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ABSTRACT

During the summer of 1986 a group of 15 racially diverse students entering 10th and 11th grades participated in an intensive 4-week course taught by members of an NSF-funded project. The project's goals were to develop science and mathematics curricula based on the principles of systems dynamics. The course was a test of materials intended to provide students with methods for problem solving by introducing them to modeling software and concepts of systems thinking. Students engaged in a study of levels and rates, causal-loop diagrams, feedback, exponential growth, exponential decay, goal-seeking behavior, s-shaped behavior, and oscillating behavior by modeling real-life problems. Some of the problems that students worked on dealt with population growth, bank balances, temperature cooling, capacitor discharging, city growth, and predator-prey relationships. Nine boys and six girls volunteered to participate in the project. Evaluation of the course consisted of pre- and posttesting, observations, administration of a student background questionnaire, and interviews with students at the conclusion of the course. All questions in the categories tested showed a significant gain in correct responses from the pretest to the posttest. It was concluded that some transfer of skills was shown. (CW)

Tools for Teaching Problem Solving: An Evaluation of a Modeling and Systems Thinking Approach*

Paper Presented at the 1988 Annual National Association for Research in Science Teaching (NARST) Conference, Lake Ozark MO, April 1988

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This paper presents an evaluation of two pilot efforts to provide high school students with math and science problem solving skills based on the principles of system dynamics. The first pilot test was held during the summer of 1986, and the second was a five-week systems units taught in two advanced 8th grade classes in the spring of 1987. The curriculum for the initial summer effort was developed and taught by members of the NSF-funded Modeling project, a two-year project whose goals are to develop innovative computer-based modeling tools to help students solve complex social and natural science problems. The curriculum for the 8th grade unit was based on the initial curriculum and was modified by project staff and the participating classroom teacher.

Objectives

The objective of the project is to allow students to apply their intuitive knowledge of mathematics and science to solve problems that would normally require knowledge of calculus. The project utilizes computer modeling tools that enable students to solve problems drawn from the natural and social sciences which involve behavior that changes over time. Students are encouraged to make predictions, to collect data, and to make computer models to test their predictions. They are presented with data as graphs of functions over time, so they develop a working understanding of graphing and graph interpretation, and become familiar with concepts such as *function* and *change over time*. Students typically have difficulty applying these concepts to problem solving without a more extensive mathematics background. Barclay and Roberts (1987) and Tinker (1987) provide a discussion and rationale of the objectives of the project.

The objective of the summer course was to test to what extent a mathematical modeling approach allows students to solve social and natural science problems. This was the first opportunity to try the new systems-based curriculum that was developed by project members. The objective of the 8th grade unit was to apply this approach in a real classroom setting to see how well students could learn systems concepts without the direct guidance of project staff.

We would like to thank Tim Barclay and Robert Tinker for their helpful comments on an earlier draft of this paper.

Summer Course Description

Project staff taught the 1986 summer course on weekday mornings from 9 a.m. to noon for a period of four weeks. During the first week, students became familiar with the Macintosh computer and a spreadsheet program, and discussed math functions, their graphing and transformations. During the second week students were introduced to a physical model of levels and rates that involved the measurement of the flow of water into and out of plastic beakers, and began to construct mathematical models of levels and rates using the spreadsheet program EXCEL (1986). Students also began to use an iconic modeling program called STELLA (1985) at the end of the second week. STELLA allows the user to define diagrams of how valves and reservoirs, the iconic representations of flow and level, are interconnected. The program then runs the underlying mathematical model over time and displays the modeled behavior on a graph or in a table. During the third and fourth weeks of the course, students used STELLA to model problems involving different types of behavior over time. Students explored models of bank balance and population (representing exponential growth), temperature cooling and electrical capacitor discharging (exponential decay/goal-seeking behavior), city growth (s-shaped behavior), and predator-prey relationships (oscillating behavior). The main concepts covered during these two weeks involved rates and levels, inflow and outflow, and the importance of causal relations and feedback. Students learned to use causal-loop diagrams to think about complex systems involving feedback. These diagrams are symbolic representations of the relationship between the elements in a system and their effects on each other.

Six Macintosh computer stations were available in the classroom throughout the course, and students typically worked on problems in groups of two or three.

The Participants

A total of 15 students, a racially integrated group of nine boys and six girls, participated in the summer course. The students had completed ninth or tenth grade and all had taken an Algebra I course, but none had taken Algebra II. Pretest results showed a wide range of ability to deal with algebraic and numerical concepts. Background questionnaires administered at the outset showed a diversity in students' science course backgrounds, level of computer experience, and types of interests. All students volunteered to participate in the course and were paid for their attendance.

Summer Course Evaluation

The evaluation of the course included pre- and post-testing, daily observations, and interviews with students at the completion of the course. The observations and student interviews provided helpful feedback to project staff about the content and process of the course, which was incorporated into further development efforts.

Observation Results

One important concept that students grasped relatively easily during the summer course concerned the difference between arithmetic and geometric growth, and how these patterns of growth are represented in graphs as linear or exponential functions. Early on in the course, students used a series of tanks and valves through which water could flow, as a way of building physical models of growth and decay. One day, the students' task was to add to or to subtract from the bucket a constant amount of water over a series of trials, and to observe the graph of the water level on a computer which measured the water level in the tank. When students were asked to explain what they saw and what the graph meant, typical responses included, "it [the level] went straight up, except for a few bumps, when we shook the bucket by mistake. ... It gets higher and higher, but it doesn't change the slope if you add in the same amount over time," or "The line was supposedly increasing. It just goes up at the same rate."

Students also modeled exponential growth and decay using the same series of tanks and valves. In this case, students did not add or subtract a constant amount of water from the bucket, but rather they added or subtracted a percentage of the water in the bucket over trials. One boy made the following observation about exponential decay: "We're taking out 31, now take out 29... as it [the level] goes down, the drop is going to be less... because as you take it out, the interest, no I mean the tax, is going to be less." This boy's response illustrates not only that he understood the concept of exponential decay, but also that he applied his understanding to a problem involving withdrawal of a certain percentage of tax out of an account.

One group of students disagreed with each other about how a graph of exponential growth would look, although they all agreed that over time, more "interest" (water) was being added to the bucket. One girl commented that "the line isn't very inclined at the beginning, but as you go on, the more you add, the more inclined it [the slope] would be." In contrast, her partner said "you are just adding a little more each time, like 17, then 18, and you are really not adding much so the slope would be the same." The girl who made this comment appeared not to be convinced by the exponential shape of the graph, perhaps because only 12 trials were displayed on the screen. It is possible that if she had seen the pattern over a longer period of time, she would have seen that the slope of the line becomes steeper over time, as more "interest" (water) is added to the bucket.

Students' understanding of exponential growth was also evident when they built models using a calculator, when using the spreadsheet Excel, when building STELLA models, and when working with causal-loop diagrams. For example, one boy explained a bank balance model of exponential growth by saying "as you have more money in the bank, the interest increases." Students often recognized that exponential growth depended on the presence of feedback. For example, when working with causal loop diagrams one day, one girl noted that "debts keep getting bigger and bigger because of interest."

During the summer course students also demonstrated an increasing understanding of levels and rates. For example, one boy graphed the number of cars in the United States from the late 1800's to the present, with the level increasing fairly steadily, except for a dip around 1929. He pointed out that the number of cars would go down during the Depression. Some class members challenged him on this point, saying that during the Depression the actual number of cars did not go down, but only the rate of buying cars would decrease. They argued that the graph of the number of cars should not have a dip, but should just level off during the Depression. Students actively discussed the difference between rates and levels among themselves, with no prompting from the teacher.

Students also recognized that different problems shared the same underlying structure. One girl suggested that the temperature model of cooling coffee was similar to a model of a toilet filling up with water. She noted that in both cases the greatest amount of change happened at the beginning, with the rate of cooling or water flowing into the tank slowing over time. She also recognized both problems as examples of goal-seeking behavior. Students also discovered the similar exponential growth patterns in population growth and bank balance examples, and recognized that exponential growth occurs when the rate of growth is defined as a function of population size or bank balance.

At certain points throughout the course, students demonstrated a general understanding of a problem or a concept, but had some difficulty constructing the appropriate model. For example, when learning about exponential decay, students used the Heat and Temperature unit from the Microcomputer-Based Laboratory (1987) software to observe water cooling. When discussing this phenomenon with the teacher, students understood that water cools fastest when the difference between the water temperature and the room temperature is greatest. They understood that as the water temperature approaches the room temperature, cooling slows down. When constructing STELLA models of this phenomenon, however, students had difficulty coming up with the equation for the rate of cooling that would take into account this notion of a gap between the room temperature and the water temperature. Once the teacher told students how to define the equation, they saw that it made sense.

In another instance, students built STELLA models of city growth and demolition. They clearly understood that for the number of buildings in the city to remain constant, the inflow must equal the outflow, or the number of buildings being built would have to equal the number of buildings being destroyed. However, students didn't realize that the number of buildings being built would have to be represented as the reciprocal of the number of buildings being destroyed in their growth equations. When students built their models, they used "brute force" to get the inflowing buildings to equal the outflowing buildings, by adjusting one fraction and then another, until the line representing the level in their STELLA graph appeared constant.

Sometimes students recognized that there was a problem in their models, and they found their own solutions to the problems. For example, one group of students had built a STELLA model of their personal budget. When they ran the model, their graph went off the screen. They decided that they could remedy this problem either by rescaling their axes to make the graph appear smaller, or by spending more money, so that the amount of money (the level) would be less, and therefore stay on the screen. This group realized that both approaches were possible, but they decided to solve the problem by rescaling their axes, rather than by modifying their model.

Summer Course Testing Results

Results of tests administered to the group before and after the four-week class support the observations of what students learned. Students improved the most on questions dealing with exponential growth and decay. A total of seven items on the test, presented in several different formats, dealt with these concepts. Students improved substantially on all of these items. There was a question that asked students to explain why a given model would show exponential growth rather than linear growth, and another that described a situation and asked students to draw a corresponding model and to sketch the corresponding graph (which was an exponential growth graph). Two questions required students to choose a graph that matched a described situation (in one an exponential decay graph was correct, in the other exponential growth). In another question the task was to match a graph with a given exponential decay equation, and in another to match a set of formulas with a diagram illustrating an exponential growth model.

On these seven questions the 15 students improved from an average 30% correct on the pretest to 67% correct on the posttest, an increase of 37 percentage points ($p<.01$). Results for this set of questions are summarized in Table 1. In order to give a flavor for the types of questions asked about exponential growth and decay, two of the questions are presented in Figures 1 and 2.

Table 1: Test Questions Dealing with Exponential Growth and Exponential Decay

QUESTION	% Students Answering Correctly Pretest	Posttest	% change	Statistical Signif. (**)
1. Describe Exp. Growth	20	53	33	p<.01
2. Exp. Growth Formulas	53	87	34	p<.02
3. Exp. Decay Graph(*)	13	60	47	p<.01
4. Sketch Exp. Growth Diagram	27	80	53	p<.01
5. Sketch Exp. Growth Graph(*)	27	67	40	p<.01
6. Match Exp. Decay Graph w/ Eq.	13	40	27	p<.10
7. Exp. Decay Graphs	60	80	20	ns.
MEAN OF 7 QUESTIONS	30	67	37	p<.01

(*) Selected questions are shown in Figures 1 and 2

(**) Paired two-tailed t-tests, n=15 (Given the small sample size, statistical significance from pretest to posttest is only achieved when differences in percent correct are substantial.)

Two other questions that showed substantial improvement dealt with feedback loop diagrams. During the course, students spent a good deal of class time drawing positive and negative feedback loops to help them understand particular types of models. This experience helped them to interpret and to draw causal loop diagrams on the posttest. A total of six questions dealt with feedbck loops. The results for these six questions are presented in Table 2, and the two questions that showed significant improvement are reproduced in Figure 3. Some of the feedback loop questions seemed intuitively obvious to students, as evidenced by high pretest scores in this section, and as a result the overall gains for the section are not statistically significant.

Table 2: Test Questions Dealing with Feedback Loop Diagrams

QUESTION	% Students Answering correctly Pretest	Posttest	% change	Statistical Signif. (**)
8. Diagram Interpretation(*)	40	87	47	p<.01
9. Diagram Interpretation (*)	53	100	47	p<.01
10. Diagram Explanation	80	87	7	ns
11. Diagram Interpretation	93	93	0	ns
12. Sketch Relationship	67	67	0	ns
13. Sketch Diagram	53	73	20	ns
MEAN OF 6 QUESTIONS	64	84	20	ns

(*) Selected questions are shown in Figure 3

(**) Paired two-tailed t-tests, n=15 (Given the small sample size, statistical significance from pretest to posttest is only achieved when differences in percent correct are substantial.)

A third set of fourteen items on the test involved more general graphing and graph interpretation concepts that were not explicitly taught in the course. Five of these items tested linear graph interpretation skills, including matching a graph with data given in a table, and cnoosing or sketching a graph that represents a depicted physical situation. Another five of these items dealt with rate change in graphs, and four additional items dealt with non-linear graphs. The results of some of the fourteen graph interpretation items are shown in Table 3, and selected questions are presented in Figures 4, 5, 6, and 7. Taken together, the questions did not show significant gains from pretest to posttest, but students did substantially better on some individual questions in the group, particularly on questions dealing with linear graphs. The fact that students were able to improve on questions that were not explicitly covered in the four-week course demonstrates that some transfer of skills in the area of graph interpretation had taken place.

Table 3: Selected Test Questions Dealing with Graph Interpretation

QUESTION	% Students Answering correctly Pretest	Posttest	% change	Statistical Signif. (**)
14. Inverse relationship	60	47	- 13	ns
15. Table to graph (*)	47	80	33	p<.05
16. Distance graph: steady speed	53	80	27	p<.05
17. Line graph (*)	13	60	47	p<.01
18. Sketch direct relationship	73	73	0	ns
19. Ball velocity (*)	47	67	20	ns
20. Constant velocity	67	67	0	ns
21. Slowing growth of GNP (*)	47	67	20	ns

(*) Selected questions are shown in Figures 4,5,6, and 7

(**) Paired two-tailed t-tests, n=15 (Given the small sample size, statistical significance from pretest to posttest is only achieved when differences in percent correct are substantial.)

It is interesting to note that students showed no gains, and in fact scored consistently, though not significantly, lower on a set of 13 items dealing with algebra skills. These questions asked students to solve algebraic equations, factor expressions, and calculate ratios and sequences. It appears that while the summer course experience did lead to improvements in students' understanding of behavior that changes over time and enhancement of students' graph interpretation skills, it did not affect their ability to solve traditional algebra problems.

The results of the summer course evaluation indicate that high school students who are taught to use computer modeling tools can solve relatively complex natural and social science problems without having knowledge of advanced mathematics, and can begin to understand the concepts involved in modeling systems behavior over time. Test results indicate that students can also improve their graph interpretation skills. Such results are preliminary, however, and are not generalizable given the small, self-selected sample, the experienced teaching staff, and the intensive summer course.

8th Grade Classroom Trials

In order to further our understanding of the applicability of this approach in a real school setting, we field tested some of the materials in two 8th grade physical science classes taught by a science teacher who had not previously used the materials and who did not usually use computers in instruction. The teacher was supported by a colleague who is a consultant to the project and who has experience in teaching mathematics using a systems thinking approach. The 8th grade classes were in a selective private school, which implies the students were by no means typical, but the setting allowed a curricular flexibility not usually attainable in public schools.

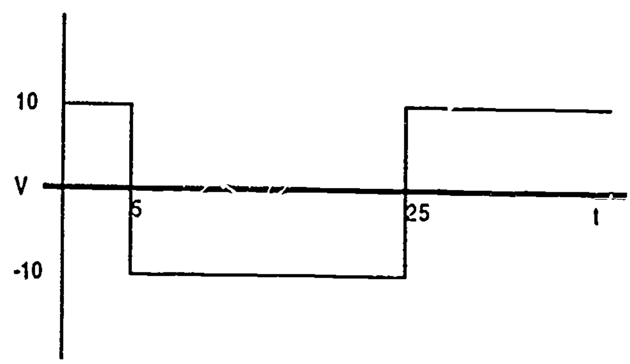
The five-week unit, carried out in the spring of 1987, focused on a study of rates and levels. The classes met four times per week, three times for 40 minutes each and once for 75 minutes. Students studied the relationship between velocity and distance curves, the physical relationship between the rate of flow of a liquid into or out of a container and the level of the liquid in the container, and bank balance compound interest problems. Students used the icon-based modeling software STELLA and a spreadsheet program to develop computer models of the examples they were studying, and they were encouraged to compare the underlying structure of different models in order to see their similarity. Five Macintosh computers were used in the unit and students worked in groups of two or three per computer. Prior to the start of this unit, students used the Motion detector of the Microcomputer-Based Labs (1987) for two weeks to familiarize themselves with graphs of velocity and distance over time.

Velocity and Distance

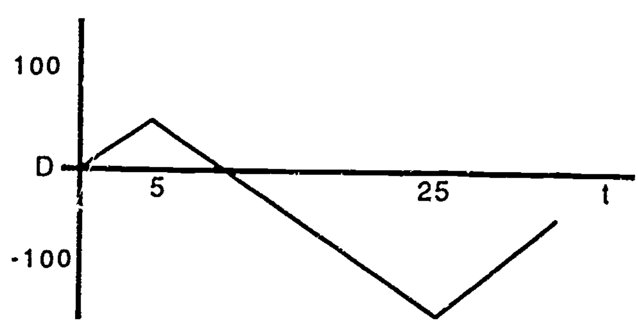
The classes began studying velocity and distance curves using a STELLA model template that was provided. A STELLA template is a prepared model that has been saved on a disk for student use. Students load the template into STELLA, then input various velocity curves into a predefined graph by using the computer's mouse. The graph used for this activity had an arbitrary velocity range from -10 to 10, and a time range from 0 to 30. Once students enter a velocity curve they can run the model, and STELLA generates a distance graph that corresponds to the given velocity curve.

Students were given a worksheet with curves of velocity over time which they input into the computer, and were encouraged to predict the corresponding distance curves. They then ran the computer model and compared their predictions with the actual distance graphs generated by STELLA.

Some of the students in the class were very impressive in their ability to predict a distance graph from a given velocity graph. A group of three students correctly predicted, before they performed the simulation using STELLA, the shape of a distance graph that corresponds to the following complex velocity graph.



These students realized that the distance graph will be negative at times, and rescaled their distance axis to include negative values before they ran the model. Their prediction, and the generated distance curve, looked like this:



A group of three girls working on the same problem did not initially predict that distance will be negative at some points along the generated graph. They ran the model and wondered why their distance graph disappeared off the bottom of the screen. After some discussion, one of the girls got up and demonstrated walking forward and then backward to explain that the distance graph will become negative. She said that the STELLA graph cannot show negative distance because it starts at zero. The teacher told the girls that they could rescale the distance axis and he showed them how to do this. They reran the model and found that their description of the velocity graph as walking forward for 5 seconds, backward for 20 seconds, and then forward was correct.

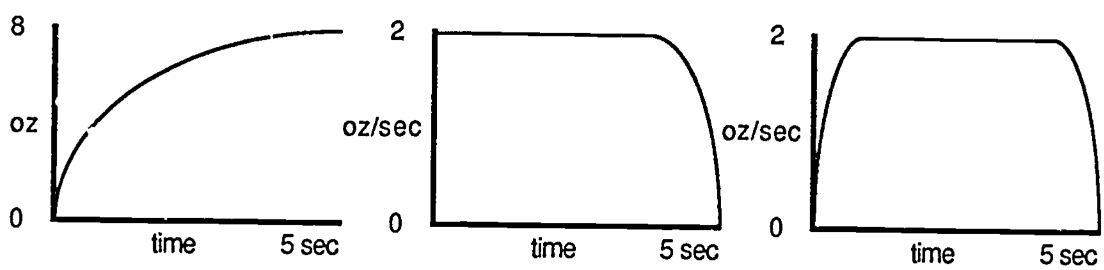
After several classes using the STELLA motion template, students received an off-computer quiz that required them to use graph paper to draw a set of distance graphs based on given velocity graphs. Given how well students did on the class worksheets described above, it was surprising how much difficulty some of the students encountered in doing the off-computer quiz. While students knew the shape of the graphs they were asked to generate, they had trouble computing exact points on a graph that had to be drawn accurately on graph paper, and not just sketched in as on previous worksheets. The off-computer quiz was a task that the students did not have to do previously, and illustrates the difference between graph interpretation and graph production skills. When students used the STELLA model, the computer generated the exact graphs, and they merely had to predict the shape and general features of the graphs. The mathematics required to compute exact points for some of the graphs was quite complicated, and was beyond the reach of many of the students in the class. The teacher and project staff agreed that students had a fundamental understanding of the relationship between velocity and distance graphs, and would have no trouble answering the questions on the quiz if they had access to the STELLA template.

Liquid Flow Rates and Levels

In this part of the unit students discussed the relationship between the rate of pouring of a liquid into or out of a container and the level of the liquid in the container. The teacher demonstrated the concept by pouring milk into a glass at varying rates, and asked students to come up with rate and level graphs that corresponded to the physical action over time. He also drank from the glass using a straw and asked students to generate the appropriate graphs. Students were given the following problem for homework:

> You pour a glass of milk, being careful to fill it right to the top and therefore pouring more carefully as you get near the top. Graph both your rate of pouring against time, and the level of your glass against time. Label your axes and choose your scales to be realistic.

The next day the students discussed the solution to the given problem. They first discussed the scales of the two axes. They agreed that the level should be from 0 to 8 oz, and the time from 0 to 5 secs. There was more discussion about the rate scale, but a consensus was reached to say it would be from 0 to 2 oz/sec. All students agreed that the level graph would look like Figure **A** below. There was discussion about what the rate graph should look like. Some students thought it should be like Figure **B** below, and others argued that since it takes some time to begin the full flow of milk into the glass, the rate graph should look like Figure **C** below.



A. LEVEL **B. RATE** **C. RATE**

Again students thought about the shapes, not exact values. The discussion reflected that the students understood the basic relationship between the level and rate graphs, and how the graphs related to the physical action. However there was no discussion of the exact rate of pouring necessary to fill up an 8 oz. glass in five seconds. For example, the rate graph shown in **B** above would make the glass overflow after 4 secs., and the first part of a corresponding level graph would be linear. Students did not have an opportunity to explore this relationship using the computer, which may have helped them formulate more precise graphs.

Students were given a similar problem on a test at the conclusion of the unit. They had access to the computer-based modeling tools for this test, but none of them chose to use the computer for this question. While most of t' responses had the correct shapes, there were some inaccuracies in the scales so that the level and rate graphs drawn by some students did not match accurately.

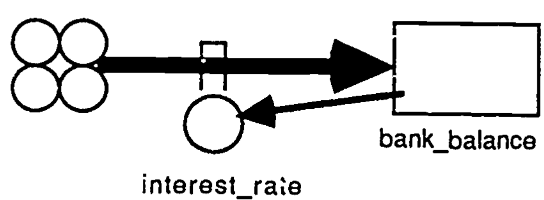
Bank Balances

Students began their study of bank balance problems using the spreadsheet program. They learned to specify formulas in the spreadsheet to calculate simple and compund interest over a given time period, and to solve a variety of bank balance problems. Students were then asked to use STELLA to begin solving financial math problems. During the first of these classes, students were given the following problem to solve using STELLA:

> You have $10,000 to invest. Do you invest at 9% simple interest or at 8% interest compounded annually? What will be the difference in balances at the end of 8 years?

One student immediately asked if he had to use the computer to solve the problem. He recognized that this was a simple problem to solve by hand, because only 8 iterations were involved. The teacher encouraged all students to use STELLA, in order to learn how to set up their own STELLA models.

Students found that setting up their own model in order to solve the problem was quite different, and more difficult, than having a model all ready to manipulate as was the case with the template prepared for the motion problems. The appropriate STELLA model is shown below.



The bank balance is represented by a rectangular "reservoir" which changes its level depending on the size of the bank balance. The initial amount, in this case $10,000, is entered as the starting level of the bank balance reservoir.

The thick interest rate arrow represents a "valve" which adds a defined amount of interest to the bank balance reservoir each year. In the simple interest case the value of the valve is constant. In this example the valve would be defined as .09*10,000, or $900 added to the bank balance each year.

In order to represent the compound interest case, a thin feedback arrow from the bank balance reservoir to the interest valve must be added to the model. The feedback arrow indicates that the amount of interest added every time period is a function of the balance. In the compound interest example, the valve would be defined as .08*bank_balance, so that 8% of the current bank balance would be added as interest each year.

Some students had trouble setting up the compound interest model correctly, with an appropriate feedback arrow for the compound interest case from the balance reservoir to the interest valve. Students also had trouble defining what to graph, how to scale, how to set the initial balance, and how to define the interest equation. With help from the teacher and the observers, they finally did create functioning models and were able to answer the posed question.

It was interesting to note that some students did not evaluate the reasonableness of their answers. Some students, due to an error in their interest formula or simulation time, got bank balances that grew at phenomenal rates. Rather than recognizing that their answer was not reasonable, students simply rescaled the balance axis so that the generated graph "looked right". One group of students used a simulation time of 96 (as in 96 months in 8 years), but still used the 8% annual compound interest rate in the interest formula, rather than .67% per month. As a result their projection was in fact for 96 years, and not 8 years as they thought.

Students were presented with a similar problem on the unit test. The question was:

> You have $1,000 in a bank account that pays 12% interest compounded monthly. Each month the bank charges you a $2 handling fee which is taken from the account, and you make no other deposits. Sketch a graph to show the bank account balance over time. What is the approximate account balance after 10 years? In how many months will the account balance double?

Most students correctly sketched an exponential growth curve to represent the inceasing bank balance over time, and nearly all students set up a STELLA model to help them answer the questions in the problem. Students now had no trouble defining the basic STELLA diagram (shown earlier), but some of the problems in correctly defining the interest formula and simulation time persisted. As a result, none of the students managed to answer the numerical questions totally correctly, even though they understood the basic concept of a bank balance growing exponentially when interest is compounded.

8th Grade Classroom Trials: Conclusions

Students who participated in this unit studied several different examples of levels and rates, and they came to understand how a feedback loop in a system leads to exponential growth. They learned to predict the behavior of systems over time, and were able to generalize their knowledge from one example to another. This transferability of knowleddge is illustrated by a population growth problem given to students on the unit test, in which they were asked to sketch the graph of the number of people in a town with 10% population growth each year. The classes did not study population growth in this unit, but all students were able to recognize and define the parallel structure of this population problem with the bank balance and motion problems.

Students had trouble in applying their general understanding of feedback systems to the calculation of particular values. They seemed to be hampered by their lack of familiarity with the STELLA modeling environment and by not checking their answers for feasibility. Project staff concluded that we need to go further in encouraging students to go beyond a general understanding of graph shapes to a more refined level of problem-solving and attention to numerical detail.

One week after this class finished the unit and took the unit test, they took their final exam. During the exam they had access to a battery of computers and modeling software which they could use to answer any of the questions on the exam. The teacher was very pleased with the performance of the students on the exam. He said that he was very pleased with the unit, and the test results matched well with his expectations. He said that all students used the computers to answer test questions or to check their predictions. When asked if students could have learned the same material without the computers, he said "Definitely not." The teacher said that he is generally against technology and the ways it is being used, but he will definitely continue using the computer for modeling. He said that using the materials was like doing a good lab and concluded, "The effect of a good lab lasts one or two days, but this was a long time of sustained interest. The kids were thinking the whole time. This has been my best term teaching since I started fifteen years ago."

A Common Theme: Students' Conceptions

One theme which has emerged from our observations and testing of students in both the summer course and in the 8th grade classrooms is that the types of problems that students worked on can be conceived of at different levels.

One level of conception involves an intuitive level of understanding, whereas another level involves a more mathematical, precise understanding of problem solutions. In general, we found that students in both settings were able to attain the first, more intuitive understanding, which involves recognizing patterns of growth and decay, and understanding rate change problems. However, given their limited math background and short exposure to the modeling approach, students often had difficulty moving beyond the intuitive level of understanding to a more precise, mathematical understanding of problem solutions. For example, in the 8th grade classroom, students could often sketch a graph of the pattern of behavior over time, but they had difficulty providing the numbers which would allow for a precise solution to the problem. Similarly, students had trouble evaluating the solution to a bank balance problem in which their conception of the problem was generally correct, but the numbers were unreasonable. Likewise, in the summer course, students demonstrated that they understood concepts such as inflow and outflow, but had difficulty constructing the proper equations to define a given problem.

Nevertheless, we believe that the intuitive level of understanding in and of itself is valuable, as it can serve as the basis for more complex understanding later on. It is possible that having an intuitive understanding of rates and levels, of inflow and outflow, and of feedback will facilitate students' progress in more advanced math courses such as calculus. Furthermore, students' understanding that different problems share the same underlying structure is also important, as it provides them with a way of approaching seemingly different problems across various natural and social science disciplines--at least at the general level.

At this point, we do not know what types of experience with these materials or what type of math background would be necessary to move students from an intuitive to a more precise level of understanding. One barrier to students' understanding is simply learning the mechanics of computer modeling, although reaching a precise level of understanding of these problems involves more than the ability to use the computer programs with ease. One goal for future research will be to identify the components of students' understanding, and to learn what types of experiences foster both intuitive and precise levels of understanding of problems involving behavior that changes over time.

Final Comments

Further research has been planned to evaluate the effectiveness of this approach in a more typical high school setting, taught by science, math and social studies teachers. This research is to focus on how students learn modeling techniques, on how these skills enhance their understanding of math and science concepts, and on the extent of the transferability of skills involved in using modeling techniques to probem solving in other contexts. In preparation, we invited a group of 20 local high school teachers to a four-day summer workshop to introduce them to systems dynamics concepts and the computer software, and we followed up with a seven-session in-service program in the fall of 1987 (See Roberts and Barclay, 1987 for further discussion).

We found that the teachers were generally very excited about the approach, but it took a long time for them to feel comfortable with the concepts and the computer modeling tools. For many of the teachers this was the first time they had considered using computers in their classes, and for most it was the first time they had actually used the Macintosh. Even when teachers became familiar with the computer, the modeling software and the basic concepts of systems dynamics, they still had to integrate appropriate units into their curriculum. Teachers found this difficult, because integrating the units meant altering their curriculum and most of the teachers did not have the flexibility to do this adequately. We believe that a modeling approach presents an opportunity for learning to occur in exciting and non-traditional ways, but it will probably take a great deal more time, effort and patience to reap the potential benefits of such an approach in mainstream classrooms.

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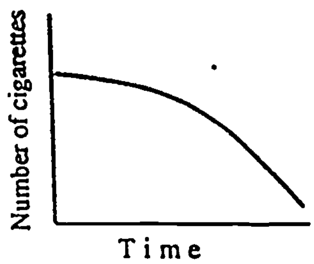
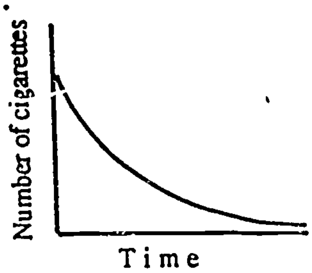
Figure 1: Question 3: Situation Describing Exponential Decay

Carolyn decides to quit smoking by cutting back by 5% each day on the number of cigarettes she smokes. Which of the following graphs best characterizes Carolyn's progress in quitting smoking?

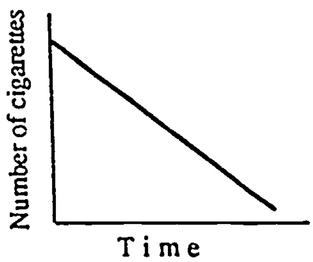
A.



B.



C.



D.

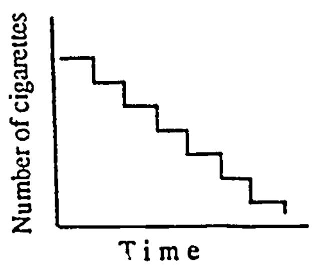


Figure 2: Question 5: Sketching Exponential Growth Function

One year Greg planted some tulip bulbs in his garden. Over the winter, each bulb divided to produce new bulbs. In the spring, the bulbs bloomed, and the following winter, each bulb divided again, and so on.

Using the tulip bulb problem in question 5, please draw a graph on the axes below of how the number of tulip bulbs changes over time.

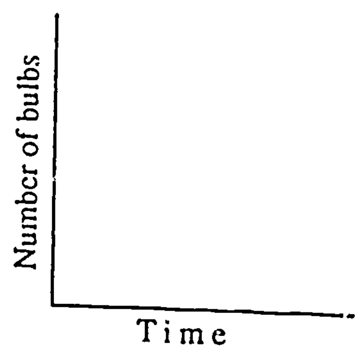
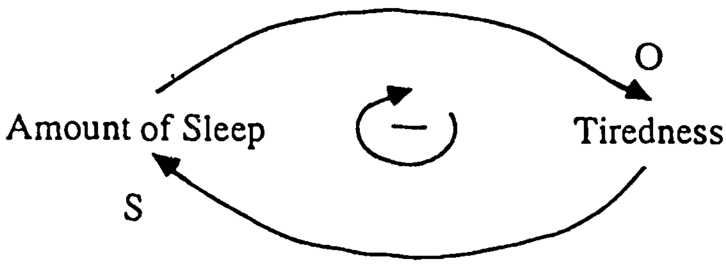


Figure 3: Questions 8 and 9: Interpreting a Causal Loop Diagram

Which of the following statements describes the causal-loop diagram below?



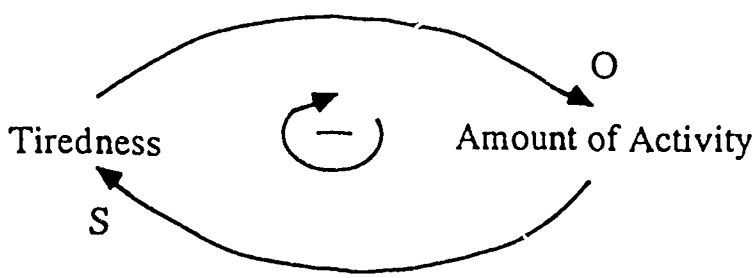
A. People who sleep a lot are often tired.

B. When you are really tired it is hard to sleep.

C. If you are tired, you need more sleep.

D. People get tired regardless of how much they sleep.

Which of the following statements describes the causal-loop diagram below?



A. People who sleep a lot tend to be inactive.

B. People who are more active tend to be more tired, usually causing them to slow down.

C. People who are tired need to be more active to keep them going.

D. People who sleep less have more time to do things.

Figure 4: Question 15: Matching data table to graph

Jean let his flashlight burn for 14 straight hours. He measured the amount of light given off (in lumens) at various times. He collected this data:

Time (hours)	Light given off (lumens)
0	9.5
2	8.5
3	8.5
5	6.0
8	4.2
14	0.6

Which graph best show his results?

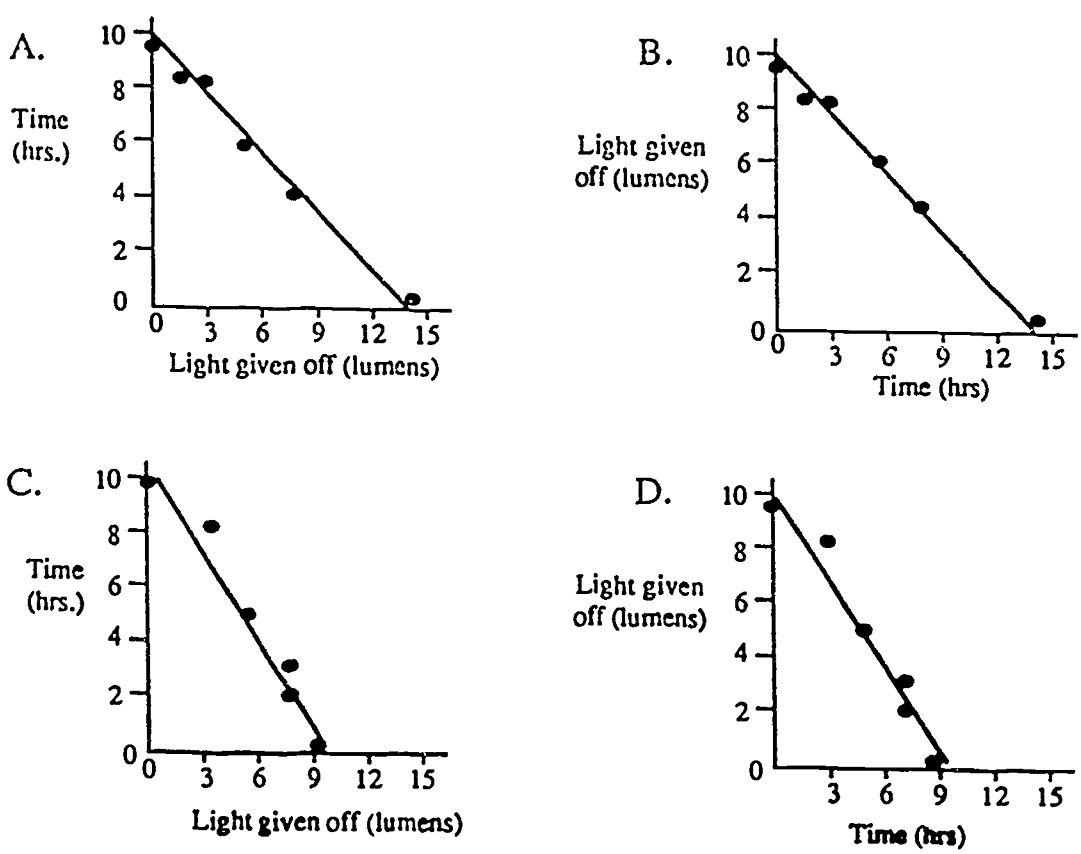
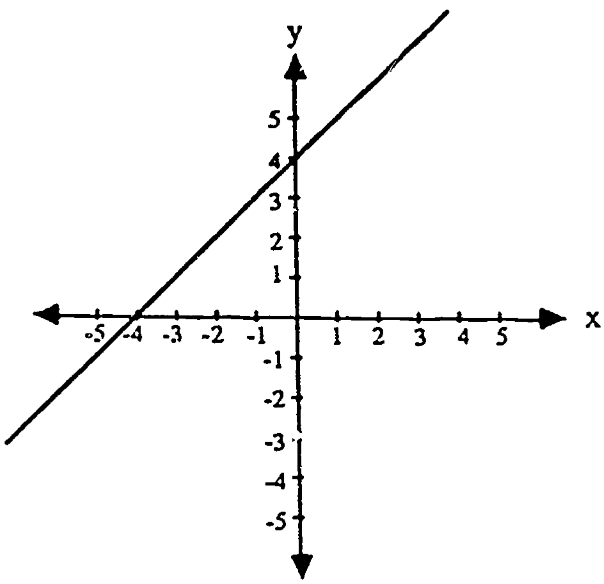


Figure 5: Question 17: Matching simple equation to graph

Which of the following equations describes the line in the graph below?



a. $y = 2x + 4$

b. $y = x + 4$

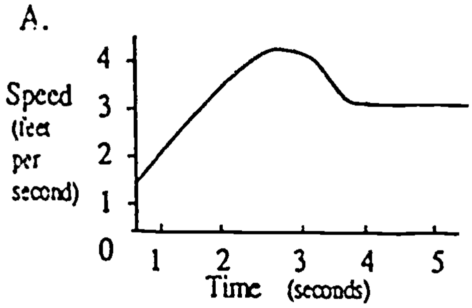
c. $y = -x + 4$

d. $y = x - 4$

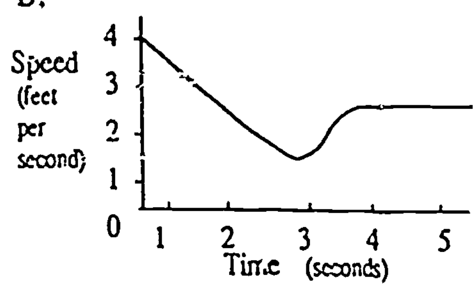
Figure 6: Question 19: Identifying graph showing speed of ball

A ball rolls down a ramp, up a hill, and along a flat surface. Which graph best shows the speed of the ball?

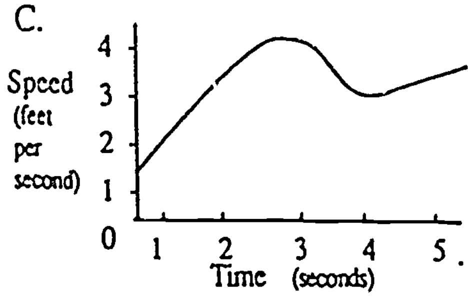
A.



B.



C.



D.

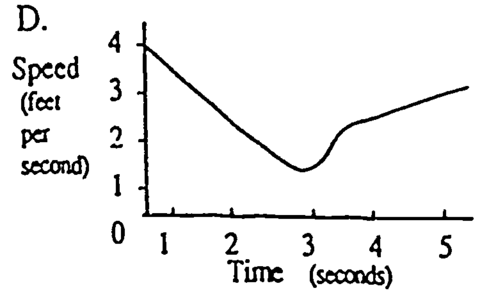


Figure 7: Question 21: Indetifying graph showing slowing rate of GNP

The president said, "The rate of growth of the GNP is slowing." Which of the following graphs best depicts this?