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ABSTRACT

Density is the first intensive physical quantity students encounter that can be understood in terms of an underlying model, the particulate theory of matter. Learning about density provides students with explanations for a range of phenomena such as sinking and floating, and changes of state. Teachers report, however, that density is a difficult concept for students to grasp. Researchers conducted pilot studies to determine whether students can understand a visual analog of density presented in computer graphics more easily than they can understand the concept of density inferred from manipulation of real world materials. Second-, fourth-, and sixth-grade students received two sets of parallel tasks: one involved manipulation or real materials and the other involved shapes presented in a computer display. Findings indicate that experience with computer models can help students to think about the difference between steel and aluminum cylinders as an intensive one--that is, stemming from the kind, not the amount of the material. Younger children, however, need help to see the computer analog as a "model" of density. (CW)

WEIGHT, DENSITY AND MATTER

A STUDY OF ELEMENTARY CHILDREN'S REASONING ABOUT DENSITY WITH CONCRETE MATERIALS AND COMPUTER ANALOGS

Technical Report

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A STUDY OF ELEMENTARY SCHOOL CHILDREN'S REASONING ABOUT

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Introduction

The concept of the density of a material has an important role in elementary and secondary school science curricula. Students are taught about a variety of phenomena which require some understanding of density to explain: for example, how various materials differ from each other, what happens to materials when they change state, why objects made of certain materials are heavier for their size than objects made of other materials, why some objects sink in liquids while others float. Further, density is the first intensive physical quantity students encounter that can be understood in terms of an underlying model, the particulate theory of matter. This theory holds that matter is composed of a finite number of discrete and uniform "bits", each of which weighs something; that the weight of an object is a function of the number of bits; and that the density of the object is a function of how closely packed the bits are. This model is a major theoretical achievement--built on both observable and unobservable properties and entities. Teaching students about density, thus, provides them with explanations for a range of phenomena as well as an opportunity to develop their understanding of an intensive quantity and to engage in real theory construction.

Teachers have reported, however, that density is a difficult concept for students to grasp. Our project explores why this should be and whether there are some simpler, more accessible notions which can serve as the basis for building a concept of density in students' minds.

One reason that the notion of the density of materials may be difficult for students to understand is that density is an intensive quantity. Intensive quantities are quantities that generally have a ratio structure: weight/unit volume; number of candies/bag; number of children/classroom. Students may simply have difficulty representing quantities with a ratio structure. In support of this hypothesis, Quintero found that students through the fifth grade had trouble giving visual depictions of simple intensive quantities, such as "14 candies per bag" or "20 children in each classroom." (Quintero, 1980). Additionally, students may have difficulty with density because it is an intensive quantity which they cannot directly perceive. Although they can perceive an object's weight, size, and the material it is made of, the density of the material is a quantity which must be inferred from knowledge of an object's weight and volume. Limitations in students' conceptions of matter, material kinds, and weight may also prevent them from constructing mental models in which the densities of different materials are directly represented. For example, if students think of matter as fundamentally continuous rather than particulate, they cannot represent the density differences of materials in terms of the crowdedness of underlying particles. Other intensive quantities, such as the crowdedness of an array of dots, may be easier for them to grasp because they can directly perceive dot crowdedness. In support of this hypothesis, Quintero found that highly visualizable intensive quantities were understood better by elementary school children than ones that were less visualizable. If this is true, then elementary school children may be helped to understand density better by providing them with an appropriate visual model.

Our work this year lays the groundwork for designing ways to teach students about density. The main goal of our studies has been to determine whether students can understand a visual analog of density (depicted in computer displays) more easily than the notion of the density of materials (inferred from manipulating real materials). If they can, then the visual analog can provide a base on which to build a notion of density. Our strategy has been to present children with two identically structured sets of problems: one with real world objects and the other with computer arrays. To date, we have completed two studies, both of which compare children's abilities with the computer displays and real world objects. The studies differed, however, in the exact way the intensive quantity was visually portrayed in the computer displays and in the way children's reasoning about intensive quantities was probed.

In our first pilot study (Smith, 1984, the first Technical Report of this group), we worked with students from grades 1, 2, and 3. The density task involved presenting them with pieces of steel and aluminum of varying sizes and weights. Students were shown that a piece of steel was heavier than a piece of aluminum the same size, and that a large piece of aluminum equaled a small piece of steel in weight, so that they could infer that steel was denser than aluminum. They were then asked to predict whether two objects could weigh the same, drawing on their knowledge of the relative sizes of the objects and the materials they were made of. Students were given a variety of problems, the most critical of which involved presenting the child with a piece of aluminum which was three times larger than a piece of steel. This latter type of problem was included to determine if they interpreted "heavier" in the generalization "steel is heavier than aluminum" as <u>absolutely heavier</u> or as <u>denser</u>. If students interpret heavier as absolutely heavier, they should predict that the steel would be heavier on these problems. If, however, they correctly interpreted it as denser, they should realize that extensive differences in the objects can compensate for intensive differences and that the two objects could weigh the same.

The computer problems in the first pilot study were similar in structure to the problems with steel and aluminum. The computer display presented two shapes--one outlined in purple, and the other in green--of varying sizes . Each shape was filled with dots uniformly spaced. The dot density in the green shape was, however, four times the dot density in the purple shape. Again, students were given preliminary experience with the shapes to learn that the green shapes were more crowded with dots than the purple shapes. They were then asked to predict from knowledge of the outline color of a shape and the relative sizes of the shape, whether the shapes would have the same number of dots in them when they were filled in.

We hypothesized that children would be able to describe and reason about dot crowdedness better than the density of materials because it was a directly perceptible intensive quantity. Our results only partially supported our hypothesis. We found that children did use more precise language for talking about the computer displays than the real objects (i.e., children referred to an intensive quantity--crowdedness, number of dots per row--in describing the differences between purple and green shapes; in contrast, children used the ambiguous word "heavier" in describing the differences between steel and aluminum). However, despite the superiority of their language for talking about the intensive quantity in the computer case, children at all ages actually did better making predictions about the weights of the steel and aluminum objects than they did making predictions about the number of dots in the purple and green shapes.

There were at least two possible explanations for the greater difficulty of the computer problems. First, one could argue that the two tasks vary in familiarity. Children may have had previous experience with steel and aluminum objects, but not with the purple and green computer shapes. If we had given children more time to familiarize themselves with the computer materials, their performance might have dramatically improved. Second, one could argue that the children used different strategies on the two tasks. In our previous technical report, this is the hypothesis we favored. In particular, we argued that children approached computer problems searching for an exact numeric solution. Their justifications indicated that they were asking themselves how many times bigger a purple shape had to be to have the same number of dots as a green shape. The problem was that they were coming up with the wrong number: they expected the shapes to have an equal number of dots when the purple shape was two times bigger rather than four times bigger. Consequently, when they were presented with a purple shape that was four times bigger than a green shape, they said that the purple shape would have more dots. This was scored as an error, but in fact seemed to result from a sophisticated line of reasoning, a measurement strategy whose only error was an inaccurate estimation of the difference in dot crowdedness of the two shapes. If children had been given a wider range of problems (in which the purple shape was two, three, and four times bigger than the green shape), they might have shown the sophisticated pattern of judging that the shapes would have the same number of dots when the shapes were in a 2 to 1 ratio, and then judging that the purple shape would have more dots for larger ratios.

In contrast, children did not seem to take a numerical approach in reasoning about the steel and aluminum objects. They noted that steel was much heavier than aluminum, but did not attempt to say how much heavier. When they were presented with a piece of aluminum three times larger than a piece of steel, they said that the two could be equal in weight. However, judging from their justifications, they did not seem to have come to the conclusion that steel was always three times denser. They seemed only to reason: the aluminum is much larger, but the steel is much heavier, so they could be equal. They might also have judged that pieces of aluminum two or four times bigger than steel could also be equal to the steel in weight. If this interpretation of their responses is correct, then children were using a more quantitative strategy with the computer problems than the steel and aluminum ones.

In our second pilot study, we modified the reasoning task we gave to the children so that we would be better able to infer the strategy children used on the computer and real world objects problems from their patterns of judgment. In particular, we now gave children a wider range of problems: problems where the large object was so much larger that it was heavier/had more dots in spite of its being less dense, problems where the two quantities exactly compensate, and problems where the larger object was not enough larger for the two quantities to compensate exactly (see Table 1). Children were exposed to all types of compensation problems in the initial exploration period on each task and were asked to find the pairs that had the same weight/number of dots. Thus, we restructured the procedures in ways that would encourage them to think about the intensive differences more quantitatively on both tasks. We predicted that with this new version of the procedures, children would be more successful with the computer displays than the steel and aluminum pieces.

In addition, we decided to develop another computer model that would overcome the limitations of the one used in the first pilot study. In the original computer model it was hard to correctly quantify the difference in dot densities of the two shapes. Each dot should be conceived to be at the center of an imaginary box, where the area of the box surrounding a purple dot was four times the area surrounding a green dot. However, if children conceived of an area bounded by the dots themselves, and then counted the number of dots in each area, they would not find a 4 to 1 ratio between the dot densities of the green and purple shapes (see Figure 1a). In this case, they get a 2.5 to 1 ratio. This is because they have not really identified comparable areas. When they consider the dots as being in the center of an imaginary box, they find that the four purple dots are included in a larger area than the nine green dots (see Figure 1b). Thus, they need to add imaginary boxes to the green shape to make the areas truly equivalent, and then they would get a 4 to 1 ratio (see Figure 1c). This may, in fact, be one reason so many of them extracted the wrong numerical rule. Further, our first computer display provided an incorrect model of the density differences of materials at an atomic level.[1] It is not the case that denser materials have more closely packed atoms than less dense materials. Rather, a denser substance has heavier atoms than a less dense substance (i.e., more mass in the nucleus).

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[1]We are indebted to Sylvia Shafto for reminding us of this problem with our model.

In our new computer model, there are three variables: number of dots/bunch (some have 3 dots/bunch; others 1 dot/bunch); total number of bunches in a shape (bunches are equally spaced, independent of bunch size), and total number of dots in a shape (see Figure 2). In this model, all three quantities including the intensive quantity are easy to quantify (3 dots/bunch ... 1 dot/bunch). Further, the model provides a more correct representation of the density differences of materials at the atomic level (bunches correspond to the nuclei of individual atoms which vary in total number of particles). Indeed this is a more powerful model than our earlier one, since intrinsic differences in material kinds are represented separately from the overall spacing of bunches. Thus, we believe this model can potentially help children understand a wider range of problems involving density.

The main purpose of our second pilot study was to test the hypothesis that children would be more successful reasoning quantitatively with the intensive quantity in our new computer model (#dots/bunch) than with the intensive quantity of density of materials. We hypothesized that because the intensive quantities in the computer display were directly perceptible, children would be more likely to attempt to quantify them and would frequently extract the correct numeric rule. In contrast, because they could not directly perceive the densities of steel and aluminum, they should make fewer attempts at explicit quantification. Further, without such a visual referent, they should have more trouble using numeric information to extract a correct rule about the ratio of the densities. That is, when they are shown that a steel object weighs 3 units and a same size aluminum object weighs 1 unit, they should still have difficulty concluding that steel is three times denser than aluminum. If these predictions are confirmed, it suggests one way in which having a visual model of an intensive quantity might aid in reasoning about density.

There were three additional purposes served by the second pilot study. First, we wanted to gather more information about how children spontaneously conceptualized weight and density. Do they think that all matter has weight? What is their concept of the internal structure of materials-- what kinds of pictures do they draw (if any) when asked to explain why steel is heavier than aluminum? Second, we wanted to sample children across a broader age range (grade 2 through 6) than we did in pilot study 1 and we wanted to include children of more diverse ability levels. Finally, we wanted to probe children's ability to see analogies between the computer model and real world objects.

Methods

Subjects

The study was conducted in a Watertown, Mass. elementary school. The school philosophy might be described as goal-oriented and progressive: the pupils were encouraged to take on responsibilities (e.g. school newspaper) and were actively involved in diverse curriculum-enrichment activities and events. Children of all grade levels also participated in a computer class. Most of the attending population seemed to be from a middle socio-economic background.

Students from grades 2,4 and 6 were selected by their respective teachers to represent a range of ability levels. Teachers were instructed to form pairs of students of equal general ability; given this constraint, they were also encouraged to include an equal number of boys and girls. The distribution of 32 subjects by grade, sex and ability level is shown in Table 2. The subgroups in Grades 4 and 6 contain only 10 pupils because 4 subjects served to pilot the testing procedures and their protocols were not included in the data analysis.

The science curriculum in this school was based on the textbook series Accent on Science (Charles C. Merrill, 1983). The grade 2 children had not been exposed in any formal way to the concepts addressed by the study. The grade 4 children had been introduced to the particulate model of matter and phase changes in the previous year ("Matter and its Changes"), but had not yet had the fourth grade unit that dealt with the densities of different rocks. The grade 6 students had received the most instruction in these concepts. In their previous unit in Grade 5 ("Classifying Matter"), density was introduced as an object's mass divided by its volume with illustrations of same-sized containers filled with different materials; in this unit the particulate model of matter was used to explain phase changes in a given material, but was not directly linked to density differences of different materials. The grade 6 unit ("Interactions of Matter") gave more details on the atomic structure of matter, the classification of elements and the combination of atoms.

Overall Design

Subjects were given three tasks in three separate sessions. The steel/aluminum and computer model tasks were parallel in structure and were given in counter-balanced order. Half of the children were given the steel/aluminum problems in the first session and the computer model problems in the second session. The other half of the children (matched for grade and ability level) were given the computer model problems in the first session and the steel/aluminum problems in the second session. There was a minimum of one day between sessions 1 and 2. In both the steel/aluminum and computer model tasks, students were given preliminary experiences to allow them to infer that steel is denser than aluminum and to learn that green shapes have more dots per cluster than red shapes. They were then given 12 test problems in which they had to predict whether two objects could weigh the same based on knowledge of the relative sizes of the objects and the material each was made of and to predict whether two shapes could have the same number of dots based on knowledge of the relative sizes of the shapes and the outline color. The playdough and probe task was the third task which always was presented in the last session. This task explored children's beliefs about weight and density and probed their ability to see an analogy between the computer model and the steel/aluminum tasks.

Stimuli

Steel and aluminum pieces

A set of steel and aluminum cylinders of varying lengths were designed for the study. The pieces were 1.5 inches in diameter for ease of handling by the child. The steel used was approximately 2.9 times heavier than the aluminum. Even though steel is darker, the two metals were also distinguished by a color sticker on top of each piece (blue for steel and yellow for aluminum) to facilitate proper identification by subjects. Each piece was also identified by a letter on the bottom for the experimenter's convenience.

The purpose of the preliminary problems was to allow children to infer that steel is denser than aluminum. Three steel pieces (2, 3, and 5 inches in length) and three aluminum pieces (2, 5.8, and 8.7 inches in length) were used. These pieces could be arranged in different pairs: for example, one same size pair made of different materials, two pairs in which the larger aluminum weighs the same as the smaller steel, one pair in which the larger aluminum is heavier than the smaller steel, and one pair in which the larger aluminum is lighter than the smaller steel. Since students were allowed to weigh and lift the objects during this phase of the task, different cylinders were used in the test phase to insure that they were not answering based on the remembered weights of specific items. Four new steel pieces (1, 2.5, 4, and 6 inches in length) and six new aluminum pieces (1, 2.9, 4 , 5 , 7 , and 11.6 inches in length) were used to construct the twelve pairs of objects used in the test phase. For each pair, the subject was asked to predict if the two pieces could weigh the same and, if not, which piece would be heavier. There were six types of pairs with two instances of each type. Table 1 lists the six pair types, along with the the exact dimensions of the instances for each type.

Computer model

Computer analogs of the steel and aluminum pieces were programmed on an IBM microcomputer. These consisted of rectangular shapes that could be called up on the screen as empty outlines or filled with dots (see Figure 2). The matrix consisted of 4 equally spaced points across the width of the rectangle. The red shape stood for the aluminum piece and was filled in with one dot at each matrix point whereas the green shape stood for the steel piece and was filled in with a bunch of 3 dots disposed in a triangle at each matrix point. Thus a green shape of equal size contains three times as many dots as a red shape just as a steel piece of equal size weighs just about three times as much as the aluminum piece.

During the preliminary phase of the task, students were allowed to call up pairs of filled-in shapes. When a filled-in shape was called for, an empty shape first appeared on the screen and the rows of dots successively filled in at a fairly rapid rate. This allowed children to check which shape had more dots either by visual inspection or by counting. Because feedback was given during the preliminary phase, different size shapes were used during the test phase. In the test phase, children were only presented with shapes in outline form to test their ability to make predictions about whether two shapes could have the same number of dots. The number of rows in the computer shapes could be varied to match the different heights of the respective metal pieces on a scale of 1 row per inch. In this way, computer pairs were constructed to be exactly analogous to the pairs used in the steel and aluminum task (see Table 1 again for description of exact dimensions of the different pair types).

Other materials

A postage scale was used in the steel and aluminum task. Numbered stickers were placed on the pound intervals to facilitate reading of weights. This scale also served as a back-up instrument in the conservation task when the child did not acknowledge the weight of the material used. In the computer task, a reminder card (showing a red shape with single dots and a same size green shape with bunches of 3 dots) was placed near the screen. During the exploration period, a card showing coloured outlines of the shapes that were available and an instruction card were posted. Another card with two empty outlines of equal size, one red and one green, was also used as a final probe at the end of the preliminaries. Commercial colorless playdough was used in the conservation task. Finally, white sheets of paper and a set of 8 colored markers were used in the drawing task.

Procedures

The subjects were seen individually by two research assistants. One assistant presented the various tasks to the children. The other assistant recorded their responses and noted some of their reactions.(The complete interview protocols are contained in the Appendix).

Steel and aluminum task

Preliminaries. The preliminaries introduced children to the tasks so that they became familiar with the materials and worked out a solution to the problems at least on a trial and error basis. This part of the procedure involved several steps: three problem presentations were followed by an exploration period and then by further probing.

After some greetings and introductory remarks ("I'll be showing you some metal pieces I want you to look at carefully"), the first pair (■□) was presented to the children. They were invited to handle the pieces and to make a prediction about their relative weights ("Do these pieces weigh the same?"); they were then allowed to check their response on the scale. Once the children acknowledged that the steel piece was heavier, the examiner probed for an explanation: "These pieces are the same size but they don't weigh the same. How can that be?" Children were then told that the pieces were made of steel and aluminum and it was explained how they could be visually distinguished. Subjects' previous knowledge of these metals was also checked. Two more problems (□■; ■■) were presented. Children were first asked to make a prediction and then were allowed to check their prediction.Their reasons for their predictions were also explored.

Next, all the preliminary pieces were placed in front of the child. Children were told that they could explore these objects as they liked (e.g., compare the weights of different pairs),and that afterwards, they would be shown new pieces of steel and aluminum that they would not be allowed to touch. Once children had exhausted their curiosity, they were asked to find the steel and aluminum pairs which were equivalent in weight (there were two such pairs). Children were allowed several trials and were helped if necessary. Once the pairs had been located, children were asked to explain how they had identified such pairs. Finally, a 2 inch piece of steel and a 2 inch piece of aluminum were placed on a scale. Children saw that the steel weighed three units and the aluminum weighed one unit. They were then asked directly how much heavier the steel object was than the aluminum object.

<u>Test phase</u>. From this point on, children were not allowed to handle or weigh the pieces but had to make predictions about their relative weights by observation of the two pieces placed in front of them. "Here is a piece of steel and a piece of aluminum. Could these pieces weigh the same? If not, which one is heavier?" The task included 12 problems, two of each of the six types previously described. On four problems, three of which were compensation ones, children were asked to explain their prediction.

<u>The Computer Model Task</u>

<u>Preliminaries</u>. The computer preliminaries served the same purpose as in the previous task and closely followed the sequence of steps described above. The first pair (■ □) was presented in filled-in form. Children were asked which shape had more dots and helped to notice that the green one had more, following which they were probed for an explanation: "The shapes are the same size but they don't have the same number of dots. How can that be?" It was then pointed out that the green one had more in a bunch.

For the next two problems (□ ■ ; ■ □), children were first invited to make predictions about the relative number of dots from the empty outlines and queried for their reasons. After the probes, the shapes were filled in with dots so that children could count them to check their answers. Children were also shown the model card beside the screen as a reminder that the red shape was always filled in with one dot per bunch and the green one with three dots per bunch.

Next, children were told they could play with some of the shapes (a card indicated which ones) and were instructed how to put them up on the screen. Once they had satisfied their curiosity, they were shown a card with outlines of different size red and green shapes and were asked to point to the pairs with equal numbers of dots. Children then checked their predictions by calling up these shapes on the computer until they found the two pairs which had equal numbers of dots. Finally, children were shown a 2 row red shape (with 8 dots) and a 2 row green shape (with 24 dots) and were asked how many more dots were in the green shape than the red shape.

The Test Phase. The corresponding computer analogs were presented in the same order and in the same manner as in the steel and aluminum task. Empty outlines were called up on the screen and subjects were asked: "Here is a red shape and a green shape. Could these shapes have the same number of dots? If not, which one has more?" The reminder card was posted during the task but children were not allowed to check their answers. Children were asked for justifications on four of the 12 problems, three of which were compensation problems.

The Playdough and Probe Tasks

The Conservation Task. A medium-sized piece of playdough was brought out. Children were asked if it weighed anything and then asked if it still weighed something when it was not held. Children helped the experimenter flatten the ball into a pancake shape and were asked if it still weighed the same when flattened as it did before and whether it still had the same amount of stuff in it. The ball was reshaped and broken into little pieces; children were asked whether the pieces together weighed the same as the ball. A tiny piece was then broken off and children were asked whether this piece would weigh anything, even if only a little bit. For all questions, children were asked to explain their judgement.

The Probe Drawing Task. A 4 inch piece of steel and a 4 inch piece of aluminum were brought out and children were asked: "What is it about the steel that makes it heavier?" Children were then invited to imagine the tiniest piece of steel and aluminum and to draw what they might look like inside. The drawing was then discussed with the subject for clarification.

The Analogy Probe. The reminder card showing a red shape filled with single dots and a green shape filled with bunches of dots was brought out and children were progressively queried about the analogy between the steel and aluminum pieces and the computer shapes. Do the shapes remind them of anything? Do they see any connections between the two tasks? Could the steel and aluminum pieces look like the display of dots inside and if so, which shapes would match with the respective metals? The plausibility of this analogy was checked by asking children if the dot display helped them understand why steel is a heavier kind of material than aluminum. A brief wrap-up explained the purpose of the study to the children and tested their acceptance of the particulate model: "Some people say that steel and aluminum pieces are made up of small bits of stuff like the dots we saw on the computer. Heavier kinds of stuff have more bits in a bunch but they have the same spaces between bunches. Does that seem like a good explanation of why steel is a heavier kind of stuff than aluminum to you?"

Results and Discussion

Predictions task: steel and aluminum pieces vs. computer analogs

Three aspects of children's performance in these two tasks were analyzed and compared: (a) the language used in describing the differences between steel and aluminum and red and green shapes; (b) the pattern of error on the predictions problems in the two tasks and (c) the justifications of predictions in the two tasks.

Language used for describing the differences between steel and aluminum objects and red and green shapes.

In the preliminary problems of the predictions task, children were asked to explain why two objects (shapes) of the same size had different weights (number of dots) and why two objects (shapes) of different size had the same weight (number of dots). In answering these questions children typically appealed to some differences between steel and aluminum objects (materials) and some differences between red and green shapes. Tables 3 and 4 show the main ways children of different ages talked about these differences in the two tasks.

In the computer model problems, there were two main ways that children talked about the differences between the red and green shapes: (a) the green shapes had dots that were closer together (or in bunches) while the red shapes had dots that were more spread out (or in singles); and (b) the green shapes had 3 per bunch (group, set, etc.) while the red shape had only 1 in a group (singles). In both cases children were referring to an intensive aspect of the arrays which could be quantified: the closeness of the dots, the number of dots per bunch. In the latter case, however, children explicitly acknowledged the ratio structure of the quantity (#

dots per bunch) and assigned a specific magnitude. Table 3 shows that grade 2, 4, and 6 children talked about the arrays quite similarly. At every grade, almost all the children characterized the difference between red and green shapes in terms of an intensive quantity (dot closeness, # dots per bunch). Further, the majority acknowledged the ratio structure of this quantity and assigned it a specific magnitude. The two children who say only "the green has 3 dots, the red has 1 dot" are probably trying to say the same thing, although they do not verbally mention the denominator (bunches). Only two of the youngest children do not make any reference to an intensive quantity. Instead, they say that the difference between the red and the green is that the green has triangles while the red has dots. Note, however, that even these children are focussing on the internal structure of the red and green shapes and describe an intensive difference.

In problems with concrete materials, there were four main ways that children talked about the differences between steel and aluminum: (a) steel is heavier than aluminum; (b) steel is stronger/fuller than aluminum; (c) steel is a heavier material than aluminum; and (d) steel is a denser material than aluminum. Table 4 shows that there are clear age changes in how children talk about the difference between steel and aluminum. Grade 2 children almost always talk about the steel as being heavier, stronger and/or fuller than the aluminum, whereas grade 4 and 6 children typically talk about steel as being a heavier, denser or more tightly packed material than aluminum (X^2=10, d.f.=2, p=<.01).

These results highlight two ways our language for talking about the density of materials is ambiguous: (1) the word "steel" can refer to objects made of steel or the material steel; and (2) the word "heavy" can refer to <u>absolutely heavy</u>, <u>heavy for size</u>, or <u>dense</u>. With increasing age, children have a less ambiguous, more precise way of talking about density: first specifying that they are referring to differences between steel and aluminum at the material level, and still later specifying that this is a difference in density and not simply weight. However, because of the ambiguity of the words they use, it is hard to know what most of the younger children are intending to refer to without further information. It is possible that children who say that "steel is a heavier material" or even that "steel is heavier" really mean to say that it is a denser material but do not yet have the language for expressing this idea. Children's actual pattern of predictions on the items where size and density vary inversely will bear on whether they distinguish weight and density.

At this point, however, we can conclude that there are at least two important differences in language used on the two tasks. Children have more precise language for talking about the intensive quantity in the computer model task; in contrast children must stretch their language for talking about the extensive quantity "weight" to talk about the intensive quantity "density" in the steel/aluminum task. Second, children are more aware of the ratio structure and the specific magnitude of the intensive quantity in the computer model task. No child talked of the steel objects as being heavier for their size in the steel/aluminum task, nor in the early stages ventured a guess as to how much denser the steel was.

<u>Patterns of judgements: steel and aluminum pieces vs computer analogs.</u>

Children made virtually no errors on the problems where size and the intensive quantity directly varied (■ □) or on the problems where one quantity was held constant while the other varied (□ □; ■ □) for either task. In particular, 28 of the 32 children had perfect patterns of judgement on these six problems in the computer model task and 29 of the 32 children had perfect patterns in the steel and aluminum task. Only one second grader made errors on these problems for both tasks: systematically judging on the basis of size in the computer model task and systematically judging on the basis of kind of stuff in the steel/aluminum task (she erred by judging that objects in the (□ ▭) pairs must have the same weight). The other five children made only one error on the six items for one task and made perfect predictions for the six items in the other. Thus, in general, children of these ages can simultaneously consider both quantities in making predictions on these simple problems.

Children made many more errors on the six compensation problems where size and density varied inversely. There could be two quite different reasons children made errors on these problems. First, these problems require that children correctly conceptualize density (dot crowdedness) as an intensive quantity and realize that extensive differences between two objects can compensate for intensive differences. If children interpret "steel is heavier" as "steel is absolutely heavier than aluminum", then the child should not realize that a large aluminum object can equal a small steel object in weight. Instead, whenever they consider the generalization about the heaviness of steel, they should judge that the small steel object will be heavier. Second, these problems require that children understand quantitatively how the two quantities compensate. To have a perfect pattern of judgment on these items, children must realize that the two quantities exactly compensate when the objects are in a 3 to 1 ratio. Careful examination of children's patterns of judgement on these six items should therefore provide clues as to the source of difficulty.

Children were categorized as showing one of four patterns on the compensation problems. Recall that compensation problems are those where the less dense piece is larger in size. There are three types of pairings in the compensation problems: (a) those where the larger member of the pair is heavier (has more dots); (b) those where both members are equal in weight (number of dots); and (c) those where the smaller member is heavier (has more dots). Children with perfect (or exact compensation) patterns correctly predicted the weights/# of dots on all six problems. Since there are two items of each type, a perfect pattern would be: LL/EE/SS. These children clearly realized that the two quantities exactly compensate only when in a 3 to 1 ratio.

A second group of children made some errors but had patterns which showed their responses varied systematically as a function of ratio differences. These patterns were called ratio sensitive compensation patterns. To be credited with this pattern, the subject's responses had to meet two criteria. At least two kinds of responses among the three that are possible had to be given and the ordering of the answers had to reflect a systematic direction (e.g., from picking the larger piece for the larger ratio differences to picking the smaller pieces as the ratio differences decreased). However, this transition could occur within a given ratio size (e.g., LL/(L)(S)/SS or LL/(L)E/(E)S) as well as between different ratio sizes (e.g., LL/(L)(L)/SS or LL/(L)(L)/(E)(E)).

A third group of children showed non-systematic compensation patterns. These children are credited with appreciating that size differences in objects (shapes) can compensate for intensive differences because they all made some "could be equal" in weight (number of dots) judgments on these items. Given that these children were perfect on the six noncompensation problems (and hence never made the judgment that "they could be equal" on any other type of item), it seems likely that these judgments reflect some understanding of compensation. However, these children did not systematically vary their responses as a function of ratio differences. Some of the children judged that the objects "could be equal" on at least five of the six compensation problems. Others used several different responses, but not in as systematic ways as the children with exact compensation and ratio sensitive compensation patterns. For example, they went back and forth between two different judgments for two different ratio sizes (e.g., L(E)/(L)E/(E)(E)) or made some clearly inappropriate judgments for the different ratio sizes (e.g., L(S)/(L)E/(E)S). Thus, their patterns provided less evidence that they knew how to adjust their response as a function of ratio differences.

The last group of children gave no evidence of realizing that size differences could compensate for intensive differences since they never said that the two objects "could be equal" in weight (# of dots) at any point. Instead, they showed one of three patterns: picking the larger object on at least five of the six items, picking the smaller object as heavier on at least five of the six items, or oscillating between these two judgments in apparently unprincipled fashion (e.g., L(S)/(L)(S)/(L)S). Consequently this pattern is called the <u>no compensation</u> pattern.

Table 5 presents children's pattern of response to compensation items for both tasks. There were two matched groups of children: those who had the steel/aluminum problems first and the computer displays second and those who had the reverse order of presentation. Results are presented separately for the two groups because order of presentation had dramatic effects for the steel and aluminum task. When children had the steel and aluminum problems first, only 5 out of 16 children showed patterns that were clearly sensitive to ratio differences with the metal pieces. In contrast, 14 out of 16 of children had exact compensation or ratio sensitive patterns on the steel and aluminum problems when they were presented after the computer problems (X^2=10.5, d.f.=1, p=<.01). Order of presentation did not affect performance on the computer problems: the majority of children had ratio sensitive or exact compensation patterns regardless of order (X^2=2.12, n.s.). For the children who had the steel and aluminum problems first, the computer problems were clearly easier: 13 out of 16 had more sophisticated patterns on the computer problems, while 3 had the same pattern on both. No child had a more sophisticated pattern on the steel and aluminum problems than on the computer problems (Sign test, $p < .01$). In contrast, there was no difference in difficulty in the two types of problems for the chidren who had the computer problems first: 9 out of 16 had the same pattern on both, 3 had more sophisticated patterns on the computer problems, and 4 had more sophisticated patterns on the steel and aluminum problems (Sign test, n.s.).

Table 6 shows children's pattern on the computer model problems as a function of grade. Children's performance on the computer problems at all ages was remarkably good. Only two children--both second graders--had patterns which indicated that they were not making compensation judgments. Significantly, these were the only two children who had not talked of the differences between red and green shapes in terms of an intensive quantity (they had said the greens had triangles, the reds dots, see Table 3). The majority of children at every age showed at least ratio sensitive patterns. Exact compensation patterns were common, however, only among the grade 4 and 6 children.

Table 7 shows children's patterns on the steel and aluminum problems as a function of grade and order of presentation. Children's performance at every grade is worse when one considers the group that had the steel and aluminum problems first. Indeed half the second graders (and one older child) had " no compensation" patterns--indicative of failure to understand the intensive quantity. All these children had talked simply of steel as being heavier in their spontaneous comments. Further, ratio sensitive patterns were rare among grade 2 and 4 children. Children's performance on the steel and aluminum problems was much better for the group which had these problems after the computer problems. In this case only 1 second grader showed a no compensation pattern and 14 of the 16 children attended to ratio differences in understanding the compensation. This suggests that experience with computer problems both helps the child to think about the differences between steel and aluminum as an intensive one and to think more precisely about the magnitudes of the intensive quantities.

Justifications of predictions

Analyses of children's justifications of compensation items provide further support for these conclusions and gives insight into the actual strategies children were using in solving the compensation problems. Justifications were found to fall into six mutually exclusive categories: (1) correct multiplicative rule (i.e., "the red needs to be three times the size of the green to be equal"); (2) incorrect multiplicative rule (e.g., "the steel is double the aluminum in weight; Al needs to be two times as big to weigh the same"); (3) direct estimation of the number of dots in both shapes either by counting (imaginary dots) or multiplying (imaginary) rows and columns (used only for computer problems); (4) comparison to a remembered standard (e.g., "this is smaller than the one before; it would have to be bigger to be equal"); (5) consideration of a trade-off between the two quantities (e.g., "even though the aluminum is bigger, the steel is heavier") and (6) consideration of only one quantity at a time (e.g., "It's bigger"; "steel is heavier"; or "the green has bunches").

Table 8 shows children's justifications on the computer model problems as a function of grade. These justifications of computer problems revealed a great deal of understanding of how the extensive and intensive quantity compensate. The most common justification, especially among older children, was to give a correct multiplicative rule. Two additional children attempted to give a multiplicative rule, but came up with the wrong magnitude. (2x instead of 3x--both had had the steel and aluminum problems first). Another common strategy on the computer problems was to attempt to directly estimate the # of dots in each shape: younger children counted imaginary dots while older children multiplied imaginary rows and columns. These kinds of justifications--all dealing with specific magnitudes or counts--involve 50% of the G2 children, 60% of the G4 and 70% of the G6. Most of the other children mention a trade-off between both quantities or make an explicit comparison to a remembered standard. Only four children simply appealed to one quantity in their justifications.

The justifications on the steel and aluminum problems are examined as a function of the order of presentation (see Table 9). Children who had the steel and aluminum problems first showed less sophistication in their justifications. No child gave a correct multiplicative rule or attempted to make a direct estimation of the magnitude of the weights. A few formulated an incorrect multiplicative rule and a couple more attempted to make a comparison to a remembered standard. Most commonly, however, they showed the least sophisticated justification of talking about only one quantity at a time.

In contrast, the children who had the steel and aluminum problems second were able to articulate greater understanding. Four gave the correct multiplicative rule, two gave incorrect rules, and four more made comparisons to a remembered standard. Only three children made reference to a single quantity. Thus, in both patterns and justifications, the children with the computer model problems first showed greater understanding of these problems.

A final analysis revealed that there was a clear relation between children's patterns of judgments and their justifications. For both tasks children with perfect patterns gave correct multiplicative rules and children with no compensation patterns referred to only one quantity. Further, on the steel and aluminum problems, children with ratio sensitive compensation patterns typically gave justifications in terms of an explicit rule or in terms of a comparison to a remembered standard, while children with non-systematic compensation patterns at best mentioned the trade-off between quantities. There was no systematic difference in justification type for these two patterns on the computer patterns; children typically gave sophisticated justifications for both patterns.

Convictions: Weight, Density, Analogies

In the final interview, children were asked questions which probed their conceptualization of weight and density. They were also asked whether they saw any similarity between the computer and steel and aluminum problems.

Children's conception of weight

Five aspects of children's conception of weight were probed. These included their belief: (1) that a large scale object (i.e., a ball of playdough) weighed something; (2) that this object weighed something even when they were not holding it; (3) that this object weighed the same amount when flattened into a pancake or (4) divided into pieces, and (5) that even a small piece of this object weighed something. Table 10 shows the number of children at each grade level who gave evidence of holding each of these beliefs.

The majority of children at each grade level gave evidence of correctly holding the first four beliefs about weight. In contrast, there were dramatic changes in answer to the question about whether a small piece of playdough weighed anything. Only 25% of the grade 2 children said that it did while 90% of the grade 6 children agreed with this this statement (X^2=9.4, d.f.=2, p=<.01). Children's justifications of their answers to these questions provide clues as to the meaning of this shift. Children who said the small piece of playdough weighed something almost always justified their answer by saying that "everything weighs something" or "it's matter" or "it's still something". Further, many of these children had articulated this viewpoint earlier in the interview as well. In contrast, those who denied it would weigh anything had not made such statements earlier in the interview. When they got to the small piece of playdough question, they simply said "it is too small to weigh anything". Thus, this question about whether a small piece of playdough weighs anything seemed to act as a probe of their belief that weight is a fundamental property of matter; older children believe that it is while younger children do not.

Children's conception of density

Children's conception of density was probed in the final interview by asking children to explain why a piece of steel was heavier than a piece of aluminum the same size. They were also asked to pretend that they could look inside the tiniest pieces of steel and aluminum and to make a drawing of what it might look like inside.

Table 11 shows the number of children at each age giving drawings of different types. A common type of drawing among the G6 children was to draw for each substance little particles which differed in terms of how densely packed they were. This kind of representation was rarely drawn by the younger children. Instead, younger children usually gave one of three other types of drawings/explanations: (1) drawings which portrayed steel and aluminum as solid but differing in color; (2) drawings in which steel is portrayed as solid and aluminum is portrayed as hollow inside, or in which steel is shown to be filled with something heavy (i.e., a heavy liquid or bricks) while aluminum is filled with something light (i.e., cotton balls); and (3) drawings in which steel is portrayed as solid while aluminum is permeated with air holes. In all these drawings, materials were portrayed as essentially solid masses which at most can be hollowed out or have air holes. There was no hint of a belief that matter is essentially particulate instead of continuous. Further, many of the children did not think of steel and aluminum as homogeneous materials. Some children made a clear distinction between the outer appearance and inner composition of materials. Steel might be solid on the outside but filled with a liquid or bricks on the inside; aluminum might be solid on the outside but hollow or filled with cotton balls on the inside. One child even explained that steel was heavier because there was a small piece of aluminum on the inside.

Overall, children at every age appealed to intensive differences between steel and aluminum in their drawings (color differences, empty/full differences, differences in dot crowdedness). However, there were three respects in which young children's conception of density differed from older children's. First, young children did not assume matter is fundamentally particulate whereas older children did. Second, young children frequently failed to represent materials as homogeneous whereas older children did not. Finally, younger children focused on intensive differences which were not quantities (red/green; empty/full) while older children focused on intensive differences which were quantifiable (crowdedness of dots). Significantly, all four children who had perfect patterns on the steel and aluminum problems (predictions task) portrayed density differences in terms of variations in particle density in their drawings. Further, all the children who gave these kinds of particulate drawings had given evidence of believing that weight is a fundamental property of matter.

Children's ability to see analogies between the two tasks.

Several children (2 second graders, and 1 fourth and and 1 sixth grader) spontaneously commented on the similarity between the two prediction tasks when the second task had been presented. At the end of the interview, all children were probed to determine if they could see the similarity between the two tasks. Children were first probed indirectly. They were asked "Do these computer shapes remind you of anything?" and then if there was no answer, "Do you see any connection between these shapes and the steel and aluminum pieces?" "What kind of connection?" They were then probed very directly: they were told that one of the shapes was like a steel piece and one like an aluminum piece and were asked to make the match.

Table 12 shows the number of children at each grade level who were able to articulate the connection between the two tasks both for the more open-ended probes and the directed match. Virtually all the children could see the analogy when asked to make a directed match. Further, the majority of grade 2 and 6 children were able to articulate the analogy in more open-ended questioning as well. However, articulation of the analogy does not always imply acceptance of the analogy. Indeed many of the grade 2 children had theories of matter which were at odds with taking the analogy seriously.

Conclusion

There were three purposes of the present study: (1) to determine how elementary children spontaneously conceptualize the density of materials; (2) to determine if they could understand the intensive quantity presented in computer displays better than the density of materials; and (3) to determine if they could understand the parallels between the computer displays and the steel and aluminum objects.

The results suggested that some second graders may have difficulty distinguishing density from weight, but by grades 4 and 6 this is generally not a problem for children. The prediction problems in which size and density inversely vary directly tests whether children distinguish weight from density. These problems require that the child realizes that the difference between steel and aluminum is an intensive one. If they realize the difference is intensive, then it follows that extensive differences between two objects can compensate for intensive differences: a large aluminum piece can equal a small steel piece in weight. However, if children think of the difference as an extensive one (i.e., steel objects are absolutely heavier than aluminum objects), then they will not understand that size differences can compensate for weight differences. Instead, children should be perplexed by these problems and forced to consider one generalization at a time. When they consider their generalization about the weight differences between steel and aluminum, they should predict that the steel piece will be heavier. When they consider their generalization that big things tend to be heavy, they should predict that the large object will be heavier. We found that half of the second graders (among those who had the steel and aluminum problems first) never made the judgment that the large aluminum piece could equal the small steel piece in weight on any of the compensation problems: one systematically picked the small steel piece as heavier, one systematically picked the large aluminum piece as heavier, and one oscillated between picking the bigger piece and small piece in no principled manner. Further, these children only referred to one quantity in justifying their judgments--the one they had based their judgment on. Thus, there is no evidence from either their justifications or patterns of judgment that they realize the two quantities can compensate. In contrast, all but one of the grade 4 and 6 children made the judgment that a large aluminum piece could equal a small steel piece in weight. Further, the majority indicated in their justifications that they were simultaneously considering both quantities and realized that they could compensate.

There were, however, limitations in the older child's conception of density. These limitations were revealed in two ways. First, in their pattern of judgments and justifications on the predictions task; and second, in their drawings portraying the essential differences between very small pieces of steel and aluminum. Consider first their responses in the predictions task. Among the children who had the steel and aluminum problems first, few grade 2 and grade 4 children had patterns that showed they were systematically taking ratio information into account in making judgments about how size and density compensate. Further, these children typically do not give justifications involving an explicit multiplicative rule. Instead, they either comment on only one quantity or they simply refer to the fact that the two quantities can compensate (i.e., "although the aluminum is bigger, the steel is heavier, so they could be equal"). Thus, they are taking a more qualitative than quantitative approach to these problems: they are noting that the aluminum is bigger than the steel but not worrying about how much bigger. Similarly, they are noting that the steel is heavier than the aluminum, but not worrying about how much heavier.

The drawings of the grade 2 and 4 children also reveal some of the limitations in their conceptions of the densities of materials. There are three main types of drawings produced by these children: 1) drawings which portray steel and aluminum as solid but differing in color; 2) drawings which portray steel as full and aluminum as hollow or filled with a lighter substance/or object; and 3) drawings which portray steel as solid and aluminum as essentially solid, but with some air holes (a la Swiss cheese). While all three types of drawings are depicting an intensive difference between steel and aluminum, none of them is explicitly representing an intensive quantity. Rather, they all are categorical differences: dark color/light color, empty/full, full/air holes. Thus, although children's models of density help them to see its intensive aspect (and hence distinguish density from weight), they do not help them see it as a quantity. These same children (those who were presented with the steel and aluminum problems first) had a much better understanding of the intensive quantity in the computer displays. This better understanding was revealed in several ways: by the way they talked about the quantity and by their patterns of performance and justifications on the predictions task. Three of the four children with no compensation patterns on the steel and aluminum problems had ratio sensitive compensation patterns on the computer problems. The seven children with non-systematic compensation patterns on the steel/ aluminum problems also moved up to ratio sensitive compensation patterns with the computer displays. Finally, three of the four children who had shown ratio sensitive compensation patterns with the steel and aluminum problems, now showed exact compensation problems with the computer displays. This more sophisticated performance with the computer problems is associated with their having a more precise way of talking about the intensive quantity. All but one of the children saw the red and green shapes as differing in an intensive quantity.

Why do these children do better with the computer problems than with the steel and aluminum problems? The improvement cannot be a nonspecific order or practice effect. If children did better with the computer problems simply because they came second, then one would expect that the children who had the computer problems first would do worse. However, the two groups performed equivalently (note: these two groups had been matched by the teacher by ability level, making it less likely that one group was simply "more sophisticated".) We propose that the reason children do better with the computer problems is that the intensive quantity is directly represented in the computer displays in a form distinct from the two extensive quantities and in a form lending itself to quantification. In contrast, the only quantities which are directly perceptually available in the steel and aluminum task, are weight and size. Thus, the child needs to construct the quantity of density from knowledge of the weights of two equal size pieces. Further, the models of density the young child has available (because of conceptions of matter in which the notion of density is embedded) do not help him to see it as a quantity. The models simply reinforce its intensive aspect. Children this age are, however, capable of understanding models which do represent density as an intensive quantity. This suggests, then, that their understanding of density could be significantly enhanced by presenting them with the computer displays and teaching them about the particulate model of matter.

In fact, the present study provides some direct evidence that experience with the computer simulation can help students with steel and aluminum problems. In particular, when children had the steel and aluminum problems after the computer problems, there was no longer an advantage for the computer display tasks. Children perform at a high level on both tasks. There are at least two reasons specific experience with the computer problems may enhance children's performance with steel and aluminum . First, it provides them with experience focusing on an intensive quantity which is easier to conceptualize (i.e. not confusable with an extensive quantity) and with practice co-ordinating this intensive quantity with an extensive quantity. This may prime children to focus on density as an intensive quantity as well. Second, it provides them with the opportunity to come up with a specific strategy for solving the problem: to check if the red shape is three times bigger than the green shape. This specific strategy is directly applicable to the steel and aluminum problems since their densities are in a 3 to 1 ratio as well. Note that in either case for transfer to occur, it is necessary that the child sees some analogy between the problems. Direct evidence that the child does have the capacity to see relevant analogies between the two systems came from the last questions in the interview. In particular, we asked children both a more open-ended question ("Do you see some connection between the computer shapes and the steel and aluminum pieces? What connection? ") and a more directed question ("Do you think the steel and aluminum piecescould look like this inside? Which one would be the steel? Why do you think that? ") to assess their understanding of the analogy. The results showed that at all ages children were aware that the green shape was more like steel, because having more dots is like being heavier. Further, both the G2 and G6 children typically articulated this analogy before they were given the more directive probe.

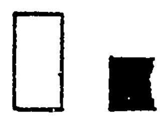
The findings of the present study suggest, then, that it is easier for children to conceptualize the intensive quantity in the computer displays than to conceptualize density. Further, there was evidence that experience with the computer displays may help children to think about density more clearly as an intensive quantity. We do not, however, think that this brief experience with the computer problems brought about a deep re-organization in children's conception of density. Children's existing conceptions of density were sufficient for them to see some analogy between these two systems. At the same time, their existing conceptions of matter and material kind probably prevented them from regarding those analogies as deep ones. In particular, most grade 2 children and many grade 4 children as well did not yet regard weight as a fundamental property of matter. Further, virtually all of th· · regarded matter as essentially continuous rather than particulate.

One question for future research is wh_ther elementary children can be taught to take the computer displays as a model of density. This would involve making basic changes in their conceptions of matter and material kind, which is notoriously hard to do. Another question is to what extent children need to take the computer displays as a model of density in order for them to be helpful as a pedagogic tool. Our present results suggest that it may not be necessary for children to take the displays as a formal model of density in order for them to be helpful since grade 2 children benefited as much, if not more from the experience than G6 children. Indeed, the reverse might even be the case: experience with using the model to successfully solve density related problems (i.e., problems such as those used in the present study; or sinking and floating problems, where the densities of objects are directly represented in computer displays and where there is a dynamic component as well) may provide the child with some motivation for taking the particulate model of density seriously.

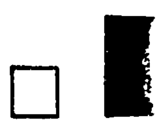
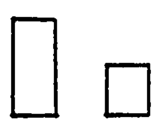
Table I

Test item types:
Steel/aluminum and computer model tasks

Compensation problems:size and density vary inversely

Type	Definition	Symbol[1]	Dimensions[2]	
I	Larger object is heavier/has more dots		b) 5:1	c) 11.6:2.5
II	Both objects equal in weight/number of dots		f) 2.9:1	g) 11.6:4
III	Smaller object is heavier/has more dots		i) 7:4	j) 4:2.5

Non-compensation problems

Type	Definition	Symbol[1]	Dimensions[2]	
IV	Size constant/ density varies		l) 1:1	m) 4:4
V	Size and density covary		o) 1:4	p) 5:6
VI	Density constant/ size varies		r) 2.5:1	s) 11.6:5

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[1]Empty boxes stand for aluminum (or red shapes filled with single dots); full boxes stand for steel (or green shapes filled with clusters of dots). The pairs depict relative size but are not to scale. Single underlining indicates heavier weight (or more dots); double underlining indicates equality of weight (or of number of dots).

[2]The letters correspond to the letter of the items on the interview protocols in the Appendix. The numbers refer to inches for the metal pieces and to rows for the computer model;the fractions were rounded to the next higher num-number in the case of rows.

Table 2

Number of subjects by grade, sex and ability level

	Grade					
	2		4		6	
Ability	Male (n=6)	Female (n=6)	Male (n=5)	Female (n=5)	Male (n=5)	Female (n=5)
Low	2	2	-	2	2	2
Average	2	2	2	-	2	2
High	2	2	3	3	1	1

Table 3

Computer analogs:
Language used in describing the difference
between red and green shapes

	Grade		
	2 (n=12)	4 (n=10)	6 (n=10)
Explanation of difference			
Green has triangles; Red has dots	2	0	0
Green has 3 dots; Red has 1 dot	1	1	0
Green is in bunches (dots closer together); Red in singles (dots spread out)	3	3	3
Green has 3/bunch; Red has single dots or 1/bunch	6	6	7

Table 4

Concrete materials:
Language used in describing the difference between steel and aluminum

Explanation of difference	Grade 2 (n=12)	Grade 4 (n=10)	Grade 6 (n=10)
Steel is heavier	8	1	0
Steel is fuller/stronger	3	3	3
Steel is a heavier material/substance	1	6	4
Steel is a denser substance/material	0	0	3

Table 5

Response patterns as a function of task and order of presentation

	Order of presentation			
	Steel and aluminum first		Computer first	
Response pattern	St & Al (n=16)	Comp (n=16)	St & Al (n=16)	Comp (n=16)
Exact compensation	1	5	3	3
Ratio sensitive compensation	4	10	11	9
Non-systematic compensation	7	0	1	3
No compensation	4	1	1	1

Table 6

Computer model task:
Response patterns as a function of grade

	Grade		
	2 (n=12)	4 (n=10)	6 (n=10)
Response pattern			
Exact compensation	1	3	4
Ratio sensitive compensation	8	7	4
Non-systematic compensation	1	0	2
No compensation	2	0	0

Table 7

Steel and aluminum task:
Response patterns as a function of
order of presentation and grade

	Order of presentation					
	Steel and aluminum first			Computer first		
	Grade					
	2 (n=6)	4 (n=5)	6 (n=5)	2 (n=6)	4 (n=5)	6 (n=5)
Response Pattern						
Exact compensation	0	0	1	0	1	2
Ratio sensitive compensation	1	1	2	5	3	3
Non-systematic compensation	2	4	1	0	1	0
No compensation	3	0	1	1	0	0

Table 8

Computer model task:
Justifications as a function of grade

Justification	Grade 2 (n=12)	Grade 4 (n=10)	Grade 6 (n=10)
Correct rule	1	5	4
Incorrect rule	1	1	0
Direct estimation of number of dots	4	0	3
Comparison to remembered standard	1	0	1
Trade-off between 2 quantities	2	4	1
One quantity only	3	0	1

Table 9

Steel and aluminum task:
Justifications as a function of order of presentation and grade

	Order of presentation					
	Steel and Aluminum first			Computer first		
	Grade					
Justification	2 (n=6)	4 (n=5)	6 (n=5)	2 (n=6)	4 (n=5)	6 (n=5)
Correct rule	0	0	0	0	1	3
Incorrect rule	1	1	1	0	2	0
Comparison to a remembered standard	0	0	2	3	0	1
Trade-off between two quantities	1	3	0	1	1	1
One quantity only	4	1	2	2	1	0

Table 10

Beliefs about the weight of piece of playdough
as a function of grade level

Belief	Grade 2 (n=12)	Grade 4 (n=10)	Grade 6 (n=10)
Large piece of playdough weighs something	9	8	10
Playdough has weight even when not holding it	8	9	9
Playdough weighs same even when flattened	8	8	10
Playdough weighs same when divided	10	8	9
Small piece of playdough weighs something	3	5	9

Table 11

Types of drawings as a function of grade level

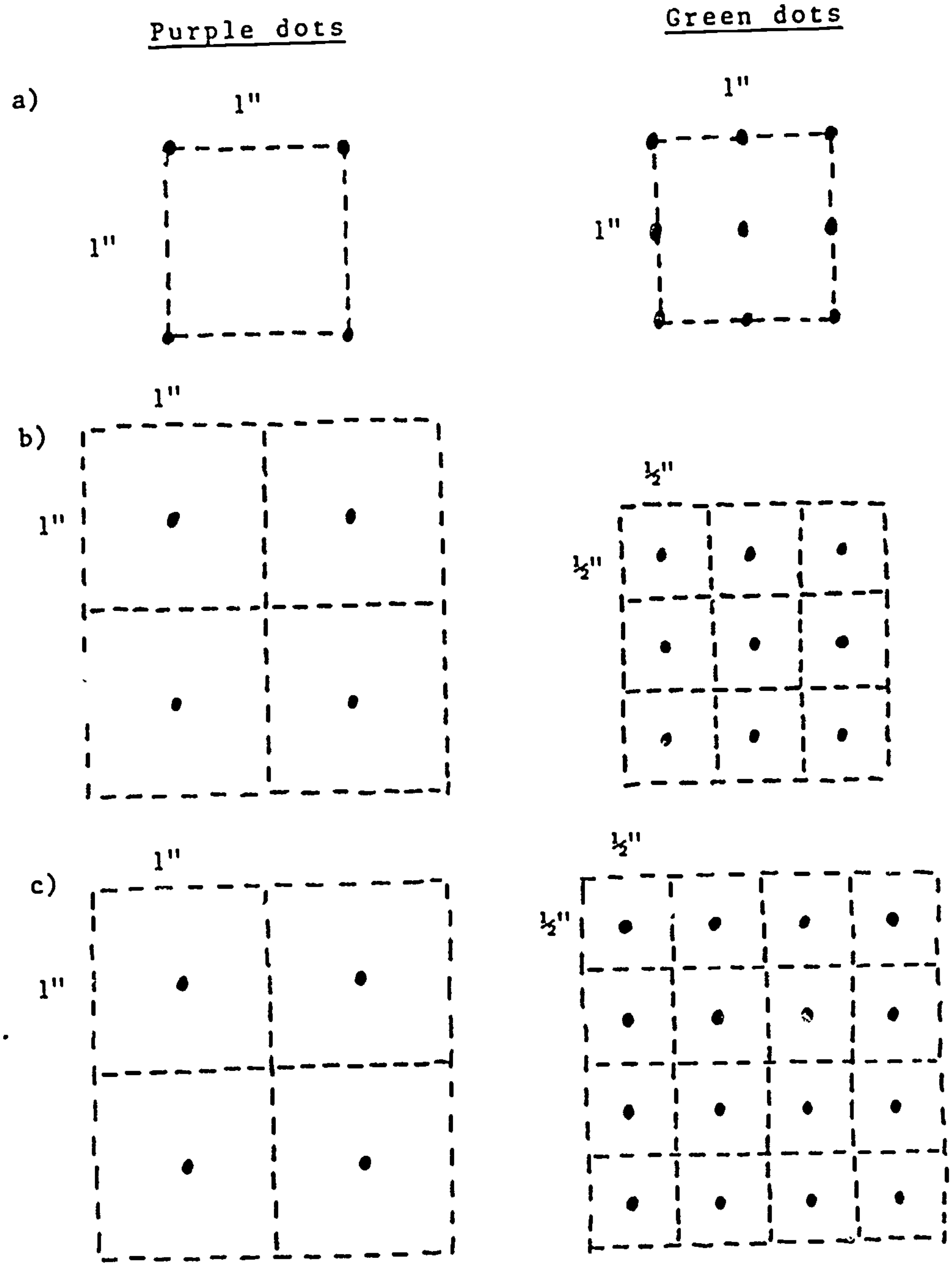
	Grade		
	2 (n=12)	4 (n=10)	6 (n=10)
Type of drawing			
Color or surface appearance	3	3	1
Empty/full	3	3	1
Solid/air holes	2	1	1
Particle density	0	2	5
Other	4	1	2

Table 12

Number of children who see appropriate analogy as a function of grade level

Type of probe	Grade 2 (n=12)	Grade 4 (n=10)	Grade 6 (n=10)
Open-ended probe	9	4	7
Directed match	10	10	9

FIGURE 1



The problem of finding comparable areas from which to extract a ratio.

a) the dots serve to define the boundaries of the area (incorrect)

b) when the dots are contained in the center of an imaginary box, the areas are seen to be not equivalent

c) the number of boxes containing green dots needs to be increased to make the areas comparable

FIGURE 2

Print-out from Computer Display:

A. Outlines and filled-in shapes
for compensation problem type I
(Red shape has more dots)

Screen 1

RED GREEN

Screen 2

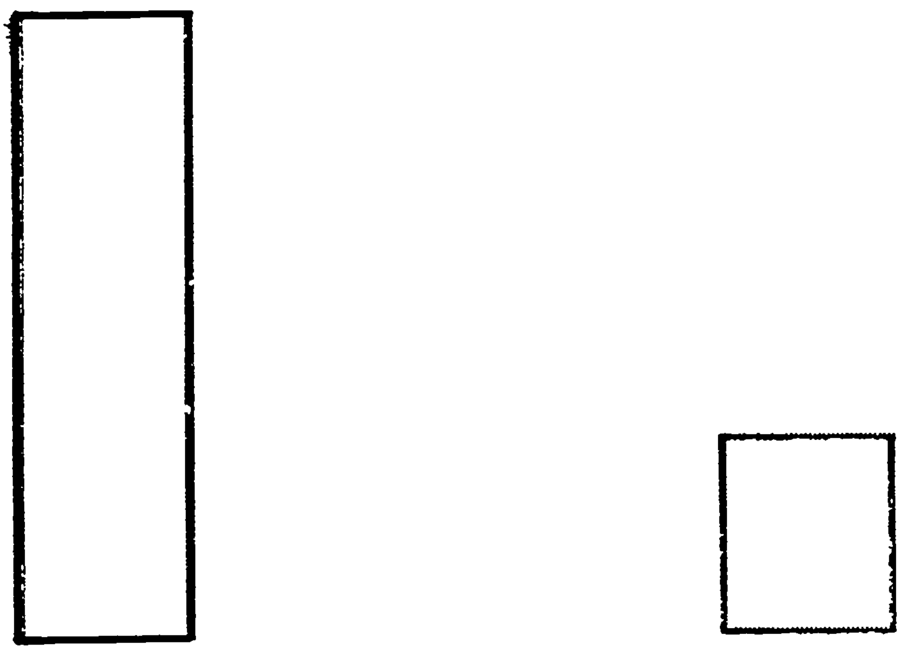
RED GREEN

FIGURE 2(ct'd)

Print-out from Computer Display:

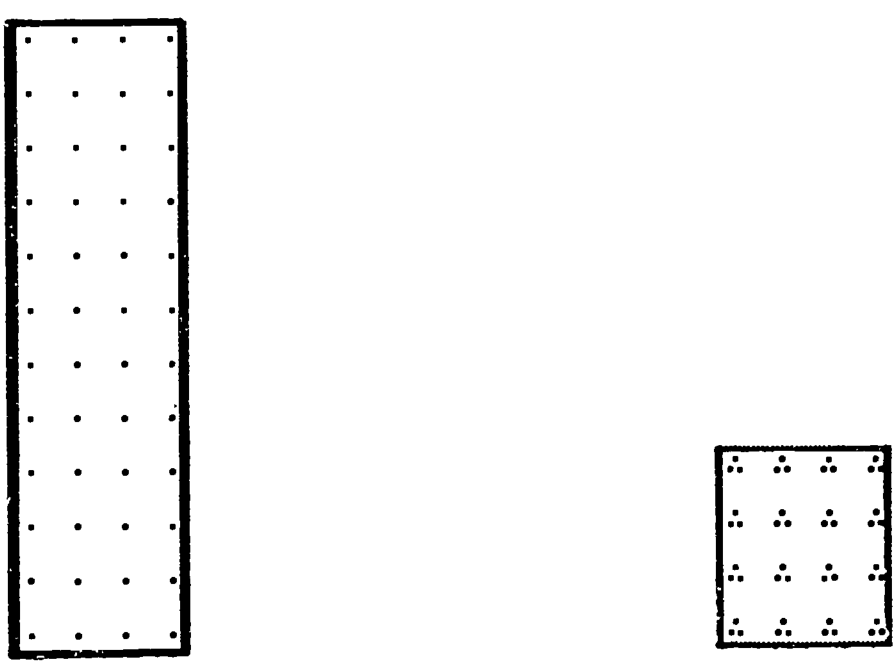
B. Outlines and filled-in shapes
for compensation problem type II
(Shapes have equal number of dots)

Screen 1



RED GREEN

Screen 2



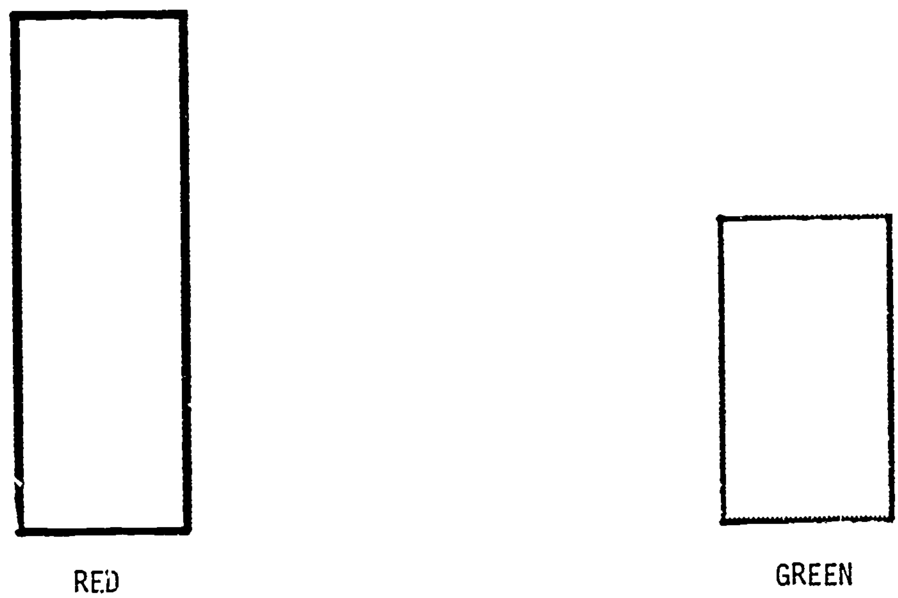
RED GREEN

FIGURE 2 (ct'd)

Print-out from Computer Display:

C. Outlines and filled-in shapes
for compensation problem type III
(Green shape has more dots)

Screen 1



Screen 2

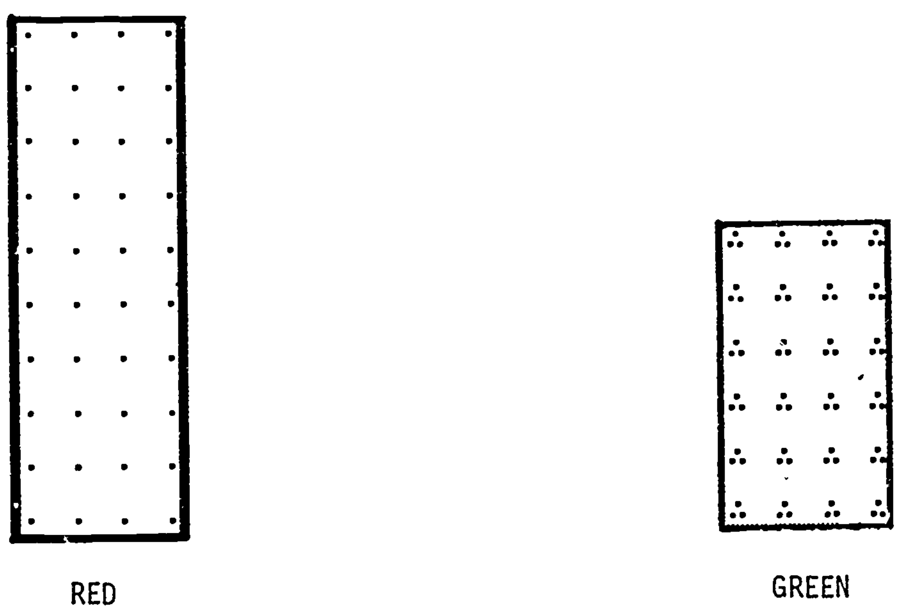


FIGURE 2 (ct'd)

Print-out from Computer Display:

D. Outlines and filled-in shapes
for non-compensation problem type IV
(Green shape has more dots)

Screen 1

RED GREEN

Screen 2

RED GREEN

FIGURE 2 (ct'd)

Print-out from Computer Display:

E. Outlines and filled-in shapes
for non-compensation problem type V
(Green shape has more dots)

Screen 1

RED GREEN

Screen 2

RED GREEN

FIGURE 2 (ct'd)

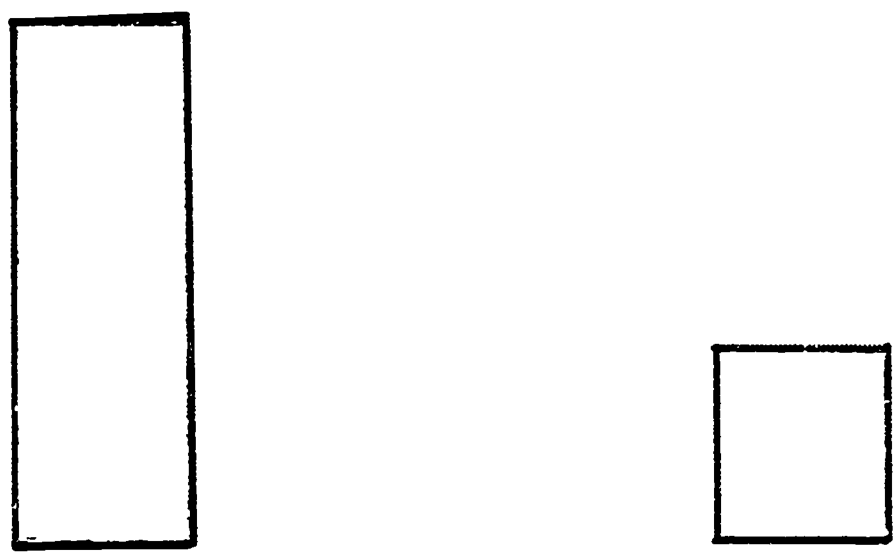
Print-out from Computer Display:

F. Outlines and filled-in shapes

(for non-compensation problem type VI

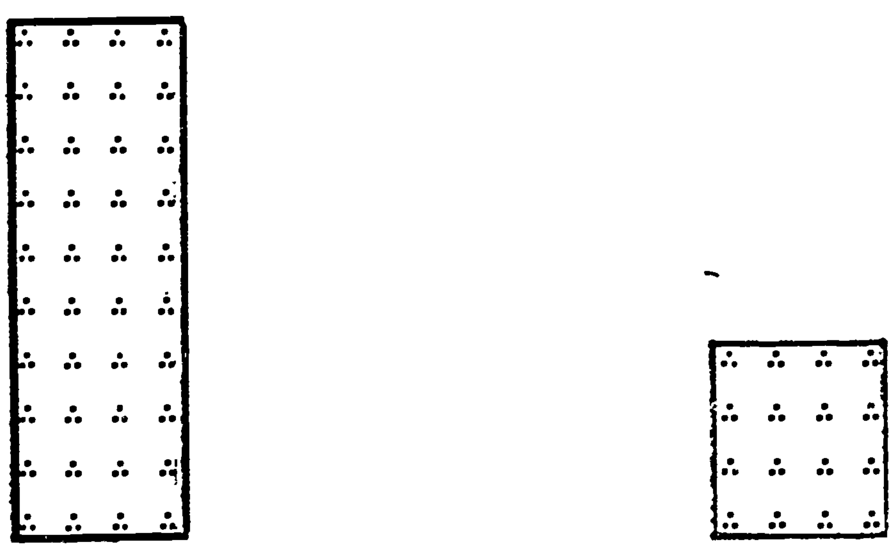
(Large shape has more dots)

Screen 1



GREEN GREEN

Screen 2



GREEN GREEN

REFERENCES

Quintero, A. (1980). The role of semantic understanding in solving multiplication word problems. Unpublished doctoral dissertation, Massachusetts Institute of Technology, Cambridge, MA.

Smith, C. (1984, November). Weight, density and matter: A preliminary study of elementary children's conception of density as an intensive quantity. Unpublished technical report, Educational Technology Center, Harvard Graduate School of Education, Cambridge, MA.

APPENDIX

List of interview protocols

A. Steel and Aluminum Preliminaries

B. Steel and Aluminum Problem Set

C. Computer Model Preliminaries

D. Computer Model Problem Set

E. Playdough and Probe.

A. STEEL AND ALUMINUM PRELIMINARIES

NAME: ________________ GRADE: ______ BOY / GIRL DATE: __________

step a.

IV (k)

2" 2"

= 8

I'LL BE SHOWING YOU SOME METAL PIECES AND I WANT YOU TO LOOK AT THEM VERY CAREFULLY.

NOW I'D LIKE YOU TO TAKE THESE TWO PIECES AND TELL ME IF THEY WEIGH THE SAME? (Hand the pieces to the child)

Yes /. No

. If no ask WHICH IS HEAVIER - (S / A)

Show balance scale and explain:

DO YOU KNOW WHAT THIS IS CALLED? WHAT IS IT USED FOR? HOW DOES IT WORK? WE USE IT TO WEIGH THINGS. THIS HAND TELLS US HOW HEAVY THINGS ARE. THE FARTHER IT GOES, THE HEAVIER THINGS WEIGH.

LET'S PUT THESE PIECES ON THE SCALE AND SEE HOW MUCH THEY WEIGH. DO THEY WEIGH THE SAME?

Yes (incorrect response, show child)

No . . WHICH IS HEAVIER . . S / A

ARE THEY THE SAME SIZE? Yes / No . . . (show they are same)

THESE PIECES ARE THE SAME SIZE BUT THEY DO NOT WEIGH THE SAME. HOW CAN THAT BE?

____

____

____

CAN YOU TELL ME WHAT YOU MEAN . . . (Use child's words).

____

____

____

THESE PIECES ARE MADE OF DIFFERENT KINDS OF STUFF (MATERIALS). DO YOU KNOW THE NAMES OF WHAT THEY ARE MADE OF ?

____

THIS ONE IS MADE OF ALUMINUM AND THIS ONE IS MADE OF STEEL.
CAN YOU TELL ME WHAT IS DIFFERENT ABOUT STEEL AND ALUMINUM?

____

____

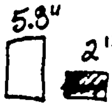
YOU CAN SEE THAT THE STEEL ONE IS DARKER AND THE ALUMINUM IS LIGHTER . WE ALSO HAVE PUT THESE DOTS ON TO HELP. THE BLUE DOT IS ON THE STEEL PIECES AND THE YELLOW DOT ON THE ALUMINUM.

II (d)

step b.

5.8"

2"



M = B

NOW, I WANT YOU TO LOOK AT THESE TWO PIECES, BUT I DON'T WANT YOU TO LIFT THEM UP UNTIL LATER.

ONE IS MADE OF STEEL AND ONE IS MADE OF ALUMINUM, JUST LIKE THE FIRST TWO WE SAW. CAN YOU SHOW ME WHICH ONE IS STEEL AND WHICH ONE IS ALUMINUM ?. (Show color marker).

DO YOU THINK THAT THESE TWO PIECES COULD WEIGH THE SAME?

Yes / No - if no ask:

.WHICH IS HEAVIER ? S / A

WHY DO YOU THINK SO?

____

____

LETS SEE (have child put them on the scale)

DO THEY WEIGH THE SAME?

Yes / No . . (if no help them see that they are the same)

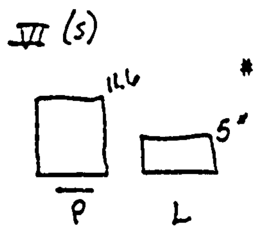
ARE THEY THE SAME SIZE ? Yes / No

B. STEEL AND ALUMINUM PROBLEM SET

NAME: ____________________ GRADE __________ BOY/GIRL DATE: __________

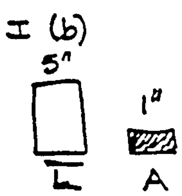
For presentation 1 through 12 ask the following

DO YOU THINK THESE COULD WEIGH THE SAME ? If NO, ask WHICH IS HEAVIER ?



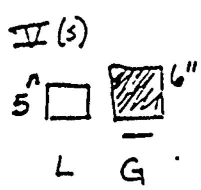
#1. HERE ARE TWO PIECES MADE OF ALUMINUM
COULD THESE TWO PIECES WEIGH THE SAME ?

Yes / No
.. if no ask WHICH IS HEAVIER tall / short



#2. HERE IS A PIECE OF STEEL AND A PIECE OF ALUMINUM. DO YOU THINK THESE COULD WEIGH THE SAME?

Yes / No
.. if no ask WHICH IS HEAVIER ? steel / aluminum

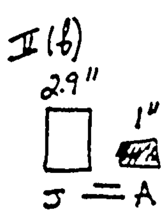


#3. HERE IS A PIECE OF STEEL AND A PIECE OF ALUMINUM. DO YOU THINK THESE COULD WEIGH THE SAME ?

Yes / No
.. if no ask WHICH IS HEAVIER ? steel / aluminum

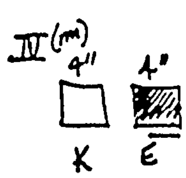
WHY ? ______________________________

____



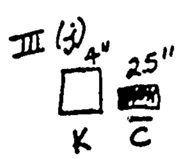
#4. HERE IS A PIECE OF STEEL AND A PIECE OF ALUMINUM. DO YOU THINK THESE COULD WEIGH THE SAME ?

Yes / No
.. if no ask WHICH IS HEAVIER ? steel / aluminum



#5. HERE IS A PIECE OF STEEL AND A PIECE OF ALUMINUM. DO YOU THINK THESE COULD WEIGH THE SAME ?

Yes / No
.. if no ask WHICH IS HEAVIER ? steel / aluminum



#6. HERE IS A PIECE OF STEEL AND A PIECE OF ALUMINUM. DO YOU THINK THESE COULD WEIGH THE SAME ?

Yes / No
.. if no ask WHICH IS HEAVIER ? steel / aluminum

Probe Question

(place preliminary pieces (S: B D F / A: I M O) in front of child)

CAN A PIECE OF STEEL AND ALUMINUM WEIGH THE SAME ? Yes / No

.. If No say :LETS MAKE SURE. CAN YOU FIND A PIECE OF ALUMINUM THAT WEIGHS THE SAME AS THIS PIECE OF STEEL?
(place piece B of steel in front of child
(help child to select S/A pair "B/M" or "D/O")
(weigh the pieces)

.. If Yes say: CAN YOU SHOW ME WHICH PAIRS OF S/A PIECES WEIGH THE SAME?
picks :S:____ :A:____

LETS WEIGH THEM AND SEE. DO THEY WEIGH THE SAME? Yes / No

.. if Yes say YOU WERE RIGHT. CAN YOU FIND ANOTHER PAIR THAT WEIGHS THE SAME ? (check on scale)
picks :S:____ :A:_____ (B/M or D/O)

.. if No say THEY DON'T WEIGH THE SAME. CAN YOU FIND ANOTHER PAIR THAT DO WEIGH THE SAME? (help child as necessary to find pair)
picks :S:____ :A:____ (B/M or D/O) (check on scale)

CAN YOU FIND ANOTHER PAIR THAT WEIGHS THE SAME ?
picks :S:____ :A:____ (B/M or D/O) (check on scale)

HOW CAN YOU KNOW THAT THIS PIECE OF ALUMINUM WEIGHS THE SAME AS THIS PIECE OF STEEL (point to first pair), AND THAT THIS PIECE OF ALUMINUM AND THIS PIECE OF STEEL WEIGH THE SAME (point to second pair)?

____

____

____

(point to 2"steel (B) and say :

YOU SAID THAT THIS ONE (B) WEIGHS THE SAME AS THIS ONE (M).
COULD THERE BE ANOTHER PIECE OF ALUMINUM THAT WEIGHS THE SAME AS THIS PIECE OF STEEL? (piece "B")

Yes / No

. if Yes say WHICH ONE(S) ______________________________

place 2"steel "B" and 2" aluminum "I" in front of child and say:

HERE IS A PIECE OF STEEL AND A PIECE OF ALUMINUM. HOW MUCH HEAVIER DO YOU THINK THE STEEL IS THAT THE ALUMINUM ? HOW DO YOU KNOW THAT?

____

THESE PIECES WEIGH THE SAME BUT THEY ARE NOT THE SAME SIZE. HOW CAN THAT BE?--

<u>(From here, child must answer questions before checking).</u>

VI (9)

<u>step c.</u>

5"

3"

F D

HERE ARE TWO PIECES OF STEEL. COULD THESE PIECES WEIGH THE SAME?

Yes / if <u>No</u>

WHICH ONE IS HEAVIER - <u>tall</u> / short

HOW COME / WHY DO YOU THINK.....(child's words)

LETS SEE IF YOU'RE RIGHT <u>(Let child handle pieces and weigh them).</u>

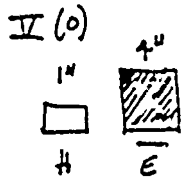
if right say: YOU WERE RIGHT

if wrong say: THE SCALE SHOWS THIS ONE IS HEAVIER. HOW CAN THAT BE?

<u>step d.</u>

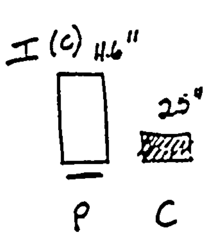
<u>Exploration period. Maximum 3 minutes. Record child's play.</u> Time: ______
<u>Use preliminary pieces only.</u>

I HAVE SOME MORE PIECES TO SHOW YOU. BUT BEFORE WE DO THAT, I'LL LET YOU LOOK AT THESE PIECES A LITTLE LONGER. YOU CAN HOLD THEM IN YOUR HAND OR PUT THEM ON THE SCALE IF YOU LIKE. (record child's responses)



#7. HERE IS A PIECE OF STEEL AND A PIECE OF ALUMINUM. DO YOU THINK THESE COULD WEIGH THE SAME ?

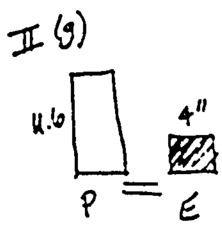
Yes / No
.. if no ask WHICH IS HEAVIER ? steel / aluminum



#8. HERE IS A PIECE OF STEEL AND A PIECE OF ALUMINUM. DO YOU THINK THESE COULD WEIGH THE SAME ?

Yes / No
.. if no ask WHICH IS HEAVIER ? steel / aluminum

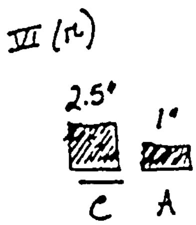
WHY ?____



#9. HERE IS A PIECE OF STEEL AND A PIECE OF ALUMINUM. DO YOU THINK THESE COULD WEIGH THE SAME ?

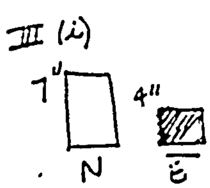
Yes / No
.. if no ask WHICH IS HEAVIER ? steel / aluminum

WHY____



#10. HERE ARE TWO PIECES OF STEEL. DO YOU THINK THESE TWO PIECES COULD WEIGH THE SAME ?

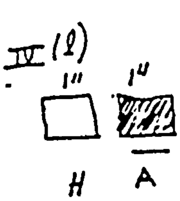
Yes / No
.. if no ask WHICH IS HEAVIER ? tall / short



#11. HERE IS A PIECE OF ALUMINUM AND A PIECE OF STEEL. DO YOU THINK THESE TWO PIECES COULD WEIGH THE SAME ?

Yes / No
.. if no ask WHICH IS HEAVIER ? steel / aluminum

WHY ____



#12. HERE IS A PIECE OF STEEL AND A PIECE OF ALUMINUM. DO YOU THINK THESE TWO PIECES COULD WEIGH THE SAME ?

Yes / No
.. if no ask WHICH IS HEAVIER ? steel / aluminum

C. COMPUTER MODEL PRELIMINARIES

NAME:______________ GRADE:______________ BOY/GIRL DATE ______________

I'M GOING TO BE SHOWING YOU SOME SHAPES AND I WANT YOU TO LOOK AT THEM VERY CAREFULLY. HERE IS THE FIRST ONE.

step a.

IV (k)

P- P+

2 ☐ ▨ 2

press (P-P+)

ONE OF THESE SHAPES HAS A RED OUTLINE AND THE OTHER HAS A GREEN OUTLINE. BOTH OF THEM ARE FILLED WITH DOTS.

DOES THE RED SHAPE HAVE THE SAME NUMBER OF DOTS AS THE GREEN SHAPE ?

Yes / No

. if No say WHICH ONE HAS MORE red / green

. if Yes say YES, THEY DO HAVE THE SAME NUMBER OF BUNCHES (CLUSTERS), BUT THE GREEN ONE HAS MORE DOTS IN A BUNCH/CLUSTER THAN THE RED ONE. DO THEY HAVE THE SAME NUMBER OF DOTS ?
(help child count the dots to see the green one has more)

ARE THEY THE SAME SIZE ?

Yes / No (if no correct child)

THEY ARE THE SAME SIZE, BUT THE GREEN ONE HAS MORE DOTS. HOW CAN THAT BE ?

____

____

THAT'S RIGHT/ THEY HAVE THE SAME NUMBER OF BUNCHES, BUT THE GREEN ONE HAS MORE DOTS IN A BUNCH/CLUSTER THAN THE RED ONE.

NOW I'M GOING TO SHOW YOU SOME MORE SHAPES. I'LL BEGIN BY SHOWING YOU JUST OUTLINES OF SHAPES (press p-p+) IF IT'S A RED ONE YOU MUST REMEMBER IT WILL BE FILLED IN WITH BUNCHES THAT LOOK LIKE THIS (point to red example on card)

IF ITS A GREEN ONE YOU MUST REMEMBER IT WILL BE FILLED IN WITH BUNCHES WHICH LOOK LIKE THIS (point to green example on card)

step b.

l- p+

6 ☐ ▨ 2

press (l-p+)

HERE IS AN OUTLINE OF TWO NEW SHAPES. REMEMBER THE GREEN ONE IS ALWAYS FILLED IN LIKE THIS (point to card), AND THE RED ONE IS ALWAYS FILLED IN LIKE THIS (point to card).

DO YOU THINK THAT THESE TWO SHAPES COULD HAVE THE SAME NUMBER OF DOTS?

Yes / No
.. if no ask WHICH ONE HAS MORE DOTS green / red

WHY DO YOU THINK SO ? ______________________________

____

LETS COUNT AND CHECK IF YOU ARE RIGHT. (press L-P+)

(help them count so they see they have the same number of dots)

ARE THE SHAPES THE SAME SIZE No

THEY ARE NOT THE SAME SIZE BUT THEY HAVE THE SAME NUMBER OF DOTS.
HOW CAN THAT BE ?

____

____

VI (?)

m+ o-

5 3

step c. press (m+n+)

HERE ARE TWO GREEN SHAPES. COULD THESE TWO SHAPES HAVE THE SAME NUMBER OF DOTS?

Yes/ No
.. if no ask WHICH ONE HAS MORE DOTS large / small

WHY DO YOU THINK THAT? ______________________________

____

LETS CHECK IF YOU ARE RIGHT press (M+O+)

____

____

step d.

I'M GOING TO BE SHOWING YOU SOME MORE SHAPES, BEFORE I DO YOU CAN PLAY WITH TH. SHAPES FIRST. HERE IS A CARD WHICH SHOWS YOU THE DIFFERENT SHAPES AND THE LETTER TO PRESS TO PUT IT ONTO THE SCREEN. (leave 5 minutes for play)

record play ______________________________

____

____

Probe Questions

(place card with outlines of shapes P O M L I in front of child)

CAN A GREEN SHAPE HAVE THE SAME NUMBER OF DOTS AS A RED SHAPE? Yes / No

.. if NO say: LETS MAKE SURE. CAN YOU FIND A RED OUTLINE THAT HAS THE SAME NUMBER OF DOTS AS THIS GREEN OUTLINE?
(place outline P+ in front of child, help child select outline G/R pair P+/L- or O+/I-; put on screen, count dots)

.. if YES say: CAN YOU SHOW ME WHICH SHAPES HAVE THE SAME NUMBER OF DOTS?
picks :G:____ :R:____

LETS COUNT THE DOTS AND SEE. DO THEY HAVE THE SAME NUMBER OF DOTS? Yes/No

.. if Yes say: YOU WERE RIGHT. CAN YOU FIND ANOTHER PAIR OF SHAPES THAT HAVE THE SAME NUMBER OF DOTS? (check on screen, count dots)

picks :G:____ :R:____

.. if No say: THEY DON'T HAVE THE SAME NUMBER OF DOTS. CAN YOU FIND ANOTHER PAIR OF SHAPES THAT HAVE THE SAME NUMBER OF DOTS?
(help child as needed to find pair P+/L- or O+I-)
picks :G:____ :R:____ (check on screen, count dots)

CAN YOU FIND ANOTHER PAIR OF SHAPES WHICH HAVE THE SAME NUMBER OF DOTS?
picks :G:____ :R:____ (check on screen, count dots)

HOW CAN YOU KNOW THAT THIS GREEN SHAPE AND THIS RED SHAPE (pt to 1st pair) AND THIS GREEN SHAPE AND RED SHAPE (pt to 2nd pair) HAVE THE SAME NUMBER OF DOTS?

____

____

(point to p+ and say):

YOU SAID THAT THIS ONE (P+) HAS THE SAME NUMBER OF DOTS AS THIS ONE. COULD THERE BE ANOTHER RED SHAPE THAT HAS THE SAME NUMBER OF DOTS AS THIS GREEEN SHAPE?

Yes / No

.. if Yes say WHICH ONE(S) ______________________

point to p+ and p- and say:

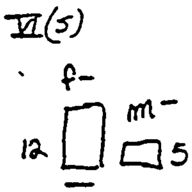
HERE IS A GREEN SHAPE AND A RED SHAPE. HOW MANY MORE DOTS DO YOU THINK THERE ARE IN THIS GREEN SHAPE THAN IN THIS RED SHAPE?

____

D. COMPUTER MODEL PROBLEM SET

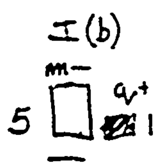
NAME: ______________ GRADE: ________ MALE / FEMALE Date: ________

For presentation 1 through 12 ask the following: DO YOU THINK THESE HAVE THE SAME NUMBER OF DOTS? If No, ask WHICH ONE HAS MORE DOTS ? -



#1 press f-m- HERE ARE TWO RED SHAPES. COULD THESE TWO SHAPES HAVE THE SAME NUMBER OF DOTS ?

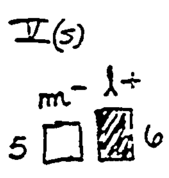
Yes / No
.. if no ask WHICH ONE HAS MORE DOTS ? big / little



#2. press m-q+ HERE IS A RED SHAPE AND A GREEN SHAPE. COULD THESE TWO SHAPES HAVE THE SAME NUMBER OF DOTS ?

Yes / No

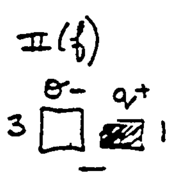
.. if no ask WHICH ONE HAS MORE DOTS ? red/green



#3. press m-l+ HERE IS A RED SHAPE AND A GREEN SHAPE. COULD THESE TWO SHAPES HAVE THE SAME NUMBER OF DOTS ?

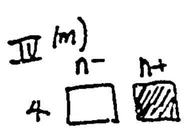
Yes / No
.. if no ask WHICH ONE HAS MORE DOTS ? red / green

WHY ? ______________________________



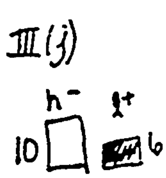
#4. press o-q+ HERE IS A RED SHAPE AND A GREEN SHAPE. COULD THESE TWO SHAPES HAVE THE SAME NUMBER OF DOTS ?

Yes / No
.. if no ask WHICH ONE HAS MORE DOTS ? red / green



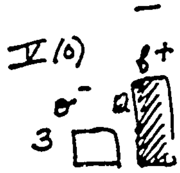
#5. press n-n+ HERE IS A RED SHAPE AND A GREEN SHAPE. COULD THESE TWO SHAPES HAVE THE SAME NUMBER OF DOTS ?

Yes / No
.. if no ask WHICH ONE HAS MORE DOTS ? red / green



#6. press h-l+ HERE IS A RED SHAPE AND A GREEN SHAPE. COULD THESE TWO SHAPES HAVE THE SAME NUMBER OF DOTS ?

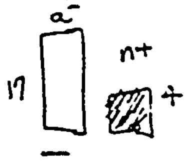
Yes / No
.. if no ask WHICH ONE HAS MORE DOTS ? red / green



#7. press o-f+ HERE IS A RED SHAPE AND A GREEN SHAPE. COULD THESE TWO SHAPES HAVE THE SAME NUMBER OF DOTS ?

Yes / No
.. if no ask WHICH ONE HAS MORE DOTS ? red / green

I (c)



#8. press a-n+ HERE ARE A RED SHAPE AND A GREEN SHAPE. COULD THESE TWO SHAPES HAVE THE SAME NUMBER OF DOTS

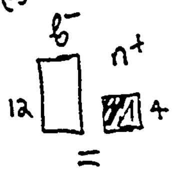
Yes / No

.. if no ask WHICH ONE HAS MORE DOTS ? red / green

WHY? ______________________________

____

II (g)



#9. press f-n+ HERE IS A RED SHAPE AND A GREEN SHAPE. COULD THESE TWO SHAPES HAVE THE SAME NUMBER OF DOTS ?

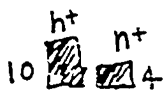
Yes / No

.. if no ask WHICH ONE HAS MORE DOTS ? red / green

WHY ? ______________________________

____

VI (r)

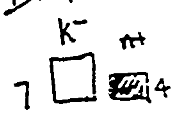


#10. press h+n+ HERE ARE TWO GREEN SHAPES. COULD THESE HAVE THE SAME NUMBER OF DOTS ?

Yes / No

.. if no ask WHICH ONE HAS MORE DOTS ? big / small

III (i)



#11. press k-n+ HERE IS A RED SHAPE AND A GREEN SHAPE. COULD THESE HAVE THE SAME NUMBER OF DOTS?

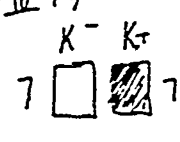
Yes / No

.. if no ask WHICH ONE HAS MORE DOTS ? red / green

WHY ? ______________________________

____

III (l)



#12. press k-k+ HERE IS A RED SHAPE AND A GREEN SHAPE. COULD THESE HAVE THE SAME NUMBER OF DOTS ?

Yes / No

.. if no ask WHICH ONE HAS MORE DOTS? red / green

E. PLAYDOUGH AND PROBE

NAME: ____________ GRADE: ____________ BOY/GIRL DATE: ____________

I. HERE IS A PIECE OF PLAYDOUGH.

a. DOES THIS PIECE OF PLAYDOUGH WEIGH ANYTHING? Y/N

WHY DO YOU THINK THAT?

____

____

(if child says that the piece isn't big enough to weigh anything, add more to the piece until the child agrees that it has weight, before going on)

b. WOULD THE PIECE OF PLAYDOUGH STILL WEIGH SOMETHING EVEN WHEN YOU AREN'T HOLDING IT? Y/N

HOW DO YOU KNOW THAT?

____

____

c. (change the shape of the playdough--flatten into a pancake--as the child watches or have the child help you) COULD YOU HELP ME FLATTEN THIS BALL OF PLAYDOUGH INTO A PANCAKE. GOOD. DOES THE PLAYDOUGH WEIGH THE SAME NOW AS IT DID BEFORE? Y/N

WHY DO YOU THINK SO?

____

____

DOES IT HAVE THE SAME AMOUNT OF STUFF IN IT AS IT DID BEFORE? Y/N

HOW DO YOU KNOW THAT?

____

____

d. (make the playdough back into a ball, and let the child hold it) CAN YOU HELP ME AGAIN? THIS TIME I WANT TO BUT THIS BALL OF PLAYDOUGH INTO LITTLE PIECES (have child help). DO ALL THE PIECES TOGETHER WEIGH THE SAME AS THE BALL OF PLAYDOUGH DID BEFORE? Y?N

WHY DO YOU THINK SO?

____

____

e. (pick up a tiny piece of playdough). DOES THIS LITTLE

PIECE OF PLAYDOUGH WEIGH ANYTHING? Y?N

HOW DO YOU KNOW THAT?

____

____

If no: Could it weigh a tiny bit or nothing at all?

II. a. Let's take a look at these 2 pieces of steel and aluminum (4"pair, E & K). Do you remember that even though these two pieces are the the same size, the steel one weighs more than the aluminum one?

What is it about the steel piece that makes it heavier/weigh more?

____

____

Can you tell me what you mean? ______________________

____

b. Lets pretend we could look inside the tiniest pieces of steel and aluminum. I want you to make me a drawing of what you think steel might look inside and what you think aluminum might look inside.

Tell me about you drawing. (code)
STEEL: (clusters / shading / particles / spacing / movement
ALUM : (clusters / shading / particles / spacing / movement

____

____

____

c. DO YOU REMEMBER THE SHAPES WE SAW ON THE COMPUTER (show card with dots x dots) The red shapes always had dots which looked like this and the green shapes always had dots which looked like this.

DO THESE SHAPES REMIND YOU OF ANYTHING? Y/N
If yes: WHAT DO THEY REMIND YOU OF? ______________

DO YOU SEE ANY CONNECTIONS (SIMILARITIES) BETWEEN THESE SHAPES AND THE STEEL AND ALUMINUM PIECES? Y/N
If yes: WHAT KINDS OF CONNECTIONS? ______________

____

DO YOU THINK THAT THE STEEL AND ALUMINUM PIECES COULD LOOK LIKE THIS INSIDE? Y/N

If yes: WHICH WOULD BE THE STEEL? 1 dot cluster/3 dot cluster
WHICH WOULD BE THE AL? 1 dot/ 3 dot

WHY DO YOU THINK THAT? ______________

____

DO YOU THINK THIS HAS ANYTHING TO DO WITH THE FACT THAT STEEL IS A HEAVIER KIND OF STUFF THAN ALUMINUM? ______________________

____

If no: WHY DO YOU THINK THAT? ______________________

____

Wrap-up: SOME PEOPLE SAY THAT STEEL AND ALUMINUM PIECES ARE EACH MADE UP OF SMALL BITS OF STUFF, LIKE THE DOTS WE SAW ON THE COMPUTER. THINGS MADE OF HEAVIER KINDS OF STUFF HAVE MORE BITS IN A BUNCH, BUT THEY HAVE THE SAME SPACES BETWEEN BUNCHES. DOES THAT SEEM LIKE A GOOD EXPLANATION OF WHY STEEL IS A HEAVIER KIND OF STUFF THAN ALUMINUM TO YOU? ______________________

____