ABSTRACT

Because density is unobservable and must be inferred from knowledge about weight and size, it is a difficult concept to teach and learn. Even after traditional instruction, many students still have an undifferentiated concept that mixes characteristics of both weight and density. In this study, researchers tested the effectiveness of a unit they created to help students make this difficult conceptual differentiation, which is crucial to understanding the particulate nature of matter. The Educational Technology Center's Weight/Density Unit uses both computer simulations and classroom activities with real materials of different weights, sizes, and densities. The simulations attempt to make density more visually accessible than it is with real objects. Researchers used the unit in one sixth-grade and one seventh-grade class. The findings suggest that providing conceptual change is both difficult and possible, and they further suggest that computer models, used in combination with hands-on materials can help students to understand an abstract and perceptually inaccessible concept such as density. Evidence from the pre- and posttest and the clinical interviews suggests that the teaching intervention brought about two kinds of change: (1) conceptual differentiation among students who initially made none; and (2) conceptual consolidation in which students who already had a beginning, fragile distinction deepened and extended their understanding. Appendixes make up the bulk of the volume. They include samples of the assessment instruments; teaching materials for three units, with worksheets; and a discussion of the computer programs. (CW)
TEACHING FOR CONCEPTUAL CHANGE
USING A COMPUTER-BASED MODELING APPROACH:
THE CASE OF WEIGHT/DENSITY DIFFERENTIATION

Technical Report
November 1987
Teaching for Conceptual Change Using a Computer-Based Modeling Approach: The Case of Weight/Density Differentiation

Technical Report
November 1987

Prepared by:

Carol Smith
University of Massachusetts/Boston

Joseph Snir
Haifa University

Lorraine Grosslight
Educational Technology Center

Group Members

Micheline Frenette
Joel Giampa
Lorraine Grosslight
Grace Marie LeBlanc
William Radomski
Judah Schwartz
Carol Smith
Sandra Smith
Joseph Snir

Preparation of this report was supported in part by the Office of Educational Research and Improvement (Contract # OERI 400-83-0041). Opinions expressed herein are not necessarily shared by OERI and do not represent Office policy.
Acknowledgements

Many individuals helped make this study possible. We thank Mr. Edward C. Francis, Jr. (Administrator), Dr. Joseph L. Carroll (Principal), and Mr. John R. Burns (Principal) who gave us permission to work in the Hosmer East and West Marshall Schools in Watertown, Massachusetts. We also thank the teacher members of the Weight and Density Group (Grace Marie LeBlanc, Sandra Smith, Joel Giampa, and Bill Radomski) who helped us in developing the lessons and who greatly facilitated our work in the Watertown Schools. Finally, we thank the students themselves for confronting the difficult problem of making conceptual changes.
# TABLE OF CONTENTS

Introduction ............................................................................................................. 1
Overview .................................................................................................................. 1
Analysis of endpoint ............................................................................................. 2
Analysis of student starting points ....................................................................... 4
Curricular issues .................................................................................................... 6
Assessment issues .................................................................................................. 7
The Study ................................................................................................................ 8
Methods .................................................................................................................. 9
Students .................................................................................................................. 9
Assessment Instruments ......................................................................................... 9
  The clinical interview: overview ......................................................................... 9
  The clinical interview: detailed description of tasks and scoring categories .......... 10
  Written test: overview ....................................................................................... 13
  The written test: detailed description of tasks and scoring categories .......... 14
The teaching intervention ....................................................................................... 19
  Overview ............................................................................................................ 19
  The computer programs ..................................................................................... 20
  Real world materials ......................................................................................... 21
Results .................................................................................................................... 21
  The clinical interview ......................................................................................... 22
    Description of the levels of understanding the distinction between density and weight .................................................................................................................. 22
    Shifts in children's understanding from pretests to posttests ................. 28
  Written test results (Group 1): Students who received clinical interviews .. 30
    Basic understanding of the distinction between weight and density ......... 30
    Performance on verbal tasks and sink/float problems ..................................... 33
  Written test results (Group 2): Children who did not have clinical interviews . 35
Discussion ................................................................................................................ 37
References ............................................................................................................... 42
INTRODUCTION

Overview

Recent work in science education has revealed that students typically fail to assimilate the scientist's framework as a result of instruction. Instead they either distort the lessons so that they fit their own intuitive frameworks (see, for example, the work of Wiser, 1986; Bell, 1985; and Tasker, 1981); or they attempt to learn the scientists' procedures and formulas as a separate system of little meaning: a system to be applied only in stereotyped classroom contexts (e.g., classroom examinations) and not outside school settings to explain everyday phenomena (see, for example, the work of Solomon, 1983). Both results essentially leave students conceptually untouched by their encounters with modern science. Given that students have alternative conceptual frameworks for some of the phenomena they will encounter (as documented in Driver and Erickson, 1983 and Driver, Guesne and Tiberghien, 1985), they need to make conceptual changes in those frameworks to understand science. Part of the problem is that making such conceptual changes is genuinely difficult. Furthermore, the way science is typically taught makes it hard for students to think through their ideas about science on a conceptual level. Giving students lists of formulas or definitions to memorize encourages rote approaches to learning and does not help students relate the new material to their existing conceptions. Our concern—and that of many others in the field—is with developing ways of teaching science which help promote the process of conceptual change (see also the work of Minstrell, 1982; Driver and Oldham, 1986; Wiser, 1986).

If one is to design a curriculum to promote conceptual change, one first needs a careful analysis of student starting points and expert end points in order to clearly define the conceptual change desired. The contemporary expert understanding of science is, of course, a system of great complexity. Thus, we propose that it is useful to consider historically earlier expert systems as well as the currently accepted expert system in order to identify routes into expert frameworks. Such knowledge, coupled with an analysis of student starting points, then helps one identify a powerful set of ideas which are within the student's grasp that will help them move from their framework toward the experts'.

In designing a curriculum to promote conceptual change, one also needs to identify a range of activities which will help students understand and make modifications to their conceptual system. Typically science curricula concerned with this goal have had students make predictions about real world phenomena, selecting situations which will be puzzling given students' current framework. While this kind of activity certainly is an important one, by itself it does not necessarily lead to conceptual change. We propose two additional kinds of activities— inventing models of phenomena and working with computer-based models developed by others—which we think can be effective in helping students make conceptual changes, especially when used in combination with thinking about puzzling phenomena. It is advantageous to have students work with modeling activities because such activities allow them to "see" their ideas and the conceptual relationships. Thus, it directs them to thinking about the theory side of science (i.e., underlying conceptual relationships) as well as data. Furthermore, such activities provide an opportunity to develop
some "metaconceptual" points about the nature of science: in particular, an understanding of the nature of scientific models and the ways they help scientists understand phenomena.

In evaluating the effectiveness of such curricula, it is necessary to develop assessment tools which are sensitive to student's ways of conceptualizing phenomena. Clinical interviews are ideally suited for this purpose, but it is important to develop group administered written tests which can serve the same function in actual teaching practice. A final purpose of the present study is to explore ways of developing more adequate tests to assess conceptual change.

The particular case we have chosen to investigate involves young children's learning about density. We have sought to understand exactly what kinds of conceptual changes are needed in assimilating this concept, and have developed a curriculum which uses a modeling approach in trying to bring about conceptual change. The goal of this case study, then, is to illustrate what is involved in designing a curriculum to promote conceptual change and what is involved in assessing the success of such curricula.

Analysis of endpoint

In this section we begin by discussing the complexities of the density concept within the framework of an expert in physical science. From there, we clarify our pedagogic goals and what it is about the concept of density that we wanted the sixth and seventh graders in our study to understand.

In approaching any physical phenomena, physicists start by defining the system in question and examining its components (e.g., objects, forces, energies, and so on.) An object within a system can be regarded in several ways, depending on what problem the physicist is trying to solve or what aspect of a phenomenon the physicist is trying to analyze. In some cases it is most important to focus on characteristics particular to the object (e.g., its shape, volume, position, function, and mass). In other cases it is more important to focus on the characteristic properties of the material from which it is made (conductivity, tensile strength, and density are all examples of characteristics of material kinds).

Given that one recognizes, intuitively, that objects are made of materials and a single object can be composed of one or several different materials, one can begin to search for and find ways to characterize the different kinds of materials objects are made of. There are two ways to characterize what is distinctive about material kinds. One is based on macroscopic properties of the substance and the other is based on its microscopic properties. The notion of density of materials can be approached on both levels.

One way to define the density of materials on a macroscopic level is to define a procedure to compare the density of two objects. Historically, this is the kind of definition of density which was used by Galileo. Galileo noted there are two ways an object can be heavier than another: in absolute weight and in specific weight. When one object is heavier than the another regardless of the two objects' size, the objects differ in absolute weight. However, when both objects are the same size but one is heavier, Galileo introduced the word "specific weight" to refer to the kind of heaviness which characterizes different materials. In Galileo's words: "I shall call equal in specific
weight those materials of which equal bulks are equal in weight; for example, if two balls, one of wax and the other of some wood, being equal in volume and also equal in weight, we shall say that that kind of wood and wax are equal in specific weight” (p. 27 in Stillman Drake’s translation of Galileo’s Bodies That Stay Atop of Water or Move in It). For qualitative understanding and reasoning about density, flotation, and other phenomena (such as thermal expansion), this definition is perfectly satisfactory. When more quantitative information about relative densities is needed (for example, for predictions about level of submergence), one can still keep this definition and choose one material as the standard. This material is usually water and the concept of specific gravity emerges. Specific gravity is defined as how much heavier an object is than the same volume of water. Specific gravity and density will always be proportional; specific gravity, however, is a dimensionless number because it gives the proportion of two densities.

The contemporary physicists’ conception of density differs from Galileo’s notion of specific weight in several important respects. Density is formally defined as the ratio of mass to volume. Whereas Galileo defined specific weight by giving a procedure for determining which of two objects is denser, the contemporary definition of density provides an abstract mathematical formulation (divide mass by volume) for quantifying the density of a single object. It should also be noted that the contemporary account uses the concept of “mass” rather than “weight”; the notion of “mass” was not available to Galileo. Within the contemporary framework of physicists, weight is no longer a property of the object or the material it is made from, but is the force of gravity applied to the object (or amount of material) because it has mass. However, since the weight applied to all objects on earth is in direct constant proportion to their mass, one can see the weight of an object as a measure of its mass. Thus, under certain predefined conditions, the distinction between weight and mass is not critical.

Another way the contemporary framework differs from Galileo’s concerns the development of extensive theories about the nature of matter on a microscopic level. The search for understanding the nature of materials at this level has been a long one that has not ended yet. However, with the advent of the periodic table and the birth of modern chemistry, scientists had a productive way of thinking about the fundamental elements or building blocks from which all materials are made and for thinking about the structure in which these building blocks are arranged. This advance, however, leads to new complications in thinking about “material kinds” and what counts as “homogeneous materials;” it also has consequences for how one measures density.

For example, at the macroscopic level, wood will be considered a homogeneous substance, although at the microscopic level it is made of different elements. How then does one determine the density of wood? The modern definition of density as amount of mass per unit volume depends on the assumption that we are dealing with homogeneous materials. Therefore, when one determines this quantity experimentally, the sample used should be of such a size for which this assumption of homogeneity is valid. Intensivity should be preserved and valid for all localities at this level of resolution. The concept of density as locally defined but having a value at each and every arbitrary point in the object is a mathematical abstraction that is derived from measurement of weight and volume. The interpretation of these numbers in terms of real
measurable properties of the materials depends on the size of the sample from which the abstract number was derived.

Finally, one can also define a notion of "average density" for objects made of two or more different materials. The concept of average density can be approached from two directions. One way is to define the ratio of total mass (or weight) to total volume (or size). The other is to perform some weighted average of the densities of the materials from which the object is made.

In our teaching efforts, our goal is to have students characterize materials at the macroscopic level, and like Galileo come to make a clear distinction between absolute weight and specific weight (which, at this point, we call density). We go beyond, Galileo, however, in giving student's not only a qualitative procedural definition of density, but also a (macroscopic) model for thinking about density, and a more formal mathematical definition of this quantity.

We feel it is important for students to have a macroscopic concept of density to consolidate their understanding of material kinds, and to deepen their theoretical explanation of weight as a function of both the amount and density of material in an object. We begin with the notion of weight, rather than mass, simply because it is more accessible, and because the contexts we investigate don't require a distinction between mass and weight. We assume the complete homogeneity of substances when we want to define their density. In other words, we assume that the substance is both completely homogeneous and continuous which was, interestingly enough, the assumption Archimedes held when he formulated his famous law on the sinking and floating behavior of objects. When (later in our units) we use objects built of several parts, each made of different material, then we introduce the concept of average density.

Analysis of student starting points

Recent research suggests that during the elementary school years students are coming to reconceptualize weight as a fundamental property of matter (see, for example, the work of Smith, Carey, and Wiser, 1985). The development of students' intuitive matter theory in turn sets the stage for their seeing the need to differentiate two quantities -- weight and density -- where previously they had only needed one. Initially they "advance" by replacing a concept of absolute weight with a concept which combines both extensive and intensive meanings (heavy and heavy for size). However, without formal instruction a number of children manage to progress to making a preliminary distinction between weight and density. In this section, we summarize the strengths in students' existing conceptions which can be built on in an instructional unit as well as some of the limitations in their conceptions which need to be overcome.

One source of strength in student conceptions is their concept of material kinds. Even four-year-olds know some words for material kinds (like "wood", "glass", "steel") and know relevant perceptual characteristics of these kinds that are useful in identifying them. During the elementary school years, students deepen their concept of material kinds not only by learning about more of them, but also by coming to define them more in terms of their being an underlying constituent of the object than in terms of their surface perceptual characteristics. So, for example, such students will assert that a cut-up rubber band must still be rubber, even though it is no longer "stretchy", because it is made of rubber. They can think of rubber objects as rubber at every
point. Thus, although they don't know yet about atoms and molecules, they have made an important advance in their thinking about material kinds.

Another strength lies in student conceptions of weight. Again, even four-year-olds know the words "heavy" and "light" and have good procedures for inferring an object's weight (i.e., hefting, balancing). Further, during the elementary school years, the student's concept of weight has been deepened in an important way: most students come to think of weight as a fundamental property of matter. In their words, "if it is something, it's got to have weight." Thus, they now assert that even a tiny object must weigh something even though it doesn't feel like it weighs anything, and they explain the weight of the whole object in terms of the weight of its parts. Although in conceiving of weight as an inherent property of matter their concept of weight shares some features with the physicists' conception of mass (indeed, children at this point don't make a clear distinction between weight and mass, and don't conceptualize weight as a gravity/mass relation), students have gone beyond defining the essence of weight as "felt weight" and have begun to embed this notion in a matter theory.

Finally, students also have developed generalizations that they are "heavy and light kinds of materials" (their choice of wording). Again, even four-year-olds have begun to form generalizations of this kind. However, the sense of "heavy" they use in these generalizations -- it is primarily the "extensive" sense of "heavy" -- is of a quantity which increases as one makes the object bigger. During the elementary school years, progress is made in developing a more "intensive" sense of "heavy" (i.e., the notion of heavy for size). This notion is intensive in that it is a quality which varies in degree (or intensity) for the kind of material. Thus, steel has more of it, wood less. At this point, many students use both intensive and extensive senses of heavy in their generalizations about material kinds -- they don't see them as distinctly different kinds of quantities. Thus, one limitation in these students' conceptions is that they have not fully differentiated a notion of density from weight.

At the same time, by embarking on developing a matter theory, the seeds for making a differentiation between weight and density have already been sown. As long as children think of weight as "felt weight", the notion of the weight of objects is unanalyzed and children need only one concept. However, as children begin to develop a matter theory, the weight of objects is seen to be a function of two factors: the "weight" of the kind of material the object is made of and the amount of material in the object. Thus, their conceptual system now needs two kinds of weight where it previously needed only one. Of course, getting these differences straight is no simple matter. It involves wrestling with the differences between intensive and extensive quantities. But there is now evidence that by the later elementary school years a number of children have come to make a beginning distinction between these quantities.

Although previous research is sparse on this point, there is reason to believe that the concept of density children spontaneously develop is a qualitative one, heavily tied to their concept of material kind. At most, they have a procedural definition for density (they know steel is a heavier kind of material than aluminum, because if you took a same size piece of steel and aluminum, the steel piece would be heavier) rather than a formal mathematical one (weight per unit volume). Indeed, although students are familiar with standard units of weight, they are much less
familiar with standard units of volume. Further, students know little about thermal expansion (Strauss et al., 1983), and hence are not aware of the conditions under which the density of materials may change. Thus, they typically think of density as an invariant property of material kinds rather than an abstract expression of weight/size relationships.

The goals of our curricular unit are to encourage students to analyze the weight of objects in terms of a matter theory (i.e., see weight as a function of the amount of material and the density of the kind of material) and to have them see the need for articulating two distinct concepts within such a theory: weight and density. We want them to use their qualitative understanding of density in formulating a predictive rule for sinking and floating. In addition, we want students to go beyond seeing density as an invariant characteristic of material kinds which is defined solely by comparative procedural tasks to understanding a mathematical formulation in which density is seen more abstractly in terms of size/weight relationships. Finally, we want them to use this more mathematical formulation to understand the phenomena of thermal expansion.

Curricular Issues

Curiously, previous approaches to teaching students about density do not attempt to first consolidate qualitatively students' understanding of density as an intensive property of material kinds before introducing more formal and quantitative definitions or more complicated atomistic models. For example, in one approach the notion of density is presented as a formal mathematical definition and students use graphing techniques to “discover” the constant size/weight relationships for a particular material kind (Rowell & Dawson, 1977). In another approach, instruction begins by simultaneously challenging the student's extensive conceptions of density as mass/weight and the student's intensive conceptions of density as crowdedness (Hewson and Hewson, 1983) and introducing an atomistic model in which density is seen as a joint function of the mass of individual atoms and the packing of the atoms. Further, in both curricula students are immediately given sinking and floating problems which require a notion of average density to understand (e.g., why some people sink and others float; why a block of steel sinks but a steel boat floats). Not surprisingly, these curricula have achieved only limited success even with 9th grade students: quite a number of students persist in holding on to their alternative conceptions of density. The conclusion is that density is a most difficult concept indeed for students.

Our approach differs from this previous work in (1) working with a much younger group of students (6th and 7th graders rather than 9th graders) and (2) trying to build on students' conceptions of material kind and develop a qualitative understanding of density before moving to more quantitative formulations. Instead of introducing the concept of density with a formal mathematical definition, we encourage students to build models in which weight and density are represented in different ways. We then present our first computer model which represents the kinds of material an object is made of in two ways: by the color of the material (orange, green, purple, etc.) and by the number of dots contained in a standard size box (green material has 1 dot/box, purple 2 dots/box, etc.). The size of the object is then simply the total number of (standard size) boxes in the object; the weight is the total number of dots contained in all the boxes. Working with these computer models, students explore size/weight relationships as well as sinking/floating phenomena working only with objects made of homogeneous materials, and are thus led to "invent" a more mathematical formulation of density. We subsequently help
students complicate their notion of density in two ways: (1) by seeing that under certain conditions the density of material kinds can vary (thermal expansion unit); and (2) by seeing that one can introduce a notion of average density to deal with defining the density of nonhomogeneous objects (average density unit). Thus, we bring students a long way towards the goals set for older students, although we stop short of introducing the distinction between mass and weight and introducing atomistic models.

The main purpose of the present study is to see whether this modeling approach is effective in bringing about conceptual change. If it is, one might argue it represents an important "bridging" curriculum appropriate for the middle school years which might prepare students for more sophisticated treatments of density in 9th grade. Another purpose (to be discussed in a subsequent paper) was to study whether such a curriculum was effective in teaching students about the nature of models.

Assessment Issues

Previous work has used both paper and pencil tests and clinical interviews to assess students' understanding of density. The paper and pencil tests have been used primarily with older students and have stressed semantic aspects of students' understanding of the word "density" (see, for example, the kinds of tests used by Rowell & Dawson and Hewson & Hewson). For example, Rowell and Dawson ask students to explain what is wrong with this statement: "Steel is a heavier density than aluminum". Such tasks reveal that students have a very poor understanding of the distinction between weight and density, even after teaching. However, it may be a mistake to place such emphasis on verbal tasks. These tasks call for skill at "analytic" language comprehension which may not be well developed in these students. Further, research in the clinical interview tradition reveals there may be much more basic comprehension among students than these tasks tap.

Research in the clinical interview tradition has had the luxury of individually interviewing students, giving them objects to handle, and posing questions for them to puzzle about (e.g., Smith, Carey and Wiser). Such interviews rarely presuppose students already know the meaning of the word "density", rather they use language that students invent for this purpose (e.g., "heavier kind of material"). Further, such interviews give students new experiences and explore their capacity to organize them; they don't just assess their knowledge at the start of the interview. For example, students may be given a chance to explore which objects in a set sink or which ones float, before being asked to formulate a predictive rule. Both features allow them to be more sensitive to differences in conceptual understandings, rather than more superficial sources of confusions. Based on such tasks, Smith, Carey and Wiser concluded that a number of ten-year-olds already give evidence of making some distinction between weight and density. Given that at most one or two 14- and 15-year-olds solve Rowell and Dawson's verbal task, one can conclude there is a disparity between the two ways of assessing children's understanding of density.

At the same time, the results from other clinical interviews suggest their may be important limits in children's understanding of density. Rowell and Dawson and Hewson gave students clinical interviews about sinking and floating. Unlike the problems set by Smith, Carey, and Wiser,
In which density covaried with material kind and in which children's understanding was assessed by examining their systematic pattern of judgments, they gave children problems with nonhomogeneous materials and boat-like shapes and looked closely at the language of children's explanations. They found that even 15-year-olds were quite confused about the distinction between weight and density. One possible way to resolve these discrepant findings is to hypothesize that there might be different levels of understanding the distinction between weight and density: a first level in which density is understood as an intensive property of homogeneous materials, and a second level in which it is conceived of more abstractly in terms of the ratio of weight to volume.

In the present study we constructed two different ways of assessing changes in conceptual understanding: a clinical interview and a written test. Both kinds of test were considerably broader in scope than previous tests, allowing us to assess different levels of understanding. For example, the clinical interview used tasks which assessed students' understanding of density as a property of material kind and their understanding of the conditions under which density and material kind do not covary (e.g., thermal expansion). The written test included items which probed both their verbal understandings of "density" and their nonverbal understandings. In addition, both tests used drawing tasks (modeling tasks) as an additional means of probing children's conceptual understanding -- an important kind of task which has previously been little used. Finally, while individual investigators have probed children's understandings of various phenomena (such as sinking/ floating, thermal expansion) separately, the same children have not been interviewed about this full range of phenomena. Thus, these new tests allow us to see how systematically students apply their concept of density to a range of phenomena and to what extent performance on a variety of tasks "hangs together." While at this point in the research the written and clinical tests were not parallel in structure, by giving two different tests to the same children we could begin to investigate issues about the extent to which different forms of questioning produce comparable results.

The Study

Two classes of students (one 6th grade, one 7th grade) were given our curriculum on weight and density during the course of their science instruction in the regular year. Two units were developed: Unit 1 had students explore sinking/floating phenomena and introduced students to the distinction between weight and density (ten 40-minute class periods); Unit 2 (presented one to four months later) introduced students to the phenomena of thermal expansion (six 40-minute class periods). (The 7th graders also received a third unit on average density, which will be reported at a later date). All students received the written test immediately prior to Unit 1, immediately after the completion of Unit 1, and immediately after the completion of Unit 2. A subsample of students also received the clinical interview at those same three times (prior to Unit 1, after Unit 1, and after Unit 2). Students were always given the written test before the clinical interviews.

The study addressed several questions. First, what are student starting points? To what extent do children conflate notions of heavy and heavy for size in one concept? To what extent are their confusions verbal rather than conceptual in nature? Second, how effective is the teaching? Do students show evidence of making genuine conceptual changes as a result of
instruction? And finally, how effective are written tests in assessing conceptual change? Can written tests diagnose the same kinds of conceptual understandings tapped in clinical interviews?

METHODS

Students

The study was done with two classes of students: (1) a sixth grade class of students at the West Marshall School in Watertown, Mass; and (2) a seventh grade class of students at the Hosmer East School in Watertown, Mass. There were 20 students in the 6th grade class: 11 girls and 9 boys. There were 17 students in the 7th grade class: 11 boys and 6 girls. One 6th grade girl moved during the course of the study, and so participated in only the first part of the study (pretest and posttest 1).

Watertown is a suburb of Boston and the students are from families of low to middle income. The West Marshall and Hosmer East are both elementary and junior high schools and the population they draw on is ethnically diverse. The 6th grade class was the only 6th grade at the West Marshall, with students of heterogeneous math abilities. The 7th grade class was a higher math ability class in the junior high school.

Assessment Issues

We probed children's understanding of the distinction between weight and density in two ways: (1) in a group administered written test; and (2) in individually conducted clinical interviews. All the students received the written test; a large subsample (12/20 6th graders; 10/17 7th graders) then received the clinical interview as well. Since our previous research had always used clinical interviews to assess conceptual understanding, one important question addressed in this work was whether written group administered tests could serve this purpose as well. (see Appendix 1 for copies of the Written Test and Clinical Interview).

The clinical interview: overview.

The clinical interview used several different types of tasks to probe students' ability to make a distinction between two kinds of heaviness: the heaviness of objects and the "heaviness"(density) of different kinds of materials. These tasks involved ordering objects by weight and density, inventing models for visually representing the two dimensions, selectively applying one concept rather than the other to organize understanding of sinking/floating phenomena, and answering questions about how adding material and thermal expansion affect the two dimensions in question. For all the problems except the ones about thermal expansion, density can be understood as an invariant characteristic of material kinds. The thermal expansion problems, in contrast, require that children can abstract the notion of density from material kind.

The interview did not require any initial knowledge of the word "density". The ordering tasks began by asking children to contrast which object is heavier and which object is made of a heavier kind of material. The word "density" was then introduced in this context: it was explained that some materials are denser than others which means that they are a "heavier kind of material".
Other tasks then used the contrast between weight and "density of material" (where density was again explained in this general fashion if children forgot its meaning). The interview also did not require any initial knowledge about sinking/floating phenomena: children were given experience investigating which objects sink and float (for objects made of four kinds of materials, of varying size) prior to the questions about sinking and floating.

All the tasks (except the tasks about sinking and floating) involved asking children contrasting questions about weight and density. Consequently, they were scored in terms of the extent to which children distinguished between both kinds of questions. Children were categorized as either (1) making no distinction between the two kinds of questions (e.g., answering both in terms of absolute weight), (2) making a beginning, but not clear distinction, between both kinds of questions (e.g., answering the weight questions correctly, but answering the density questions with density/weight patterns), or (3) making a clear distinction between both kinds of questions (i.e., weight patterns on weight questions; density patterns on density questions). The questions about sinking/floating phenomena were scored in terms of whether children showed weight patterns, mixed weight/material patterns, or clear density patterns.

The clinical interview: detailed description of tasks and scoring categories.

1. The ordering tasks. There were two types of ordering tasks (pairwise ordering and four-object ordering), which were ultimately scored as one unit.

The first ordering task involved a series of pairwise comparisons. Children were first asked to judge if one object in a pair was heavier than another for six object pairs. Scales were available for them to check their felt weight judgments. For two of the object pairs, the heavier object was also made of the denser material, while for the other four, weight and density did not covary. These four critical comparisons are central to detecting any weight/density confusions. Children were then presented with 6 object pairs for which they had to judge if one of objects was made of a heavier kind of material. When asked the "heavier kind of material questions", children were first shown two pairs of same size objects which differed in weight (same size wood and aluminum objects; same size aluminum and steel objects). These two comparisons allow children to make the inference about the relative densities of wood, aluminum, and steel, but are not critical comparisons since weight and density covary. These two items were followed by four critical comparisons: two in which the item made of denser material was in fact lighter, one in which the items had the same density (they were made of the same kind of material) although they differed in weight, and one in which the items had the same weight although they differed in density. For each of these four pairs, children were asked: Is one object made of a heavier kind of material? If they thought one was, they were further asked to specify which one. At issue was whether children could ignore the absolute weight of the objects in making judgments about relative densities of materials.

The pairwise ordering task was followed by a four object ordering task. The four objects to be ordered by weight and density were: a 1 cc steel cube, a 1 cc aluminum cube, a much larger and heavier aluminum cylinder, and a taller lucite cylinder (covered with blue contact) paper which weighed the same as the aluminum cylinder. The lucite cylinder was covered with blue contact paper so that children could not tell what kind of material it was made of. We wanted to test
whether children would realize it had to be made of a less dense material than the aluminum from the fact that it was the same weight but larger in size, and didn't want students to assume it must be least dense because it was the "lightest" in color. This task was potentially more demanding in two ways: (1) children had to order a set of four objects by weight and then by density (not just two); and (2) children had to be able to infer the relative densities of two materials when presented with different size objects which weighed the same (not just infer relative densities from same size comparisons or relative darkness in color).

As in the pairwise ordering task, we could identify certain critical comparisons (comparisons for which density and weight orders differed) in the four-object ordering task. These critical comparisons were: (1) the steel cube is made of a denser material than the aluminum cylinder, but the aluminum cylinder is heavier; (2) the steel cube is made of a denser material than the lucite cylinder, but the lucite cylinder is heavier; (3) the two objects made of aluminum have the same density, but different weights; (4) the lucite cylinder is made of a less dense material than the aluminum cylinder, but they weigh the same; and (5) the lucite cylinder is made of a less dense material than the aluminum cube, but weighs more.

Children's ordering patterns across both tasks were analyzed to determine whether they made any density intrusions on weight questions and any weight intrusions on density questions. All children were correct on the weight questions (they were encouraged to check their answers with a balance scale) and they were credited with absolute weight judgments on these questions. Children showed one of three patterns on the density questions: weight patterns (answering at least 7 of the 9 critical comparisons on the basis of absolute weight); density/weight patterns (answering at least 4 of the 9 critical comparisons on the basis of density and at least 1 of the 9 critical comparisons on the basis of absolute weight); and density patterns (answering all of the critical comparisons on the basis of density. Children with weight patterns essentially made no distinction between the weight and density questions. Children with density/weight patterns made a beginning distinction. And children with density patterns made a clear distinction.

2. Modeling. The four-object ordering task was followed by asking children to make a pencil/paper model of the four objects in question. In particular, they were challenged to invent a way of depicting the size, weight, and density of the four objects and to explain their representation. At issue was whether children would represent weight and density as separate dimensions in their model and if they did so what kinds of code they would use for each. This is potentially the most demanding of the tasks in the series since it requires the child to reflect on concepts and represent them.

All children managed to represent size as a distinct dimension in their models — hence this dimension is not considered further. Instead, children's models were scored for whether weight and density were represented as distinct dimensions. Some children managed to represent only one of these dimensions in their models. For some, this dimension was weight; for others it was density; and for still others it was a mixture of weight and density. All those children who represented only one dimension (for weight and density) were scored as making no clear distinction between weight and density in their models. Other children tried to represent two distinct dimensions, but in representing the density of the objects, in fact represented a mixture
of density/weight. These children were scored as beginning to distinguish the two dimensions in their models. Finally, some children represented weight and density as distinct dimensions in their models. These children were scored as making a clear distinction between the two dimensions.

The kinds of codes used for weight and density in children's models were also analyzed. Of special interest was whether children used extensive codes for extensive dimensions like weight (e.g., number of dots), intensive codes for intensive dimensions like density (e.g., shading or color intensity, or number of dots/box) or neutral codes for these dimensions (e.g., an ordering, a summary number). This also relates to whether children's models captured the interrelatedness as well as the distinctness of weight and density. In our grid and dots model, an intensive code is used for density (dots/box), and extensive codes are used for size and weight (total number of boxes, and total number of dots). Further, the relationship between weight and density is also directly represented since the weight of the object "falls out of" the correct representation of its size and density.

3. Adding material. In this task, children were questioned to see whether they realized that adding a small piece of clay to a clay ball increased the amount of clay in the ball and the weight of the ball, but did not change the density of the ball. Justifications were also elicited for these judgments. Children were scored in terms of whether they made no distinction between the two types of questions in justifications and judgments (both increase, because more is added), whether they made a beginning distinction (e.g., weight increases because more is added, not sure what happened to density, vacillated in judgments), or whether they made a clear distinction (e.g., weight increases because more is added, density is unchanged because it is still the same material).

4. Sinking/floati9ng tasks. The sinking/floati9ng tasks probed children's ability to use a concept of density in understanding sinking and floating. Children were allowed to do sinking/floati9ng experiments with a small set of homogeneous objects of different sizes and materials before being asked the questions in this set (there were large and small objects made of four different materials). We reasoned if students had a concept of density readily available, they might be able to use it to make sense of this experience. Their ability to use a concept of density in thinking about sinking and floating was assessed in two ways: in their predictions about what objects would sink or float, and in their inferences about the relative densities of objects from knowledge of their behavior in the water.

There were two prediction problems. Children were initially shown a small (light) piece of wax which floated, and a larger (heavier) piece of aluminum which sank. They were then asked to predict whether a big piece of wax (bigger and heavier than the aluminum piece) would sink or float, and to predict whether a small piece of aluminum (smaller and lighter than the wax piece) would sink or float. In both problems, they were also asked to explain their predictions. No feedback was given for these problems (that is, children were not allowed to put the large wax or small aluminum in the water). At issue was whether children realized it was the relative density of material, not absolute weight of the object, that was relevant to sinking and floating.
The inference problem was presented as follows. Children were first shown three objects: a small (very light) piece of clay, a medium size object made of a mixture of wax and clay, and a larger (heavier) piece of wax (but not nearly as large as the piece of wax, used in the above problems). They weighed the three objects to determine that the wax piece was the heaviest followed by the clay/wax piece followed by the small clay piece. Then they put the three objects in the water and observed: the clay sank, the clay/wax floated, and the wax floated at the highest level. Their task was to order the objects by weight and by density. At issue is whether children would use information about sinking/floating behavior (rather than absolute weight) in inferring density.

There were three patterns of response on the sink/float predictions & density inference problems: (1) density patterns; (2) material/weight patterns; and (3) weight patterns. Children credited with density patterns made predictions about sinking and floating consistently on the basis of material kind and also correctly ordered the clay and wax by density. Children with material/weight patterns generally ordered the clay and wax by weight (or a mixture of density/weight) rather than material kind. Some, however, were able to make predictions about sinking and floating consistently on the basis of material kind while others sometimes predicted on the basis of material kind and other times predicted on the basis of weight. Children with weight patterns always predicted on the basis of weight and ordered on the basis of weight.

5. Thermal Expansion. Children were also questioned about their understanding of the effects of thermal expansion on the quantities in question. In particular, they were shown how alcohol expands when a thermometer is placed in hot water (and contracts when placed in cold water), and they were questioned to see whether they understood that the amount of alcohol and the weight of the alcohol in the thermometer remained the same, but the density of alcohol in the thermometer changed. Again, justifications were elicited. Answers were scored for whether they made no distinction between weight and density questions (e.g., both increase with heating, because the liquid rises; or both stay the same because none has been added), whether they made a beginning, but incorrect distinction (e.g., the weight stays the same because nothing has been added, the density increases because the water rises; or the weight stays the same because nothing has been added; the density stays the same because it is still the same kind of material), and whether they made a clear and correct distinction (e.g., the weight stays the same because no alcohol has been added, but the density decreases because the alcohol has expanded, stretched out, etc.).

Written test: overview.

The written test also used a variety of tasks to determine whether children have a clear concept of density. These tasks involved having children make predictions about the relative weight of object pairs (gallon/lidium task), predictions about whether two objects could be made of the same kind of material (mystery material task), and predictions about whether objects would sink or float (sinking/-floating tasks). They also involved having children give a definition of the word "density", compute the "density" of objects from knowledge of their volume and weight, and make judgments (and draw models) of what happens to the "density" of a material when more of the same kind of material is added or when a fixed amount of the material expands with heating.
Like the clinical interview tasks, two tasks in the written test (the galt/lidium task and the mystery material) probed children's understanding of density (and ability to distinguish weight from density) without requiring understanding of the word "density". Unlike the clinical interview tasks, however, they probed children's understanding of density in different ways: by asking them to predict the relative weights of object pairs or asking whether two objects could be made of the kind of material, rather than asking them to order objects by the two different dimensions. At issue, then, is whether the two ways of assessing understanding of density produce comparable results.

The written test was also like the clinical interview in having questions about understanding sinking/floating and the effects of adding material and thermal expansion on density. However, it probed students' understanding of these phenomena in different ways, and under different conditions. On the written test, children's understanding of sinking and floating was assessed without letting them initially experiment with a set of objects; in addition their understanding of level of submergence was also probed explicitly. There were several differences in the ways that children's understanding of how adding material and thermal expansion affected density were probed. First, in the written test, success on these tasks presupposed an understanding of the word "density", and no attempt was made to teach that word in the test itself. Second, in the written test, children were only asked about density (not weight), and were asked to draw models which explained their judgment. Finally, in the written test problems on adding material, a large amount of material was added (instead of a small piece). This may make the problem harder because children may think that when only a small amount is added, the density is essentially the same, but that when a large piece is added, the density changes. Thus the written test addition-transformation problem is perhaps a more stringent test of their understanding.

Two tasks directly assessed children's knowledge of the word "density" in the written test: they were asked to define the word and to make explicit computations of "density". In the clinical interview children were introduced to the meaning of the word "density", but no such help was given in the written test (the written pretest always came before the first clinical interview). This allows us to assess their initial understanding of the word "density".

Table 1 (next page) lists the main tasks used in both the clinical interview and written test.

The written test: detailed description of tasks and scoring categories

1. The galt and lidium and mystery materials tasks. The galt and lidium task and the mystery materials task were scored as a unit to determine children's initial level of understanding of density.

The first task on the written test was the galt and lidium task. It was used to assess student's conceptual understanding of the distinction between weight and density, but did not require verbal understanding of the word "density". The task began with a schematic drawing of two same-size objects made out of different materials. The object made of galt was shown to weigh 3 kg, while the object made of lidium was shown to be only 1 kg. Children were asked whether one object was made of a heavier kind of material than the other. Then they were shown an outline of
TABLE 1. Comparison of Main Tasks in Clinical Interview and Written Test

I. Tasks which do not require an understanding of the word "density" or any previous experience with sinking/floating

<table>
<thead>
<tr>
<th>Clinical Interview</th>
<th>Written Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>ORDERING</td>
<td>GALT &amp; LIDIUM</td>
</tr>
<tr>
<td>MODELING</td>
<td>MYSTERY MATERIALS</td>
</tr>
<tr>
<td>ADDING MATERIAL</td>
<td></td>
</tr>
<tr>
<td>THERMAL EXPANSION</td>
<td></td>
</tr>
<tr>
<td>SINK/FLOAT</td>
<td></td>
</tr>
</tbody>
</table>

II. Tasks which do require an understanding of the word "density"

<table>
<thead>
<tr>
<th>Clinical Interview</th>
<th>Written Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>DEFINITION</td>
<td></td>
</tr>
<tr>
<td>COMPUTATION</td>
<td></td>
</tr>
<tr>
<td>ADDING MATERIAL</td>
<td></td>
</tr>
<tr>
<td>THERMAL EXPANSION</td>
<td></td>
</tr>
</tbody>
</table>

III. Tasks which require previous experience with sinking/floating

<table>
<thead>
<tr>
<th>Clinical Interview</th>
<th>Written Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>SINK/FLOAT</td>
<td></td>
</tr>
</tbody>
</table>

another size piece made of galt and were asked to draw the outline of a piece of lidium that would weigh the same as the galt piece. Finally, they were shown 5 additional pairs of objects made of galt and lidium, but of different sizes. Their task was to judge whether the object made of galt was heavier, the object made of lidium was heavier, or whether the two objects weighed the same. In all 5 comparisons, the students were told how many times bigger one object was than the other (and the drawings also represented size information). In one case, the galt and lidium were the same size. In another case, the galt was two times larger than the lidium. And in three other cases, the lidium was bigger than the galt (two times, three times, and four times).

The second task used to assess an ability to make a conceptual distinction between weight and density was the mystery materials task. Again, this task did not use the word "density". Instead, children were shown four objects and given information about their sizes and weights: two were the same weight, but different sizes; two were the same size, but different weights; and two were the same weight per size unit but different overall weights. Students were asked: could any of these objects be made of the same material? This question was followed by four statements: (1) that objects A and B could be made of the same material because they were the
same weight, (2) that objects C and D could be made of the same material because they were the same size, (3) that objects A and C could be made of the same material because they were the same weight per size unit, and (4) that none of the objects could be made of the same material. Students were to circle all the statements with which they agreed. At issue was whether students would correctly distinguish between weight and weight per size unit in this context.

Children's patterns of response on these problems were analyzed in two ways: one to determine whether they had a good qualitative understanding of density and the other to determine whether they had a good quantitative understanding of density.

Children were credited with a good qualitative understanding of density if they realized that the lidium had to be larger than the gait to be equal in weight (in the drawing task), judged that the two objects made of gait and lidium could be equal weight only when the object made of lidium was larger (in the judgments task), and judged that only the two objects that were the same weight per size unit could be made of the same material (mystery materials task). These children clearly can coordinate two distinct senses of weight in a problem solving task. They realize that objects made of a heavier and lighter kinds of materials can be equal in weight (gait and lidium task) and that objects of equal density can differ in absolute weight (mystery materials task).

Some children's responses reflected difficulty coordinating these two senses of weight. In the gait and lidium task, the most common error was to draw (or judge) lidium as the same size as the gait when it was equal in weight. This pattern reflects suppression of the generalization that gait is a heavier kind of material in solving these problems. In the mystery materials task, there were two kinds of errors: judging that objects could be made of the same material if they were the same weight or same size (at times in addition to judgments that they could be the same material if they were the same weight per size unit), and judging that no objects could be made of the same material. Again these reflect difficulties distinguishing two different senses of weight.

Children who had consistent difficulty with all these problems were scored as using weight patterns; children who got some problems correct but others incorrect were scored as using a weight/density concept; and children who got all the problems correct (on a qualitative level) were scored as using a density pattern.

Children's response patterns on the gait and lidium task were also scored for their ability to think about density quantitatively (how many times denser gait is than lidium). Children could show they had made this inference in three separate ways: (1) in their pattern of judgments (only judging the pieces to be equal weight when the lidium was exactly three times larger); (2) in their spontaneous drawings (making the lidium three times larger to be equal in weight); and (3) in their justifications of their judgments (explicitly saying that lidium has to be three times larger to be equal in weight). Four patterns were distinguished: no attempt at quantification, an incorrect quantification, an inconsistent quantification, and a consistent and correct quantification.

Children were scored as showing consistent and correct quantification of density if their pattern of judgments was consistent with the assumption that a piece of lidium must be exactly three times bigger to weigh the same as a piece of gait. Their justifications and drawings also had
to be consistent with this quantification. Most typically, they also explicitly said that lidium must be 3 times bigger to be equal in weight, and drew a piece of lidium three times bigger than a piece of gait. Many children even made size markings in their drawings showing they explicitly intended the lidium to be three times bigger.

Children with inconsistent quantifications invariably formed a 3x rule in at least one of the subtasks (drawing, judgments, or justification). However, performance in another subtask was inconsistent with this rule. For example, the child might say or draw the lidium three times bigger to be equal in weight but then show a judgment pattern in which it was two times bigger when it weighed the same.

Children with incorrect quantifications gave some evidence of formulating a rule, but it was not the correct one. Most typically, these children had a judgment pattern that was consistent with a belief that lidium had to be two times bigger than the gait to weigh the same, but they were vaguer in their justifications and drawings. The systematicity of their judgments argued that they were establishing some cut-off point and making some attempt at quantification.

Children who were scored as showing no quantification did not establish a clear cut-off point in their judgments, below which size the gait was heavier than the lidium and above which point the lidium was heavier than the gait. Nor did they make explicit attempts at quantification in their drawings or words.

2. Verbal tasks. Four tasks required understanding of the word “density” to be correct.

a. Two tasks directly assess students’ understanding of the word “density” (the computation and definition tasks). In the computation task, they were asked to compute the densities of objects given information about their sizes and weights. In the definition task, students were asked whether they had heard of the word “density” and to explain what they thought it meant.

Children's computations were scored as correct if they systematically inferred the density of a material by dividing weight by volume in four different objects. Children's definitions were scored for whether they thought density referred to an extensive quantity (e.g., the amount of weight, the amount of material, the amount of space taken up), or whether they thought it referred to an intensive quantity. There were two general ways children tried to talk about it intensively: (1) by using a global adverb to define the intensive dimension in question (e.g., how tightly packed or how smashed together the material is) or (2) by relating an extensive quantity to a standard space (e.g., the number of things in a box, how much mass in one area, weight units per size unit, how much packed in one place). In addition to these two clear categories, there were two categories characteristic of children in transition from extensive to intensive definitions. In one, children begin to relate an extensive quantity to a space, but the space is not clearly a standard (e.g., instead of simply talking about the weight, they begin to talk about the amount of weight (material, space) in something. These definitions were scored as “extensive?" to indicate they were still extensive in nature but suggested some beginning awareness of the need to relate the weight to something in developing a definition. In the other transitional answer children
combined talk of packing (an intensive aspect) with talk of how much is packed, which seems more extensive (e.g., how much weight, material, or hardness, is packed into the object). This was scored as "intensive?" because they weren't unambiguously intensive. Finally, some children's definitions resisted categorization into any of these four categories (equating density with toughness, empty space, depth, solidness, thickness, hardness, gravity, width x length x weight, etc.). These were categorized as "other".

b. The other two tasks probed children's understanding of how adding material and thermal expansion affected density. First, children were shown two beakers filled with the same kind of liquid at the same temperature. One beaker had much more in it. They were asked to compare the densities of the material in both beakers and then draw a model explaining what they meant. Children were credited with a good understanding of this problem if they judged that the material in both beakers had the same density, and developed a model which was consistent with their judgment (e.g., depicted both liquids with the same shading intensity or same number of dots/box).

Then children were shown two beakers filled with the same amount of the same kind of liquid at the same temperature. They were told that the liquid in the second beaker was heated, and shown that the liquid in the beaker rose to a higher level when heated. They were then asked to compare the density of liquid in the unheated beaker with the density of that in the heated beaker. Children were credited with a good understanding if they judged that the liquid in the heated beaker was less dense and developed a model which was consistent with their judgment (e.g., depicted the beaker with more liquid as less intensively shaded or having fewer dots per box).

3. Sinking/floating problems. Finally, there were two kinds of multiple choice problems about sinking and floating. One set of four problems (basic prediction problems) explored children's ability to make predictions about sinking and floating using knowledge of the kind of material an object was made of (rather than its weight) and using knowledge of the relationship among densities of the object and liquid (rather than considering an object a sinker or floater independent of the liquid it is immersed in and rather than considering the relationships between the absolute weight of object and fluid). Children were credited with using a concept of density to understanding sinking and floating if they were correct on all four problems.

The other problem probed their understanding of the phenomena of level of submergence. In particular, they were told a large iceberg floated with 9/10 of its bulk below water, and were asked if they could predict the sinking/floating behavior of a small piece of the iceberg. Children were scored as showing an understanding of the phenomena of level of submergence if they were correct on this problem as well as the other four basic predictions problem.
The Teaching Intervention

Overview

The teaching was designed to promote students' understanding of the distinctness, and interrelatedness, of the concepts of density and weight. We did this by linking the density concept to specific real world phenomena. We emphasized density's role in the phenomena and its relationships to other dimensions via a series of computer-based models.

Three units were developed. In the first unit (10 lessons) we were primarily concerned with students' differentiation of weight from density as it relates to material kind and sinking and floating phenomena. The second unit (6 lessons) was primarily concerned with applying this concept in a different context, namely in connection with thermal expansion. The third unit (4 lessons) had students look at boats and objects made of a mixture of materials and introduced them to the idea of average density. More detailed descriptions of the lessons and worksheets are given in Appendix 2. Only the 7th graders were given unit 3 and the results from that unit will be reported in a separate paper at a later date.

Several guiding principles governed our curriculum development efforts. Our first principle was to build on student preconceptions. Since our previous research had suggested that students had a clear understanding of the intensive nature of material kind, we decided to begin by considering situations where density could be viewed as an invariant property of material kinds. Further, since many students had intuitions about the relevance of material kind to sinking/floating phenomena, we began by considering a limited range of objects (bulky objects made of homogeneous materials), to allow them to distinguish predictions made on the basis of weight from predictions made on the basis of the density of material kind. Only subsequently did we move on to the phenomena of thermal expansion. Not only did we plan our curriculum efforts with student frameworks in mind; we also conducted teaching in such a way as to make students aware of their frameworks -- having them confront and evaluate their ideas often in the context of a challenging problem.

The second principle was to move students from a qualitative understanding of a particular phenomenon to a more quantitative understanding. So, for example, students might first note that steel is denser than aluminum and aluminum is denser than wood, without being asked "how much denser". Only later would they work out that steel is approximately three times denser than aluminum, and aluminum approximately four times denser than the particular kind of wood we gave them. Similarly, when exploring sinking/floating phenomena, students are first challenged to discuss the qualitative rule: objects sink if made of a material more dense than the liquid in which they are immersed, and float if made of a material less dense than the liquid. Only later, are they challenged with the problem of predicting the level of submergence at which an object will float. Finally, in presenting thermal expansion, students are first encouraged to reason qualitatively -- if an object has the same weight spread over a larger area, it must be less dense -- and only later, are asked to worry about quantitative details.

A third principle was to use a modeling approach to develop student understandings. This entailed: (1) starting by presenting students with a puzzling phenomenon; (2) asking them to
construct models of the phenomenon; (3) presenting suitable and appropriate models to students; (4) implementing the models on the computer and allowing them to explore consequences of the models; (5) relating the computer models to real world phenomena and (6) discussing the process of modeling as a part of science.

Some of the points about models that we include in the first unit are: that models can be used as thinking tools; that it is important for models to be consistent with real world phenomena; and that one can have more than one model consistent with a single phenomenon. Our second unit devotes even more attention to getting students to reflect on the uses and nature of models. An additional theme we develop in this unit is that models can be revised to account for new phenomena (in this case thermal expansion) and that models need to be evaluated in terms of the range of problem they can help solve. Throughout the curriculum student models are not considered wrong. Rather we work with students to discover the strengths and limits of models, theirs as well as ours. It is important in our curriculum that we actually present multiple computer models and revise them in light of new phenomena encountered. Thus, the “taught” models are no more sacred than the students’.

The computer programs

The computer programs offered a range of features in terms of the phenomena covered and the presentation of different models and simulations. These features will be briefly discussed here, but are more fully described in Appendix 3.

The phenomena covered in Units 1 and 2 were: (1) weight, size, and density of objects made of different materials presented; (2) sinking and floating phenomena including level of submergence; and (3) thermal expansion phenomena depicting the distribution of weight before and after heating an object made of a given material.

The different programs can be thought of as a progression of models. In our basic “grid and dots” model, each box stands for a standard unit of volume (size unit), each dot stands for a weight unit, and the number of dots per box corresponds to the density. Material kind is represented in two ways: by the color of the material (orange, blue, green, etc.) and by the density of the material (green material has 1 dot/box, purple 2 dots/box, and so on). When students build an object, they first select the kind of building block (material) they will use, and then determine the number of building blocks used. Students can thus build objects and explore consequences of adding and removing material. They can count the number of boxes to determine an object’s size, count the number of dots to determine its weight, and count the dots per box to determine its density. Alternatively, they can request specific data about an object they have constructed; the computer displays the data requested and automatically adjusts the data as the student changes the object.

The second model is the sink/float model which situates objects represented in the first way in the context of sinking/floating. Using this program, students can do simulations of sinking and floating experiments. They can construct the object they want to use in the experiment (selecting its size and density), select the kind of liquid they want to use in the experiment (the amount of liquid in the container is always fixed), request data about the size, weight, and density of the
object and liquid, and perform the experiment. Students are challenged to discover the rule the computer uses to determine if the object will sink or float in a given liquid—a rule based on the relative densities of object and liquid, that is consistent with reality. Students can also make discoveries about level of submergence.

The third program offers two different models for thermal expansion. The first model is like the basic grid and dots model, except the individual boxes get bigger when the material is heated. Thus, the child can see that the same amount of weight is now distributed in a larger volume. This is basically a revision of the first “grid and dots” model. In this revision, dots per box still stands for material kind, but no longer stands for density since the size of the boxes is not constant. This motivates a discussion of the need for standard units. The second model returns to standard units and is more “atomistic” in flavor. It represents material kind by dots per circle, but shows the circles some distance apart (connected by squiggles). With heating, the circles stay the same size, but move further apart.

The advantages of using computer-based rather than hand-drawn models include: (1) students can interact with the models and see the consequences in the representation (e.g., what happens to density when one adds or removes material); (2) students can interact with the models and try to discover the rules embodied in them (e.g., the formula for density in the Weight and Density program, a rule for predicting whether objects will sink or float and their level of submergence in the Sink the Raft program); and (3) students can begin to link multiple representations for the same situation (real world cartoons, a grid and dots conceptual representation, data representations).

Real World Materials

Throughout the lessons, students worked with a variety of hands-on activities and saw a variety of demonstrations. They were then asked to explore these situations, discover regularities, and try to model these situations. The kinds of real world materials we used included: (1) a set of objects made of different materials (in different sizes) for doing sink/float experiments with water; (2) same-size vessels of oil, water, clay, etc. for purposes of comparing densities; (3) a set of 1 cc steel and aluminum cubes, so that objects could be constructed of different numbers of cubes, and children could see how many aluminum cubes equaled one steel cube in weight; (4) a similar set of wood and aluminum rods; (5) brass ball and ring, for demonstrating thermal expansion; (6) demonstration showing the change in density of water with heating; and (7) a demonstration showing that an object which floats in cold water, sinks in warm water.

RESULTS

In reporting results, we first consider the picture of conceptual change which emerged from analyses of the clinical interview. We next consider how the same group of children responded on the written test. Finally, we consider the performance of children who received only the written test.
The Clinical Interview

There were five tasks in the clinical interview which assessed children's capacity to distinguish between weight and density: the ordering tasks, the modeling task, the adding material task, the sink/float tasks, and the thermal expansion task. In each task, three levels of understanding of density were distinguished: (1) making no distinction between weight and density (i.e. pure weight patterns); (2) making a beginning, but uncertain distinction between weight and density (i.e., weight/density or weight/material patterns) and (3) making a clear distinction between weight and density (i.e., weight patterns on weight questions and density patterns on density questions).

The first important result is that children's responses across the different tasks were highly correlated. Hence one could identify three levels of understanding density, based on children's pattern of performance across the five tasks. At the same time, certain tasks were more difficult than others, allowing us to form subgroups within levels. In particular, the modeling task was more difficult than the ordering task, allowing us to form two subgroups within the level of weight/density lack of differentiation. And the thermal expansion task was the most difficult of all, allowing us to form two subgroups within the level of weight/density differentiation.

Table 2 (next page) shows the main patterns of responding that we observed across the five tasks, and their organization into levels and sublevels. In reporting the results, we shall first describe each level or sublevel and then discuss how children moved through these different levels from the pretest to the posttests.

Description of the levels of understanding the distinction between density and weight.

1. Level 1: Density Absent (Absolute Weight). The first level involved making no distinction among the weight and density questions across all the tasks. Only two children scored at this level. These children essentially ordered the objects by absolute weight for both the weight and density questions. When asked to model the objects they had ordered, they either represented only the weight of the objects or failed to model any dimension at all. These children knew that adding more clay to a ball of clay would make it heavier (although one thought much more clay was needed to make it heavier); however, they thought adding changed the heaviness of the kind of material as well. Taken together, these children give evidence of simply using a notion of absolute weight in answering these questions; the notion of heavy for size is not used at all.

These children go on to be quite confused by the sinking/float and thermal expansion phenomena as well. In reasoning about thermal phenomena, one child could articulate no justifications for her answers and was judged to be guessing. The other treated the density of alcohol extensively, as varying with the amount of alcohol. Both children failed to use knowledge of an object's sinking or floating behavior in making inferences about the density of material (instead they used knowledge of absolute weight). One child also used absolute weight in making predictions about whether something would sink or float, while the other focused on kind of material.
### TABLE 2
Five Levels of Understanding Density: 
The Clinical Interview

<table>
<thead>
<tr>
<th>TYPE OF TASK</th>
<th>ORDERING</th>
<th>MODELING</th>
<th>ADDING MATERIAL</th>
<th>SINK/FLOAT</th>
<th>THERMAL EXPANSION</th>
<th>OVERALL LEVEL OF UNDERSTANDING</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No distinction: W; W</td>
<td>No distinction</td>
<td>No distinction</td>
<td>Weight or Material/ Weight</td>
<td>No distinction</td>
<td>1, Weight (Density absent)</td>
</tr>
<tr>
<td></td>
<td>Beginning distinction W; W/D</td>
<td>No distinction</td>
<td>Beginning distinction W; D/W</td>
<td>Material/ Weight</td>
<td>Beginning distinction</td>
<td>2a, Weight/Density Lack of Differentiation (Strong)</td>
</tr>
<tr>
<td></td>
<td>Clear Distinction W; D</td>
<td>Clear distinction</td>
<td>Beginning distinction or Clear distinction</td>
<td>Material/ Weight</td>
<td>Clear Distinction</td>
<td>2b, Weight/Density Lack of Differentiation (Transitional)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Clear distinction</td>
<td>Density</td>
<td></td>
<td>3a, Density (Material Kind)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Clear distinction</td>
<td>Density</td>
<td></td>
<td>3b, Density</td>
</tr>
</tbody>
</table>

---

**Weight & Density** 23
2 Level 2a: Strong Weight/Density Lack of Differentiation. At the next level, children are beginning to make a distinction between weight and density on the ordering tasks, but their understanding of this distinction is at best fragile as evidenced by their failure to represent or talk about this distinction in the modeling task, their failure to make any distinction between weight and density in the adding material or thermal expansion tasks, and their confusion of these dimensions in thinking about sinking and floating.

More specifically, these children are beginning to make some distinction between weight and density in the ordering tasks. They correctly order objects by absolute weight (at least when a balance scale is available and they are encouraged to use it before making a final "weight" judgment). Further, in ordering objects by "density", they do not rely simply on the object's absolute weight, but go back and forth between different criteria (an object's heaviness, its heaviness for size, its kind of material). Thus, although they have not yet clearly articulated a separate dimension of density, they are beginning to articulate a dimension of density/weight.

There are several ways such a child might go about ordering objects by "density." The child might begin by comparing the same size steel and aluminum cube and judge the steel to be denser (using either a notion of heavy for size, or absolute weight). Then the child might put the large aluminum cylinder with the small aluminum cube (ignoring the weight differences and focusing on the sameness of kind of material). Finally, the child might put the large lucite cylinder with the two aluminum objects (perhaps even concluding the mystery material is aluminum, because it is the same absolute weight as the smaller aluminum cylinder).

Alternatively, the child might first compare the same weight lucite and aluminum cylinders and correctly conclude the aluminum is denser than the lucite (using a notion of heavy for size); then compare the 1 cc steel and aluminum cubes and correctly conclude that the steel is denser (using either a notion of heavy or heavy for size), but fail to place the two aluminums together. Such a child produces an overall order of (from densest to least dense): the large aluminum cylinder, the large lucite cylinder, the small steel cube and the small aluminum cube. Such an order seemed in part a result of making two separate comparisons, not a conscious attempt to produce an overall order. Before we accepted this as the child's final order, then, we would explicitly draw the child's attention to his overall order by saying: "so you mean, this is the object made of the densest material, this is the next densest, this the next densest," etc. If the child left the above mentioned order after the probe, we concluded the child had some absolute weight intrusions in his or her order, having placed the lucite as denser than the steel and the large aluminum as denser than the small aluminum. Occasionally, children correctly ordered the objects by density, but made weight intrusions on some of the pairwise comparisons (e.g., when shown a large piece of aluminum which was heavier than a small piece of steel, they concluded the aluminum was the heavier kind of material because it was heavier).

Although these children showed some beginning distinction between weight and density in the ordering tasks, they were able to represent only one dimension when asked to model the weight and density of objects of these same objects. For some the dimension represented was the absolute weight. For others, the dimension represented was either "density/weight" (e.g., the lucite was grouped with the two aluminums, and the steel was the densest) or "density" (the
steel object was represented as the densest, followed by the two different size aluminum objects which were represented as having the same density, followed by the lucite). In either case, they interpreted this dimension as both weight and density, in response to specific interviewer probes (e.g., if they initially described the dimension as "density", the interview asked, have you represented the "weight" in your drawing? and students would comment that they had — the same code stood for the weight). Thus, although these children give evidence of having two senses of weight available in the ordering task, they give little indication of realizing that these two senses define different concepts. At the time of pretest and first posttest, the most common code children use for this dimension was an intensity shading code.

Children's performance on the other three tasks also reveals confusions about the distinction between density and weight. These children uniformly have material/weight patterns on the sinking and floating problems. On the addition-transformation problems, these children at best make only a beginning distinction between the questions (being clear on one problem and vacillating on the other). For example, they might initially judge that the weight has increased when a small piece has been added, but when justifying their answer switch and say that the weight is still the same because the piece still sinks. They then go on and judge the density as still the same because it is the same material. Many of these children fail entirely to distinguish the two questions: they say both the weight of the ball and the density of the clay increases because more has been added. Similarly, on the thermal expansion problems, these children frequently make no distinction between questions about weight and density (judging that both the weight and density of the alcohol in the thermometer has increased when the temperature rises because the alcohol has risen in the thermometer). At best, they make only a beginning, but incorrect distinction on the thermal expansion problems (e.g., the weight is the same because no alcohol has been added, but the density has increased because the temperature has increased the alcohol level has risen). [Note: Those children who scored as making a beginning distinction on the adding material problems are no more likely to be making a beginning distinction on the thermal expansion problems than those scored as no distinction; this is one reason we lumped these categories together.]

Finally, many of these children (as well as the level 1 children) reveal a less sophisticated way of thinking about weight itself in their patterns. In particular, their notion of weight seems more perceptually bound than tied to an underlying matter theory. Thus, a number of these children judge that one needs to add a large piece of clay to a clay ball in order to make it heavier, a small piece won't change the weight. In addition, these children typically think that the alcohol in the thermometer weighs more when it rises, because it looks larger. More sophisticated children say that all matter has weight, thus even a small piece changes the weight of an object. Similarly, they note that the alcohol is enclosed and nothing has been added when it expands; hence, they argue it still must weigh the same because there is the same amount of alcohol.

3. Level 2b: Transitional Weight/Density. Lack of Differentiation. These children still have not clearly differentiated weight from density. This lack of differentiation is reflected in numerous ways: in their ordering patterns, in their models, in their predictions about sinking and floating, and in reasoning about transformations (especially thermal expansion). However, based on their performance in the modeling task, these children show greater awareness than level 2a children
that there are two distinct dimensions. Further, a number of these children even make a clear distinction between weight and density when reasoning about the effects of adding material. Finally, these children think about weight in a more sophisticated way than the level 1 and level 2a children. Thus, they are more prepared to make the differentiation between weight and density than the other children.

More specifically, these children are beginning to distinguish weight and density in their ordering patterns. They order objects correctly by absolute weight (with the help of a balance scale) for the weight questions, and then show weight/density patterns when ordering by density. The specific types of patterns they show when ordering by density are similar to those of the level 2 children (described above). That is, they might group the lucite cylinder and the two aluminum pieces together (as being of the same density but less dense than the steel) or they might make some errors in the pairwise comparisons, occasionally judging a heavier object to be made of a heavier kind of material.

In addition, all these children show awareness that two dimensions are called for in their models. However, the dimensions they represent are "weight" and "density/weight" -- the second dimension has not been fully freed from weight. Nonetheless, they talk about them as distinct dimensions, and thus show some insight that two distinct concepts are needed. In the pretest, they frequently draw separate models for each dimension -- for example, two different orderings of objects, two separate pictures of objects in some physical situation (on scales, in water), two different relative fullness codes. Thus, although these children are working to see these dimensions as distinct, they do not see them as interrelated. In the posttests, children were more apt to draw one model, but still the representation for weight and density was not intrinsically related: for example, an intensity shading code for "density/weight", and numeric values for weight.

A further indication of these children's beginning awareness of weight and density as distinct dimensions comes from their performance on the addition-transformation and thermal expansion problems. All are quite clear that adding even a small piece of clay will make something weigh more, and most believe that the weight of the alcohol remains the same when it expands. Thus, they give more evidence of tying their notion of weight to a matter theory. Further, at least half make a clear distinction between weight and density on the adding material problems: arguing that adding material changes weight but not density, because it is still the same kind of material.

Evidence that these children still do not firmly grasp the distinction between weight and density comes from their responses on the sinking and floating tasks and problems about thermal expansion. These children still typically have material/weight patterns on the sinking and floating problems, and many make no distinction between weight and density when reasoning about thermal expansion. Thus, although they argue that the weight is the same because no alcohol has been added, they also maintain that the density is the same because nothing has been added.

4. Level 3a: Density/Material Kind. These children are capable of making a clear distinction between weight and density, for the problems where density covaries with material kind (ordering,
modeling, sinking and floating, adding material). However, they do not yet understand what happens to weight and density in thermal expansion.

Children credited with these patterns are now perfect in ordering the items by weight and density (for both the pairwise comparisons and the 4-object ordering task). They also typically represent two distinct dimensions in their models, know that adding a small piece changes the weight of the object but not the density of the material, and can use a notion of density to organize their understanding of sinking and floating.

However, these children are quite confused about the thermal expansion questions. Sometimes (especially on the pretest) they lapsed into making no distinction between weight and density at all. In these cases, they might judge that the weight and density of the expanded alcohol was the same because nothing had been added. Or they might judge that both had been lessened because the alcohol had been stretched [Note: this kind of argument is observed only among children who give evidence of making a distinction between weight and density in other tasks]. More typically, however, they made a distinction but an incorrect one between weight and density. A common argument (especially by the time of the posttest) is that the weight is still the same because nothing had been added, and the density is still the same because it is still the same material. This reflects mastery of the principle that density remains invariant with material kind, a kind of argument fostered in the first unit. Another pattern is to judge that the weight is the same because nothing has been added, and the density has increased because the alcohol level has risen.

The kinds of codes children used for weight and density were not used to classify them as a level 3a pattern. Nonetheless, it is interesting to note that the kinds of codes they used at the time of both the pretest and posttest were quite limited. Some used the same kind of code for both dimensions, thus failing to capture differences between intensive and extensive quantities in their code (for example, an intensity shading code for density, and a different intensity shading code for weight which correctly depicted the weight ordering of the objects). Few were able to invent or use a code which showed the relationships between quantities even at the time of the posttest. That is, their representation of size and density did not allow one to determine the weight of objects.

5. Level 3b: Density. These children not only distinguish between weight and density when density covaries with material kind, but also maintain their understanding of the distinction in thinking about thermal expansion. In justifying their answers to what happens to the weight and density of alcohol in the thermometer, they most commonly noted that the weight of the alcohol was still the same because none had been added, while the density had lessened because the alcohol had "stretched", "expanded", "thinned", "spread out". It should be noted that these were the kinds of words chosen by the children; atomistic schema were typically not invoked.

At the time of the pretest, the few children who showed this pattern were able to represent weight and density as distinct dimensions in their models, but did not capture the interrelation among these dimensions in their models. By the time of the 2nd posttest, however, these children (with one exception) had all assimilated the grid and dots model, a model which gives
them a way of representing the relationships as well as the distinctness of size, weight, and density.

Summary. Taken together these levels represent different steps in the acquisition of the distinction between weight and density. Initially, children use only absolute weight. Then they begin to have two senses of weight, but don't clearly see them as distinct dimensions, defining different concepts. As they become ready to make the differentiation, they show a clearer understanding of weight within a matter theory and are beginning to represent two dimensions in their models. Finally, they are able to make a clear distinction between weight and density. However, they first understand the intensive aspect of density for problems where density covaries with material kind; only later do they extend their understanding (using "stretching" schemas) to understand thermal expansion.

Shifts in children's understanding from pretests to posttests.

Children's patterns on the entire clinical interview were analyzed, according to these levels. Children were credited with a certain level if they conformed to the specified criteria for each of the five tasks. In certain cases, children were also assigned to a pattern if they departed from its specification on only one task. The only time that such departures were common was on levels 3a and b (the density level) in the pretest. There it was common for children to make minor errors on one of three tasks (modeling, addition-transformation, sink/float), but to be correct on others as well as on the ordering tasks. Since it varied from child to child on which task the errors occurred, and since they made only one slip, assigning them to the density category seemed to capture the bulk of their responding. By the time of the posttests, most children assigned to the density categories were perfect across all four tasks.

Children who made more than 2 departures from a pattern (or one very significant departure) were put in the category of "other". No child was in the other category at the time of the pretest. And only 2 were so scored in each of the posttests. Thus, the responding of the majority of children in the clinical interview hangs together in organized patterns. Further, these patterns are respected as they make progress as a result of teaching, suggesting that the teaching is producing underlying conceptual changes and not simply rote learning.

Table 3 (next page) shows how children's level of responding changed from the pretest to posttests 1 and 2. At the time of the pretest, children ranged across the different levels, but the majority (63%) were in levels 1 and 2, and did not yet clearly differentiate weight and density. After the first teaching unit (posttest 1), the majority (66%) had reached level 3, thus giving evidence of clearly differentiating weight and density. Most of these children showed the pattern of understanding density when it covaried with material kind (level 3a), but not in the thermal expansion problems (level 3b). The second teaching unit on thermal expansion had its main effect in moving students to an understanding of density not tied to material kind. Thus, in posttest 2 students maintained their capacity to differentiate weight and density (70% had one of the two density patterns), and the majority of these children were able to move to an understanding of thermal expansion as well.
TABLE 3. Clinical Interview: Percent of Students Showing a Given Level of Understanding on Pretest and Posttests

<table>
<thead>
<tr>
<th>Time of Testing</th>
<th>Level of Understanding</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Other</td>
</tr>
<tr>
<td>Pretest</td>
<td>0</td>
</tr>
<tr>
<td>Posttest1</td>
<td>10%</td>
</tr>
<tr>
<td>Posttest2</td>
<td>10%</td>
</tr>
</tbody>
</table>

A finer-grained analysis of the movement among levels from test to test is provided in Table 4 and Table 5 (next page). These data provide further confirmation of the orderly nature of children's progression among levels. When students change, they typically move to the level (or sublevel) one higher than the one they were in. Indeed, only two clear regressions were observed at any point: both occurred between the pretest and posttest 1 where two children at level 3b (density patterns) at the pretest regressed to level 3a (density/material kind patterns) by posttest 1. Given that unit 1 stressed the relationship between density and material kind, their confusion on the thermal expansion problems is understandable. Both children went back to level 3b by posttest 2. Three other children moved to a pattern of responding across the five tasks which did not neatly fit the categorization scheme ("other" patterns). Significantly, all three

TABLE 4. Clinical Interview: Comparison of Level of Understanding on Pretest and Posttest 1

<table>
<thead>
<tr>
<th>Pretest Level</th>
<th>Level of Understanding in Posttest 1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Other</td>
</tr>
<tr>
<td>W/1</td>
<td>1</td>
</tr>
<tr>
<td>W/D(2a)</td>
<td>1</td>
</tr>
<tr>
<td>W/D(2b)</td>
<td>0</td>
</tr>
<tr>
<td>Den(3a)</td>
<td>0</td>
</tr>
<tr>
<td>Den(3b)</td>
<td>0</td>
</tr>
</tbody>
</table>
TABLE 5. Clinical Interview: Comparison of Level of Understanding on Posttest 1 and Posttest 2

<table>
<thead>
<tr>
<th>Pretest 1 Level</th>
<th>Other</th>
<th>Wt(1)</th>
<th>W/D(2a)</th>
<th>W/D(2b)</th>
<th>Den(3a)</th>
<th>Den(3b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Other</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Wt(1)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>W/D(2a)</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>W/D(2b)</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Den(3a)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>Den (3b)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3</td>
</tr>
</tbody>
</table>

children were ones who had begun with lower levels of understanding (levels 1 or 2a). Their "other" patterns showed uneven performance on the posttest, and may be a sign of "rote" learning on one task rather than integrated conceptual responding.

Written Test Results (Group 1): Students Who Received Clinical Interviews

These same children also received a written group-administered test. We could not assign one global score to the written test (as we did for the clinical interview) since some tasks on the written test required understanding of the word "density" and some did not. At the time of the pretest, some students gave evidence of being able to make a distinction between weight and density even though they did not know the meaning of the word "density". By the time of the posttest, however, the two types of tasks were more likely to hang together. Thus, we present the results for these two kinds of tasks (density nonverbally assessed and verbally assessed) separately. The sink/float tasks are also presented with the verbal tasks, since they presupposed some familiarity with the phenomena. At the time of the pretest, students may not have had such familiarity.

Basic understanding of the distinction between weight and density

Children's pattern of performance on the galt and lidium task and the mystery materials task was used to assess their understanding of the distinction between weight and density. In one task (the galt and lidium task), they had to predict the weights of different size pieces of galt and lidium from knowledge of the weights of two pieces of galt and lidium that were the same size. In the other task (the mystery materials task), they had to infer whether two objects could be made of the same material from knowledge of the sizes and weights of the objects. Each task called for children to simultaneously co-ordinate knowledge of size, weight, and density in solving a problem.
**Qualitative distinctions.** Children's performance on these tasks was first scored to assess their qualitative understanding of the distinction between weight and density. If children consistently coordinated their knowledge of size, weight, and density in all the problems they were credited with having a differentiated concept of density. If they were correct on some problems, but also sometimes lapsed into using only one sense of weight, they were scored as having density/weight lack of differentiation patterns. And if they consistently used only the notion of absolute weight on all the problems, they were scored as weight patterns.

Table 6 shows the percentage of students who have weight, density/weight, and density patterns on the written test at three different times of testing. At the time of the pretest, children were rather evenly split between those who had density/weight patterns and those who had density patterns (weight patterns are always quite infrequent). By the time of the posttest 1, the balance had shifted: the majority of children had clear density patterns. Posttest 2 saw some slight regressions, but a sizeable group still showed a good grasp of the distinction between weight and density.

**TABLE 6. Comparison of Written Test and Clinical Interview: Percent of Students at Each Level of Understanding on Pretest and Posttests**

<table>
<thead>
<tr>
<th>Time of Test</th>
<th>Level on Written Test</th>
<th>Level on Clinical Interview</th>
<th>Percent Agreement</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>W</td>
<td>WD</td>
<td>D</td>
</tr>
<tr>
<td>Pretest</td>
<td>5%</td>
<td>50%</td>
<td>45%</td>
</tr>
<tr>
<td>Posttest 1</td>
<td>0</td>
<td>23%</td>
<td>77%</td>
</tr>
<tr>
<td>Posttest 2</td>
<td>5%</td>
<td>33%</td>
<td>62%</td>
</tr>
</tbody>
</table>

Table 6 also compares the picture of conceptual change which emerges from the written test with that produced from the clinical interview (grouping children just in the three main levels of understanding, analogous to those used in the written test). The percentage agreement in categorization between both measures was tabulated.

This analysis revealed that there was a high level of agreement between both classifications at the time of the pretest and the first posttest. In both cases, the written test produces higher estimates of those who understand density than the clinical interview. Given that the patterns in the clinical interview were based on much more intensive probing of children and wider range of items, we conclude that the written test is slightly over-estimating understanding of density. By the time of the 2nd posttest, there is less agreement between both assessments. The clinical interview reveals strong understanding of density, but performance has flagged on the written
test. We think that repeated use of the same written measure may have led some students to be more careless at the time of the 2nd testing. All in all, then, the written test has some capacity to detect the kinds of patterns we uncovered in the clinical interview, but seems a bit less reliable. Some revisions to the written test might improve it on this score.

Quantitative understandings. Children’s judgments on the galt/lidium problems were scored in another way to assess their ability to think quantitatively about relative density. In particular, we examined whether children were able to infer that galt was three times denser than lidium (from knowledge that one piece of galt weighed 3 kg and another piece of lidium the same size weighed 1 kg). Four levels of quantification were identified based on children’s pattern of performance across drawings, judgments, and justifications: (1) no attempt at quantification; (2) incorrect quantification (e.g., about 2x); (3) inconsistent quantification (e.g., sometimes 3x and sometimes 2x); and (4) correct and consistent quantification.

Table 7 shows the changes in the percentage of students in each of these categories from the pretest to the posttests. As can be seen, there is a strong improvement in the ability to infer the correct quantitative relations from the pretest through to the second posttest. At the time of the pretest, only a few students (27%) consistently make the correct quantitative inference; while at the time of the second posttest the majority (57%) do.

<table>
<thead>
<tr>
<th>Time of Testing</th>
<th>Quantification Pattern</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No Quant.</td>
</tr>
<tr>
<td>Pretest</td>
<td>27%</td>
</tr>
<tr>
<td>Posttest 1</td>
<td>14%</td>
</tr>
<tr>
<td>Posttest 2</td>
<td>19%</td>
</tr>
</tbody>
</table>

Children with weight patterns make no attempt to think quantitatively about density. Many children with density/weight patterns were beginning to make some quantitative inferences, but these inferences were typically either incorrect or inconsistent. Finally, the majority of children with density patterns were able to think consistently and correctly about density (at least by the time of the 2nd posttest).
Performance on verbal tasks and sink/float problems.

In this analysis, children were grouped by their level of understanding of the distinction between weight and density (as assessed on the galt/lidium and mystery material problems in the written test). Then their percent correct on the verbal tasks and the sink/float problems was computed. This analysis was done three times: at the time of the pretest, posttest 1, and posttest 2 (see Table 8, next page).

At the time of the pretest, few children were correct on the verbal tasks and sink/float problems, regardless of their level of understanding of density. Although virtually all the children had encountered the word "density" before, most had incorrectly or incompletely mapped its meaning. Spontaneous definitions were analyzed and assigned to one of five categories: (1) extensive; (2) extensive?; (3) intensive?; (4) intensive; and (5) other. Table 9 (page 34) reveals that at the time of the pretest, most children gave extensive definitions or leaned in that direction (e.g., it's the weight of an object, the volume of an object, or the amount of weight in an object). The poor performance on the sink/float predictions at the time of the pretest also indicates that children need some experience to abstract a generalization focusing on kind of material rather than weight.

By the time of the posttests, however, children are beginning to show more organized patterns of responding depending upon their level of understanding of density. In particular, children who give evidence of making a conceptual distinction between weight and density (on the galt and lidium and mystery materials problems), perform at high levels on the verbal and sink/float tasks. In contrast, children with weight/density patterns on the galt/lidium and mystery materials problems perform much more poorly on the verbal and sink/float tasks. The one exception (for this latter group) is the verbal computation problem. All students (regardless of level of understanding) do quite well on this type of problem -- perhaps an indication that computational formulas can be memorized even if they are not understood.

Significantly, students who give evidence of making a distinction between weight and density improve on the verbal thermal expansion problem at the time of the first posttest, even though thermal expansion has not been explicitly taught. This is another indication that the students with density patterns are genuinely making a conceptual distinction between weight and density (and not just memorizing certain answers). While many of these students are still confused about thermal expansion (they give an incorrect answer on the clinical interview version of the thermal expansion problem focusing on the invariance of density with material kind), it seems that by the time of posttest 1 these students have several schemes available for thinking about thermal expansion (the thinning scheme, and the invariance with material kind scheme). After instruction (posttest 2), these children come to be consistently correct on these problems in both the written test and clinical interview.

Conclusions. The written test was useful in diagnosing children's level of understanding of the distinction between weight and density. Although the written test was quite different from the clinical interview, the two instruments had a fairly high level of agreement, at least for the pretest and first posttest. Thus, each of these test instruments was capable of detecting conceptual change in students from the pretests to the posttests. Given that the type of change observed
### TABLE 8. Written Test (Group 1) Percent of Students Correct on the Verbal and Sink/Float Questions as a Function of Level of Understanding of Density

#### A. At the time of the Pretest

<table>
<thead>
<tr>
<th>Level of Understanding</th>
<th>Verbal Tasks</th>
<th>Sink/Float</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Def</td>
<td>Comp</td>
</tr>
<tr>
<td>1 (WT) (N=1)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2 (W/D) (N=11)</td>
<td>9%</td>
<td>0</td>
</tr>
<tr>
<td>3 (Den) (N=10)</td>
<td>10%</td>
<td>30%</td>
</tr>
</tbody>
</table>

#### B. At the time of Posttest 1

<table>
<thead>
<tr>
<th>Level of Understanding</th>
<th>Verbal Tasks</th>
<th>Sink/Float</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Def</td>
<td>Comp</td>
</tr>
<tr>
<td>2 (W/D) (N=6)</td>
<td>33%</td>
<td>67%</td>
</tr>
<tr>
<td>3 (Den) (N=16)</td>
<td>75%</td>
<td>81%</td>
</tr>
</tbody>
</table>

#### C. At the time of Posttest 2

<table>
<thead>
<tr>
<th>Level of Understanding</th>
<th>Verbal Tasks</th>
<th>Sink/Float</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Def</td>
<td>Comp</td>
</tr>
<tr>
<td>1 (WT) (N=1)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2 (W/D) (N=7)</td>
<td>14%</td>
<td>85%</td>
</tr>
<tr>
<td>3 (Den) (N=13)</td>
<td>46%</td>
<td>84%</td>
</tr>
</tbody>
</table>

After each unit reflected the focus of the unit, and that the units themselves were brief, it is likely that the change was triggered by the teaching unit rather than some general developmental improvement. For example, most of the movement from weight/density lack of differentiation to initial weight/density differentiations occurred after Unit 1 which specifically addressed this issue and was three weeks in length. And most of the movement from understanding density tied to material kind to understanding density more abstractly in terms of size/weight relationships occurred after the 2-week unit on thermal expansion.

The written test also provided valuable information which complemented the information we obtained from the clinical interview in three ways. First, we learned that children are beginning to think quantitatively about density prior to clearly differentiating weight from density (although their
TABLE 9. Written Test (Group 1) Changes in Percent of Students Giving Different Kinds of Definitions of "Density" Before and After Teaching

<table>
<thead>
<tr>
<th>Time of Test</th>
<th>Other</th>
<th>Extensive</th>
<th>Ext.?</th>
<th>Int.?</th>
<th>Intensive</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pretest</td>
<td>23%</td>
<td>41%</td>
<td>18%</td>
<td>9%</td>
<td>9%</td>
</tr>
<tr>
<td>Posttest1</td>
<td>14%</td>
<td>9%</td>
<td>5%</td>
<td>36%</td>
<td>36%</td>
</tr>
<tr>
<td>Posttest2</td>
<td>14%</td>
<td>5%</td>
<td>5%</td>
<td>43%</td>
<td>33%</td>
</tr>
</tbody>
</table>

quantifications are invariably incorrect or inconsistent). And children who clearly differentiate weight from density can frequently make consistent and correct inferences about relative densities. This suggests that quantitative thinking may play an important role in consolidating the differentiation of weight from density. Second, we learned that although children have already heard of the word "density" they incorrectly think of it extensively -- as corresponding to the object's weight or size. This was true for the children who clearly had a concept of density as an intensive quantity as well as for those who didn't. This suggests that something about the situation in which they encountered the word did not help them make the correct "mapping" of word meaning. Finally, we learned that most children do not have a good understanding of sinking and floating without having some preliminary experience with the phenomena -- even those with a grasp of the distinction between weight and density. A brief experience, however, is all that is necessary for the children who understand density -- as indicated by their excellent performance on the clinical interview questions about sinking and floating after such a brief experience.

Written Test Results (Group 2): Children Who Did Not Have Clinical Interviews

Approximately one-third of the children received only a written pre/posttest (with no intervening clinical interview). Curiously, the teaching seemed less effective for these children than for those who had also had the clinical interview. That is, the Group 2 children improved less on the written test than did the Group 1 children.

Table 10 (next page) shows shifts in the "basic" level of understanding of density from the pretests to posttests for these children who received only the written tests. The majority of children in this group start out by making a distinction between weight and density (60%). In this respect, they are initially performing better than the children who were selected for clinical interviews. However, there is little improvement in this group: 67% of the children show a basic understanding of the distinction between weight and density by the time of the second posttest. (Actually, these overall percentages underestimate the degree of movement in this group, since a couple of students regressed and several more improved; some of this movement may also reflect lack of reliability in the assessment procedure).
TABLE 10. Written Test (Group 2) Percent of Children Showing Different Levels of Understanding Before and After Teaching

<table>
<thead>
<tr>
<th>Time of Testing</th>
<th>Level of Understanding</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 (Wt)</td>
</tr>
<tr>
<td>Pretest</td>
<td>13%</td>
</tr>
<tr>
<td>Posttest1</td>
<td>0</td>
</tr>
<tr>
<td>Posttest2</td>
<td>0</td>
</tr>
</tbody>
</table>

Turning now to changes in children's approaches to quantifying density (in an unstructured task), we find more evidence for improvement. Table 11 shows that the number of children with correct quantification patterns doubled by the time of posttest 2. The numbers of students with no quantification or incorrect quantification were essentially unchanged.

TABLE 11. Written Test (Group 2) Percent Students with Different Quantification Patterns on the Galt/Lidium Task Before and After Teaching

<table>
<thead>
<tr>
<th>Time of Testing</th>
<th>Pattern of Quantification</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No quant.</td>
</tr>
<tr>
<td>Pretest</td>
<td>14%</td>
</tr>
<tr>
<td>Posttest1</td>
<td>20%</td>
</tr>
<tr>
<td>Posttest2</td>
<td>14%</td>
</tr>
</tbody>
</table>

Finally, there was also evidence of improvement in children's verbal understandings and in their understanding of sinking and floating phenomena as a result of the teaching. As in the case of children who had the clinical interview, the amount of improvement was related to the child's level of understanding of density. However, the performance of the children who made a distinction between weight and density was not as integrated across the verbal tasks as it was for the children with the experience of the clinical interview.
In particular, Table 12 (next page) shows that at the time of the pretest, there was little understanding of the word “density” or sink/float phenomena for any of the children despite their initial level of understanding. By the time of the 2nd posttest this had changed. By then, the children who gave evidence of making a distinction between weight and density, knew how to compute “densities” and predict sinking/float while those who showed weight/density confusions did not. Further, only the children who showed understanding of density were able to understand thermal expansion (though not all of them did). Curiously, all the children were weak at understanding the effects of adding material on density. This is the main indication that children in this group had less fully integrated their understanding of density than those in the clinical interview group.

Another interesting difference between the pattern of results for Group 1 and Group 2 on the written test concerns the timing of their improvement. Group 1 children made the biggest gains on the written test (verbal and sinking and floating items) between the pretest and posttest 1; in contrast, Group 2 children made less progress by posttest 1 but continued to improve in posttest 2. This was particularly striking in terms of the data for their spontaneous verbal definitions (compare pattern of change for Group 1 in Table 9 and for Group 2 in Table 13 (next page).

Summary. There was some evidence of change in understanding among the children who received only the written test, but the changes most often involved consolidation of their understanding of the differentiation of weight from density, rather than coming to make the differentiation in the first place. These changes included: increased ability to quantify density, increased understanding of the word “density”, increased understanding of thermal expansion, and increased understanding of sinking and floating. These children were slower to change, than children who had had the clinical interview, they made less radical conceptual changes, and their final level of performance was more uneven, as evidenced by their worse performance on spontaneous definitions and understanding of the effects of adding material; the occurrence of regressions on some tasks, etc.

DISCUSSION

The present study clearly documented different levels of understanding of the distinction between weight and density in 6th and 7th grade students. At the lowest level, students make no distinction between weight and density at all since they appeal only to the notion of absolute weight. Next, they have both intensive and extensive senses of weight available (heavy, heavy for size), but they combine both notions in one undifferentiated weight/density concept. When they begin to be aware that two concepts are needed in their conceptual framework, the concepts are at first not kept entirely distinct (i.e., they use weight in one context and weight/density in another). Then they manage to distinguish weight and density, but do so first in a qualitative way in situations where density is seen as an invariant property of material kinds. Only later do they develop a notion of density which can support an understanding of how materials vary in density with thermal expansion.
TABLE 12. Written Test (Group 2) Percent of Students Correct on the Verbal and Sink/Float Questions as Function of Level of Understanding of Density

<table>
<thead>
<tr>
<th>Level of Understanding</th>
<th>Verbal Tasks</th>
<th>Sink/Float</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Def</td>
<td>Comp</td>
</tr>
<tr>
<td>A. At the time of the Pretest</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 (WT) (N=2)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2 (W/D) (N=4)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3 (Den) (N=9)</td>
<td>11%</td>
<td>11%</td>
</tr>
<tr>
<td>B. At the time of Posttest 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 (W/D) (N=6)</td>
<td>0</td>
<td>67%</td>
</tr>
<tr>
<td>3 (Den) (N=9)</td>
<td>22%</td>
<td>55%</td>
</tr>
<tr>
<td>C. At the time of Posttest 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 (W/D) (N=5)</td>
<td>20%</td>
<td>20%</td>
</tr>
<tr>
<td>3 (Den) (N=10)</td>
<td>50%</td>
<td>80%</td>
</tr>
</tbody>
</table>

TABLE 13. Written Test (Group 2) Percent of Students Giving A Different Kinds of Definitions of "Density" Before and After Teaching

<table>
<thead>
<tr>
<th>Time of Testing</th>
<th>Kind of Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Other</td>
</tr>
<tr>
<td>Pretest</td>
<td>26%</td>
</tr>
<tr>
<td>Posttest1</td>
<td>7%</td>
</tr>
<tr>
<td>Posttest2</td>
<td>13%</td>
</tr>
</tbody>
</table>
Prior to teaching, students had a range of starting points which spanned these five levels of understanding density. Thus, while many students in our sample began the study having difficulty differentiating weight and density, it was also clear that many did not. This latter point is particularly important since virtually all children, when probed verbally, gave evidence of being confused about the meaning of "density". Thus teachers, who frequently rely on direct questioning techniques, may overestimate the conceptual confusions students have and underestimate the conceptual resources students can draw on.

Further study would be needed to understand why students are at such different points in their understanding of density. One possibility, hinted at in the present study, is that students may be at different points in their abstraction of a matter theory in which weight is analyzed as a function of the kind of material and the amount of material an object is made of. In particular, many of the children with lower levels of understanding gave evidence of thinking of weight simply as a property of an object, its felt weight. For example, they sometimes thought one had to add a large piece to change the weight of an object, because a small piece did not feel like it weighed anything. And they frequently judged an expanded object to be heavier, because it rose to a higher level (again focusing on a surface perceptual characteristic of the whole object). In contrast, those who more clearly differentiated weight and density, seemed to think of weight within a matter theory. All matter has weight; thus, adding even a small bit adds weight. Further, if no alcohol has been added to a warmed thermometer, they judge its weight must remain the same. However, since the present study did not systematically pursue the issue of the extent to which student's had abstracted a matter theory, more work would be needed to test this hypothesis. It would also be of interest to investigate why students are at different points in their abstraction of a matter theory: different interests, experiences, or math ability might all play some role. It should be noted that the 7th graders were all of high math ability, and the majority of them had made a differentiation between weight and density at the time of the pretest. In contrast, the 6th graders were more heterogeneous in math ability and fewer had made the differentiation at the start of the study.

The present study also strongly suggested that our teaching intervention was successful in bringing about two kinds of change: (1) conceptual differentiation, in which students who did not initially differentiate between weight and density were led to make this differentiation; and (2) conceptual consolidation, in which students who already made a basic distinction between weight and density, deepened their understanding of this distinction. However, the teaching was not equally effective with all children, and it is useful to consider what factors affect its overall impact on students.

Two factors seemed to affect student's ability to differentiate weight and density: their starting level of understanding and their participation in a clinical interview. In general, the majority of students who moved to making a differentiation between weight and density were those who were only one step away from making this differentiation. That is, they were at level 2b (i.e., they had begun to be aware that two concepts were needed although they still conflated weight and density when thinking about density.) Further, the curriculum was most effective for this group if they had a clinical interview, perhaps because it succeeded in revealing the tensions (puzzles) implicit in their conceptualization. Of course, students with lower levels of understanding...
frequently made some progress in their level of understanding as a result of teaching, but they did not progress as far as level 3a -- making a distinction between weight and density when density covaries with material kind. These children perhaps needed to make a more radical change in their framework: from a framework in which weight was analyzed as a property of objects to a framework in which it is analyzed as a property of matter. If this is the case, then the present study reveals that such framework changes are hard to make, even with the help of models.

The teaching intervention was highly effective in bringing about conceptual consolidation. In general, those students who had a concept of density were prepared to learn a verbal label for "density", learn appropriate quantification, and learn about sinking/floatin and thermal expansion. Further, after teaching they were more consistent in their performance across tasks. Again, however, the clinical interview may have played some role in promoting conceptual consolidation, since those with the clinical interview progressed more quickly and thoroughly than those who did not.

In contrast, those students who didn't achieve conceptual understanding of density had difficulty with many tasks: spontaneous definitions, sinking/floatin, qualitative reasoning about adding material and thermal expansion. Indeed, the only problems they had any success at all on were the problems involving formal computation. The fact that many students could memorize a formula for density -- in the absence of any conceptual understanding -- was striking. It should alert teachers to the dangers of "overinterpreting" the success on this type of task. Many traditional tests, however, rely more on this type of task than the more qualitative and conceptual tasks.

What about the teaching intervention made it effective for many students? While the present study was not designed to systematically investigate this issue, our results suggest that at least two aspects may be particularly important: (1) engaging student initial conceptions; and (2) using modeling activities.

Some evidence for the importance of engaging student conceptions comes from the superiority of the group which had the clinical interview. This group made more conceptual change and achieved a more integrated conception of density. While we tried to engage student conceptions throughout the teaching, clearly this can be most dramatically done in one-on-one situations where the student talks and the interview probes and listens (i.e., clinical interviews). Indeed, the experience of one-on-one interview can function as a valuable teaching tool. We are currently trying to understand more fully the nature of the effect: in particular, we want to know when these two groups began to diverge. We are also exploring ways we can structure the teaching situation to make it more responsive to individual needs and differences: for example, using a small-group rather than whole-class format. The computer can also be more fully exploited as a tool for challenging students and helping reveal puzzles in their conceptualizations.

There was also some evidence that our models were helpful for students, especially when dealing with problems like thermal expansion. Many students were capable of inventing qualitative models of density on their own (e.g., intensity shading codes), just as many are able to
make some differentiation between weight and density before instruction. However, these models do not show how weight and density are interrelated. The power of our "grid and dots" model is that it shows weight and density as distinct, yet interrelated quantities. Significantly, we found a high level of use of our grid and dots model in our "converts" to weight/density differentiation. Further, there was a high correlation between those students who assimilated our model and those who were able to understand the phenomena of thermal expansion. (It should be noted students do not always immediately jump to using our grid and dots model; frequently they first adopt models which show density more qualitatively, via an intensity shading code.)

Overall, we found our newly developed clinical interview very effective in diagnosing conceptual change. The written test was also effective, although as constructed it was not as sensitive as the clinical interview, and performance was less stable. Some changes could be made in the written test to improve its sensitivity and to make it more comparable to the clinical interview. In general, it would be good to have more items which do not presuppose an understanding of the word "density". One could also incorporate sink/float demonstrations in the testing procedure, so that the test does not require prior experience with sinking/floating phenomena. It also is undesirable to give the same test three times, as motivation wanes, and carelessness increases.

In summary, the present study re'aaled the potential of the teaching intervention for bringing about conceptual change and consolidation. At the same time, it alerted us to issues in the implementation of the teaching intervention to which we need to pay more attention: how to make students more aware of their initial conceptions and the puzzles inherent in them, and how to exploit work with the computer and small group activities to promote more active dialogue.
References


Appendix 1

ASSESSMENT INSTRUMENTS

A. Written Test

B. Clinical Interview
A. Written Test
PART A.

1. Here are two solid objects made of different materials. The yellow one is made of GALT and the white one is made of LIDUM. Both are the same size but weigh different amounts.

* WHICH OBJECT IS MADE OF A HEAVIER KIND OF MATERIAL?  
  GALT  LIDUM

* HOW DO YOU KNOW?  EXPLAIN YOUR ANSWER.

2. Here is another object made of GALT.

* IMAGINE AN OBJECT MADE OUT OF LIDUM THAT WEIGHS THE SAME AS THE OBJECT MADE OF GALT.

Draw a picture of what it would look like in the space above.
3. Here are some more pairs of objects made of GALT and LIDIUM. Decide if the objects in each pair weigh the same or if one of them is heavier. Circle the answer.

a. The object made of GALT is 2 times the size of the object made of LIDIUM.

Circle one: THIS IS HEAVIER THIS IS HEAVIER THEY WEIGH THE SAME

b. The object made of LIDIUM is 2 times the size of the object made of GALT.

Circle one: THIS IS HEAVIER THIS IS HEAVIER THEY WEIGH THE SAME

c. The object made of LIDIUM is 4 times the size of the object made of GALT.

Circle one: THIS IS HEAVIER THIS IS HEAVIER THEY WEIGH THE SAME

d. The object made of LIDIUM is 3 times the size of the object made of GALT.

Circle one: THIS IS HEAVIER THIS IS HEAVIER THEY WEIGH THE SAME

e. These two objects made of GALT and LIDIUM are both the same size.

Circle one: THIS IS HEAVIER THIS IS HEAVIER THEY WEIGH THE SAME

f. How did you tell if the objects weighed the same or different amounts?

_________________________
1. Consider the following three objects made of different materials: wood, aluminum, and steel. The objects are all the same size. The one made of steel is heavier than the one made of aluminum, and the one made of aluminum is heavier than the one made of wood.

* THE OBJECTS ARE THE SAME SIZE BUT THEY HAVE VERY DIFFERENT WEIGHTS.

WHAT DO YOU THINK COULD BE DIFFERENT ABOUT THESE THREE MATERIALS THAT WOULD EXPLAIN THIS?

* Draw a picture which shows what you mean.

**WOOD**  **ALUMINUM**  **STEEL**
1. * HAVE YOU HEARD OF THE WORD "DENSITY"?  
   Yes  No

2. * WRITE DOWN WHAT YOU THINK IT MEANS.

3. Here are some statements about density.  
   * Circle the statements you think are correct.
   
   a. Objects made of denser materials are always heavier than objects made of less dense materials.
   
   b. Objects made of denser materials are always smaller than objects made of less dense materials.
   
   c. Objects made of denser materials are always heavier for size than objects made of less dense materials.

4. Here is a block of wood which is cut into two pieces.

   - IS THE DENSITY OF BLOCK "b" THE SAME AS THE DENSITY OF BLOCK "a"?
   
   Yes  No

5. Here are some more statements about density. * Circle the statements you think are correct.

   The density of a material may be changed by:
   
   a. taking a small piece off.
   b. heating it.
   c. a chemical reaction with another material.
   d. nothing.
1. Here are four objects which have the following sizes and weights:

- **A**: Size: 4 cube units, Weight: 12 grams
- **B**: Size: 5 cube units, Weight: 12 grams
- **C**: Size: 2 cube units, Weight: 6 grams
- **D**: Size: 2 cube units, Weight: 8 grams

Think about whether any of these objects could be made of the same material. Circle the correct statements.

a. Objects A and B **could** be made of the same material because they are the same weight.

b. Objects C and D **could** be made of the same material because they are the same size.

c. Objects A and C **could** be made of the same material because they have the same weight per size unit.

d. None of the above **could** be made of the same material.

2. What is the density of the material in each object?

a. The density of the material in object A is: __________

b. The density of the material in object B is: __________

c. The density of the material in object C is: __________

d. The density of the material in object D is: __________
Part E

Here are two pieces made of WAX and ALUMINUM placed in a tub of water. The ALUMINUM piece weighs 150 grams and the WAX piece weighs 50 grams. When they are placed in water, the WAX floats while the ALUMINUM sinks.

Circle the correct statements

1. IF A VERY SMALL PIECE OF ALUMINUM WEIGHING ONLY 2 GRAMS WERE PUT IN THE WATER, IT WOULD:
   a. Definitely float
   b. Definitely sink
   c. Can't tell whether it would sink or float from the information given.

2. IF A VERY LARGE PIECE OF WAX WEIGHING MORE THAN 200 GRAMS WERE PUT IN THE WATER, IT WOULD:
   a. Definitely float
   b. Definitely sink
   c. Can't tell whether it would sink or float from the information given.

3. IF THE 50 GRAM PIECE OF WAX WERE PUT INTO A LIQUID OTHER THAN WATER, IT WOULD:
   a. Definitely float
   b. Definitely sink
   c. Can't tell whether it would sink or float from the information given.

4. IF THE 2 GRAM PIECE OF ALUMINUM WERE PUT INTO A TUB OF WATER AS BIG AS AN OLYMPIC SIZE SWIMMING POOL, IT WOULD:
   a. Definitely float
   b. Definitely sink
   c. Can't tell whether it would sink or float from the information given.

4. EXPLAIN YOUR ANSWERS TO THE ABOVE QUESTIONS. HOW CAN YOU TELL IF AN OBJECT WILL SINK OR FLOAT?
5. Here are some statements about sinking and floating. Circle the statements you think are correct.
   a. Objects always sink when they are heavy.
   b. Objects made of a material less dense than water float.
   c. Objects float only if they have air in them.
   d. The shape of an object never affects whether it will sink or float.
   e. If a solid cube weighs more than an equal amount of liquid, it will sink in a big tub of that liquid.

6. Here is a large iceberg floating; 9/10ths of it is below the water.

* A SMALL PIECE OF THE ICEBERG BREAKS OFF.

Circle the correct statement: (Draw a diagram showing how the little piece looks in the water if it helps you find the correct statement.)
   a. The little piece will float with 9/10ths of it above the water
   b. The little piece will float with 9/10ths of it below the water.
   c. The little piece will float with 1/2 of it below the water.
   d. The little piece will sink.
   e. Can't tell the from information given.
Part F

1. Beaker A and B are both filled with the same liquid at the same temperature. There is more liquid in Beaker B than Beaker A.

2. Draw a picture that shows the density of the liquid in beakers A & B.

Circle the correct answer:

- The liquid in beaker A ___________ the liquid in beaker B.
  a. is less dense than
  b. has the same density as
  c. is denser than

* EXPLAIN HOW YOU HAVE SHOWN THE DENSITY OF THE LIQUID IN YOUR PICTURE.
3. Beakers C and D are now filled with the same amount of liquid at the same temperature. The liquid in Beaker D is then heated, causing its level to rise in the tube:

Circle the correct answer

* After beaker D is heated, the liquid in it _____ the liquid in beaker C.

a. is less dense than
b. has the same density as
c. is denser than

4. Draw a picture that shows the density of the liquid in beakers C and D after heating beaker D

* EXPLAIN HOW YOU HAVE SHOWN THE DENSITY OF THE LIQUIDS IN YOUR PICTURE.
Bill and Bobby's class is studying watering and how seeds grow. They want to answer the question: Does more water help seeds grow faster?

Bill and Bobby did the following experiments:

(a) Bill takes pumpkin seeds and sunflower seeds and puts each kind in a different box. He waters one kind every day and the other kind once a week.

(b) Bobby takes only pumpkin seeds and puts them in two different boxes. He waters one box every day and the other box once a week.

* HAS ONE OF THE STUDENTS DONE A BETTER EXPERIMENT? Yes No

* (If yes): WHICH ONE? Bill Bobby

* EXPLAIN WHY OR WHY NOT.
Part G

1. Mr. X is in a soundproof room.
   * WHICH PICTURE DO YOU FEEL CONVEYS THIS IDEA BEST? A B

   Why did you choose that picture?

2. Here is a box filled with black and white butterflies. As the box gets bigger, more black and white butterflies are added, as shown.

Here is a tank with black and white fish. Fill in the missing black and white fish in the last tank, in the same way.
3. Here are some pictures of cylinders. Look at the pictures and decide which are the best ones to help you solve the following problem:

**PROBLEM:** Cut the longer cylinder so that you have a piece that is the same size as the short cylinder.

Which picture or pictures would you use to solve the problem?

Circle here: A B C D

* EXPLAIN WHY THE PICTURE OR PICTURES YOU CHOSE ARE BETTER FOR SOLVING THE PROBLEM THAN THE OTHERS.
5. Here are some statements about scientific models. Circle the statements you think are correct.

a. Models which contain lots of information are always the most helpful.

b. The more a model looks like the real thing looks, the more useful it is.

c. A model which is useful for solving one type of problem is not always useful for solving another type of problem.

d. The information put into a model should depend on the problem being solved.

e. Models are a tool for thinking.

f. A scientific model is an accurate model which never changes.

Part G

Betty and Jill's class is doing an experiment to find out about weight and sinking and floating. They want to answer the question: Does the weight of an object determine if it will sink or float?

Betty and Jill did the following experiments:

(a) Betty took a heavy rock and a light piece of wax. She put them in the water to see which would sink or float.

(b) Jill took a heavy piece of clay and a light piece of clay and put them in the water to see which would sink or float.

* DID ONE OF THEM DO A BETTER EXPERIMENT? Yes No

*(If yes): WHO? Betty Jill

* BRIEFLY EXPLAIN WHY OR WHY NOT.
B. Clinical Interview
PRE/POST CLINICAL INTERVIEW

NAME____________________ M/F GRADE_______ DATE__________________

I. SORTING BY MATERIALS.

Set of cylinders (1 1/2 in. diameter) made of wood, aluminum, and steel). WOOD: 1, 2, 3; AL: H, L, O; STEEL: A & B.

"Some of these objects are made of different materials and some are made of the same material. Can you sort them into groups according to the kind of material they are made of?"

wood aluminum steel

Other________________________

(Tell them names and correct any mistakes.)

II. PAIRED COMPARISON OF WEIGHTS OF OBJECTS.

POSTAGE SCALE. Same set of cylinders.)

"Now I'm going to ask you some questions about the weights of these objects. I'll show you two objects at a time and ask you whether one of them is heavier or whether they are the same weight. I want you to think carefully about your answer. So for each problem, I want you to take these objects in your hands, and put them on the postage scale before giving your answer.

Q: Is one of these objects heavier or do they weigh the same? (If one is heavier, ask: Which one is heavier? (Repeat question as needed)

(1) W2-L □ X (same size WD & AL) __________________________

(2) W2-W3 □ (big & med WD) __________________________

(3) O-N □ = (equal wt ST & AL) ?? __________________________

(4) W3-H □ (big heavy WD & small AL) ?? __________________________

(5) O-A □ (big heavy AL & small ST) ?? __________________________

"Very good. How did you know when an object was heavier?

__________________________________________________________

__________________________________________________________

__________________________________________________________

__________________________________________________________
III. PAIRED COMPARISON OF DENSITIES OF MATERIALS

(POSTAGE SCALE. Same set of cylinders).

"Now I'm going to ask you different questions about these objects. You've already sorted these objects by the kind of material they are made of: some are wood, some aluminum, some steel. Now I'm going to ask you about the heaviness of the kind of material an object is made of.

Q: Is one of these objects made of a heavier kind of material, or not? (Repeat question as needed)

(1) W2-L □ □ (same size WD & AL) __________________________
(2) H-A □ □ (same size AL & ST) __________________________
(3) N-D □ □ (equal weight ST & AL) ??______________________
(4) N-A □ □ (big heavy AL & small ST) ??____________________
(5) N-H □ □ (big and small AL) __________________________
(6) W3-H □ □ (big heavy WD & small AL) ??_________________

"Very good. How did you tell which object was made of a heavier kind of material?

________________________________________________________________________

IV. MYSTERY MATERIALS.

BALANCE SCALE. Three regular objects: 1" wood, 1" al, 1" steel. Mystery materials: 1" RED (wood), 1" BLUE (lucite)

"Here is a balance scale (check that you know how it works) and three new pieces of wood, al, and st. There are also two objects that are not wood, al, or st. Your job is to figure out what kind of material they are made of.

RED (the wood): "Could this object be made of wood, aluminum or steel, or must it be made of something else? How do you know? (note strategy and explanation)

________________________________________________________________________

BLUE (lucite): "Could this object be made of wood, aluminum or steel, or must it be made of something else? How do you know? (note strategy and explanation)

________________________________________________________________________
V. ORDERING BY WEIGHT AND DENSITY.

BALANCE SCALE: Small steel cube, small Al cube, Medium Al (II) equal in weight to mystery lucite (BLUE).

"I'd like you to order these four objects according to their WEIGHT. Put the lightest object here, the next heaviest here, and so on, if they are the same weight, put them together. Think about it as carefully as possible."

Strategy:  

"Now I'd like you to order these objects in a different way. Now order these four objects according to the DENSITY of the material they are made of. That is, put the object (or objects) made of lightest material on the left; next heaviest kind here; and so on. If some objects are made of materials which have the same density, put them in order."

Strategy:  

"How did you know where to place them?"

VI. MODELING

(Same four objects as above: pencil & markers; 8 1/2" x 11" unpaped)

"You've ordered these four objects by weight and by density. Can you now draw a picture (make up a picture rule) which gives information about the size, weight, and density of each object?"

How have you shown their size?  

How have you shown their weight?  

How have you shown their density?"
VII. SINK AND FLOAT

1. (Set of 8 objects: one kind of floating wood (of two different sizes), large and small pieces of apple; large and small piece of clay, large and small pieces of lucite)

Here are different objects. Why don't you put them in the water to see if they sink or float.

"What types of things sink and what types of things float? Can you make up a general rule which allows us to predict what will sink and what will float?"

2. (Bring out: large & small WAX; large & small AL. State materials. Order by relative weights using BALANCE SCALE. Put small WAX & large AL in water.)

On the large AL sinks and the small WAX floats. Now if we were to put the big WAX and the small AL in the water, what do you think would happen?

Large WAX: S F Reason: ____________________________

Small AL: S F Reason: ____________________________

3. (Bring out lucite and jar of fresh and salt water. Show them that one lucite floats in one jar, but not the other)

"Here is a piece of lucite. If I put it in here it floats. But if I put it in here, it sinks. How can that be?"

(Bring out glass of oil.)

"This glass has oil in it. If I put this lucite in the oil, do you think it will sink or float?"

S F Reason: ____________________________

67 BEST COPY AVAILABLE
(Show them that lucite sinks in oil.)

(IF WRONG IN PREDICTION: "In fact, the lucite sinks in oil. How can that be?

4. (Bring out 2 same size pieces of CLAY & 2 small pieces of WAX)

"Here are two same size pieces of clay and they weigh the same. Put on BALANCE SCALE. Now I'll put one of these little pieces of clay in-between these two pieces of wax. Show them that the clay/wax piece clearly weighs more than the small clay piece alone.

When I put the small clay in water, it sinks. Show them.

If we put this heavier object in water (clay stuck between wax pieces), do you think it would float or sink?

F S F Reason: _____________________________

(Do experiment and show that the clay/wax floats).

IF S PREDICTED IT WOULD SINK: The clay ball sinks, but the heavier object made of clay and wax floats. How can that be?

_________________________ __________________________

5. a) Small clay, b) clay/wax, c) candle.

Could you order these objects by how much they weigh?

_________________________ __________________________

Now could you order these objects by the density of their material?

_________________________ __________________________
VIII. **EFFECTS OF TRANSFORMATIONS ON OBJECTS AND MATERIALS**

1. **CLAY**

   a). "Did I change the AMOUNT of clay in the ball?"

<table>
<thead>
<tr>
<th>Yes (more)</th>
<th>Yes (less)</th>
<th>Reason: __________________________</th>
</tr>
</thead>
<tbody>
<tr>
<td>No</td>
<td></td>
<td>Show me how much to add to change the amount of clay? (have them tell you the amount, and then use that amount for the rest of the questions)</td>
</tr>
</tbody>
</table>

   b). "Did I change the WEIGHT of the clay ball when I added that little piece?"

<table>
<thead>
<tr>
<th>Yes (heavier)</th>
<th>Yes (lighter)</th>
<th>Reason: __________________________</th>
</tr>
</thead>
<tbody>
<tr>
<td>No</td>
<td></td>
<td>Show me how much more clay I need to make it heavier?</td>
</tr>
</tbody>
</table>

   c). "Did I change the DENSITY of the clay in the ball when I added that (little) piece?"

<table>
<thead>
<tr>
<th>Yes (denser)</th>
<th>Yes (less dense)</th>
<th>Reason: __________________________</th>
</tr>
</thead>
<tbody>
<tr>
<td>No</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. **EXPANSION**

   Bring out thermometer and glass of warm and cold water. Have them observe what happens to thermometer in hot and cold water.

   When you put the thermometer in the warm water, the alcohol in the thermometer rises like this and when you put it into cold water, it goes down like this.

   **SAME-LESS**

   Is there MORE alcohol in the thermometer when it's in warm water? **Y** **N**

   How do you know that?

   **SAME-LESS**

   Does the alcohol WEIGH more when it's in warm water? **Y** **N**

   How do you know that?

   **SAME-LESS**

   Does the alcohol have the same DENSITY when it's in warm water? **Y** **N**

   How do you know that?
Appendix 2

TEACHING MATERIALS

UNIT 1: Sinking and Floating Phenomena and Introduction to the Density of Materials

A. Description of Lessons
B. Worksheets

UNIT 2: Thermal Expansion

A. Description of Lessons
B. Worksheets

UNIT 3: Average Density

A. Description of Lessons
B. Worksheets
UNIT 1:

Sinking and Floating Phenomena and Introduction to the Density of Materials
UNIT 1: DESCRIPTION OF LESSONS

LESSON 1: Introduction to sinking and floating phenomena

Objectives: preliminary experience with materials, sorting and classification, and exploring ideas for sink and float criteria; explore meaning of rules and the kind of predictive rule we are looking for

I. Students are given a tub of water, scale, and kits with various objects such as different size pieces of clay, pumice, wood, and stone. They are asked to group them according to whether they sink or float and describe them on a worksheet.

II. Discussion follows along two lines. First, students agree on which objects sink or float. Yet, they do not all have the same way of describing them; there are several ways to describe a given object. We have students begin to think about the different characteristics of the objects and their materials, paying particular attention to what it might be about the objects or materials which makes some sinkers and some floaters. They are urged to formulate some general rule about what kinds of things sink and what kinds of things float.

III. We hand out another kit that has items which challenge students to formulate their rules. Included in the kit are small pieces of metal, ceramic, lignam vitae (a dense, sinking wood) and a large piece of floating wood. Students must predict whether each item will sink or float. Those students who use the criteria of weight or size alone for their predictions must come to terms with the small, light things which sink and the large heavy item which floats. Students are to state their predictions and reasons before testing each item in the water.

LESSON 2: Investigating properties or characteristics of materials

Objectives: to investigate properties of materials and extend range of materials which will be studied; further work with classifying and sorting; preliminary exploration of the difference between intensive and extensive properties; preliminary testing of rules

I. We list the ideas students had about what will sink or float on the board. Students do not generally come up with one basically consistent rule or criterion to apply for all objects, but rather each object will have its own reason. The reasons why objects sink include: if it's big, heavy, solid, thin, made of clay, heavy material, sinking material, dense material. Objects float if small, light, thin, hollow, if there is air inside, or if made of light material, floating material, or wood.

II. We discuss again the meaning of a general rule and the role of experimentation. Our goal will be to come up with a rule that will work for all the objects we have considered and could easily apply to objects we have not yet seen. We consider experiments in two ways: first, for
exploration, to see what will happen, and to try something new to get information or ideas; secondly, to test out an idea (hypothesis) that we already have in a systematic way. The teacher refers to the list on the board and states that we have already generated ideas and now we need to find a thoughtful way of testing our ideas and to see if we were right about certain ideas in particular.

III. We structure the experimental phase by first having students classify objects according to their material kind. They test for the conclusion that it is material kind, not size or weight alone that makes a difference. Hence, some materials sink and others float. If we know what some amount of material will do we know what other quantities of that material will do.

IV. The teacher does a few demonstrations which enrich understanding of the new rule. Other size pieces of the familiar materials are placed in water. Finally two pieces of white soap are placed in water. One sinks while the other floats. Students discuss whether or not they could be made of the same material.

V. Students are encouraged to try out objects at home and make their predictions. Sometimes it is necessary to discuss the limits of our investigation. At this point we must limit our discussion to objects which are made of one material only, are not shaped like a boat, and do not act like sponges.

LESSON 3: Investigating weight and developing and evaluating predictive rules for sink/float

Objectives: to work more explicitly with the weights of objects and determine their effect on sinking and floating; to reflect on what makes a meaningful experiment in light of our investigations and discuss counter-examples, discovery of variables, and control of variables

I. We start with four objects: a large and small piece of pumice and a large and small stone. Students will order the objects in two distinct ways: by their weight and then by the density of their materials. It was useful to review that materials are characterized in many ways and that one of these characteristics is that some materials are heavier than others. That is to say that some materials are denser than others and that we have an easy procedure for determining the denser of two materials. If we compare the weights of objects which are identical in size but made of different materials, we say that the heavier object is made of denser material. We note that the small stone weighs less than the large pumice although it is made of denser material.

II. More objects are given to the students: equal size pieces of wood, copper, and hard rubber as well as larger and smaller pieces of the same materials. Again they will order these objects in two distinct ways: by their weights and by the densities of their materials. In addition, they use their orderings to conduct systematic experiments. Since some people had the idea that objects sank or floated because of their weights, this idea is now tested. Students see, by ordering their objects by weight and placing each in the water, whether or
not weight alone is a valid predictor of the objects' sinking and floating behavior. They then order objects by the density of their materials and test whether this is a more useful parameter for predicting sinking and floating behavior. We conclude that density, not the weight of an object alone, is the relevant parameter.

LESSON 4: Introduction to using models

Objectives: explore student models visualized or imagined ideas of what makes some materials sink and some float and further what makes some objects weigh more than others, even though they are the same size; begin to focus on the idea of crowdedness, packedness, intensity

I. We first review our procedure for determining whether one object was made of a denser material than another by comparing the weights of equal amounts of each material.

II. We now propose to students that drawing models or pictures sometimes helps make ideas clearer. We continue our investigation of objects and materials by having students draw models of five objects. Three of them are the same size but are made of different materials - wood, brass or copper, and rubber pegs which weigh distinctly different amounts. The other two objects are a larger piece of wood and smaller piece of metal (either a penny or brass gram weight.) Students are asked to represent the objects on a worksheet which supplies the outline of the small size pieces. They are asked to draw the other two objects free-hand in the space provided underneath.

III. As the teacher circulates, students are asked about the kinds of things they can tell from their pictures. Can they tell whether or not two objects are made of the same material? Do their models let them make judgements about the relative sizes of the objects? They are also asked to think about whether they can tell anything about the objects' weights or about the relative densities of their materials.

IV. We use the above questions in a follow-up discussion. We have the class look at several different models and evaluating which ones could or could not be used as meaningful representations. At this time, we focus on how the information is represented or communicated and whether the code used is internally consistent as well as consistent with reality. Later, we will focus more specifically on the relationships of the three quantities: size, weight, and density.

LESSON 5: Using the model to find out about sinking and floating (Including role of liquid)

Objectives: preparation for Archimedes program and extending the idea of density of liquids as well as solids; to discover the rule built into the Archimedes program and think of how that would translate or what it would mean in the real world.
I. The densities of various solids and liquids (such as wood, clay, water, and oil) are compared by weighing equal portions of each. Students see that we can talk about the density of liquids as well as of solids.

II. Relating to the experience students had the previous day with modeling, they are shown how solids and liquids are represented by the computer. The teacher draws several objects on the board as they would appear on the computer screen and points out how each dimension is represented: Size - boxes; Weight - dots; Density - dots in each box.

III. The teacher guides the class in how to use the Archimedes program, talking through the functions of the various commands and how to create objects by first selecting the material from which they are made. The range of materials is given on a menu which shows a single building block of each material. Each building block consists of a square box (size unit) with a characteristic number of weight units (dots) within it. Since objects are made up of identical building blocks, the theoretical size of each block is arbitrary so long as the condition of homogeneity is valid. That is, the size of the real world material sample, which this building block represents, is arbitrary, so long as its weight is consistent no matter from where in a given substance or object we draw the sample.

IV. Students now use the Archimedes program to perform experiments about sinking and floating. The program simulates the placement of various objects in different liquids. All the objects are the same size. By noting which objects sink and which float, students are asked to form a general rule which would allow them to predict the behavior of any given object in any given liquid.

V. Discussion has students articulate their rules. We look for appropriate terminology (i.e., referring to "dots/box" as opposed to just "dots" when necessary) and generality (i.e., a relative density rule as opposed to specific case citing: "green material floats in orange liquid")

LESSON 6: Testing the rule for sinking and floating using the sink the raft program in which we can change the size of objects

Objectives: to have students gain practice using the simulation and confirm the notion that density, not weight, is the relevant dimension for sinking and floating phenomena

I. The students continue to work with the computer model. This time they use the Sink the Raft program, in which the size of objects can be changed. Students are challenged to see if they can make a floating object so big and thus heavy that it will sink. Similarly they try to make sinking objects float by removing material from them.

II. Discussion of conclusions and consequences for real world objects and materials.
LESSON 7: Using the computer model to quantitatively represent the sizes and weights of objects and the densities of their materials (the computer program as quantitative modeling tool)

Objectives: become familiar with the computer model as a way of representing real objects and materials; begin to focus on the quantitative aspects of the model - specifically, seeing weight and density as two distinct quantities; obtain another procedure for finding relative densities by comparing the sizes of equal weight objects.

I. Students are shown some qualitative models of objects and asked to make judgements about their relative weights. For the qualitative model we represent an object's relative density by how darkly it is shaded. We propose that although qualitative models are useful, they are sometimes ambiguous. We highlight this ambiguity by showing a qualitative model of an object which could correspond to two or more real objects. Consider a large grey object. Is it heavier or lighter than a smaller black object? Certainly a large piece of wood weighs more than a penny. Yet the same piece of wood could be lighter than a small piece of lead. We could also find a piece of wood larger than a penny which will be lighter in weight than the penny. This demonstration makes it clear that a weight judgement based solely on qualitative representations can be quite arbitrary and unreliable. An explicit rationale emerges for shifting from qualitative models to more quantitative models and motivates the need for more precision.

II. Students are guided through a series of activities which lead to an understanding of how to model an arbitrary object on the computer using the Weight and Density program. In a step-by-step progression, they are urged to focus on real objects' relative sizes, weights, and densities by selecting appropriate representations of these dimensions on the computer. We start with having them represent different groups of cubes of the same material. Then they have to represent cubes of different materials. In a demonstration, students see that three cubes of aluminum are equal in weight to one steel cube. They must represent the steel and aluminum pieces on the computer by selecting appropriate materials - two materials with a density ratio of 1:3. By the end of the lesson students are expected to accurately model other pieces of steel and aluminum which are equal in weight.

III. For homework, students fill out a worksheet which has them focus on the amount of one material needed to balance the weight of a specified amount of a different material. They also need to think about how these materials, their weights and amounts could be represented on the computer.

LESSON 8: Exploration of the computer model as a way of exploring ideas about real materials

Objectives: use the computer model to explore aspects of objects with regard to weight, shape, size, and density of material and compare findings with real objects and materials; gain experience working with the three interdependent quantities (2 of which are extensive and
Appendix 2

1 of which is intensive) and seeing their relationships to each other mathematically and to real objects and materials.

I. Short review of how steel and aluminum pieces were modeled on the computer. Students are asked to think about whether or not the density of a small piece of aluminum is the same as a large piece of aluminum and to make judgements about the relative weights and densities of a small piece of steel compared to a large piece of aluminum. They also should think about how much more aluminum than steel would be needed if their weights are to be equal.

II. During this lesson, students use the Weight and Density program to explore ideas about real materials. For example, students add and remove material from objects and observe the consequences this has for the objects' weight and density. There is also further emphasis on clearly distinguishing the dimensions and focusing on their precise quantities. Students construct objects according to given specifications. For example they must create two objects which are the same size but different weights, then copy the objects and data.

III. We also begin to explore the meaning of a size unit through the use of the computer program and real 1cc cubes made of various materials. Students drop 1 cc cubes into a graduated cylinder and see how the principle of water displacement as a way to measure volume works. They also see that an object made of clay can be distorted in shape but its volume as measured by water displacement will be preserved. They are given the chance to guess the volume of some objects. We point out that the computer program counts the total volume of objects and gives that information as size units, but cannot show the three dimensional shape of objects.

IV. Students also work with objects represented in the "filled in" mode on the computer whereby reliance on numerical data is encouraged.

LESSON 9: Emphasizing quantitative aspects of the model in order to explore level of submergence and the formal definition of density

Objectives: further experience with ordering and to focus on finer quantitative differences in the weights, sizes, and densities of various objects and materials; review procedures for finding densities; using the model to find out about level of submergence

I. We begin to formalize the definition of density. The students are challenged to figure out the density of the material an object is made of if the only information available is the object's size and weight. We emphasize that density is a relationship between size and weight, not simply one or the other. We review three procedures for obtaining information about density: comparing equal size pieces, comparing equal weight pieces, or dividing the weight of one object by its size. II. Next students are shown in a demonstration that equal size pieces of different floating materials, such as wood and styrofoam, float at different levels. They use the Sink the Raft program to explore the behavior of different size objects.
made of various materials in order to discover the "rule" associated with a floating object's level of submergence (i.e. if the density of the object is 'o' and the density of the liquid is 'l' then 'o/l' of the object's volume will be submerged.) This further highlights the notion that the quantitative model is more useful in certain circumstances than is a qualitative one (such as the shading intensity model.)

LESSON 10: Wrap-up

Objectives: review; relate what has been learned to some new contexts - how Archimedes solved the crown problem, panning for gold

I. We review the points that have been made throughout the unit as a prelude to handing out a reading concerning Archimedes. We review that materials can be characterized by their densities and the various procedures for finding information about what their densities are. Students read the Archimedes story and try to think of how he solved the problem of the crown. (The problem is: Archimedes knew how much a chunk of gold weighed, and he knew that the crown weighed this precise amount, but how could he be sure that the crown was pure gold?)

II. We also use a 3-2-1 Contact television show segment to cap this unit. The segment shows how the density concept is related to the practice of panning for gold, how the density concept is similar to the idea of packing, and introduces the principle behind the way submarines work, i.e. by increasing or decreasing the proportion of air in the ship. We pick up on this notion of mixed materials and average density in the third unit.
Worksheet #1

NAME: ____________________________________________

<table>
<thead>
<tr>
<th>THINGS THAT SINK</th>
<th>THINGS THAT FLOAT</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>#1</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>#2</td>
<td>#2</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>#3</td>
<td>#3</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>#4</td>
<td>#4</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

80
<table>
<thead>
<tr>
<th>DESCRIBE (What is the object?)</th>
<th>PREDICT Will it Sink or Float?</th>
<th>GIVE REASON Why is it floating or sinking?</th>
<th>CHECK Put in water. Does it sink or float?</th>
<th>CHANGE REASON Do you have a new reason to explain why it is floating or sinking? (If yes: What is it?)</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>#2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>#3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>#4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>#5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
**Worksheet #2**

<table>
<thead>
<tr>
<th>MATERIAL: ____________</th>
<th>MATERIAL: ____________</th>
</tr>
</thead>
<tbody>
<tr>
<td>Describe Object</td>
<td>Describe Object</td>
</tr>
<tr>
<td>Does it Float or Sink?</td>
<td>Does it Float or Sink?</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>MATERIAL:</td>
<td>MATERIAL:</td>
</tr>
<tr>
<td>---------------------------------</td>
<td>---------------------------------</td>
</tr>
<tr>
<td>Describe Object</td>
<td>Does it Float or Sink?</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**WHAT WAS THE PURPOSE OF THE EXPERIMENTS?**

**WHAT DID YOU FIND OUT?**
**Worksheet #3**

Name: ________________________________

I. Ordering Objects by Weight

<table>
<thead>
<tr>
<th>Heaviest</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th>Lightest</th>
</tr>
</thead>
<tbody>
<tr>
<td>Describe Object</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sink or Float?</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

II. Ordering of Materials by Density

<table>
<thead>
<tr>
<th>Heaviest Kind of Material</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th>Lightest Kind of Material</th>
</tr>
</thead>
<tbody>
<tr>
<td>List Objects made of Given Material</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sink or Float?</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
1. Model the three pegs of wood, hard rubber and copper (or brass). Your model should show that they are all made of different materials and show the heaviness of each kind of material.

2. Model the large piece of wood and the penny (or small brass piece). You should be able to tell what material each object is made of and how big it is from your model.
### Finding a rule for floating and sinking

1. When do objects sink or float? Trying different kinds of liquids.

   a) Does an object made of **GREEN** material float or sink in these liquids?

<table>
<thead>
<tr>
<th>Float or Sink</th>
<th>Green</th>
<th>Purple</th>
<th>White</th>
<th>Orange</th>
<th>Blue</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   b) Does an object made of **PURPLE** material float or sink in these liquids?

<table>
<thead>
<tr>
<th>Float or Sink</th>
<th>Green</th>
<th>Purple</th>
<th>White</th>
<th>Orange</th>
<th>Blue</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   c) Does an object made of **WHITE** material float or sink in these liquids?

<table>
<thead>
<tr>
<th>Float or Sink</th>
<th>Green</th>
<th>Purple</th>
<th>White</th>
<th>Orange</th>
<th>Blue</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   d) Does an object made of **ORANGE** material float or sink in these liquids?

<table>
<thead>
<tr>
<th>Float or Sink</th>
<th>Green</th>
<th>Purple</th>
<th>White</th>
<th>Orange</th>
<th>Blue</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   e) Does an object made of **BLUE** material float or sink in these liquids?

<table>
<thead>
<tr>
<th>Float or Sink</th>
<th>Green</th>
<th>Purple</th>
<th>White</th>
<th>Orange</th>
<th>Blue</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. What is a general rule for floating and sinking?  
   (Hint: Does the kind of liquid matter?)

   - 
   - 
   - 
   - 
   - 

---
Testing the Rule for Floating and Sinking

1. **Experiment**: Set up an ORANGE object in PURPLE liquid.

   It is sinking. Can you change the size (and the mass) of this ORANGE object to make it float in this PURPLE liquid? Why or why not? (Try to predict first)

2. **Experiment**: Set up a WHITE object in BLUE liquid.

   It is floating. Can you change the size (and the mass) of this WHITE object to make it sink in this BLUE liquid? Why or why not? (Try to predict first)

3. Does changing the mass of an object by making it bigger or smaller change whether it will sink or float?
1. KEL-F and ACRYLIC are two different kinds of plastic. One (1) bar of KEL-F plastic weighs the same as two (2) bars of ACRYLIC plastic. (All the bars are the same size).

Which plastic is denser, KEL-F or ACRYLIC? ________________

Make a picture to show how you would model one bar of KEL-F and one bar of ACRYLIC on the computer.

2. Here is a model of two objects made of different materials, wood and vulcanite.

Which one is made of a denser material? ________________

Add on or take away some wood in the drawing to make the two objects weigh the same.
3. One (1) liter of mercury weighs the same as 5 liters of gold paint. Which is denser?

Make a picture to show how you would model a liter of mercury and a liter of gold paint on the computer.
1. Make 2 objects on the computer that are the same size, but one is made of a denser material than the other. Copy the data below.

   **Object A**  
   (made of denser material)

   SIZE=____ units  
   WEIGHT=____ units  
   DENSITY=____ units

   **Object B**  
   (made of a less dense material)

   SIZE=____ units  
   WEIGHT=____ units  
   DENSITY=____ units

2. Make 2 objects on the computer that weigh the same, but one is smaller than the other. Copy the data below.

   **Object A**  
   (smaller object)

   SIZE=____ units  
   WEIGHT=____ units  
   DENSITY=____ units

   **Object B**  
   (larger object)

   SIZE=____ units  
   WEIGHT=____ units  
   DENSITY=____ units

3. Make 2 objects on the computer with material of the same density, one weighs more than the other. Copy the data below.

   **Object A**  
   (heavier object)

   SIZE=____ units  
   WEIGHT=____ units  
   DENSITY=____ units

   **Object B**  
   (lighter object)

   SIZE=____ units  
   WEIGHT=____ units  
   DENSITY=____ units

4. Construct an object that is made of material twice as dense as the material in this object (Object A), but which weighs the same as Object A.

   **Object A**

   SIZE=____ units  
   WEIGHT=____ units  
   DENSITY=____ units

   **Object B**

   SIZE=____ units  
   WEIGHT=____ units  
   DENSITY=____ units
1. Construct these 2 objects on the computer. Then, make them weigh the same by changing only the size of one of them.

Solution:

2. Construct these 2 objects on the computer. Then, make them weigh the same by changing only the material of one of them.

Solution:
Homework for Lesson #8

1. Object A

   a. Order these 3 objects by SIZE:
      LARGEST

   b. Order these 3 objects by WEIGHT:
      HEAVIEST

   c. Order these 3 objects by DENSITY of the material they are made of:
      MADE OF DENSEST MATERIAL

2. Compute the DENSITY of the material each object is made of:

   SIZE = 8 S/u  WEIGHT = 8 W/u  DENSITY = ????
   SIZE = 1 S/u  WEIGHT = 5 W/u  DENSITY = ????
   SIZE = 3 S/u  WEIGHT = 9 W/u  DENSITY = ????

   (You may fill in the boxes with the right number of dots)
WORKSHEET # 9

NAME: __________________

Looking for a rule to predict the level of submersion.

1. a) Set up some BLUE LIQUID on the computer. Construct a FLOATING OBJECT. Get all the DATA and VIEW BOTH the object and the liquid.
   What material did you pick? _______________
   How much of the object is submerged? _______________
   
   b) Make the object very BIG. Again, VIEW BOTH the object and the liquid.
   How much of the object is submerged? _______________
   
   c) Now, make the object very SMALL. VIEW the object and the liquid again.
   How much of the object is submerged? _______________
   
   d) What have you noticed about the level of submersion for objects made of this material floating in blue liquid? ________________________________

2. a) Now, using the SAME BLUE LIQUID, construct a FLOATING object made of a DIFFERENT material. Get all the DATA and VIEW BOTH the object and the liquid.
   What material did you pick? _______________
   How much of the object is submerged? _______________
   
   b) Make the object very BIG. VIEW the object and the liquid again.
   How much of the object is submerged? _______________
   
   c) Now, make the object very SMALL. VIEW the object and the liquid again.
   How much of the object is submerged? _______________
   
   d) What have you noticed about the level of submersion for objects made of this material floating in blue liquid? ________________________________
3. How can we tell the level of submergence for a floating object? What do you think the rule is?

________________________________________________________________________

4. a) Now set up some ORANGE LIQUID. Predict how much of a small green object will be submerged. __________ Try it. How much of the object is submerged?

________________________________________________________________________

b) Predict how much of a large green object will be submerged in this orange liquid. __________ Try it. How much of the object is submerged?

________________________________________________________________________

5. Did your rule work? YES _____ NO _____

If you need to change it, what is your new rule?

________________________________________________________________________

6. See if your rule works for all floating objects.
The Archimedes Story

Students read a story about Archimedes, in which he solves a problem for the king. Students are to try to solve the problem themselves based on what they know about density and material kinds.

The story begins when the king gives some gold to a goldsmith to have his crown made. When the king gets his crown back, it weighs the same as the original amount of gold, but he suspects that the goldsmith tried to trick him and that the crown is not all gold. The king thinks that it might have been made from a different material which looked like gold.

Archimedes was a clever person who the king trusted. The king asked Archimedes to tell him if the crown was really gold or not. Archimedes thought about the problem for several days. The answer came to him suddenly one day while he was in the bathtub.

If you understand the following, you have enough information to solve the problem.

- Density is a property of materials - that is to say it is one way that we can tell one material from another, and the density of a material is the same whether we have a big piece or a little piece of it.
- There are ways of telling whether one object is made of a denser material than another one:
  - If a clump of one sinks and a clump of the other floats, the sinker is made of denser material.
  - If both clumps float, then the one which floats at a lower level of submergence is made of denser material.
  - If we take two equal size objects, then the heavier one is made of denser material.
  - If two objects weigh the same, then the smaller one is made of denser material.
- We can figure out the density of the material an object is made of by finding out how big the object is, finding out how much it weighs, and computing its weight for size or weight over volume. (That is, weight divided by size).
- We can get information about how big a sinking object is by putting it in a container of water and observing the way the water level changes.
UNIT 2:
Thermal Expansion
LESSON 1: Review

Objectives: assess (through written test and discussion) where students are and what they have retained from last unit; give review of: how objects were modeled on the computer; difference between weight and density; density formula and obtaining weight and volume measurements

I. Students are given a review test (see Appendix 2b). The items most related to modeling are an 'interpretation' task and a 'translation' task. For the former, students are shown a range of objects as they would look on the computer screen and asked to make judgements regarding their size, weight and density. For the latter, students are shown a range of objects which are more realistic in appearance and for which verbal information is given. They are asked to represent these objects as they would look on the computer screen.

II. The test is discussed in class.

LESSON 2: Beginning to model the phenomenon of thermal expansion

Objectives: formally introduce phenomenon; have students create a model for it and elicit their thoughts about it; begin to have students consider and reason about what happens to a material's density when heated; introduce some metaconceptual points about models: models represent important information - they give us information which is important for understanding something; models give us a picture or a way of thinking about things or events; it is important for models to be consistent with phenomenon

I. Students copy words from the board: thermal, density, volume, weight, phenomenon, expand. They are to find their definitions for homework.

II. Students witness a demonstration in which a brass ball is heated until it expands and can no longer pass through a metal ring. They also see that the brass ball's weight remains constant. Students are asked to draw models of the phenomenon. In guiding this activity, the teacher briefly reviews the intended meaning of the word model (i.e. a way of showing a thing or event which gives a clear picture of important information and helps us think about it) and suggests that information about the ball's material kind, size, weight, and temperature are represented. After drawing their spontaneous models they are asked to think of how the phenomenon might be represented on the computer. They are invited to revise the computer model if necessary.
Appendix 2

Examples of student models are then discussed with a focus on how material kind, weight, size and temperature were represented. Students are also asked to start thinking about what happened to the material's density and how that can be seen in their models. One goal is to have them recognize that the most useful or complete models in this case give accurate information about material kind, weight, and size simultaneously. In the past, some students have offered suggestions for revising models under discussion. For example, some students who drew models that conserved total number dots (i.e. weight) by decreasing the number of dots/box and increasing the number of boxes after heating have suggested lightly coloring in the objects to show that they are the same materials.

LESSON 3: Using the computer programs to explore more models of thermal expansion

Objectives: introduce computer programs which give two different qualitative models of thermal expansion - one continuous ("swelling") and one particulate ("atoms and springs" or "circles and squiggles"); introduce metaconceptual points about the need to revise models and that more than one model can describe the same phenomenon; have students think about what happens to the material's density with expansion

I. In this lesson students explore the thermal expansion phenomenon via the computer simulation. At first the teacher guides them through the use of the program and they become familiar with its similarities to and differences from the previous programs. In this way their attention is drawn to how the model was revised. Furthermore, they observe that this computer program now provides two separate ways to represent the same object on the screen. They are free to switch from one representation (grid and dots) to the other (circles and squiggles).

II. Following the directions on a worksheet, students then go through a series of computer based activities the purpose of which is to use the models and data to discover what happens to the size, weight, and density of materials with an increase in temperature. With the new model, the visual image of density is liberated from its previous fundamental tie with material kind. Hopefully students are ready to conceptualize density as a separate aspect.

LESSON 4: Quantitative aspects of the new phenomenon and model

Objectives: motivate reliance on formal definition of density; gain experience distinguishing quantities of weight, volume, and density; stress importance of standards

I. Go over homework.

II. Students continue to work with the computer simulation following the directions on a worksheet. Whereas the previous lesson was designed to involve them in interpreting the model on a qualitative level, this lesson's activities have them focus on the actual quantities and how they are calculated. We spend a little time on the meaning of decimals since the densities of some objects' materials will be given in decimal notation. This is done briefly - to
the extent that students should recognize a decimal number less than 1, between 1 and 2, and so on.

III. Since the boxes which stood for size units in previous programs are now expanding, we discuss the meaning of the data related to size. We can now no longer simply count boxes. Students can experiment with sets of 1 cc cubes to see that when the length of one side of a cube doubles or triples, its volume increases 8 and 27 fold respectively.

LESSON 5: The effect of thermal expansion on sinking and floating and the presence of the phenomenon around us

Objectives: observing and reasoning about the effects of thermal expansion on the density of liquids and how that relates to sinking and floating; presentation of the ice "anomaly"; (other things that could be worked into this lesson include: gaining a qualitative understanding of the fact of different rates of expansion for different materials and discussion of technology as a way of using physical phenomena to do useful work for people.)

I. Students are again asked to come up with their own models of a particular phenomenon and think of ways to apply the computer model to what they see. A demonstration is given in which an object made of styrofoam covered in clay is immersed first in hot water and then in cold water. The object sinks in the hot water and floats in the cold water.

II. One purpose of this activity is to see whether they can both adequately explain and represent the phenomenon. A number of pieces are thus brought together: the students' understanding of thermal expansion and the relative density rule for the behavior of objects in liquid, as well as the ability to portray them.

III. We note that usually objects expand when they are heated. There is a very common exception to this rule: ice. Ice cubes float in water. How would the world be different if ice were denser than water?

IV. Other topics for discussion: Why must we allow for thermal expansion in construction? How can we use thermal expansion to do work for us? (Thermostat demonstration)

LESSON 6: Maps and modeling

Objectives: emphasize metaconceptual aspects of modeling; have students gain practice evaluating models especially in relation to their consistency, usefulness, likeness to or interpretation of reality

I. This lesson is designed to make some explicit points about models in general. First, characteristics of models are examined via discussion about maps. Various maps of the Boston area (such as a subway map, street map, surrounding highway map, and souvenir of buildings map) are distributed among the students. Students describe their maps and the points which are made during discussion include: each map tells us something different
Appendix 2

about the same area - different kinds of information are given in each map; one map is not better than another - a map's value is dependant on its usefulness for a given purpose; the map should be reliable and accurate for its purpose; the map has a consistent code, key, or method of representation.

II. Students then fill out a worksheet which asks them to evaluate some different models related to thermal expansion. Students are told that the explicit purpose of the model is to accurately show what happens to a brass chunk's size, weight and density after heating. They are to decide whether or not this information has been portrayed.

III. A good follow-up activity this would be to have students make further judgements about the adequate models and decide which of them might be the most fruitful or helpful. IV. Closing discussion invites students to think further about why people make models and why we have used models during class. Their ideas have included that models are used for: reference; to see your ideas; for fun; to see if something works. These points are reinforced by the teacher who adds that: sometimes we can do experiments with models that would be more difficult to perform on a different scale; the computer programs were developed to help make some difficult ideas easier to see and work with; sometimes by working with a model and exploring its implications, one confirms or disconfirms his or her ideas about what something is and how it works and in that process gets more new ideas.

104
1. According to the computer model, which is the BIGGEST object? ________

   What is its size? ________

   Which object WEIGHS the most? ___

   How much does it weigh? ________

   Which is made of the DENSEST material? ________

   What is its density? ________
2. Here are three cylinders.

A and B are the same size. B and C are made of the same material.

A and C weigh the same amount. Draw a picture which shows how these objects would be represented on the computer.

B. Carefully describe some of the differences between weight and density.

________________________________________________________________________

________________________________________________________________________

________________________________________________________________________

________________________________________________________________________
Here are five objects.

They are made from two different materials.

How could you find out which object weighs the most?

How could you figure out which material is denser?

C. 1. This object weighs 20 grams.

Its size is 5 cc's. What is the density of the material it is made from?

2. The density of water is 1 gram per cc. How much does 100 cc's of water weigh?

3. This beaker has 50 cc's of water in it.

A chunk of gold is put into the water and the level rises to 60 cc.

What is the volume of the gold chunk?

What is the volume of the water?
WORKSHEET for 3rd class

DATE: ____________________

NAME: ____________________

1. Create an object.

Change the object's SIZE by ADDING or REMOVING MATERIAL.

As you do this what happens to the object's WEIGHT?

How do you know?

Is it still made of the SAME MATERIAL as before?

How do you know?

Does it still have the SAME DENSITY as before?

How do you know?

2. Create another object.

Change its SIZE by changing the TEMPERATURE. (Make it bigger.)

As you do this what happens to the object's WEIGHT?

How do you know?

Is it still made of the SAME MATERIAL as before?

How do you know?

Does it still have the SAME DENSITY as before?

How do you know?
3. Now create an object and change its REPRESENTATION from dots and boxes to dots, circles and squiggles.

Change the object’s SIZE by ADDING or REMOVING MATERIAL.

As you do this what happens to the object’s WEIGHT?

How do you know? ____________________________

Is it still made of the SAME MATERIAL as before? ________

How do you know? ____________________________

Does it still have the SAME DENSITY as before? ________

How do you know? ____________________________

4. Create another object and use the dots, circles and squiggles representation.

Change its SIZE by changing the TEMPERATURE. (Make it bigger.)

As you do this what happens to the object’s WEIGHT?

How do you know? ____________________________

Is it still made of the SAME MATERIAL as before? ________

How do you know? ____________________________

Does it still have the SAME DENSITY as before? ________

How do you know? ____________________________
5. Does one of the representations (models) make more sense to you?

Explain your answer.
WORKSHEET for 4th lesson

DATE: ______________________
NAME: ______________________

1. BUILD two identical objects on the screen. (They should be the same size, made of the same material, and be at the same temperature.)

Get all the DATA and copy it below:

(LEFT SIDE)                                        (RIGHT SIDE)
Temperature: ________              Temperature: ________
Total Weight: ________        Total Weight: ________
Volume: ________                  Volume: ________
Density: ________                  Density: ________

Increase the temperature of the object on the LEFT SIDE and copy the data for it below:

(LEFT SIDE)
Temperature: ________
Total Weight: ________
Volume: ________
Density: ________

Did the WEIGHT change? ( yes  no )  Is it greater or less? ________
Did the SIZE change? ( yes  no )  Is it greater or less? ________
Did the DENSITY change? ( yes  no )  Is it greater or less? ________
Did the kind of MATERIAL change? ( yes  no ) What kind of material is it? ________

2. BUILD an object that has this data:

    Temperature: 20 Tu
    Weight: 5 Wu
    Volume: 1 Su
    Density: 5 Wu/Su

Draw a picture of the object below:
3. BUILD two objects that are the SAME TEMPERATURE, but one is DENSER than the other. Draw the two objects below:

Can you make the denser one LESS dense by cutting away or removing some of its material?

How do you know?

Can you make the denser one LESS dense by increasing its temperature?

How do you know?

4. Find the object (or objects) in the program with the LEAST density. What is the density?

What material is the object made of?

5. Find the object (or objects) in the program with the MOST density. What is the density?

What material is the object made of?
6. BUILD two objects that WEIGH the same at different temperatures.

Copy the objects below:

BONUS QUESTIONS:

7. BUILD the two objects that look like these:

```
| | |
```

```
• •
```

Are they the SAME SIZE? 

Why or why not? 

Check the data. Explain how the computer finds the VOLUME.

8. BUILD two objects that look like these:

```
•
```

```
• •
```

Do they have the SAME DENSITY? 

Why or why not? 

Check the data. Explain how the computer finds the DENSITY.
The models below are supposed to show what happens to a chunk of brass after it is heated.

The PURPOSE OF THE MODELS is to ACCURATELY SHOW what happens to the brass chunk's SIZE, WEIGHT, and DENSITY.

You are to decide whether the models shown below serve the purpose well.

Circle the models you think serve the purpose well.

(In models 1-5, a DOT stands for a weight unit.)
(In model 6, a DOT stands for a temperature unit)

6. Before After

(In model 7, the SHADING stands for density)

7. Before After

(In model 8, the SHADING stands for temperature)

8. Before After

(In model 9, the DARK COLOR means greater)

9. Before After
UNIT 3:

Average Density
UNIT 3: DESCRIPTION OF LESSONS

Overall objectives: experience with boats and objects with mixed materials, experimenting, gaining first hand knowledge, checking intuitions, discovering new questions; distinguishing between a material's density and an object's density; getting to some more formal definitions or principles concerning boats - two ways to describe the volume of a container: there is the volume of the container itself and the volume of the container plus the volume of what is contained (even if that is just air.); explore the meaning of concave, catching, enclosing; arriving at some understanding of how density is affected when materials are mixed; how to order objects according to their densities when they are made of various proportions of different materials; how much of a sinking material is needed to make a floater sink? how much of a floating material is needed to make a sinker float? moving toward the notion of averaging and not adding/subtracting densities.

LESSON 1:

I. On a pre-test, students are asked to make judgements about the density of some objects composed of a mixture of materials. These objects are represented in two different ways. For some tasks they are represented with the grid and dots model: an object is composed of building blocks that have two different numbers of dots/box. For other tasks the objects are represented with the shading intensity model - that is, they have differing proportions of lightly colored and darkly colored materials.

II. Students are now given a kit with some clay and odd pieces of styrofoam, wood, and ceramic. They are to create objects which they think will meet the challenge. The following day, they will test and revise their ideas.

LESSON 2:

I. Students try out their ideas by placing objects in pans of water. They note which ideas worked and which didn't and any new ideas that come up.

II. They should see that combining the clay with another material such as styrofoam or fashioning the clay into a boat or "bubble" will enable it to float.

III. Using the computer model as a reference, students are encouraged to describe ways of representing the mixture of materials and use this model along with their empirical knowledge to come up with a meaningful way to consider and talk about the overall (average) densities of the objects and the effect that changing the proportions of a mixture can have on the objects' densities.
LESSON 3:

I. Students use the second part of the Sink the Raft program along with a worksheet. This is a new version of the sinking and floating simulation used in S/F Lesson 6. This version allows them to explore the consequences of putting a "hole" or air bubble inside of solid objects and immersing these objects in liquid. Students use the visual model and the on-screen numerical data to find patterns about average density and sinking and floating behavior related to changing the ratio of material to "air".

II. Go over worksheets and hand out homework assignment. The assignment asks students to judge whether a range of objects made of mixed materials would sink or float, and to calculate and order them by their average densities. All the objects were represented using the grid and dots model.

LESSON 4:

I. Go over homework.

II. A model of a boat is proposed which begins by taking an object with a hole in it (as portrayed on the computer) and then moving this hole closer and closer to the top of the object until it causes a break in the object's surface.

III. During discussion, students figure out which of a series of boat-like objects drawn on the board will sink or float based on the average density of objects (or the relative amounts of material and air.) They then create their own sinking boats and floating boats made out of clay.
UNIT 3:

Worksheets
Here are three objects.

A
B
C

According to the computer model,

a) Which object has the greatest density? _____
   How do you know?

b) Which object has the greatest size? _____
   How do you know?

c) Which object has the greatest weight? _____
   How do you know?

Here is an object made of two different materials.

Its average density is...

a) less than 1    b) between 1 and 2    c) 3    d) 18

(circle your answer)
Here is a chunk of very dense material.

Here is a chunk of not so dense material.

The following objects were made by combining the two materials in different proportions as shown.

A
B
C
D

Do these objects have the SAME average density? (Circle the answer)

a) Yes, ALL of them do.

b) Yes, SOME of them do. (Which ones?__)

c) No, NONE of them do.

How do you know? Briefly explain your answer. ____________

If they do not all have the same average density which one or ones have the GREATEST average density? ____________
Here are models of six different objects.

Here is a model of a tub of liquid.

Predict whether each object would sink or float in the liquid. Circle the S for SINK or the F for FLOAT underneath each object.
Worksheet for Lesson 1

Write down at least 4 or 5 ideas for making the clay float. You can describe objects that you have made, or describe ideas that you would like to try out.

1) 

2) 

3) 

4) 

5) 

Use the back of the page if necessary.

Challenges: Can you make all the clay float? Can you make the clay float without using any other material?
NAME: ___________________________ DATE: _______________

Worksheet for Lesson 3

1) BLUE IS VERY DENSE MATERIAL. A CHUNK OF BLUE MATERIAL WILL SINK IN EVERY OTHER COLOR LIQUID.

CAN YOU FIND A WAY TO MAKE BLUE MATERIAL FLOAT BY USING THE COMPUTER PROGRAM?

DESCRIBE THE BLUE OBJECT OR OBJECTS WHICH FLOAT.
(Draw a picture, write down the data, describe the object and the liquid in which it is floating.)

WRITE SENTENCE OR TWO ABOUT HOW YOU COULD GET BLUE MATERIAL TO FLOAT. WHY DO THE OBJECTS FLOAT?

2) FIND THE BLUE OBJECT OR OBJECTS WITH THE GREATEST DENSITY.

What is the density?________
Does it sink or float in orange liquid?________

DESCRIBE. (What does it look like? How big is it? How much does it weigh? Are there more than one?)
FIND THE BLUE OBJECT OR OBJECTS WITH THE LEAST DENSITY.

What is the density? ________
Does it sink or float in orange liquid? ________

DESCRIBE. (What does it look like? How big is it? How much does it weigh? Are there more than one?)

3) CAN YOU MAKE AN ORANGE OBJECT FLOAT IN WHITE LIQUID?

WHY OR WHY NOT? (Describe or copy objects below)

4) STATE A GENERAL RULE ABOUT SINKING AND FLOATING. AN OBJECT WILL FLOAT IF ...
5) WE CANNOT GET A PURPLE OBJECT TO FLOAT IN GREEN LIQUID. Why can't we get it to float?

WHAT WOULD WE NEED TO DO IN ORDER TO GET A PURPLE OBJECT TO FLOAT IN GREEN LIQUID? If you could revise the computer program, what would you do?

6) EXTRA CHALLENGE: You have already found the densest and least dense blue objects. Experiment with the computer and work with all kinds of blue objects - different sizes, with and without holes. Find a way to order the objects according to their density. Draw a picture of the objects ordered by their density below.
(Lesson 3 homework)

Here are models of eight objects.

INSTRUCTIONS:

1) Decide if each object would sink or float in a tub of liquid that has a density of 3 wu/su.

Circle the S for SINK or the F for FLOAT under each object.
2) Order the objects according to their average density. Write their letter names here:

| densest object | | least dense object |

If any objects have the same average density, be sure to say that.

3) CHALLENGE: Compute the average density of each object. Write down the average density of each object on the line underneath it.

Use the space below for notes or calculations.
NAME: ____________________  DATE: __________

Part A

1. WRITE DOWN WHAT YOU THINK THE WORD "DENSITY" MEANS.

________________________________________________________________________

________________________________________________________________________

2. Here is a block of wood which is cut into two pieces.

   (a) (b) (c)

   IS THE DENSITY OF BLOCK "b" THE SAME AS THE DENSITY OF BLOCK "a"?

Yes       No

3. Here are some statements about density. Circle the statements you think are correct.

The density of a material may be changed by:

a. taking a small piece off.
b. heating it.
c. a chemical reaction with another material.
d. nothing.
Part B

1. Here are four objects which have the following sizes and weights:

<table>
<thead>
<tr>
<th></th>
<th>SIZE:</th>
<th>WEIGHT:</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>4 cube units</td>
<td>12 grams</td>
</tr>
<tr>
<td>B</td>
<td>6 cube units</td>
<td>12 grams</td>
</tr>
<tr>
<td>C</td>
<td>2 cube units</td>
<td>6 grams</td>
</tr>
<tr>
<td>D</td>
<td>2 cube units</td>
<td>8 grams</td>
</tr>
</tbody>
</table>

Think about whether any of these objects could be made of the same material. Circle the correct statements.

a. Objects A and B COULD be made of the same material because they are the SAME WEIGHT.

b. Objects C and D COULD be made of the same material because they are the SAME SIZE.

c. Objects A and C COULD be made of the same material because they have the SAME WEIGHT PER SIZE UNIT.

d. NONE of the objects above could be made of the same material.

2. WHAT IS THE DENSITY OF THE MATERIAL IN EACH OBJECT?

a. The density of the material in object A is: 

b. The density of the material in object B is: 

c. The density of the material in object C is: 

d. The density of the material in object D is: 

3. Here is an object made of two different materials.

Its average density is...

a) less than 1  b) between 1 and 2  c) 3  d) 18
Part C

Here is a chunk of very dense material.

Here is a chunk of not so dense material.

The following objects were made by combining the two materials in different proportions as shown.

1. Do these objects have the SAME average density? (Circle the answer)
   a) Yes, ALL of them do.
   b) Yes, SOME of them do. (Which ones? _____)
   c) No, NONE of them do.

2. How do you know? Briefly explain your answer. ________________

3. If they do not all have the same average density which one or ones have the GREATEST average density? ____________
Part D

Here are models of six different objects.

Here is a model of a tub of liquid.

1. Predict whether each object would sink or float in the liquid. Circle the S for SINK or the F for FLOAT underneath each object.

2. Briefly explain how you could predict if the object would sink or float.
Part E

1. A piece of iron weighs 20 kg. at room temperature.

When it is heated, it expands, but still weighs 20 kgs. What has happened to the density of the iron?

How do you know?
Part F

The following questions ask you to think about two different ways to represent density.

Here is a model of 3 objects. They are shaded. The darker the shading is, the denser the material.

1. Can you tell the weight of each object using this shading model?
   Why or why not?

Here is another model of the same three objects as they would appear on the computer.

2. Can you tell the weight of each object using this computer model?
   Why or why not?
Part G

Here is a shading model of an object and liquid.

1. Using this shading model, can you tell whether the object would sink or float? ________________
   Why or why not? ____________________________________________

2. If you think it floats, can you also tell how much of it would be submerged?
   Why or why not? ____________________________________________

Here is a computer model of an object and liquid.

3. Using this computer model, can you tell whether the object would sink or float? ________________
   Why or why not? ____________________________________________

4. If you think it floats, can you tell how much of it would be submerged?
   Why or why not? ____________________________________________
Part H

1. Do you think one type of model (the shading type or the computer type) is more helpful than the other? __________

   Why or why not? __________________________________________________________________________

   ______________________________________________________________________________________

2. In general, did you find working with models in class useful for learning about the difference between weight and density? ______

   Why or why not? __________________________________________________________________________

   ______________________________________________________________________________________
THE COMPUTER PROGRAMS

The computer programs we have developed provide an environment where children can manipulate different elements that play a role in the notion of density and its relation to the phenomena of sinking and floating and thermal expansion.

Sinking and floating are rich, interesting and puzzling phenomena. Because they are governed by a limited number of independent variables, it is possible to build a compelling microworld in which children can investigate and learn not only about the specific phenomena, but about scientific inquiry and experimentation as well.

Our thermal expansion model was intended to help students conceptualize and envision density within another phenomenological context. The thermal expansion programs highlight how density is affected by volume change when the weight remains constant.

We describe here the relevant concepts and variables, the graphic representations we chose for these concepts, the ways of interacting with these representations on the screen (the menu options), and the basic activities that the computer program can support.

The Physical Concepts and their Representation In the Computer Programs

In devising a computer simulation, many decisions must be made about what is relevant to represent, how information should be represented and the kind of accuracy which is desirable. In what follows, we discuss the particular choices we made as well as our rationale for such choices. We believe some of these choices and assumptions are important to discuss with students as well if they are to understand how models correspond to the real world.

In discussing size, weight, and density, we see that any two of these variable parameters can be thought of as independent. The two will then determine the third. In the real world, we may perceive and take weight and size measurements for objects and then deduce, infer, or calculate their density. These two extensive parameters of weight and size define, through a mathematical relation, the intensive quantity of density, which is the center and focus of our teaching effort. On the other hand, when creating objects or building from materials, we can use knowledge of the density of a material to figure out how much an object will weigh. An object's weight is a consequence of what material was chosen and how much of that material was used in fashioning the object. Furthermore, there are circumstances in the context of thermal expansion, by which density changes are effected by changes in the volume parameter only.

In later paragraphs we note which variables were chosen to appropriately function as independent or dependent in the computer programs. First we explain how each variable parameter is represented.
Appendix 3

Size

When the programs refer to size or when we speak of the size of an object in teaching, the pertinent physical parameter is volume. As a three-dimensional quantity (length to the third power), volume must be represented in a symbolic way on the two-dimensional computer screen. We decided against designing the program to show perspective and three-dimensionality because we wanted the model to depict only the information that is directly relevant to the phenomena or topic at hand.

In the model, a unit of volume is represented by a two-dimensional square. Hence, there is a simple relation between the volume of an object and the abstracted representation of its size by the number of square units on the screen. This representation of volume can be used for an object of any shape, so long as one bears in mind that it is a symbolic and not a pictorial representation of size. We are concerned with volume and not with shape. All shapes are reduced to their rectangular (or cuboid) volume equivalents.

In class we discuss the meaning of size units by having students portray eight 1 cc blocks on the screen. In reality, these blocks can be arranged in several ways, including a 2x2x2 cube. We explain that we designed the computer program to count blocks but not necessarily to show shape, since size and not shape is more relevant to the notion of density. (We may, however, change this or add more options to the program later.) In any case, the program can be useful even before the concept of volume is fully discussed in class.

Weight

The weight of an object is represented visually (in a quantitative and consistent way) by the total number of dots displayed within the object's perimeter. This is not an atomistic picture of the solid. It allows the concepts of weight and density to be well-defined without any atomistic theory of matter. This symbolic representation could, however, be interpreted later in atomistic terms. As we now interpret it, each dot represents one unit of weight. (Later we might interpret the number of dots in a cluster as being proportional to the number of nucleons.) The total weight of an object is thus represented by the total number of dots that represent its weight in some arbitrary weight units.

Density

Density is represented as the number of dots in each size unit. This visual representation helps connect the notions of increasing crowdedness with increasing density. Since, at the moment, all objects created in the model are homogeneous, the number of dots per size unit is constant for any given object, thus conveying the notion of density as an intensive property of kinds of materials.
Material Kind

The computer allows users to define material kind in two independent ways: by density shown as dots per size unit or by color. So far we have described the dots per size unit option. When choosing the color option, the representation of weight and density by dots and dots per size unit are not visible. The object is presented as a solid color within its perimeter. Each material is a different color so that materials are distinguished by color, rather than dots per size unit.

In this mode there is no visually accessible representation of the variables of weight and density, but the specificity of materials is emphasized through another local property, color. The user can switch easily from one mode of representation to the other.

Multiple Representational Features of the Programs

Four representational devices are used in the microworld: verbal, pictorial, conceptual, and numeric. All can appear simultaneously on the same screen. In order to manipulate programs, students have to use the appropriate scientific terms to articulate their desired changes. These changes are then reflected on the screen in three modes: the pictorial representation of the phenomena, similar to an animation of an event; the representation of relationships among the conceptual entities involved (such as a visual depiction of weight/size as dots/box); and the numerical representation for the quantitative aspects. To show the link between the visual displays and the values of the variables for each constructed object, the program allows the user to "collect" data about the size, weight, and density of any object displayed on the screen. When in the data mode, the data are displayed and updated as the user interacts with the program.

Process and Interaction

The microworld environment is divided into three main parts. Each part consists of two or more programs. The user can move from one to the other through a common menu at any time. Our programs are grouped to depict:

A) Weight, size and density characteristics of objects made of different materials;
B) Sinking and floating behavior of objects including objects of the same size, different sizes, depiction of the objects' levels of submergence in different liquids, and what would happen if a hole were put in the center of a given object;
C) Thermal expansion.

Part A: Building Objects

The first part is designed for manipulating the weight, size and density of objects. The shapes of objects are limited to rectangles.

Program 1: Modeling With Dots / Weight and Density

These programs are designed for building and manipulating up to three different objects in three separate windows on the screen. This can be done in the dots or color mode, with or without displaying the numerical data (see Figure 1, next page). As seen in Figure 1, the student
Figure 1: Screen from the Weight and Density program

Build an object LESS DENSE and HEAVIER.

Choose material: Green Purple White Orange Blue

Figure 2: Screen from the Game
can "Build" an object (in one of three windows) and "Change" its material and size. One defines the object's mode of presentation (dots or color) through the "View/Hide" command. The "Collect data" command allows the user to display numeric information about the three variables independently. The user can also "Exchange" objects between windows.

This program thus lets students explore the relationship among the three parameters and perform tasks that involve ordering, building, or modeling real life objects according to their different dimensions. From the user's point of view, weight emerges from density, since the user first selects a kind of material for an object and then determines the object's size. Thus, in these computer programs, the independent variables are density and size. These are the variables that can be selected and modified. The weight is determined by manipulating these two quantities.

"Modeling with Dots" and "Weight and Density" are actually two versions of the same program. The only distinction between them is found when asking for data. The "Dots" program gives data with the labels "dots", "size units", and "dots per size unit", while the other version gives data in terms of "weight", "size units" and "weight per size unit." Thus the Modeling with Dots program affords some flexibility in designing activities which can deal with intensive quantities other than density (e.g., number of beads in a cup, number of pennies in a pile).

Program 2: A Game

The game part of the computer program is designed to give students practice with the terms weight, size, and density and to train them to distinguish these by using the correct language. In the game, the computer randomly displays an object of a certain size, weight, and density and challenges the student to build another object while complying with certain restrictions. The constraints posed at random include: make a smaller object, make a smaller and heavier object and so on (see Figure 2). The student can pass to the next challenge after fulfilling each task requirement correctly. The answers are checked by the computer. A scoring system might be added to increase students' motivation.

Part B: Sinking and Floating

Because we wanted these programs to react to user input from the keyboard in a way that would truly simulate the behavior of real objects and liquids, the relevant principles were embedded into the program. In other words, the computer model is scientifically accurate; the mathematical rules that the computer uses in calculating and portraying experimental results are the same rules that govern the phenomena of sinking and floating. Thus the learner can become familiar with the underlying principles and abstract mathematics of the phenomena through interaction with their dynamic numerical and visual representations on the screen.

The sinking and floating simulation programs we devised deal only with objects in the solid or liquid state, and assume constant temperature. Under these conditions, only three variables--size, weight, and density--are relevant to the phenomena of sinking and floating.

Since our main concern is to facilitate understanding of the principles and rules involved in sinking and floating and not to build a tool for exploring every real-life possibility, we have limited
ourselves to a subclass of objects that are well-suited to our current purpose. For the time being, we have also limited the objects to rectangles in their screen appearance; these rectangles stand for three dimensional objects, with the unseen dimension held constant. The objects consist of homogeneous materials or homogeneous materials with an air bubble in the middle of the object where noted.

The sinking and floating program shows two-dimensional representations of solid objects as well as of liquid in a container. The assumption is that both the object and the container of liquid, in three dimensions, would extend back away from the screen to the same extent. Of course, in real life, the object and container could not be exactly equal in this respect. We chose to ignore this discrepancy, however, because we wanted to keep any and all measurable size (volume) quantities visible and to avoid having hidden liquid or container volume behind the object. When the user gets numerical data, it corresponds directly to what he or she sees in the visual representation. Furthermore, this numerical data remains consonant with abstract principles as well as with actual (physical) measurement using suitable containers.

Program 1: Archimedes

In this program the screen shows two distinct elements: an object of fixed size and a tub of liquid, also of fixed size. Students can perform "experiments" in which the object is immersed in the liquid (see Figure 3). This is a continuation of the first part of the program and enables the student to choose and manipulate several elements: (1) the object and the liquid in the container by changing the materials; (2) the modes of presentation; (3) data collection; and (4) when to perform "experiments".

The results of the experiments are shown visually on the screen. The object submerges to a depth that takes into account the relative densities of the material and the liquid. Liquid displacement follows accordingly. Numerical information about the level of submergence is also available.

The experiments can be done with both the object and the liquid represented in solid colors or in the dots mode. Once an object is immersed in liquid, however, it is represented as a solid color. This is to ensure a clear distinction between object and liquid borders. Even though the object is seen as a solid color, the "View" command enables the user to view the dot distribution in a small subsection of the object (see Figure 4).

Additionally, once an object is submerged, the rise in the level of the liquid is portrayed in a solid color. Since, in most cases, the increase in liquid level will not be an integer number of units, we felt it best not to complicate the screen display with partial or "open" squares (size units).

"Archimedes" is designed to enable students to explore the role of an object's and a liquid's densities in defining the outcome of a sinking and floating experiment. The approach we adopted was to keep the size parameter constant, thus concentrating student attention on the density parameter only.
Data collection, View/Hide structure, Change materials, Experiment

Figure 3: Screen from the Archimedes program

View, Scale, Object out

Figure 4: Screen from the Archimedes program
Program 2: Sink the Raft

Part 1. In this part of the program, students can repeat the experiments afforded by the Archimedes program with one additional option. They can, in addition to all the other actions, change the size of the submerged objects and observe the effect such changes have on the outcome of the the sink-float experiments (see Figure 5). The data are continually updated, indicating the size, weight, density, and the portion of the object submerged as the user experiments. All objects and liquids in this section are portrayed in solid color. If the user wishes, a small section of the object or liquid may be viewed in the grid and dots type of representation.

Part 2. We now lift a restriction from the system to allow students to explore increasingly complex situations. We take the first step toward lifting the restriction on the homogeneity of the objects. The student can form an empty space in the middle of the object when she or he chooses to do so (see Figure 6).

At this stage of development, the size of the empty space is constant but we plan to add an option that will let the student control the size of the hole within the object's limits. When this empty space is created the density of the object changes. Accordingly, its behavior in the liquid can be observed. By choosing the right proportion of object size (this is within the student control) and the size of the hole the student can create a situation when an object made of a certain material can float in a liquid made of less dense material. That is because the average density of the system object plus empty space is below the density of that liquid. The program shows in the data window its average density and the student can follow the changes in these numbers when this proportion (between object size and space size) changes. This program therefore can be used as an introduction to the concept of average density and as a model by which one can explain how boats made of steel can float in water. The basic model behind this simulation is identical to that behind Archimedes and sink the raft, except that one restriction, i.e. the homogeneity of the object is relieved.

Part C: Thermal Expansion and Considerations Regarding Models

In this part of the microworld concerned with thermal expansion, we have lifted the restriction of constant temperature for objects built by the user. In the thermal expansion programs the screen is split into two windows. The student can build two objects similarly to the way this is done in the Weight and Density program, with an extra option. That is, the user can change the temperature of the objects and observe any changes that consequently result in the weight, the density, and the size of the objects.

In order to accommodate the new phenomena, we had to change our basic model somewhat. We will now elaborate some of the considerations we had when choosing the new model. Eventually the decision about which of several models to choose is arbitrary and the decision reflects a compromise between the need to be accurate in regard to the scientific explanation and the need to keep the model simple and in keeping with children's abilities. As we will see there is not one unique correct way to model this or any other phenomena. In order to convey this message of the possible existence of several models for the same phenomena - which is an important message for science education - we have decided to develop two representations.
2/3 OF OBJECT SUBMERGED

DATA VIEW Object-out Change

Press space to continue.

Figure 5: Screens from Sink the Raft, part 1
### Object Information

<table>
<thead>
<tr>
<th>Property</th>
<th>Value 1</th>
<th>Value 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height</td>
<td>48 µm</td>
<td>81 µm</td>
</tr>
<tr>
<td>Size</td>
<td>10 µm</td>
<td>27 µm</td>
</tr>
<tr>
<td>Density</td>
<td>4 µm/µm</td>
<td>3 µm/µm</td>
</tr>
</tbody>
</table>

### Liquid Information

<table>
<thead>
<tr>
<th>Property</th>
<th>Value 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height</td>
<td>28 µm</td>
</tr>
<tr>
<td>Size</td>
<td>10 µm</td>
</tr>
<tr>
<td>Density</td>
<td>2.8 µm/µm</td>
</tr>
</tbody>
</table>

---

**Figure 6: Screens from Sink the Raft, part 2**

Press space to continue
based on two different models for the thermal expansion phenomena. The student can thus build objects and change them, change the temperature, and change the model by which the objects and the changes are represented.

The equation we use for thermal expansion is: \( dV = V_0 \cdot dT \cdot C_v \) where \( dV \) is the change in the object volume, \( V_0 \) is the original volume, \( dT \) is the temperature change and \( C_v \) is the coefficient for volume expansion for that material.

The main problem was how to represent this change of size without introducing the concept of volume quantitatively. In the physical world, when an object expands each of its "size units" expands proportionally and therefore a natural choice is to represent the expanded object made of bigger squares. In this case, the number of squares is conserved and so are the number of dots (weight units) per square. One can see in a natural way that the density changes. The same weight is in a bigger box or square. This change in density can be seen easily on the screen because the dots are spread out in a larger area. That is, they are distributed farther apart in each square and thus throughout the total object (see Figure 7).

There is one drawback to this representation. That is, the square loses its significance as a symbol for a standard unit size. The number of squares, which previously gave the object's absolute size, has remained constant. The student can see that the object gets bigger or smaller and that its density changes. However, he or she cannot find the new size and density from this conceptual model in the clearly quantitative terms that were previously available.

Since the main point of this part of the programs was to show the changes in density with temperature in a qualitative way, we felt that it was justified to give up this quantitative option and to use this model as it is. The only way to overcome this problem was to superimpose on the expanded object a grid made of "unit squares" composed of the original size squares. This new grid cannot be used without a comprehensive discussion about the way to measure size, i.e. length, area, and volume. We wanted to avoid this in this stage, but it is perhaps, a direction that should be taken in the future since its need arises naturally from the unit. The introduction of the "super-grid" will also cause another problem. That is, the representation of the density will no longer be the number of dots per unit square.

In order to facilitate discussion about the quantitative changes that occur when the temperature changes we had two more versions of the thermal expansion program. In one version, one can see in the data, the new calculated size (which is calculated correctly as the volume expanded) and the new density. In another version, the changes in size of the units with temperature are designed so that in each step the square side multiplies. This way we get squares that represent 1, 8, and 27 units of volume. Our assumption is that the square expands in all dimensions as it is in reality including the dimension that goes into the screen. The round number for sizes and density that one gets in these cases make it possible to discuss more clearly the quantitative aspects of thermal expansion and their influences on the density.

Until now the size of the square that represents a unit size was kept constant. The number of dots per unit square represent the density and one can learn about the size of the object by
Figure 7: Screen from the Thermal Expansion program

Change size by:
Temperature

Figure 8: Screen from the Thermal Expansion program

Change Material kind Representation Size
counting the number of squares from which the object is constructed. When the temperature changes then physically the size of the object changes linearly in each dimension in proportion to the temperature change and a given coefficient specific to the material.

As a first approximation we ignored the differences in the coefficient $C_v$ for different materials and made all the objects expand and contract with the temperature by the same rate. (We had another version of the program in which each material kind has its own coefficient but this version was not actually used.)

In addition to that we developed another model in which there is a distinction between the material kind and the box. In each box there is a circle with dots. The number of dots per circle represents different material kinds and densities. When the object expands the circle does not change but the distances between each two circles gets larger (see Figure 8).

In one version of this program there are "springs" between the circles. This model is isomorphic to the other and students can switch back and forth between the two. In a way this model is a pseudo-representation for the atomic theory of matter that includes not only information about thermal expansion as a phenomena but also about the mechanism by which this expansion happened. This version can be used later as a bridge to other thermal phenomena, but this is beyond the scope of our current project.

The idea of modeling and representations should be introduced to those involved in any attempt to use computer simulations as tools in science. The model's assumptions (what is relevant, how to best to represent, accuracy and compatibility with real phenomena) should be discussed and made explicit as well. Even if many of these ideas are not discussed with students, the model builder and the teacher should be aware of them. Clarity about the assumptions built into the model gives users the possibility of modifying or giving up some features as needed or desired.