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ABSTRACT

Third of a four-part series, this article presents an instructional plan for teaching the place value notation concept, base ten, to preschool and elementary school children. After a brief review of the place value notation concept, contents describe an integrated and progressive series of four game-like activities which are fundamentally overt organizational and naming schemes. The activities are termed "child's-play constructions," because they are transparently simple and are designed to be conducted in a spirit and attitude of constructive play. The activities are entitled: (1) The Cup Game; (2) The Ten-Count Game; (3) Number Towers; and (4) The Flip-card Counter, Base Ten. Each construction is designed to display and confirm particular components of the place value notation concept, base ten, as it is used when children exercise initiative and control in performing patterned interactions with the materials provided. Guidelines for teachers implementing the games in classrooms are provided in the discussion. Concluding remarks restate requirements for an instructional plan to teach the place value notation concept to preschool and elementary school children and indicate means by which the child's play method fulfills the requirements. (RH)

III Making the Place Value Notation Concept Child's Play: Constructions for Displaying and Confirming Component Elements of the Place Value Notation Concept with Elementary and Preschool Children

ABSTRACT

Third of a four-part series this article presents the author's instructional plan for teaching the place value notation concept, base, ten, to elementary and preschool children. The plan is patterned after the Geneva research model, which is discussed in articles #1 and #2. Bentley coins the name child's-play constructions, because: (1) the constructions are transparently simple; and (2) they are to be conducted in a spirit and attitude of constructive play. The four child's-play constructions form a progressive and interrelated series of organizational and naming schemes. Bentley's conjecture is that children will internalize the schemes naturally and spontaneously as a consequence of volitional opportunities to practice and perfect them.

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III Making the Place Value Notation Concept Child's Play: Constructions for Displaying and Confirming Component Elements of the Place Value Notation Concept with Elementary and Preschool Children

A. Introduction

This article is the third in a four-part series. The first article contained a synopsis of a report of Geneva research (1979, Sinclair, Unpublished) and my interpretation of it as an educator. The report described the onset and evolution of organizing procedures spontaneously practiced and perfected by ten to twenty-four month old children. The second article presented my analysis of the descriptive data. My objective in performing the analysis was to derive pedagogical insights specifically applicable to the task of communicating the place value notation concept to elementary and preschool children. The purpose of this paper is to describe how I applied the insights.

Accordingly I describe four gamelike activities. To these I have ascribed a techanical name, child's-play constructions, because: (1) the constructions are transparently simple; and (2) they are designed to be conducted in a spirit and attitude of constructive play. The activities are entitled: (1) The Cup Game; (2) The Ten-Count Game; (3) Number Towers; (4) The Flip-card Counter, Base Ten. Each construction is designed to display and confirm particular components of the place value notation concept, base ten, as children exercise initiative and control in performing patterned interactions with the

materials provided. The teacher's role is that of friendly, interactive facilitator and observer.

The four child's-play constructions are overt organizational and naming schemes. My pedagogical conjecture is that children will internalize these schemes naturally and spontaneously as a consequence of volitional opportunities to practice and perfect them. The first child's-play construction is supportively related to the second; the first and second are supportively related to the third; the first, second, and third fully undergird and support the fourth. The fourth is designed as an integrative, representational experience which unites the concepts of the concrete analogies and the naming schemes in a culminating display of continuous, integrated harmony.

My reasons for developing this interrelated and progressive series of child's-play constructions are the subject of the fourth and final article of this series.

B. A Brief Analysis of the Place Value Notation Concept, Base Ten

The place value notation concept can be thought of as a method of organizing number and naming successive organizations. The naming we do in the process of organization is called counting when we do it orally; for each successive count there is a unique oral name. Counting, when it is written, is called notation. In order to create a concise visual representation of number we make use of just ten different digit symbols and a rule called the place value convention. This convention

accords to digits a different value based on their horizontal position relative to each other. For example:

1.	.1
10.	.01
100.	.001
1000.	.0001
10000.	.00001

The digits to the left of the period represent one, ten, one hundred, one thousand, and ten thousand. The same digit symbols arranged as illustrated to the right of the period represent one tenth, one hundredth, one thousandth, one ten thousandth, one one hundred thousandth.

The place value convention is extremely puzzling to children. Their befuddlement over it issues in a predictable pattern of errors (1), My particular concern is with the introduction of this concept to children. Therefore the child's-play constructions are designed to address the place value notation concept as it applies to whole number numerosities. My purpose in contriving the constructions is to clarify the puzzling aspects of place value notation and enable children fully to understand, enjoy and accurately use the place value convention, base ten.

C. Overview of the Four Child's-play Constructions

The first construction (The Cup Game) involves children in imposing structural organization (1:10 and 1:10:100) on 100 paper cups. The game involves no counting. I have played the entire game in a single session with children as young as three years.

The second construction (The Ten-count Game) involves organizing and counting from one to ten by ones and from ten to one hundred by tens. Tokens are placed in or emptied from containers while appropriate oral counting is simultaneously conducted. I have played this game also in a single session with children as young as three years.

The third construction (Number Towers) uses 1000 paper cups. It is a simple extension of the cup game but adds a physical constraint. This addition is necessary in order to maintain a harmony of analogy between the concrete model and place value notation. This vital physical constraint assures that construction takes place in a leftward, horizontal direction. To accomplish this I simply arrange the setup of materials in a corner of the play space so that construction can freely occur, but only in a leftward direction. I have played this game with children three to eight years old. The number towers model cannot be reasonably constructed in a single session.

The fourth construction (The Flip-card Counter, Base Ten) involves the children in imposing organization on papers and applying number symbols to the papers. The papers are pierced and set in fixed positions relative to each other and to the child-builder's own body. The steps of procedure in this construction as well as the others have been planned with a specific objective in view: namely to create concrete displays of the component elements and relations of the place value notation concept which are valid to children and verifiable by them.

D. Child's-play Construction #1: The Cup Game

The Geneva research team presented an environment of manipulable objects limited in number and kinds. The three sets of objects were always presented in disarray. Likenesses and differences of form and structure were noticed by the children, and these became the basis of identity patterns which were the first step of their organizational procedures. Similarly in the cup game children are presented with a limited number of forms and manipulable objects. While the 100 paper cups can be arranged to give concrete display to 1:10 ratio structure and 1:10:100 ratio progression, this structural organization is not manifest when the cups are presented.

The Geneva children handled objects one after another (sequence A). They placed objects one by one in containers until the containers were full; then they removed the objects one by one until the containers were empty (sequence B). They did this over and over again. These were the first procedures of the Geneva children. They are also the first procedures of the cup game. To the basic filling-emptying procedure the cup game adds two simple strategies. These simple additions make it possible to give obvious display to the 1:10 ratio structure and 1:10:100 ratio progression.

Following are steps of procedure which I have used effectively in playing the cup game with children as young as three years. Notes of explanation and rationale are enclosed in brackets.

1. In a basket or interesting carrying case present 100 paper cups randomly stacked. (Stacked cups appear as a single form. Step 2 promotes individualization of the cups. Providing a rug with a border along which to place individual cups enhances interest in the task and helps to guide the children's focal attention.)

2. Model the procedure and invite the children to join you in setting overturned cups along the border of a small rug. When all the cups have been placed contiguously along the border, proceed to step 3. (A contiguous lineup of cups also appears as a single form. Invite the children to stand up and admire the form created. They often have called it a "house" and the breaks with no cups, they have called "doors".)

3. Using masking tape create a triangular form in the center of the space framed by the contiguous lineup of cups. The triangle should be of a size to hold exactly ten overturned cups within its borders.

4. Model the filling procedure. One by one take cups from the lineup and place them within the triangular frame until it is full.

5. Model the emptying procedure. One by one take cups from within the triangular frame; stack the cups one on top of the other as you remove them.

6. Set the stack outside the triangular frame, and invite a child to take cups from the line up, fill the frame, and then remove and stack the cups one after another. Invite the child to place his stack beside (but not on top of) the first stack.

7. Continue the filling-emptying-stacking procedure until all the cups are arrayed in stacks outside the triangular frame. Present short strips of paper (9 are needed). Demonstrate draping a single strip over the top of a stack before bringing a second stack down on top of it. (The strip should protrude below the bottom lip edge of the second stack. This preserves the individual integrity of the first stack. This is essential as it is the first step in creating a concrete display of the 1:10:100 ratio progression.)

8. Assist the children as needed to drape the topmost stack before bringing down another stack on top of it.

9. Continue the draping-stacking procedure until all the stacks have been consolidated to form a single tall stack. The tall stack may be placed within the triangular frame.

10. Completion of the tall stack signals the conclusion of the game. The stack may be dismantled or left intact.

E. Child's-play Construction #2: The Ten-count Game

The ten-count game uses the one by one filling procedure (sequence B) with one important variation: the child, not the size of the container, determines when a container is <u>full</u>.

Ten paper cups, 100 small tokens (e.g. hickory nuts), and 10 poker chips (or the equivalent) are needed for this game. Steps of procedure follow:

1. Select a cup and count ten tokens into it. Suggest that you and the children consider a cup full when there are ten tokens in it.

2. Place a poker chip in the first cup on top of the ten tokens to indicate that the cup is full.

3. Invite the child (or children) to count tokens, fill cups, and place the poker chip lids as the count of ten is reached.

4. When the supply of cups, lids, and tokens has been exhausted, suggest that you and the children count lids. Introduce the ten-to-one hundred counting scheme. Remove the lids one by one in coordination with the enunciation of the counting scheme: ten, twenty, thirty, forty and so forth.

5. Repeat the ten to one hundred counting scheme as you replace the lids.

6. Repeat the pattern two or three times; encourage the children to chime in as they pick up on the pattern of words.

7. Finally, repeat the scheme to conclude the game. As you remove each lid, pour the tokens into the original container and count the emptying by tens.

F. Child's-play Construction #3: Number Towers

Number towers is a simple extension of the cup game. It carries the game further in order to give concrete display to the ratio progression 1:10:100:1000. The basic strategies are the same as those of the cup game. In order to maintain a harmony of analogy with place value notation, however, construction must take place in a leftward horizontal direction. Therefore, set up in a corner of the play space so that construction can freely occur, but only in a leftward direction.

The following materials are needed for the number towers construction: 1000 paper cups; 4 triangular frames of a size to hold exactly 10 overturned paper cups; 90 strips of paper (for maintaining the integrity of the ten-sized stacks); 10 clear plastic sheaths (to wrap the tall stacks of one hundred cups); masking tape. Steps of procedure follow:

1. In the righthand corner of the play space create a triangular frame (using masking tape) of a size to hold exactly ten overturned paper cups.

2. Place paper cups within the frame one by one until it is full.

3. Remove the cups one by one, stacking them as you do so.

4. As soon as the cups have been stacked, immediately provide a triangular frame contiguously to the left of the first. Model placement of the stack in the second frame.

5. Repeat steps 2 - 4 until second frame is full.

6. Consolidate the stacks in the second frame as in steps 8 and 9 of the cup game. Immediately provide a third frame contiguously to the left of the second; and model or invite placement of the tall stack in the third frame.

7. Present a clear plastic sheath and assist children to wrap the sheath snugly around the tall stack. Secure sheathing with masking tape. (This preserves the integrity of the tall stack of stacks, guards against accidental dissolution by toppling, and gives concrete display to the ratio progression 1:10:100)

8. Repeat steps 2 - 7 until the third frame is full.

9. Immediately provide a fourth frame contiguously left of the third. Lay the sheathed stacks on the floor and tape them together to form a single tower of stacks.

10. Erect the tall, tall tower if you can. Or make a representation of it using adding machine tape, and erect that while leaving the concrete proof prone on the floor. (Second graders enjoy making a poster to take home using adding machine tape and crayons. In the light of these questions: How tall is ten? How tall is one hundred? How tall is one thousand?--the coils of adding machine tape make an impressive display.)

(At this point the supply of paper cups has been exhausted. The final tower gives concrete display to 1:10 ratio structure and 1:10:100:1000 ratio progression. All the way along in the construction process double definition is displayed and confirmed at critical points of transition. For example: ten ones is the exact equivalent of one ten; ten tens is the exact equivalent of one hundred; ten hundreds is the exact equivalent of one thousand. There is a continuous harmony of analogy between place value notation, base ten, and the number towers model. The construction of this model gives children a solid basis of practical experience which can serve them reliably as procedural tools when they are faced later on with probl ms of interpretation, application, and computation involving place value notation.)

G. Child's-play Construction #4: The Flip-card Counter, Base Ten

For the Geneva children Sequence F was a culminating event in which all the identity patterns previously practiced were perfectly reviewed in a single event of continuous succession. The flip-card counter is designed to fulfill a similar role for children in relation to the identity patterns which have given display and confirmation to the component relations of the place value notation concept, base ten.

Two versions of the flip-card counter can be made. The primary version is constructed of card stock and secured with a plastic binding. It can be used to display place value notation in coordination with oral counting by ones or tens to one hundred. I normally introduce this version after the ten-count game so young children can wee what counting looks like when it is written down. Five year olds enjoy manipulating this counter in coordination with one-to-one oral counting.

The advanced version of the flip-card counter is not introduced until after construction of the number towers model. The advanced version can be constructed by children using lighter stock paper. The advanced version can be used in coordination with oral counting to 9,999 by ones, tens, hundreds, or thousands.

Errorless manipulation of the flip-card counter in coordination with one-to-one correspondent counting displays place value notation, base ten, accurately and continuously. At the same time it also tracks step by step and summarizes the entire construction of the number towers model. After the number

towers materials have been put away, the completed flip-card counter remains a permanent summary representation of the number towers construction procedures.

The following supplies are needed to make the advanced version of the flip-card counter, base ten. 40 papers approximately 1½ x 2½ inches; a stamp pad and set of numeral stamps (0 - 9); 2 cover papers approximately 3 x 6 inches; 4 openable rings (I use small chicken leg bands); a piercing device which can be safely used by young children; glue.

To arouse curiosity and stimulate interest in construction you may wish to ask the children if they would like to make a counting machine. I've always received an affirmative response to this suggestion. Then introduce the materials and the following order of procedures:

1. Present the two 3 x 6 cover papers, four 1½ x 2½ papers, and the piercing tool.

2. Demonstrate centering the 1½ dimension of a small paper on the piercing element; depress the cover mechanism to pierce the paper.

3. Remove the pierced paper stamp it with a zero, and place it on the righthand side of the cover papers (Stack and position the cover papers so that the four small papers can be positioned in a horizontal row.)

4. Follow the same procedure with the second, third, and fourth papers. Place the second to the left of the first, the third to the left of the second, and the fourth to the left of the third.

5. Arrange the four papers in an even horizontal row. Glue them in place.

7. Pierce both cover papers through the holes in the glued down papers. Set the top cover aside.

8. Present the remaining 1½ x 2½ papers. (Now the rules of the ten-count game apply.) Invite the child to position the papers one at a time, depress the cover mechanism, and count each paper. After each count, select the proper numeral and stamp the pierced paper.

9. After stamping the 9, remove the stack, point to the glued down zero in the rightmost position and announce "ten". Set the stamped 1 - 9 papers on top of that zero.

10. Repeat this procedure until the child has pierced and stamped a second set of 1 - 9 papers. Counting can be done by ones or tens. Let the child choose. Place the second set of numerals on top of the zero to the left of the first stack.

11. Repeat this procedure for the third and fourth set of papers.

12. Place the top cover on top of the four stacks of numerals and insert a ring through each set of holes.

13. When each position has been secured between the two covers, write the name of the counting machine on the outside.

14. Open the front cover. On the inside which faces the numerals 9999 write these words: This is not the beginning; this is the end.

15. Close the front cover and turn the flip-card counter over. On the outside of the back cover write these words:

This is not the end.

16. On the inside of the back cover below the zeros write these words: This is the beginning.

(Writing these words on the covers of the counter provides children a comical and dependable reminder of how and where to begin the counting-manipulating coordination.)

The flip-card counter is now complete and the counting demonstration modeled . The steps of procedure are listed below:

1. Set the counter to display the four zeros.

2. Beginning at the rightmost position, flip forward one card for each count, 1 - 9.

3. Just prior to the count of ten, make ready for a two-handed coordination. As the count of ten is enunciated, flip back the cards in the units position while flipping forward toward the body the first card in the tens position. This will display 0010 as the count of ten is enunciated.

(As the children emptied the triangular frame of single cups, so they also empty the counter of moveable papers in the units position. As the stacked cups were set as a singular form in the second position, so the numeral "1" stands to signify the singular stack-form in the second position. The zero in the units position signifies the emptied rightmost frame. The numeral 1 in the contiguous left position signifies the stacked form of ten cups. The two zeros to the left of the numeral 1 signify that the analogical positions in the number towers model are empty of cups.)

4. Count continues as cards in the units position are flipped forward. Oral count is pronounced as the place value numerals are displayed. For example:

0010 (ten)
0011 (eleven)
0012 (twelve)
0013 (thirteen)
...
0019 (nineteen)

5. Just prior to the count of 20 the papers in the units position are again flipped back to expose the zero while the second paper in the tens position is flipped forward.

0020

6. Coordinated manipulation and oral counting can be conducted by ones, tens, hundreds, or thousands.

(The flip-card counter performs a summary representation of the number towers and ten-count game procedures. It is both comprehensive and integrative, for every step of procedure in the concrete models has an analogical counterpart in the manipulations and displays of the flip-card counter.)

H. Summary and Conclusion

I have described an integrated and progressive series of four child's-play constructions. These present the component elements and relations of the place value notation concept, base ten for notice and confirmation. The instructional plan is patterned after the Geneva model. Therefore, construction of the concrete analogies uses forms of action and rules of

practice which are similar to those implemented spontaneously by the ten to twenty-four montn old Geneva children. The forms of action used are: (1) individualizing of objects by handling them one after another; (2) one by one filling and emptying of containers; (3) nesting of objects; (4) establishment of identity patterns; (5) exhaustive imitation of identity patterns.

In article #2 I conjectured requirements for an instructional plan to address the place value notation concept for elementary and preschool children after the manner of the Geneva model. These requirements are restated below along with the means which the child's-play method uses to fulfill them:

(1) Environmental setups must be equipped so that component relations of the place value notation concept can be given obvious concrete display. Using paper cups the 1:1, 1:10 and 1:10:100 relations are given obvious display in the cup game; obvious display is extended to include 1:10:100:1000 in the number towers model. Appropriate coordination of counting-naming schemes is given obvious display in the ten-count game and manipulation of the flip-card counter.

(2) Environmental setups must be staged so as (a) to stimulate children's interest and (b) to prompt and facilitate child-motivated, child-directed demonstrations of the component relations of the place value notation concept. My informal testing of the child's-play constructions indicates that children 3 - 8 years of age enjoy imposing the suggested forms of action on paper cups which create the obvious concrete displays.

The counting coordinations of the ten-count game are a stimulating challenge to three year olds and are enjoyed by the older children also. Five year old Chris was excited and energized by the tricky coordinations of the primary and advanced versions of the flip-card counter. He also enjoyed constructing the advanced version.

(3) Equipping and staging should be planned so that the two principles (one:one and imitation) will function to guide the children's focal attention and motivate pertinent acts of initiative which reveal specific component relations of the place value notation concept. The cup game and the number towers model give concrete display to the 1:10 ratio structure and progression which underlie place value notation. The procedures and structures of these games rely exclusively upon these two principles. The coordinations required by the flip-card counter require continuous implementation of the one:one principle and specific applications of the principle of imitation. The ten-count game introduces rudimentary counting schemes in coordination (one:one) with specific forms of action. It is well to note that all that is needed beyond the one to ten and ten to one hundred schemes is the eleven to twenty scheme and the category words such as thousand, million, billion and so forth. The flip-card counter demonstrates place value notation in coordination with the counting patterns as far as 9,999.

(4) Revelatory procedures must conform to the six rules of practice which characterized the effectual method of the Geneva children. The rules of practice are listed in article #2.

Rules #1 - #3 are used in the cup game. Rules #1 - #5 are used in the number towers model. Rule #6 governs the accurate use of the flip-card counter.

(5) Procedures conducted must lead toward and prepare the children to perform independently a comprehensive review of the component relations of the place value notation concept. In describing the number towers model I explained the necessity, for example, of the physical constraint which assures that construction of the number towers proceeds in a leftward horizontal direction. Every feature of the child's-play constructions has been contrived in order to prepare children to conduct a comprehensive review of all the component relations of the place value notation concept which they have previously displayed and confirmed by concrete analogy. The device for facilitating this comprehensive and integrative review is the flip-card counter, base ten. Only objective testing of the child's-play method can prove the adequacy of the preparation. Such testing is beyond the scope of this paper. I would welcome opportunity to cooperate with researchers and schools (public or private) in order to test the sufficiency of the child's-play constructions.

(6) The culminating review must demonstrate the integrated harmony of all the component relations which have previously been displayed and confirmed. The flip-card counter fulfills this requirement by demonstrating the continuous, harmonious integration of one-to-one oral counting, imposition of the 1:10 ratio structure and progression, and coordinated display of

place value notation, base ten.

My pedagogical conjecture is a two stage projection: (1) The progressive series of child's-play constructions which I have described will be internalized as schemes by children who are given volitional opportunity to practice and perfect them. (2) The internalized schemes will function for children as effective procedural tools which will enable them to comprehend and accurately use place value notation, base ten.