This paper posits a cognitive model for understanding and learning physics that is an alternative to the formal deductive system. Recent findings of Fuller, Karplus, Lawson, and others are cited in arguing against using a formal deductive approach to teaching physics. The model demonstrates types of action-oriented knowledge structures as they actually exist and operate in a person, including practical knowledge, qualitative physical models, concrete mathematical models and written symbol manipulation. A major aspect of the theory is that the ability to link together structures from these different domains is crucial to understanding a topic in physics. Each of these structures is defined and examples of student learning are given to support the theory. It is stated that some links are simple associations learned by rote while others are formed when one domain assimilates and interprets a structure from another domain. Finally, the paper provides a list of five pedagogical implications derived from this theory that provide a framework for discussing some interesting pedagogical problems. (CW)
There is a growing concern among researchers in the area of physics teaching that there has been an overemphasis on teaching introductory physics as a set of equations and principles, linked together in a formal, deductive system. It has been suggested that this approach may contribute to the problem of students learning physics by memorizing large numbers of formulas with little real understanding of the principles behind them.

This paper outlines an alternative to the formal deductive system as a model for the nature of understanding in physics.

At Berkeley, Fuller, Karplus, and Lawson argue in a recent article in Physics Today that many college students do not possess the formal reasoning skills required to learn physics directly in terms of a deductive system expressed in symbolic equations. Instead, students learn to manipulate formulas in a superficial manner using the less sophisticated forms of reasoning available to them. They argue for the development of introductory courses for this population of students, that "focus on the development of reasoning rather than the mastery of content." Larkin, also at Berkeley, compared the problem solving behavior of novices and experts. She finds different problem solving styles even when both novice and expert are familiar with the same set of equations. She proposes the hypothesis that experts have complex knowledge structures, in addition to principles in the form of equations, that allow them to apply relevant equations to a problem in a more organized fashion.

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In a report from the LOGu group at M.I.T., diSessa writes:

In the past, axiomntics or other formal systems have, principally by default, served as model representations of knowledge for pedagogical purposes. But while such systems which stress internal simplicity and coherence may serve useful roles for some purposes, they are not good models for learning. ... We must better take into account intuitive and other formally ill-formed knowledge that students already possess.

Thus diSessa also argues against using a formal and deductive approach to teaching introductory physics.

But if physics principles summarized in the form of symbolic equations do not by themselves constitute an effective understanding of physics, then the question arises: "What constitutes a more valid cognitive model of what it means to understand a topic in physics?" If the equations are not the only thing one needs to know, then what are the other key ingredients for understanding?

A Model for Understanding

The diagram on the following page is an attempt to model the several types of knowledge needed for a person to understand a topic in physics. The four large areas above the horizontal line represent four domains of internal knowledge structures in the person, while items below the line represent objects and events in the external world. Thus I want to model types of action-oriented knowledge structures as they actually exist and operate in a person; I do not want to assume, a priori, that these are equivalent to the body of knowledge statements—in the form of expressions that can be written down on paper—that comprise a formal exposition of the discipline of physics.

The characteristics of each domain can be introduced by referring
SOME TYPES OF KNOWLEDGE USED IN PHYSICS

INTERNAL KNOWLEDGE DOMAINS

PRACTICAL KNOWLEDGE

QUALITATIVE PHYSICAL MODELS

CONCRETE MATHEMATICAL MODELS

WRITTEN SYMBOL MANIPULATIONS

OBSERVATIONS OF EVENTS, PRACTICAL ACTIONS

DRAWINGS, VECTOR DIAGRAMS

COUNTING, MEASURING, GEOMETRIC CONSTRUCTION, GRAPHING

EQUATIONS, WRITTEN CALCULATIONS

OBJECTS, EVENTS, AND ACTIONS IN THE EXTERNAL WORLD
to the situation described in the following problem:

The electrical energy used by a battery powered water heater varies according to the formula:

$$E = \frac{\Delta t \cdot V^2}{R}$$

where:
- $E$ = energy used
- $\Delta t$ = time period of use
- $V$ = voltage supplied
- $R$ = resistance of the heater (assumed to be a constant for a given heating coil)

The heating coil is changed, so that $R$ is cut to $1/3$ of its original value. $V$ and $\Delta t$ are kept the same. What will be the size of the resulting effect on $E$? Or is this impossible to predict without knowing the specific values of $\Delta t$, $V$, and $R$? Give a short reason for your answer.

If a person were dealing with a real water heater, and had some knowledge about how to recognize one, turn it on, change the filament, etc., these would be examples of practical knowledge. These manipulations could also be performed mentally in a thought experiment with an imagined water heater in the absence of a real one.

An example of a knowledge structure in the qualitative physical models domain would be a conception of electrons being "pushed" through the heating element and causing the element to heat up by crashing into its molecules. One could represent and manipulate this model in the real world by using drawings or diagrams. These models are often action-oriented and causal -- in them are embedded anticipations like: "If the electrons are pushed harder, they'll come into the element faster, they'll hit the..."
molecules harder, and more heat will be produced.²⁻⁵

At this point one can already begin to model what one means by one level of understanding. If "pushing the electrons harder" is connected mentally with a practical knowledge structure for "how to turn the voltage up" on the real water heater, then one has a qualitative model for understanding that practical aspect of the heater. Notice that one might have this understanding without using any quantitative conceptions.

One crosses into the concrete mathematical models domain when using a conception like "the energy released is probably proportional to the push on the electrons; if I double the push, I'll double the energy released." This kind of mathematical model relating scaled variables via the concept of proportionality can be represented in terms of operations on sets of objects or operations on measured line segments, or in a graph. There are many species of idealized, concrete, mathematical objects used in mathematical models, such as the length of a line segment representing the magnitude of a certain physical variable, or the cutting of an object of a certain size into a certain number of equal parts representing a division relationship between two variables. These conceptions can become activated to represent quantitative aspects of the way the water heater behaves.

Finally, a knowledge of memorized equations and rules of algebra and arithmetic resides in the symbol manipulations domain. An equation can itself be treated as an object capable of being transformed via the rules of algebra and related to other equations by knowledge structures in this domain. For example, the equations

\[ \text{Energy} = \frac{At \cdot V^2}{R} \quad \text{and} \quad \text{Power} = \frac{\text{Energy}}{At} \]

could be combined algebraically to yield an expression for the power used
by the heater. Given the two formulas this could be (and in courses apparently often is) done using symbol manipulation rules without making any connection to the other types of knowledge mentioned—without an appreciation for any underlying meaning.

As another example, consider the solution one sophomore, Student A, gave for the water heater problem. He wrote:

"E = \frac{tv^2}{R}

if R + 1/3 R, then \( E + \frac{tv^2}{1/3R} = \frac{3tv^2}{R} \)

E becomes bigger times three."

One can account for this student's behavior by assuming that the only knowledge structures participating in the solution are symbol manipulation structures. In contrast, another sophomore, Student B, said: "The energy would probably be more because if you're cutting down the resistance by 1/3, the energy is going to be able to flow in more freely -- it'll go in faster -- so you should get 3 times the energy." These solutions are interesting because they indicate the use of two entirely different types of knowledge to solve the same problem.

Student A uses a knowledge structure in the symbol manipulation domain. He knows that the equality can be conserved when a variable is changed in an equation by changing the other side of the equation in the same way. This method does not depend on the qualitative situation portrayed in the problem.

Student B uses his knowledge of a qualitative physical model for the situation. He imagines a reduction in the resistance causing an increase in energy flow: "... the energy is going to be able to flow in more
freely -- it'll go in faster ..." The second student's method does depend on the qualitative situation portrayed.

The symbol manipulation method is useful (and highly efficient) if the student is working from a given formula or set of formulas. However, the physical model is essential when one is attempting to construct a formula or to select an appropriate formula for a new situation. This suggests that someone who can bring both kinds of knowledge to bear on problems understands the subject more deeply than someone who uses either method alone.

Student B first predicts that more energy will be used, then predicts that three times more energy will flow into the bulb. One can account for this behavior by assuming that he also uses a conception of an inverse proportion in his mathematical models domain. Thus he is able to link together structures from at least two domains in bringing them to bear on the problem.

A major aspect of the theory being proposed here is that the ability to link together structures from these different domains is crucial to the understanding of a topic in physics. Some of these links are simply associations learned by rote -- such as the association of a quantity in the mathematical models domain with a particular letter used to symbolize it in the symbol manipulation domain. Other, more significant links are formed when a structure in one domain assimilates a structure in another domain and provides an interpretation for it. An example of such a link was given earlier, where a qualitative physical model involving a conception of "pushing the electrons harder" assimilated a practical knowledge structure for "how to turn the voltage up on the heater." These links are
what cause a model at one level to "make sense" as an interpretation of knowledge at another level.

In terms of this model, then, "understanding energy use in the water heater" consists of a knowledge of symbol manipulations that can be performed on the formula, \( E = \frac{\Delta t \cdot V^2}{R} \), connected to knowledge structures in the other three domains -- concrete mathematical models, a qualitative physical model, and practical operations one could perform on a real water heater.

**Pedagogical Implications**

This theory of understanding involving interacting knowledge domains is supported by a large number of observations made by the author while tutoring physics students. The theory is undoubtedly oversimplified, and many detailed analyses of clinical interviews need to be conducted in order to refine the theory and establish its validity. But it does provide a framework for discussing several interesting pedagogical problems:

(1) To return to the issue of whether the formal exposition of physics content is a sufficient model for what the physics student needs to learn, one can see that formal expositions emphasize heavily the use of written formulas in the symbol manipulations domain. The danger here is that a student may get "stuck" in the symbol manipulation mode -- he may learn a certain set of equations, but not understand their meaningful interpretation in the form of physical models, mathematical models, or practical actions. Making sure that these connections are made is a worthwhile goal and a real pedagogical challenge.
(2) At the University of Washington, George Monk and others have been developing the student's ability to translate freely between modes of describing physical events: from an equation to a graph to a picture to descriptions of a situation in English and back again. This appears to be a promising approach to increasing the student's ability to make connections between knowledge domains.

(3) Knowing a formula is not the same as knowing when to use it. How does one determine when a formula is applicable to a certain practical situation? The ability to do this is crucial for being able to apply one's knowledge of physics to problems in the real world. It is suggested here that qualitative physical models can play a critical role in providing the connection between practical situations and appropriate equations.

(4) One way to increase the emphasis on understanding in a course is to develop the student's ability to answer 'why' questions like: "Why does the energy used depend on the voltage applied to the water heater?" Satisfying answers to these questions often involve qualitative physical models.

(5) Knowledge structures in the qualitative physical models domain can be formal (developed in the school setting) or intuitive. Intuitive conceptions students enter courses with can be deeply seated and difficult to change. Unless a course puts emphasis on dealing with the physical models domain and takes into account the student's intuitions there, the student may have great difficulty in attaching physical meaning to the equation he is learning. This presents another challenging direction for course improvement.
Notes


4. The four knowledge domains discussed relate to the levels of abstraction discussed in:

5. Similar kinds of direction-of-change relations between ordinal concepts like acceleration, velocity and momentum would also be included in this domain. These are distinguished from practical knowledge by the fact that they focus on particular features of a system selected for purposes of analysis.