respects from those studies. First, we did not specify a program of instruction for the teachers. Teachers designed their own programs of instruction. A goal of the study was to investigate whether and how teachers applied knowledge about children's thinking and problem solving in their own classrooms. Second, we hypothesized that the most critical influence on teachers' instruction would be their knowledge and learning, including the knowledge about students that teachers gained during the school year as they taught their own students. Thus, the year of classroom instruction following the initial teacher workshop was not conceptualized as a separate implementation phase; it was part of the treatment. Whereas we did not continue intensive work with the teachers during the instructional year, we assumed that teachers' knowledge and beliefs would continue to change as teachers gathered more knowledge about their own students. We collected classroom observation evidence not to assess fidelity of treatment implementation, but rather to obtain quantifiable data that would help us understand what the treatment actually was.

<u>Research Questions</u>

In this study we addressed the following questions about teachers and their students.

1. Did <u>teachers</u> who had participated in a program designed to help them understand children's thinking:

a) Employ different instructional processes in their classrooms than did teachers who had not participated in the program?

b) Have different beliefs about teaching mathematics, about how students learn, and about the role of the teacher in facilitating that learning than did teachers who did not participate in the program?

c) Know more about their students' abilities than did teachers who did not participate in the program?

2. Did the students of teachers who participated in a program designed to help them understand children's thinking:

a) Have higher levels of achievement than did the students of teachers who did not participate in the program?

b) Have higher levels of confidence in their ability in mathematics than did the students of teachers who did not participate in the program?

c) Have different beliefs about themselves and mathematics than did students of teachers who did not participate in the program?

Method

Overview

Forty first-grade teachers participated in the study. Half of the teachers (N = 20) were assigned randomly by school to the treatment group. These teachers participated in a four week summer workshop designed to familiarize them with the findings of research on the learning and development of addition and subtraction concepts in young children and to provide teachers with an opportunity to think about and plan instruction based on this knowledge. The other teachers (N = 20) served as a control group who participated in two 2-hour workshops focused on non- routine problem solving. Throughout the following school year, all 40 teachers and their students were observed during mathematics instruction by trained observers using two coding systems developed especially for this study. Near the end of the instructional year, teachers' knowledge of their students was measured by asking each teacher to predict how individual students in her class would solve specific problems. Teachers' predictions were then matched with students' actual responses to obtain a measure of teachers' knowledge of their students' thinking and performance. Teachers' beliefs were measured using a 48-item questionnaire designed to assess teachers' assumptions about the learning and teaching of addition and subtraction. Students in the 40 teachers' classes completed a standardized mathematics achievement pretest in September, and a series of posttests in April and May. The posttests included standardized tests of computation and problem solving as well experimenter-constructed scales which more precisely assessed students' problem solving abilities. At posttest time, students were also interviewed as they solved a variety of problems to assess the processes that they used to solve different problems. Finally, students completed several measures of attitudes and beliefs developed for this study.

Subjects

The subjects in the study were the teachers and their students in 40 classrooms in 24 schools located in Madison, Wisconsin, and in four smaller communities near Madison. The schools included two Catholic schools and 22 public schools. All the teachers in the sample volunteered to participate in a four week inservice program during the summer, to be observed during their classroom instruction in mathematics during the following year, and to complete questionnaires and interviews in May of 1986 and 1987. Each participating teacher received $100 as an honorarium for each year of the study. The mean number of years of elementary teaching experience for the teachers in the sample was 10.90, and the mean number of years teaching first-grade was 5.62. Two of the teachers had just completed the first year of teaching. None of the teachers reported participating in any training in which recent research on addition and subtraction was discussed. During the instructional year of the study, 36 teachers taught first grade classrooms, and 4 teachers taught first/second grade combinations.

Teachers were assigned randomly to treatments by school. Whenever possible, twelve first grade students--six girls and six boys--were selected randomly from each class to serve as target students for observation and the interviews. While all first grade students in each class completed the written posttests, the analyses were based on data from target students so that all comparisons of treatment effects were done on the same sample of students.

Cognitively Guided Instruction (CGI) Treatment

Goals of the Workshop. The workshop for the Cognitively Guided Instruction (CGI) group was conducted during the first four weeks of the teachers' summer vacation. Because word problem solving was the organizing focus of CGI, much of the workshop was devoted to giving teachers access to knowledge about addition and subtraction word problems and how children think about them. The initial goal of instruction was to familiarize teachers with research on children's solutions of addition and subtraction problems. Teachers learned to classify problems, to identify the processes that children use to solve different problems, and to relate processes to the levels and problems in which they are commonly used. Although the taxonomy of problem types and the models of children's cognitive processes were simplified somewhat, each teacher gave evidence of understanding of the problem types and related solution strategies. This knowledge provided the framework for everything else that followed, and $1\text{-}^1/_2$ weeks of the four-week workshop was spent on it.

During the remainder of the workshop, teachers discussed principles of instruction derived from research and designed their own programs of instruction based upon those principles. Although instructional practices were not prescribed, the broad principles of instruction presented above were discussed. Specific questions were identified that teachers needed to address in planning their instruction, but teachers were not told how they should answer them. These questions included the following: 1) How should instruction build initially upon the informal and counting strategies that children use to solve simple word problems when they enter first grade? 2) Should specific strategies like "counting on" be taught explicitly? and 3) How should symbols be linked to the informal knowledge of addition and subtraction that children exhibit in their modeling and counting solutions of word problems?

Another goal of the workshop was to familiarize teachers with curricular materials available for instruction. Teachers were encouraged to evaluate these materials on the basis of the knowledge and instructional principles that they acquired earlier in the workshop.

Format of the CGI Workshop/Treatment. The workshop was taught by Professors Carpenter and Fennema with the assistance of three graduate students, two mathematics supervisors from the Madison Metropolitan School District, and one curriculum supervisor from the Watertown, WI Unified School District. The workshop involved five hours of participation each day, four days a week for four weeks. Although teachers were told that they could complete all work during the 20 workshop hours each week, some teachers did take work home with them.

Teachers were provided with readings prepared for the workshop that presented the problem type taxonomy, synthesized the results of research on children's solutions of addition and subtraction word problems, and discussed how these findings might be applied in the classroom. A number of videotapes of children solving problems were used to illustrate children's solutions strategies, and teachers had the opportunity to interview one or two young children. A variety of instructional materials were also available for the teachers to review including textbooks, manipulatives, and enrichment materials.

A typical day included an hour lecture/discussion led by Carpenter or Fennema. During the first six days, these discussions focused on the findings from research on addition and subtraction. Discussions during the next four days explored ways that these findings might be implemented in the classroom. In the remaining sessions, the topics discussed included general problem solving, time on task, and equity issues. Each day the teachers could also participate in a small group session led by one of the graduate students. The purpose of these sessions was to examine different curricula or enrichment materials and to discuss how these materials might be used to facilitate children's problem solving following principles of CGI. During the rest of the time, teachers were free to read; to plan the following year's instruction; to study videotapes of children solving problems; to talk with other participants and the staff; and to examine textbooks, manipulatives, or enrichment materials.

Teachers were given a great deal of freedom to monitor their own progress, and to select and work on activities that facilitated their own learning. Although they were given no specific written assignments, teachers were asked to plan a unit to teach during the following year, as well as a year-long plan for instruction based on principles of CGI. Each week teachers met with one of the staff to discuss their progress for the week and to clarify their ideas about their plans. Teachers either worked alone or with others as they desired.

Because we hypothesized that the teacher's knowledge about each of her student's thinking about addition and subtraction would develop during the instructional year, we conceptualized the treatment as including the following instructional year. However, after the workshop, our formal contact with the CGI teachers was limited. We met one time with them in October when teachers discussed with us what they had done to that point with CGI. One of the staff also served as a resource person and responded to any questions that CGI teachers posed to her throughout the year. Each teacher who participated in the workshop received 3 university credits and was given $50 to buy materials to implement CGI.

Control Group

Teachers in the control group participated in two 2-hour workshops that were held in September and February during the instructional year. The control group workshops were in no way comparable to the CGI workshops in duration or extent of coverage, and the purpose was not to provide a contrasting treatment. The goal was to provide control teachers with some sense of participation in the project and give them some immediate reward for their participation. The problem solving focus of the control group's workshops was different from that of the CGI workshop. While the problem solving emphasis of CGI was on story problems that could be based on the children's personal experience, the problem solving emphasis of the control group's workshop was on mathematical problems that are intriguing and of a more esoteric nature. Such problems are often designated as nonroutine problems. During the Control Group workshops, no discussion occurred about how children think as they solve problems nor was any specific framework given for how to understand children's cognitions. Rather, the discussion focused on the importance of children learning to solve problems and the potential use of nonroutine problems to motivate students to engage in problem solving.

The control workshops were taught by a graduate student who was a member of the CGI staff. Teachers were first asked to solve a nonroutine problem themselves, and then they discussed the various heuristics that they used as they found a solution. This problem solving activity was followed by a discussion of various heuristics that children might use to solve a problem such as: charting, making a diagram, drawing a picture or making a list. Teachers were given access to materials that gave examples of nonroutine problems and to trade books that included some problem solving activities for children. They also discussed how they might use their own mathematics textbook to provide problem solving experiences for children.

Classroom Observations

We constructed two observation systems: one that focused on the teacher and one that focused on the student. In both systems, observers used a 60-second time-sampling procedure in which they observed for 30 seconds, and then for the next 30 seconds, they coded the behavior and activities of either teacher or the target student, depending on the system. The teacher observer focused on the teacher from the beginning of the time that the teacher taught mathematics to the end of the mathematics period. The student observer focused on each of the target students, in turn, until all 12 students had been observed. The observer then returned to the first target student and rotated through the same 12 students as long as class time allowed. Target students were observed in a different order each day.

Observation Categories. Observation categories were selected carefully by considering the literature and the purpose of the study. From a previous study (Peterson and Fennema, 1985) we adapted the major category of setting (*whole class; medium group; small group; teacher/student*; and *alone*) because we thought that CGI teachers might vary the instructional setting to adapt to students' knowledge and abilities. To compare possible differences in CGI teachers instruction with the lesson phases of active mathematics teaching (Good, Grouws, and Ebmeier, 1983), we coded the phases of *review, development, controlled practice*, and *student work* (seatwork) in the student system.

The mathematics content was a primary category in both systems. The major categories of addition and subtraction content were *number facts, represented problems, word problems* and *other addition and subtraction. Number facts* focused on the knowledge and use of simple computations using addition number facts up to 9 + 9 or corresponding subtraction number facts. Represented problems were problems presented pictorially or with counters whose solution did not require any additional representation by the students. Included in this category were typical textbook problems in which pictures show children or animals joining or leaving a group to illustrate an addition or subtraction problem. *Word problems* were problems presented either verbally or written in story form that required the student to use addition or subtraction in order to solve them. *Other addition and subtraction* included problems or activities, other than those described thus far, that required the student to use addition or subtraction. Student non-engagement with the mathematics content was coded in the student system (Peterson & Fennema, 1985).

Subcategories under teacher behavior were derived from findings by Fennema and Peterson (1986) that students' problem solving was positively related to teachers' feedback to the process of obtaining the answer rather than to the answer so *feedback to process* and *feedback to answer* was coded. In addition, both cognitive research on children's learning and CGI principles suggest that it may be important for the teacher to *pose problems* to students and to *listen to process* which involves listening to a student working a problem aloud or listening to a student describe the way he or she solved a problem.

Building on the research on children's learning and cognitions, we coded both the strategy that the teacher expected or encouraged the students to use in solving problems and also the strategy that students actually appeared to be using. Strategies coded in both systems included *direct modeling, advanced counting, derived facts*, and *recall*. In the teacher system if the teacher appeared to expect no one clear strategy, then *no clear strategy* was coded. If the teacher encouraged or expected use of more than one of these strategies, *multiple strategies* was coded. In the student system, if the observer could not determine what strategy the student was using, then *no clear strategy* was coded.

In summary, in the teacher system for each 60-second time sampling interval when addition and subtraction instruction was occurring, the observer checked one subcategory within each of the following major categories: setting, content, teacher behavior and strategy. In the student system, for each 60-second time sampling interval when addition and subtraction instruction was occurring, the observer checked one subcategory within each of the following categories: setting, lesson phase, content (including non-engaged with content) and strategy. Table 2 shows the specific observation categories in each system.

Insert Table 2 about here.

Inter-Observer Agreement was estimated by having a reliability observer code with the assigned observer at specified times during the study. A sample of these data were used to calculate inter-observer agreement. For the teacher system, agreement estimates were calculated for a pair of observers who coded the same two teachers on each of four days during the study. For the student system, estimates were based on the coding of three teachers on each of two days by the regular coder and a reliability coder. An estimate of agreement for each category was made by comparing the coding of the two observers for each coding interval. Thus, for each coding category and interval, observers might have agreed or not agreed to check or not to check a given category. Percent inter-observer agreement was calculated for each category by summing the total number of agreements over the total agreements plus disagreements for the intervals coded. Table 2 presents the estimates of inter-observer agreement for each coding category in the teacher and student observation systems.

Observation Procedures. In September and October observers were hired and trained in a 2-week training session following procedures used previously by Peterson and Fennema (1985). Observation manuals were developed for each system. (Peterson, 1987) Training involved a week of coding transcripts and videotapes followed by a week of practice coding in first-grade mathematics classrooms that were not part of our study. At the end of the training period, each observer completed a written test that assessed the observer's content knowledge, and coded a videotape of a first-grade classroom that assessed the observer's coding ability. Observers who achieved the criterion levels on both tests were judged sufficiently knowledgeable and skilled to begin actual classroom observations.

Each teacher and class was observed for four separate week-long periods (a minimum of 16 days) from November through April. During each scheduled observation period, a student observer and a teacher observer coded together in the same classroom. During the observations, the teacher wore a wireless microphone, and the teacher observer listened to the teacher through earphones to aid in understanding the teacher's private interactions with students.

Teachers' Knowledge Measures

Three separate measures were constructed to assess teachers' knowledge of their own students. Procedures were adapted from a similar test used in an earlier study (Carpenter et al., in press). For each of the three measures, teachers were asked to predict their target students' performance on specific problems. The teachers' knowledge scores were based on the match between their predictions for each target student and that student's actual response to each given item on the corresponding student achievement test described below.

For the knowledge of Number Fact Strategies, teachers were asked to predict the strategy that each of their target students would use to answer each of the 5 items on the students' Number Facts Interview. The teachers' scores were calculated by comparing each teacher's predicted strategy to the strategy that the given target student actually used. For each item for each student the teacher received 1 point for each correct match between predicted and actual strategy, and 0 points for no match.

For the knowledge of students' Problem Solving Strategies, teachers were asked to predict the strategy that each of their target students would use to solve each of the 6 problems on the students' Problem-Solving Interview. Because students frequently use several different strategies to solve a given problem at different times, teachers were allowed, but not required, to identify as many as three strategies that a given student might use for a particular problem. Teachers were encouraged to identify only strategies that they thought the student might actually use. The teacher's response was scored as a match and given 1 point if the student solved the given problem on the Problem-Solving Interview using any one of the strategies identified by the teacher.

For the knowledge of Problem Solving Abilities, teachers were asked to predict whether each of the target students could answer correctly specific problems on the written tests. Teachers did not have to specify what strategy the student used. Eight problems were selected from the Complex and Advanced problem scales described below. The teacher's response for each student for each item was scored on the basis of the match between the teacher's prediction and the target student's actual answer to the given problem. The teacher received 1 point for each correct match for each student and each item. A teacher's prediction could match the students' actual response by predicting accurately whether or not a student would answer a given problem correctly.

The Number Fact Strategies and the Problem Solving Strategies tests were administered individually by trained interviewers using similar procedures and coding criteria to those used for the student interviews described below. For the Problem Solving Abilities test, interviewers gave teachers a list of the target students and a list of the eight problems. Interviewers asked teachers to predict which of the problems each of the target students could solve correctly. The Cronbach alphas were .57, .86, and .47 (N = 40) for teachers' Number Fact Strategies, Problem Solving Strategies, and Problem Solving Abilities tests, respectively.

Teachers' Belief Instrument

Teachers' beliefs about the learning and teaching of addition and subtraction were assessed using four 12-item experimenter-constructed scales. Peterson, Fennema, Carpenter, and Loef (in press) have provided a complete description of these scales and demonstrated the reliability, construct validity, and predictive validity of teachers' scores derived from them.

For each item teachers responded on a five-point Likert scale by indicating, "strongly agree," "agree," "undecided," "disagree," or "strongly disagree." Half of the items on each scale were worded such that agreement with the statement indicated agreement with a cognitively guided perspective. The remaining six items were worded so that agreement with that item indicated less agreement with a cognitively guided perspective. Scale 1, The Role of the Learner, was concerned with how children learn mathematics. A high score indicated a belief that children construct their own knowledge while a low score on the scale reflected a belief that children receive knowledge. Scale 2, Relationship between Skills, Understanding and Problem Solving, dealt with the interrelationships of teaching various components of mathematics learning. A high score indicated a belief that all components should be taught as interrelated ideas while a lower score indicated a belief that lower level skills are prerequisites to teaching understanding and problem solving. Scale 3, Sequencing of Mathematics, assessed teachers' beliefs about what should provide the basis for sequencing topics in addition and subtraction instruction. A high score indicated a belief that the development of mathematical ideas in children should provide the basis for sequencing topics for instruction, while a low score indicated a belief that formal mathematics should provide the basis for sequencing topics for instruction. Scale 4, Role of the Teacher, assessed teachers' beliefs about how addition and subtraction should be taught. A high score reflected a belief that mathematics instruction should facilitate children's construction of knowledge while a low score reflected a belief that instruction should be organized to facilitate teacher's presentation of knowledge.

Sample items included: "Most young children can figure out a way to solve simple word problems" (Scale 1); "Children should not solve simple word problems until they have mastered some basic number facts" (Scale 2); "The natural development of children's mathematics ideas should determine the sequence of topics used for instruction" (Scale 3); "Teachers should allow children to figure out their own ways to solve simple word problems" (Scale 4). Cronbach's alpha internal consistency estimates for teachers' scores in this sample (N = 40) were .93 for the total scales combined and .81, .79, .79 and .84 for Scales 1 through 4, respectively.

Student Achievement Measures

At the beginning of the year, students' achievement was measured using a standardized achievement test as a pretest. Posttests, given near the end of the school year, included three written tests and individual interviews. Written posttests included: a standardized test of computational skills, a standardized test of problem solving, and an experimenter-developed problem solving test. From the two problem solving tests, three scales were constructed: Simple Addition and Subtraction Word Problems, Complex Addition and Subtraction Word Problems, and Advanced Word Problems. Posttest interviews assessed each student's recall of number facts (a computational skill) and identified the strategies that were used to solve addition and subtraction word problems. Thus, there were two posttest measures of computational skills--one based on a written test and one from the interview; and there were five posttest measures of problem solving--the standardized test, the three problem solving scales, and the interview of problem solving strategies. The internal consistencies of the tests were estimated using Cronbach's alpha and are reported in Table 3.

Insert Table 3 about here .

Written Achievement Tests. The Mathematics subtest of the Iowa Test of Basic Skills (ITBS), Level 6, was used as the pretest. The Computation subtest of the ITBS, Level 7, was used as the written posttest of computation. Three posttest problem solving scales were constructed by using items selected from the Mathematics Problems subtest of the ITBS, Level 7, and experimenter designed items which included a broad range of more difficult problems. The three scales, listed below, represent different levels of problem solving ability.

Simple Addition and Subtraction Word Problems. This scale included 11 word problems involving simple joining and separating situations with the result unknown.

Complex Addition and Subtraction Word Problems. This scale included 12 more difficult addition and subtraction word problems based on the analysis of problem types by Carpenter and Moser (1983) and by Riley et al. (1983). Problems from this scale are not included in many first-grade mathematics programs.

Advanced Word Problems. This scale included multi-step problems, grouping and partitioning problems, and problems involving extraneous information.

Table 4 shows representative examples from each scale.

Insert Table 4 about here .

Student Interviews. At posttest time, target students were interviewed individually to determine the strategies that they used to solve certain problems and to assess their recall of number facts. The Number Facts Interview involved five addition number facts with sums between 6 and 16. The Problem-Solving Interview consisted of six addition and subtraction word problems involving simple joining situations and missing addend situations with the change unknown. Responses were coded by the interviewer using a coding system developed by Carpenter and Moser (1983). The strategy of interest on the Number Facts Interview was recall of number facts.

Students' Confidence and Beliefs

Students' Confidence. Students were asked to indicate if they thought they could do 12 word problems. They were asked to circle YES if they thought they could solve the problems or to circle NO if they didn't think they could solve the problem. One item was, "Dorothy has 6 stickers. How many more stickers does she need to collect to have 14 altogether?" The items were read to children in a group setting. Students were given 1 point for a YES answer and 0 points for a NO answer. Cronbach's alpha internal consistency estimate for students' scores was .91 (N=40).

Students' Beliefs. Students were interviewed about their beliefs on the same four belief constructs described above for teachers. Items from the teachers' belief scales were reworded to make them understandable to children. A 16-item interview scale was constructed with 4 items for each scale. Students were asked to respond to the interviewer on a 3-point Likert scale of "Yes", "Maybe" or "No." Students' responses were scored, "Yes" = 3, "Maybe" = 2, "No" = 1 for positively stated items, and the opposite for negatively stated items. Example items included: "The teacher should tell kids exactly how to solve story problems in math;" (Scale 4) and "Most kids can solve easy story problems by counting their fingers or something else before the teacher teaches them how to solve the problem." (Scale 2). (See Teacher Beliefs Instrument above for descriptions of each construct.) Cronbach's alpha internal consistency estimates for students' scores on each of the four belief scales were .63 (N = 40 classes) for the Total scale, and .52, .19, .45, and .68 (N = 40 classes) for Scales 1, 2, 3, and 4, respectively. Because of the low internal consistency for student beliefs on individual scales, which may be due to the small number of items per scale, only the Total scale score was used in subsequent analyses.

Students' Attention and Understanding. Students' reports of attention and understanding were assessed in an interview using questions developed and used previously by Peterson, Swing, Braverman (1982); Buss (1982) and Peterson, Swing, Waas, and Stark (1984). Peterson et al. (1982, 1984) provided empirical data that demonstrated the reliability and validity of students' reported attention and understanding on these questions. The interviewer asked students to "think back to the last time you were in math class at school" and answer the following questions:

> "During that math class, when you were supposed to be paying attention to the teacher or your work, were you paying attention all of the time, most of the time, some of the time, or not very much of the time?" (Repeat the question if necessary.)

> "During that same math class, how well were you understanding the math that you were doing? Were you understanding all of it, most of it, some of it, or not very much of it?" (Repeat the question if necessary.)

Students' responses were scored as follows: "All" = 4; "Most" = 3; "Some" = 2; "Not very much" = 1.

Testing and Interview Procedures

Student Data. The students' written tests were administered by trained testers following written protocols. Target students who were absent on the day of testing were tested after they returned to school. The pretest was administered during September. In each class the written posttests and student confidence and belief measures were administered on two consecutive days in April or May, 1987. On the first day the following tests were administered in the following order: the ITBS problem-solving test and the ITBS computation test. On the second day the experimenter designed problem solving items and the Confidence Scale were given.

The student interviews were also conducted during April and May, 1987, by trained interviewers. The Problem Solving Interview was conducted first, followed by the Number Fact Interview, followed by the Beliefs Scales, and finally by questions on student's attention and understanding.

Teacher Data. In May 1986, before the workshop began in June, teachers completed the belief questionnaire, and these data are reported in Peterson et al. (in press.) In May 1987, after the student data were collected, we again assessed teachers' beliefs. Within 1-2 days after their children's interviews were conducted, trained graduate assistants conducted the Teacher Knowledge Interview and administered the Teacher Belief Instrument to individual teachers.

Results

In this section, we describe the results of the four major analyses that we conducted. First, we examine how CGI and control classrooms differed in the content, activities, behavior, learning, and instruction in which teachers and students were observed to be engaged. Second, we describe differences between CGI and control teachers in their knowledge and beliefs. Then we examine effects on students' achievement, including students' problem solving and knowledge of number facts at a recall level. Finally, we describe effects on students' confidence, beliefs and reported attention and understanding.

Within a classroom,a student's behavior and learning is not independent from other students' behavior and learning. Because of this interrelationship, students' scores can not be considered independent for purposes of statistical analyses. Thus, the class or the teacher served as the unit of analysis in all analyses and results that we describe.

Classroom Observations

We computed means, standard deviations, and t tests between groups for each of the categories on the teacher observation system and the student observation system. Table 5 presents the results for the teacher system, and Table 6 presents the results for the student system. For most measures in both tables the numbers within each major category represent the mean proportion of time spent on that activity within the total time spent on addition and subtraction instruction. Thus, total time spent on addition and subtraction was used as the denominator for most of the proportions in these two tables.

Insert Tables 5 and 6 about here .

Although the focus of the observation--the teacher or the student--differed between the two observation systems, the results showed consistent and complementary patterns between CGI and Control teachers for similar observation categories in the two systems. Although some differences were significant in one system but not in the other, the same trend appeared consistently in both systems. The differences that exist can generally be explained by the fact that the focus of the observation, teachers or individual students, were not always engaged in the same activity. For example, for the *alone* category under setting the teacher was seldom observed to be working alone (approximately 2% of the time as shown in Table 5), but students frequently worked alone (about 34% of the time as shown in Table 6). Similarly, although the teacher engaged in one-to-one interaction with students about 20% of the time, individual students were observed to be engaged in one-to-one interaction with the teacher only about 1% of the time.

Setting. No significant differences appeared between CGI and control teachers in their grouping patterns as measured by the coding of the setting in the two systems.

Content and Lesson Phase. CGI students and control students did not differ in the proportion of time in which they were engaged with addition and subtraction content, but when teaching addition and subtraction, teacher observations showed significantly different content emphases between CGI and control teachers. During addition and subtraction instruction, CGI teachers spent significantly more time on word problems than did control teachers. In contrast, control teachers spent significantly more time on number facts problems than did CGI teachers (Table 5). A similar pattern was observed for the student observations although the difference in time spent on problem solving by students was not statistically significant (Table 6).

In the student observations the differences in content coverage varied over different phases of the lessons (Table 6). Control students were more likely to be given word problems to work on during seatwork, which was more apt to be done alone, while CGI students were more likely to work on word problems during review, development, and controlled-practice which was generally done in a large-group setting. In contrast, during these large-group lesson phases, control students were more likely than CGI students to be working on number facts. Although CGI and control teachers did not differ in time spent on *development, controlled practice*, or *seatwork*, control teachers and students did spend more time on review. Taken together, these findings for review and for content by lesson phase suggest that control students were more likely than CGI students to be spending time in a large-group setting on drill and review of number facts, while CGI students were more likely to be solving problems in the large-group setting.

Teacher Behavior. CGI teachers more often posed problems to students and more frequently listened to the process used by students to solve problems. In contrast, in giving feedback on students' solutions to problems, control teachers focused more frequently on the answer to the problem than did CGI teachers.

Strategy. Although no overall differences appeared between CGI and control classes in the strategies that students actually used during class (Table 6), CGI teachers allowed students to use a variety of different strategies during instruction more often than did control teachers (Table 5). Control teachers appeared to provide more explicit instruction in a particular strategy and expected the students to use that strategy. This pattern is illustrated by the different results for the advanced counting strategy on the two observation systems. Although results from the teacher observation system showed that control teachers expected students to use advanced counting strategies significantly more often than did the CGI teachers, results from the student observations showed no difference in the students' actual use of advanced counting strategies. It appears that CGI teachers provided as much opportunity for students to use advanced counting strategies, but they allowed the opportunity for students to use other strategies as well.

Teachers' Knowledge

Table 7 presents means, standard deviations, and t tests between groups for scores on the tests of teachers' knowledge. CGI and control group teachers differed significantly in their knowledge of student strategies for both number facts and problem solving. However, CGI and control teachers did not differ significantly in their knowledge of students' problem solving abilities in which teachers predicted students' performance on complex addition and subtraction word problems and on advanced problems.

Insert Table 7 about here.

Generally, control group teachers overestimated the use of number fact recall by their students by factors of two or three to one. In contrast, CGI teachers' predictions for the level of recall of number facts by their students generally did not deviate by more than 10% to 20% from the actual level of use by their students. Although CGI students actually used recall strategies significantly more than did control group students, control group teachers predicted higher levels of recall of number facts for their students.

Teachers' Beliefs

Table 8 presents the means and standard deviations for CGI and control teachers' beliefs at pretest and posttest. Group by Time analyses of variance (ANOVA) were computed to examine treatment effects on each of the teacher belief scales from pretest (before the workshop) to posttest (after a year of teaching). Table 8 summarizes the ANOVA results. A significant time by treatment interaction indicated that after the treatment for scales 2, 4 and total, CGI teachers were significantly more cognitively guided in their beliefs than were control teachers. Both groups of teachers increased significantly in their agreement with the perspective that children construct mathematical knowledge (Scale 1), and at posttest the CGI and control teachers did not differ in their agreement with this perspective.

Insert Table 8 about here.

Student Achievement

Analyses of covariance (ANCOVA) between groups were computed on each of the student achievement measures controlling for prior mathematics achievement as measured by the pretest. The ANCOVA was not computed on scores on the Simple Word Problem Scale because tests for homogeneity of regression revealed a significant prior achievement by treatment interaction. Table 9 presents means, standard deviations, adjusted means and t values for the ANCOVAs for the posttest student achievement measures. Although students' in CGI teachers' classes and students in control teachers' classes did not differ significantly in their performance on the ITBS-computation test, CGI students demonstrated a higher level of recall of number facts on the Number Facts interview than did control students.

Insert Table 9 about here.

A significant prior achievement by treatment interaction was found on the Simple Addition and Subtraction Word Problem scales. For classes who scored at the lower end of the scale on the ITBS pretest, CGI classes scored higher on the Simple Addition and Subtraction Word Problem Scale than did control classes. For classes at the very top of the scale on pretest achievement, control group classes scored higher than CGI classes. Figure 1 shows the interaction and the regions of significance that were computed using the Potthoff (1964) extension of the Johnson-Neyman technique. The regions of significance included 6 CGI classes and 6 control group classes at the lower end on pretest achievement and 3 CGI classes and 1 control class at the upper end on pretest achievement.

Insert Figure 1 about here .

On the test of Complex Addition and Subtraction Word Problems, students in CGI teachers' classes outperformed students in control teachers' classes. CGI and control classes did not differ significantly in their achievement on Advanced Problems. On the Problem

Solving interview, students in CGI classes used correct strategies significantly more often than students in control classes. No significant differences appeared between the groups in the students' use of any strategy.

Students' Confidence, Beliefs, Understanding, and Attention

Table 10 presents the means, standard deviations, and t tests between groups for students' confidence, beliefs, and reports of understanding and attention. CGI students were more confident of their abilities to solve mathematics problems than were control students. Like their teachers, students in CGI classes were significantly more cognitively guided in their beliefs than were students in control teachers' classes. In addition, CGI students reported significantly greater understanding of the mathematics than did control students. CGI and control students did not differ in the extent to which they reported that they paid attention during mathematics class. These latter findings are consistent with the classroom observations that showed that CGI and control students did not differ in the amount of time they were judged to be engaged with the addition and subtraction content.

Insert Table 10 about here.

Discussion

The results of this study provide a coherent picture of teachers' knowledge and beliefs, classroom instruction, and students' achievement and beliefs that is consistent with the assumptions and principles of Cognitively Guided Instruction. Two major themes are reflected in the guiding principles of CGI. One is that instruction should develop understanding by stressing the relationships between skills and problem solving with problem solving serving as the organizing focus of instruction. This suggests that CGI classrooms would be characterized by a greater emphasis on problem solving than would be found in traditional classrooms. The second major theme is that instruction should build upon students' existing knowledge. This implies that teachers regularly assess students' thinking and the processes that students use to solve different problems so that teachers understand students' knowledge and capabilities and can adapt instruction appropriately. We use these two themes in our discussion of the results.

The Role of Problem Solving

CGI and control teachers differed significantly in their beliefs about the relationship between skills and problem solving with CGI teachers' beliefs being more consistent with the basic assumptions of CGI. In particular, CGI teachers agreed more than control teachers with the belief that skills should be based on understanding and problem solving. Further, in contrast to control teachers, CGI teachers agreed more with the belief that instruction should facilitate children's construction of knowledge. Given that findings from research on young children's problem solving skills was the basis of the CGI workshop, this belief would also be consistent with a greater emphasis on problem solving during instruction.

The instruction of CGI and control teachers represented by the classroom observation data reflected the picture portrayed by the belief scales. CGI teachers focused on problem solving about 1-1/2 times as much as did control teachers, while control teachers focused on number facts about 1-1/2 times as much as did CGI teachers. Looking at these results from a slightly different perspective, CGI teachers spent over twice as much time teaching problem solving as they did teaching number facts; control teachers spent more time teaching number facts than problem solving.

Although the different measures of student computation and problem solving achievement provided slightly different perspectives, the emphasis on problem solving in CGI classes was reflected in student achievement. No overall difference appeared between the CGI and control classes in their ability to solve addition and subtraction problems with the result unknown (Simple Addition and Subtraction Word Problems). Performance on the Simple

Addition and Subtraction Word Problem scale was near the ceiling for both groups (means of 9.87 and 9.63 out of a maximum score of 11) so there was little room for treatment effects. However, a significant prior-achievement by treatment interaction appeared. In classes with low levels of achievement on the pretest, CGI students scored higher at posttest on the test of Simple Addition and Subtraction Word Problems than did control students. This interaction may have been due to the fact that the test allowed more room for lower achieving classes to move up on the test than for higher achieving classes to move.

The type of problems on the Simple Addition and Subtraction Word Problem scale represents the treatment of word problems typically found in most first-grade mathematics textbooks and programs. The Complex Addition and Subtraction Word Problem Scale included problems from the more comprehensive analysis of addition and subtraction problems that was discussed in the CGI workshop, and such problems are not typically included in a first-grade program. Performance of CGI classes was significantly higher on these complex problems than was performance of control classes. These significant differences did not transfer, however, to the advanced problems that involved multiple steps, extraneous information, or grouping and partitioning. It might be argued that the differences in problem-solving performance were simply a matter of more time spent solving problems. Increasing instructional time devoted to problem solving is not, however, a simple matter. In the past, teachers generally have been reluctant to sacrifice time traditionally devoted to teaching computational skills to teach more problem solving or to spend more time developing understanding. This is confirmed by results of national and international assessments which suggest that emphasis in mathematics instruction in American mathematics classes has been on low level skills and that the result has been that most students cannot apply the skills they have learned to solving even relatively simply problems (Carpenter, Lindquist & Matthews & Silver, 1983, McKnight, 1987).

Recent curriculum recommendations have called for a shi. in emphasis so that problem solving becomes the focus of instruction (National Council of Teachers of Mathematics, 1980; 1987). Previous reform efforts have failed, partly because teachers have not embraced the goals of the curriculum reformers (NACOME, 1975; Romberg & Carpenter, 1986) and because reformers have not attended to teachers' beliefs, thinking, and decision making (Clark & Peterson, 1986). Thus, although the content taught may have changed, t.. emphasis on skills has remained. Thus, convincing teachers to adopt a problem solving approach to teaching mathematics and to spend more time on problem solving is not a trivial matter.

Although both the treatment and control workshops in this study focused on children's problem solving performance, in neither workshop did we specify a program of instruction for teachers or dictate an amount of time that should be devoted to problem solving. The decision to spend more time on problem solving was made by the CGI teachers but not by control teachers. Teachers may be reluctant to place a greater emphasis on problem solving for several reasc ,. One is teachers may fear that many problems are beyond the capabilities of their students, and that students must master number facts first in order to solve word problems (Peterson, Fennema, Carpenter, & Loef, in press). CGI teachers knew that this was not so. The research they studied during the summer workshop provided convincing evidence that a wide variety of problems are not beyond the abilities of most first-grade students. Furthermore, this knowledge provided CGI teachers with a basis for assessing their own students to find out their students' actual problem solving capabilities. Consequently, CGI teachers may not have been reluctant to spend more time on problem solving.

Another related concern of teachers may be that time spent on proble.n solving will detract from their students' learning of computational skills. First-grade teachers and students are held accountable for students' learning addition and subtraction number facts to a requisite level of automaticity. The results of this study clearly document that a focus on problem solving does not necessarily result in a decline in performance in computational skills. In spite of the emphasis on problem solving in CGI classes and the corresponding decrease in time spent on computational skills, CGI classes and control classes performed equally well on a standardized test of computational skill. Furthermore, students in CGI classes actually had a higher level of recall of number facts than did control students. Because CGI teachers were able to accurately assess their children's knowledge, they may have been aware that their children were learning number facts. Thus, the concern of not meeting expectations for their children's competency in computation was alleviated.

Because most first-grade children have a variety of counting and modeling strategies that they can use to generate number facts, standardized tests often provide a better measure of the speed with which children can apply counting and modeling strategies than of children's actual recall of number facts. On tests such as the ITBS computation test, children often use these counting and modeling strategies so rapidly that it appears that they are recalling as they solve number fact items. Because CGI students actually used a recall strategy during the interview more than did control children, it might be argued that CGI students actually demonstrated higher levels of number fact knowledge and skills than did control students.

In summary, in contrast to control teachers, CGI teachers expressed beliefs that were more consistent with the principle that problem solving should be the focus of instruction in mathematics. CGI teachers spent more time than did control teachers on problem solving and less time teaching number facts. Differences in students' achievement on both problem solving and recall of number facts favored the CGI group.

Assessing Students' Thinking and Building Upon Their Knowledge

In contrast to control teachers, CGI teachers' knowledge, beliefs, and instructional practices were more consistent with the principle that it is important to assess children's thinking. The observation data showed that CGI teachers posed problems and listened to the processes that students used to solve problems significantly more often than did control teachers. CGI teachers also allowed students to use a variety of strategies to solve a particular problem more frequently than did control teachers. Both posing problems and listening to process provided the opportunity for teachers to assess students' knowledge. By allowing students to use any strategy that they chose, the teacher was able to assess how each student was thinking about the problem rather than requiring the student to imitate one strategy that the teacher specified. This approach was also more consistent with the belief expressed by CGI teachers that instruction should facilitate children's construction of knowledge rather than present information and procedures to children.

A typical activity that was observed in CGI classes was for a teacher to pose a problem to a group of students. After giving some time for students to solve the problem, the teacher would ask one student to describe how he or she solved the problem. In posing the problem, the teacher's emphasis was on the process for solving the problem, not the answer. After this student explained his or her problem solving process, the teacher would ask if anyone had solved the problem in a different way and give another student a chance to explain her or his solution. The teacher would continue calling on students until no student would report a way of solving the problem that had not already been described. This approach might have served at least two purposes. First, the CGI teacher was able to assess the problem solving processes of a number of students in the group, thus giving the teacher knowledge of individual student's problem solving abilities and strategies. Second, students were allowed to solve the problem at a level that was appropriate for them. In other words, the teacher facilitated students' learning by encouraging each student to construct a solution to the problem that was meaningful to him or her.

Control teachers' instruction was characterized by more control over the content of instruction and less assessment of students' thinking. In contrast to CGI teachers, control teachers posed problems less often, listened to students' strategies less, and less often encouraged the use of multiple strategies to solve problems. They spent more time reviewing material covered previously, such as drilling on number facts, and more time giving feedback to students' answers.

What CGI teachers learned by posing problems and listening to their children solve problems was reflected in their knowledge of their students. CGI teachers identified the strategies that their students would use to solve a problem or generate a number fact significantly more accurately than did control group teachers. Control group teachers consistently overestimated their students' ability to recall number facts. By better understanding the processes that children were using, CGI teachers may have been able to adapt instruction so that more appropriate activities were provided to children who were ready to learn number facts at a recall level. Recall of number facts was higher in CGI classes.

Although we had anticipated that the observation data might show treatment differences in grouping practices because CGI teachers would attempt to adapt instruction to individual students, we found no significant differences between CGI and control teachers in their use of medium groups, small groups, or individual instruction. Thus, while CGI teachers may have greater knowledge of the individual differences between students, they did not change their grouping practices. Most teachers in both groups continued to use whole class and individual seatwork as the primary instructional settings which tend to predominate in most elementary mathematics classrooms (Romberg & Carpenter, 1986; Peterson, in press).

Conclusions

The results of this study suggest that one effective approach for using the results of research on children's thinking and problem solving to improve classroom instruction is to help teachers to understand the principal findings of the research so that teachers can use this knowledge to evaluate more effectively their students' knowledge and make more informed instructional decisions. Such an idea is not new. Nearly a century ago, William James (1900) proposed the same general theme.

> You make a great, a very great mistake, if you think that psychology, being the science of the minds' laws, is something from which you can deduce definite methods of instruction for immediate classroom use. Psychology is a science, and teaching is an art; and sciences never generate arts directly out of themselves. An intermediate inventive mind must make the application (pp. 7-8).

Our findings also illustrate the fruitfulness of building on knowledge derived from two distinct paradigmatic approaches to research --classroom-based research on teachers and teaching and laboratory-based research on children's learning and cognition. Not only did important new knowledge emerge from the integration of these two paradigmatic approaches, but also the success of this research effort may herald the transition to a new, integrated paradigm that has been proposed (Romberg & Carpenter, 1986; Fennema, Carpenter & Peterson, 1988; Peterson, 1988). This study provides concrete evidence that knowledge from research on children's thinking and problem solving can make a difference in teachers' knowledge and beliefs which are reflected in teachers' classroom instruction and in students' achievement. The study also provides a perspective on what kinds of changes this knowledge leads to. In particular, teachers in this study seemed to use their specific knowledge they acquired about children's problem solving to assess their own students and to provide instruction that built upon their students' existing knowledge and skills.

In this study, we gave first-grade teachers access to specific research findings that portrayed children's addition and subtraction concepts which merge from children's informal problem solving. Teachers' subsequent classroom instruction reflected a problem solving emphasis. The question remains as to whether similar results would be obtained by giving teachers access to research based knowledge on children's thinking that focused less on specific aspects of problem solving. But the results of this study suggest that providing teachers with access to specific content related knowledge about students' problem solving can increase significantly teachers' emphasis on problem solving in their own classes. Thus, one question that requires further study is: What kinds of knowledge about students' thinking and problem solving can teachers use most effectively? We speculate that giving teachers knowledge of broad principles of learning and problem solving would have had less effect on teachers' instruction than giving teachers' access to more specific knowledge about children's problem solving in the particular content domain that was the basis for this study. In the workshop we provided teachers with explicit examples of children's problem solving in addition and subtraction that the teachers could relate directly to their own students, and also discussed examples of assessment techniques that the teachers could apply in their classrooms. As one of the CGI teachers in our study commented: "I have always known that it was important to listen to kids, but before I never knew what questions to ask or what to listen for."

A key feature of this study that distinguishes it from other studies of classroom teaching was that the mathematics content was a critical variable in the study. Cognitive researchers' analysis of content, that was the basis for the research on addition and subtraction, provided a link between the psychology of children's thinking and the mathematics curriculum so that teachers could apply what they learned about children in their teaching. The cognitive analysis also provided us, as researchers, with a framework for thinking about and for assessing teachers' knowledge, beliefs, classroom instruction and for evaluating students' achievement and beliefs. Although many unanswered questions remain, our results suggest that giving teachers access to research based knowledge about students thinking and problem solving can affect profoundly teachers' beliefs about learning and instruction, their classroom practices, and their knowledge about their students, and most importantly, their students' achievement and beliefs.

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Table 1

Classification of Word Problems

Problem Type	Result Unknown	Change Unknown	Start Unknown
Join	1. Connie had 5 marbles Jim gave her 8 more marbles. How many does Connie have altogether?	2. Connie has 5 marbles. How many more marbles does she need to win to have 13 marbles altogether?	3. Connie had some marbles. Jim gave her 5 more. marbles. Now she has 13 marbles. How many marbles did Connie have to start with?
Separate	4. Connie had 13 marbles. She gave 5 marbles to Jim. How many marbles does she have left?	5. Connie had 13 marbles. She gave some to Jim. Now she has 5 marbles left. How many marbles did Connie give to Jim?	6. Connie had some marbles. She gave 5 to Jim. Now she has 8 marbles left. How many marbles did Connie have to start with?
Combine	7. Connie has 5 red marbles and 8 blue marbles. How many marbles does she have?		8. Connie has 13 marbles. Five are red and the rest are blue. How many blue marbles does Connie have?
Compare	9. Connie has 13 marbles. Jim has 5 marbles. How many more marbles does Connie have than Jim?	10. Jim has 5 marbles. Connie has 8 more than Jim. How many marbles does Connie have?	11. Connie has 13 marbles. She has 5 more marbles than Jim. How many marbles does Jim have?

Table 2

Inter-Observer Agreement on Teacher and Student Observation Categories

Teacher Observation Category	% Interrater Agreement	Student Observation Category	% Interrater Agreement
Setting		Setting	
Whole Class	98	Whole Class	90
Medium Group	96	Medium Group	99
Small Group	98	Small Group	97
Teacher/Student	96	Teacher/Student	99
Alone	99	Alone	88
Content		Content	
Number Fact Problem	97	Represented Problems	94
Represented Problem	96	Word Problems	97
Word Problems	93	Number Fact Problems	94
Other Addition/Subtraction	99	Other Addition/Subtraction	99
		Non-Engaged with Content	95
Expected Strategy		Strategy Used	
Direct Modeling	90	Direct Modeling	90
Advanced Counting	98	Advanced Counting	99
Derived Facts	100	Derived Facts	100
Recall	91	Recall	82
Multiple	82	Not Clear	89
Not Clear	90		
Teacher Behavior		Lesson Phase	
Poses Problem	87	Review	95
		Development	94
Process Focus		Controlled Practice	99
		Student Work (Seatwork)	88
Questions Process	95		
Explains Process	96		
Gives Feedback to Process	95		
Listens to Process	98		
Answer Focus			
Questions Answer	87		
Explains Answer	95		
Gives Feedback to Answer	92		
Listens to Answer	88		
Checks/Monitors	98		

Table 3

Reliabilities of Student Achievement Measures (N = 40)

Test	Cronbach's alpha
ITBS (Level 6)	.84
Computation Posttests	
ITBS (Level 7) - Computation	.89
Number Facts Interview	.83
Problem-Solving Posttests	
ITBS (Level 7) - Mathematics Problems	.90
Scale 1: Simple Addition and Subtraction Word Problems	.72
Scale 2: Complex Addition and Subtraction Word Problems	.91
Scale 3: Advanced Word Problems	.90
Problem-Solving Interview	.66

Table 4

Representative Items from Problem-Solving Scales

Scale 1: Simple Addition and Subtraction Problems

Maria had 5 guppies. She was given 7 more guppies for her birthday. How many guppies did she have then?

Garcia had 13 balloons. 5 balloons popped. How many balloons did he have left?

Scale 2: Complex Addition and Subtraction Problems

Pat has 6 baseball cards. How many more baseball cards does she need to collect to have 14 altogether?

Larry had some toy cars. He lost 7 toy cars. Now he has 4 cars left. How many toy cars did Larry have before he lost any of them?

Jimmy has 12 rings. Amy has 7 rings. How many more rings does Jimmy have than Amy?

Advanced Problems

Jan needs 11 dollars to buy a puppy. He earned 5 dollars on Saturday and 2 dollars on Sunday. How much more money does he need to earn to buy the puppy?

Ann had 11 pennies. Candies cost 3 pennies each. Ann spent 6 pennies on candies. How many pennies does Ann have left?

Mary has 3 packages of gum. There are 6 pieces of gum in each package. How many pieces of gum does Mary have altogether?

Table 5

Means and Standard Deviations of Proportions of CGI and Control Teachers' Time Spent on Addition and Subtraction Instruction for Each Teacher Coding Category (N = 40)

Teacher Observation Category	CGI		Control		
	Mean	(SD)	Mean	(SD)	t(38)
SETTING[a]					
Whole Class	59.96	(22.97)	61.29	(18.07)	-0.20
Medium Group	11.31	(18.47)	3.00	(4.47)	1.95
Small Group	6.20	(8.23)	5.12	(5.45)	0.49
Teacher/Student	20.60	(12.85)	28.03	(14.15)	-1.74
Alone	1.93	(3.10)	2.55	(4.17)	-0.53
CONTENT[a]					
Number Fact Problems	25.95	(13.63)	47.20	(21.22)	-3.77**
Represented Problems	9.70	(8.33)	8.18	(5.50)	0.68
Word Problems	54.58	(18.84)	36.19	(21.92)	2.84**
Other Addition/Subtraction	9.77	(8.97)	6.44	(6.38)	0.54
TEACHER BEHAVIOR[b]					
Poses Problem	16.96	(6.22)	10.43	(4.64)	3.76**
Focuses on Process	24.96	(7.10)	22.78	(4.43)	1.17
Questions Process	5.91	(2.96)	4.85	(2.95)	1.13
Explains Process	6.78	(2.49)	8.24	(3.44)	-1.54
Gives Feedback to Process	4.81	(2.89)	5.71	(2.48)	-1.05
Listens to Process	7.46	(3.88)	3.97	(2.11)	3.54**
Focuses on Answer	28.63	(5.06)	30.85	(5.58)	-1.32
Questions Answer	11.94	(3.32)	11.34	(4.26)	0.49
Explains Answer	1.67	(1.07)	2.33	(1.24)	-1.80
Gives Feedback to Answer	8.04	(3.10)	10.51	(3.27)	-2.45*
Listens to Answer	6.97	(3.10)	6.66	(2.67)	0.34
Checks/Monitors	10.89	(6.21)	13.11	(6.35)	-1.12
EXPECTED STRATEGY[c]					
Direct Modeling	29.85	(16.10)	29.05	(16.46)	.16
Advanced Counting	4.65	(4.18)	10.72	(12.67)	-2.04*
Derived Facts	3.19	(4.38)	1.43	(3.21)	1.44
Recall	14.82	(8.83)	18.48	(11.24)	-1.15
Multiple	32.29	(15.51)	21.93	(10.89)	2.45*
Not Clear	5.43	(5.91)	9.95	(10.11)	-1.73

*$p < .05$
**$p < .01$
[a]Proportions within this category sum to 100% because one of each of the subcategories was coded for each time the teacher engaged in addition and subtraction instruction.

[b]Proportions within this category do not sum to 100% because for each time the teacher engaged in addition and subtraction instruction, some additional subcategories of teacher behavior were coded but are not reported here.

[c]Proportions within this category do not sum to 100% because the additional content category of "other addition and subtraction" might have been coded when the teacher engaged in addition and subtraction instruction.

Table 6

Means and Standard Deviations of Proportions of CGI and Control Students' Time Engaged in Addition and Subtraction Instruction/Activities for Each Student Coding Category (N = 40)

Student Observation Category	CGI		Control		
	Mean	(SD)	Mean	(SD)	t(38)
SETTING[a]					
Whole Class	51.79	(24.38)	53.25	(18.58)	-0.21
Medium Group	6.56	(12.91)	1.67	(2.83)	1.65
Small Group	5.92	(4.39)	7.72	(7.25)	-0.95
Teacher/Student	1.47	(1.18)	1.59	(1.07)	-0.34
Alone	34.26	(16.67)	35.76	(16.45)	-0.29
LESSON PHASE[b]					
Review	4.49	(2.78)	8.04	(4.44)	3.03**
Development	19.10	(12.06)	19.71	(10.20)	-0.17
Controlled Practice	37.50	(18.93)	30.14	(14.51)	1.38
Student Work (Seatwork)	38.44	(17.88)	41.42	(17.18)	-0.54
CONTENT[a]					
Represented Problems	7.56	(5.99)	7.23	(5.82)	0.13
Word Problems	39.93	(15.90)	31.44	(18.64)	1.55
Number Facts Problems	26.90	(13.52)	37.19	(17.58)	-2.08*
Other Addition/Subtraction	7.80	(6.76)	6.81	(5.33)	0.51
Non-Engaged with Content	17.81	(7.10)	17.23	(8.00)	0.24
CONTENT X LESSON PHASE[b]					
Review, Development & Controlled Practice					
Represented Problems	4.84	(4.72)	5.49	(5.34)	-0.40
Word Problems	36.13	(15.13)	23.27	(13.00)	2.88**
Number Facts	9.03	(5.61)	18.81	(10.81)	-3.59**
Other Addition/Subtraction	4.61	(5.78)	4.92	(4.20)	-0.20
Student Work					
Represented Problems	2.67	(2.73)	1.79	(2.18)	1.13
Word Problems	3.67	(4.33)	8.06	(8.15)	-2.13*
Number Facts	17.78	(12.58)	18.05	(11.09)	-.07
Other Addition/Subtraction	3.19	(2.83)	1.85	(2.14)	1.65

Table 6 continued

Student Observation Category	CGI Mean	(SD)	Control Mean	(SD)	t(38)
STRATEGY USED[c]					
Direct Modeling	24.40	(12.70)	26.46	(12.24)	-.52
Advanced Counting	8.35	(6.58)	8.93	(7.95)	-.25
Recall	19.91	(5.91)	20.72	(9.89)	-.31
Derived Facts	1.63	(2.06)	1.25	(2.36)	.54
Not Clear	20.10	(10.58)	18.60	(10.99)	.44

*$p < .05$
**$p < .01$

[a]Proportions within this category sum to 100% because one of each of the subcategories was coded for each time the student was expected to be engaged in addition and subtraction instruction or activities.

[b]Proportions within this category do not sum to 100% because for each time the student was engaged in addition and subtraction instruction or activities, some additional subcategories were coded but are not reported here.

[c]Proportions within this category do not sum to 100% because the additional content category of "other addition and subtraction" and "non-engagement" might have been coded for content.

Table 7

Means, Standard Deviations, and t-tests for Teacher Knowledge Tests

(N = 40)

Test	Maximum Possible	CGI Mean (SD)	Control Mean (SD)	t(38)
Number Fact Strategies	5	2 81 (.46)	2.25 (.58)	3.51**
Problem-Solving Strategies	6	2.97 (.87)	2.08 (.71)	3.56**
Problem-Solving Abilities	8	5.40 (.75)	5.25 (.51)	.74

**p < .01

Table 8

Pretest Means, Posttest Means, Standard Deviations, and ANOVA Results for Teachers' Beliefs (N = 39[1])

Belief Scale		CGI Mean (SD)	Control Mean (SD)	F Tests Group(G)	Time(T)	GXT
Scale 1	Pretest	40.15 (7.52)	39.05 (6.24)			
	Posttest	45.20 (4.82)	42.11 (7.08)	1.18	27.65**	1.68
Scale 2	Pretest	48.75 (5.80)	50.26 (6.62)			
	Posttest	53.80 (4.48)	50.16 (6.85)	0.38	9.01**	9.79**
Scale 3	Pretest	45.60 (5.83)	45.00 (6.01)			
	Posttest	47.75 (5.12)	44.63 (5.06)	1.28	1.87	3.74
Scale 4	Pretest	44.00 (6.10)	44.79 (5.42)			
	Posttest	50.85 (5.78)	47.37 (5.51)	0.37	25.68**	5.26*
Total	Pretest	178.50 (22.11)	179.11 (20.30)			
	Posttest	197.60 (16.27)	184.26 (20.95)	1.13	27.73**	9.16**

*p < .05
**p < .01

[1]Pretest Belief Scores unavailable for one teacher.

Table 9

Means, Standard Deviations, Adjusted Means, and t-values for Student Achievement Measures (N = 40)

Test	Maximum Possible	CGI Mean (Adjusted Mean)	CGI SD	Control Mean (Adjusted Mean)	Control SD	t(37)[1]
Computation						
ITBS (Level 6)	27	20.95 (20.91)	2.08	20.05 (20.10)	1.81	1.40
Number Facts	5	2.26 (2.25)	.49	1.80 (1.81)	.78	2.23*
Problem Solving						
ITBS (Level 7) -Problems	22	17.28 (17.20)	1.83	16.42 (16.50)	1.89	1.62
Simple Addition/ Subtraction[2]	11	9.87	.40	9.63	.67	
Complex Addition/ Subtraction	12	8.60 (8.53)	1.56	7.80 (7.87)	1.51	1.99*[3]
Advanced	13	8.40 (8.32)	2.02	8.05 (8.13)	1.29	.52
Interview	6	5.62 (5.61)	.28	5.37 (5.38)	.37	2.51*

*p < .05

[1] t-values calculated following ANCOVA approach.
[2] Interaction present so ANCOVA could not be calculated.
[3] One tailed test.

Table 10

Means, Standard Deviations, and t-tests for CGI and Control Students' Confidence, Beliefs, Reported Attention, and Understanding (N = 40)

Student Measure	CGI Mean (SD)	Control Mean (SD)	t(38)
Confidence	10.34 (.87)	9.77 (1.14)	1.79*[1]
Student Beliefs	31.12 (2.01)	29.61 (1.09)	2.97**
Reported Understanding	3.30 (.28)	3.16 (.22)	1.73*[1]
Reported Attention	3.43 (.30)	3.50 (.24)	-0.82

*p < .05
**p < .01

[1]one tailed test

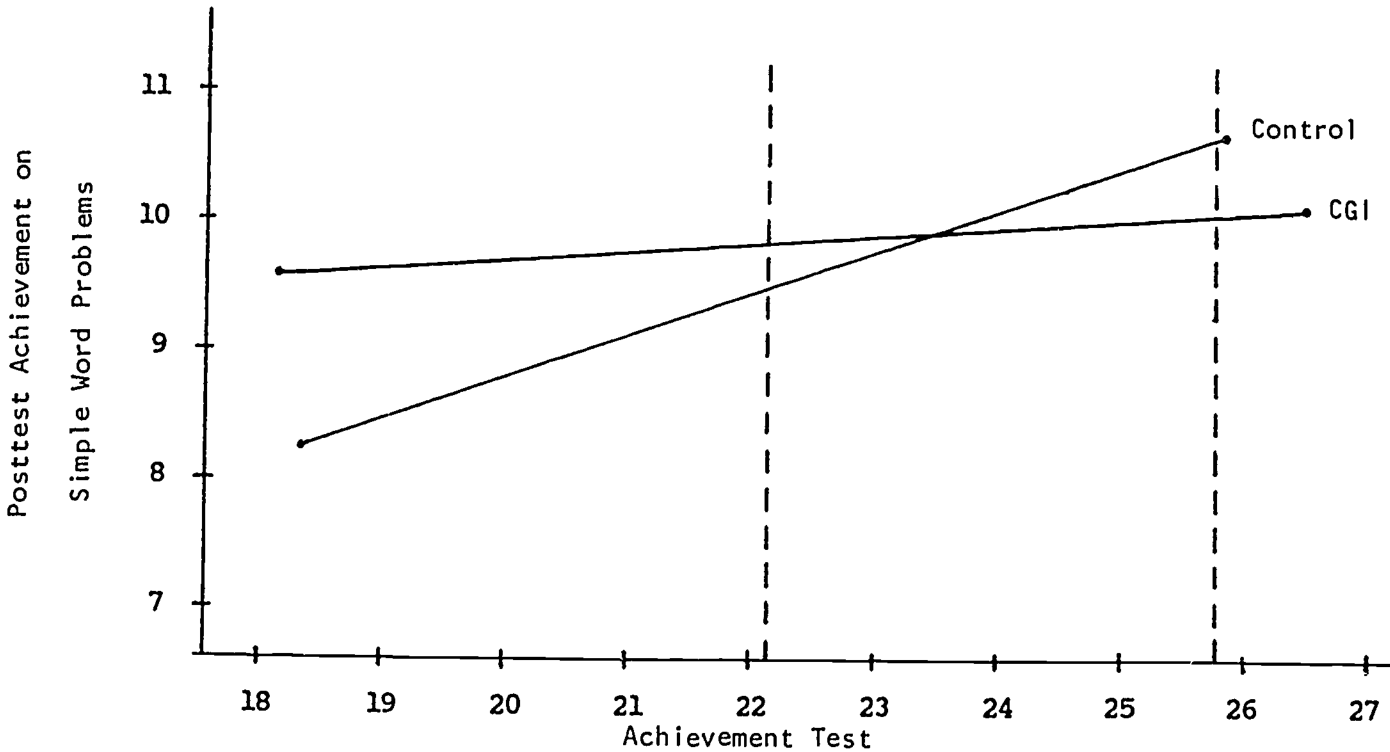

Figure 1. Regression slopes of Simple Addition and Subtraction Word Problem Scale on Achievement Pretest and regions of significance as defined by broken vertical lines at 22.14 and 25.80.