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ABSTRACT

These materials are intended to provide meaningful mathematical experiences for pre-algebra students. These experiences emphasize the development of computational skills, mathematical concepts, and problem-solving techniques. This bulletin may be used as the basis for the second term of a one-year course, or for the second year of a two-year course. The complete course is divided into 12 basic units, the last six of which are in this bulletin. These are: (1) solving simple equations; (2) tables, graphs and coordinate geometry; (3) banking; (4) areas; (5) indirect measure and scaling; and (6) solid geometry. Each individual lesson includes the aim, performance objectives, essential vocabulary, specific teaching suggestions, applications, and a summary. The mathematical skills and concepts are arranged in a sequential order allowing for the spiraling of skills among units and semesters. (PK)

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FUNDAMENTALS OF MATHEMATICS PART II

Preparing Students for the RCT

pyramid
solid figures
Indirect measurement
$2X + 7 = 15$
Product of the extremes
Find the volume of a rectangular prism. Distinguish between prisms and cylinders.
$A = lw$
Interest rate $I = Prt$
Meaning of ratio and scale
Read the scale on a pictograph
Calculate the sales tax
7% effective annual yield
$V = lwh$
circle graph

Experimental Edition, December, 1981

BOARD OF EDUCATION OF THE CITY OF NEW YORK
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Foreword

This edition of Fundamentals of Mathematics--Part II is designed to provide appropriate mathematical experiences in the development of computational skills, mathematical concepts, and problem-solving techniques for pre-algebra students. For some students, this course will contribute to later success in algebra; for others, it represents part of a program in everyday mathematical literacy.

This bulletin may be used either as the basis for the second term of a one-year course, or for the second year of a two-year course. The complete course is divided into 12 basic units, the last six of which are in this bulletin. Each individual lesson includes:

Aim
Performance Objectives
Essential Vocabulary
Specific Teaching Suggestions
Applications
Summary

The mathematical skills and concepts are arranged in sequential order allowing for the spiraling of skills within units and between semesters. Challenging and practical mathematical experiences are provided to stimulate student interest and to help overcome arithmetic deficiencies students may have.

The units in this new edition are consistent with those in the New York State Bulletin General High School Mathematics published August, 1978, the first seven of which make up the curriculum on which the New York State Regents Competency examination is based. A cross-reference between the topics of the two sets of units is included. There is a strong emphasis on the skills, concepts and problem-solving techniques which are measured by the Regents Competency Test. The actual Regents Competency items are marked by asterisks.

Teachers and supervisors using the curriculum bulletin are urged to fill out the questionnaire at the back of the book and return it to the Office of Curriculum Development and Support by the appointed date.

Charlotte Frank

CHARLOTTE FRANK
Executive Director

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Irwin Kaufman, Director of Mathematics, and Marvin Itzkowitz, Assistant to the Director of Mathematics, provided the overall superivsion.

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Contents

UNIT VII. Solving Simple Equations

LESSON 1

AIM: To solve open sentences by trial and error

PERFORMANCE OBJECTIVES: Students will be able to:

- distinguish between a mathematical phrase and a mathematical sentence.
- state whether or not a sentence is an open sentence.
- solve an open sentence using trial and error.

VOCABULARY:

New terms: phrase, open sentence, solution

Review: substitution, variable, verb

CHALLENGE PROBLEM:

Write in symbols the expression "any number increased by four."

DEVELOPMENT:

1. Discuss the challenge problem. Recall that a variable is a symbol used to represent any number. The expression $x + 4$ is called a phrase. Review the definition of phrase meaning a part of a sentence.

2. Evaluate the phrase for several values of the variable x including negative values ($x = 0, 1, 3, 4, -7$) Review substitution as the means of evaluating phrases. (See Unit I, Lesson 2).

3. Have students consider the problem: For what value of x will the phrase $x + 4$ be equal to -3? Students should be able to determine from item #2 that -7 is the value of x required.

4a. Translate the problem above into symbols:

$$x + 4 = -3$$

Have students recog..ze the difference between the phrase "$x + 4$" and the sentence "$x + 4 = -3$." Note that the " = " symbol represents the verb of the mathematical sentence.

4b. The sentence above is called an open sentence because the value of x is open for discussion.

5. Do Application A.

6. Pose the problem: "For what value of x, will x increased by 4 be equal to -5?"

   a. Have students translate this sentence into symbols: $x + 4 = -5$.

b. Have students determine that none of the values used previously resulted in $x + 4$ being equal to -5. This should be demonstrated as follows:

When	$x = 0$	$0 + 4 = 4$
	$x = 1$	$1 + 4 = 5$
	$\vdots$	$\vdots$
	$x = -7$	$-7 + 4 = -3$

c. Allow students to discover by <u>trial and error</u> that when $x = -9$, $-9 + 4 = -5$. Therefore, -9 is the only value of x which makes the sentence $x + 4 = -5$ a true sentence.

7. Define the <u>solution(s)</u> of an open sentence as the value(s) of the variable which make the open sentence true.

8. Do Applications B and C.

APPLICATIONS:

A. 1. Indicate by the letter "P" or "S" whether each item below is a <u>P</u>hrase or <u>S</u>entence:

a. $8 + 7 = 15$

b. $6 + 2 = 10 - 3$

c. $\frac{18}{9}$

d. the product of 4 and 6

e. the sum of 12 and 4 is less than the product of 5 and 6

f. the capital of New York State

g. A meter is the basic unit of length in the metric system

h. five meters

2. Determine which of the following are examples of open sentences and explain why they are open sentences.

a. He is the president of the United States.

b. Washington, D.C. is the capital of the United States.

c. The capital of New York State is New York City.

d. $10\ (25) = 250$

e. $10\ x = 70$

f. $5 + 2 = 10$

g. $4 - y = -6$

B. Determine whether the value of x, in the parentheses, is or is not a solution of the open sentence.

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1. $4x = 24$ $(x = 6)$

2. $2x + 7 = 15$ $(x = 3)$

3. $\frac{1}{3}n = 9$ $(n = 3)$

4. $30 = s - 50$ $(s = 80)$

5. $\frac{1}{10}x = 2$ $(x = 20)$

C. Using trial and error, find the solution for each open sentence below:

1. $a + 4 = 4$
2. $b + 4 = 0$
3. $x - 3 = -2$
4. $z - 8 = -9$
5. $7x = 0$
6. $7x = 28$
7. $x + \frac{3}{4} = 1$

SUMMARY:

Review new vocabulary.

CLIFF-HANGER:

If x is a positive integer less than 10, for what values of x is $x + 4 > 8$?

LESSON 2

AIM: To solve equations and inequalities given replacement sets

PERFORMANCE OBJECTIVES: Students will be able to:

- categorize a sentence as an equation or an inequality.
- state whether a replacement for a variable makes an open sentence true or false.
- use a given replacement set to find the solution set of an equation or inequality by trial and error.

VOCABULARY:

New terms: equation, inequality, replacement set, statement

Review: solution

CHALLENGE PROBLEM:

Explain the difference between the following open sentences:

a) $x + 4 = 8$

b) $x + 4 > 8$

DEVELOPMENT:

1. Discuss the challenge problem. Elicit from students that a) has an " = " sign whereas b) has a symbol of inequality.

   Define: A sentence using the equality symbol is called an equation. A sentence which uses a symbol of inequality ($<$ or $>$) is called an inequality.

2. Consider the problem: If x is a positive integer less than 10, for what values of x is $x + 4 > 8$?

   a. Have students list positive integers less than 10. These values comprise the replacement set. The replacement set is all possible numbers which may be substituted for the variable in the open sentence. $\{1, 2, \ldots, 9\}$

   b. Have students substitute each value from the replacement set for the variable in the open sentence $x + 4 > 8$ and determine whether the resulting statement is true or false. (Note: Once a replacement has been made for the variable, the sentence is no longer open. It becomes a statement which can be judged true or false).

When $x = 1$	$1 + 4 > 8$	false
$\vdots$		
$x = 9$	$9 + 4 > 8$	true

c. Have students list all values of x from the replacement set which make the statement true. $x = 5, 6, 7, 8, 9$ These numbers comprise the solution of the inequality from the given replacement set.

3. Do applications.

APPLICATIONS:

A. Using integers from - 5 to + 5, find the solutions to the following sentences. (If there are no solutions, write "none.")

1. $x + 2 = 5$
2. $x + 2 > 5$
3. $x + 2 = 10$
4. $y - 3 = -6$
5. $y - 3 = 2$
6. $y - 3 < -1$

B. Solve (find the solutions for) each of the following open sentences using the given replacement set.

1.	$6x = 24$	$\{1, 3, 4, 6\}$
2.	$\frac{y}{3} > 2$	$\{4, 5, 6, 7, 8\}$
3.	$x + \frac{1}{4} = 7$	$\{6\frac{1}{4}, 6\frac{1}{2}, 6\frac{3}{4}, 7\frac{1}{4}\}$
4.	$3a + 7 < 11$	$\{0, 1, 2, 3\}$

SUMMARY:

Have students explain the difference between an equation and an inequality.

Of the open sentences discussed in this lesson, which type may have several values as a solution?

CLIFF-HANGER:

Solve the equation:

$x + 327 = 714$

LESSON 3

AIM: To solve equations using addition and subtraction

PERFORMANCE OBJECTIVES: Students will be able to:

- use the appropriate inverse operation and the addition and subtraction properties of equality to find the solution to equations of the form $x + a = b$ and $x - a = b$ where a and b are integers.
- check the solution of an equation.
- state that addition and subtraction are inverse operations.

VOCABULARY.

New terms: addition property of equality, subtraction property of equality, inverse operation, check

CHALLENGE PROBLEM:

Find the solution to the equation $x - 25 = 47$.

DEVELOPMENT:

1. Elicit from the class that the challenge problem may not be solved easily by trial and error. Therefore, a technique must be developed. Introduce the addition property of equality in the following manner. (Note: A double-pan balance or some other similar visual aid may be helpful in developing this concept.)

   a. Have students consider the statement $25 - 10 = 15$. Elicit that this is a true statement.

      If 5 (or any other number) is added to both sides of this statement, what can be said about the resulting statement? [$25 - 10 + 5 = 15 + 5$ is also true.]

      Similarly, if 10 is added to both sides of the statement, the resulting statement remains true. $25 - 10 + 10 = 15 + 10$ is also true.

      Have students generalize that if the same number is added to both sides of an equation, the resulting equation will have the same solutions as the original. Explain to the class that this is called the addition property of equality.

   b. Demonstrate this property with open sentences.

      For example: If $x = 12$, then $x + 4 = 12 + 4$

      or $x + 5 = 12 + 5$, etc.

      Have class determine that all of these equations have 12 as their solution.

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2. Demonstrate how the addition property of equality can be used to solve an equation.

   For example: $x - 2 = 4$

   Since the same numb[illegible] be added to both sides of an equation without changing i[illegible]uth or falsity, elicit that 2 may be added to both sides of this equation so that the resulting equation is the solution.

$$\begin{aligned} x - 2 &= 4 \\ x - 2 + 2 &= 4 + 2 \\ x &= 6 \end{aligned} \qquad \text{or} \qquad \begin{array}{r} x - 2 = 4 \\ \underline{+\ 2 = +\ 2} \\ x = 6 \end{array}$$

3. Using the above method, have the class solve the challenge problem.

$$\begin{aligned} x - 25 &= 47 \\ x - 25 + 25 &= 47 + 25 \\ x &= 72 \end{aligned}$$

   Since the solution was not obvious, how do we know that it is the correct solution? Indicate that the solution should be checked by substituting the value for x (72) in the original equation to determine if the statement is true.

   Check:
$$\begin{aligned} x - 25 &= 47 \\ \underbrace{72 - 25} &= 47? \\ 47 &= 47 \end{aligned}$$

4. Do Application A.

5. In a similar manner, develop the <u>subtraction property of equality</u>. If the same number is subtracted from both sides of an equation, the resulting equation will have the same solutions as the original.

6. Have the class solve the cliff hanger from the previous lesson, $x + 327 = 714$, as follows:

$$\begin{aligned} x + 327 &= 714 \\ x + 327 - 327 &= 714 - 327 \\ x &= 387 \end{aligned} \qquad \text{or} \qquad \begin{array}{r} x + 327 = 714 \\ \underline{-\ 327 = -\ 327} \\ x = 387 \end{array}$$

   Have students check the result by substituting the value for x (387) in the original equation.

7. Do Application B.

8. Pose the question:"How do you determine which property of equality to use in solving an equation?" Have students see that, since the solution of an equation means undoing what has been done to the variable, we must use the <u>inverse operation</u>.

9. Do Application C.

APPLICATIONS:

A. Solve each equation using the addition property of equality. Check your results by substitution.

1. $x - 19 = 43$
2. $y - 21 = -6$
3. $27 = z - 41$
4. $-43 = p - 15$

B. Solve each equation using the subtraction property of equality. Check by substitution.

1. $x + 19 = 43$
2. $y + 21 = -6$
3. $27 = z + 41$
4. $-43 = p + 15$

C. Solve and check each equation using the addition or subtraction property of equality as needed.

1. $x - 15 = 7$
2. $x + 15 = 7$
3. $31 = y + 9$
4. $31 = y - 9$
5. $z - 37 = 92$
6. $92 = z + 37$

SUMMARY:

1. What is the advantage of using the properties of equality over the trial and error method of solution?

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LESSON 4

AIM: To reinforce solution of equations using addition and subtraction properties of equality

PERFORMANCE OBJECTIVE: Students will be able to:

- use the appropriate property of equality to find solutions to equations of the form $x \pm a = b$, where a and b are fractions and decimals.

VOCABULARY:

Review: addition property of equality, subtraction property of equality, inverse operation, check

CHALLENGE PROBLEM:

Find the solution to the equation $x + 5.5 = 6$.

DEVELOPMENT:

1. Discuss the challenge problem. Elicit that subtraction is the inverse operation which will undo the indicated addition in this equation. Stress the importance of rewriting 6 as 6.0 before using the subtraction property of equality. Review the procedure for adding or subtracting decimals.

   Thus:
$$\begin{array}{rcr} x + 5.5 & = & 6.0 \\ -\ 5.5 & = & -5.5 \\ \hline x \qquad & = & 0.5 \end{array}$$

2. Check the solution

$$x + 5.5 = 6$$

$$\text{Does } \underbrace{0.5 + 5.5}_{6.0} = 6?$$

$$6.0 = 6$$

3. Solve and check:

$$7 = y + 2\frac{1}{3}$$

   a. Elicit that $2\frac{1}{3}$ should be subtracted from both sides of the equation.

$$\begin{array}{rcl} 7 & = & y + 2\frac{1}{3} \\ -\ 2\frac{1}{3} & & \quad -\ 2\frac{1}{3} \\ \hline \end{array}$$

b. Review the procedure for subtracting a mixed number from a whole number.

$$\begin{array}{rcr} 7 & = & 6\frac{3}{3} \\ -\ 2\frac{1}{3} & = & -\ 2\frac{1}{3} \\ \hline & & 4\frac{2}{3} \end{array}$$

Therefore: $4\frac{2}{3} = y$. Check the result.

4. Do applications, giving particular attention to lining up decimals and changing fractions to a common denominator. Emphasize checking each solution AND , if the check yields a false sentence, redoing the problem.

APPLICATIONS:

A. Solve and check:

1. $n + \frac{1}{4} = \frac{3}{4}$
2. $C + .2 = 4.6$
3. $b - 2\frac{3}{4} = 9$
4. $8\frac{1}{8} = \quad - \frac{1}{8}$
5. $d + 1\frac{2}{3} = 5\frac{1}{3}$
6. $z - 6 = 3.7$
7. $12 = p + 1.8$
8. $t - .5 = 2.4$

SUMMARY:

1. How do you determine which property of equality to use to solve an equation?
2. How do you determine which number to add or subtract?

CLIFF-HANGER:

Solve the equation: $\frac{x}{15} = 9$.

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LESSON 5

AIM: To solve equations of the form $ax = b$ and $\frac{x}{a} = b$ where a and b are positive

PERFORMANCE OBJECTIVES; Students will be able to:

- state that multiplication and division are inverse operations.
- demonstrate the multiplication and division properties of equality in numerical sentences.
- use the appropriate inverse operation to solve equations of the form $ax = b$ and $\frac{x}{a} = b$ where a and b are positive.

VOCABULARY:

New terms: multiplication property of equality, division property of equality

Review: addition property of equality, subtraction property of equality, inverse operation

CHALLENGE PROBLEM:

Which property of equality should be used to solve each of the equations below:

a. $y + 4 = 12$ b. $y - 4 = 12$ c. $4y = 12$

DEVELOPMENT:

1. Elicit from the class that (a) and (b) in the challenge problem are solved using the subtraction and addition properties of equality. In (c) the variable is multiplied by the number 4. To solve equation (c), it is necessary to "undo" the multiplication by using the inverse operation.

2. Have students determine that division is the inverse operation for multiplication.

   Example: $4\ (6) = 24$. Therefore, $6 = \frac{24}{4}$

3. Based on their experience with the addition and subtraction properties, have students state the division property of equality. If both sides of an equation are divided by the same (non-zero) number, the resulting equation will have the same solutions as the original.

   Elicit several numerical examples such as:

   $4\ (6) = 24$ $4\ (6) = 24$

   $\frac{4\ (6)}{2} = \frac{24}{2}$ $\frac{4\ (6)}{4} = \frac{24}{4}$

   Emphasize that it is never possible to divide by zero. Illustrate

why by asking the class to compute $\frac{12}{0}$, and showing that any suggested answer must be wrong since it wouldn't pass the multiplication check. If $\frac{12}{0} = *$, then * times 0 should = 12, but no number times 0 will = 12.

4. Have students determine a procedure for using the division property to solve the equation in the challenge problem.

   $4y = 12$

   Elicit that since the variable was multiplied by 4, dividing by 4 will "undo" this operation yielding the value of y.

   Thus, $4y = 12$

   $$\frac{4y}{4} = \frac{12}{4}$$

   $$y = 3$$

   This solution should be checked.

   $$4y = 12$$

   Does $4(3) = 12$?

   $$12 = 12$$

5. Do Application A.

6. In a similar manner, have students state the multiplication property of equality: If both sides of an equation are multiplied by the same number, the resulting equation will have the same solutions as the original. Apply this property to the solution of equations such as:

   $$\frac{P}{7} = 8$$

   $$7\left(\frac{P}{7}\right) = 7(8)$$

   $$P = 56$$

   Check the solution·

   $$\frac{P}{7} = 8$$

   Does $\frac{56}{7} = 8$?

   $$8 = 8$$

7. Do Application B.

APPLICATIONS:

A. 1. Solve and check each of the foll ing equations:

a. $7a = 63$
b. $72 = 9b$
c. $5c = 9$
d. $2n = 0.6$
e. $.05r = 2$
f. $.75 = .5a$
g. $12x = 2436$

2. Write the equation that could be used to solve the following problem. Then, solve the equation and check the problem.

If 6 bars of candy cost $2.10, find the cost of one bar of candy. (Use x to represent the cost of one bar of candy.)

B. 1. Solve and check each of the following equations:

a. $\frac{m}{3} = 20$
b. $\frac{x}{4} = 24$
c. $12 = \frac{x}{7}$
d. $\frac{y}{7} = 4.2$
e. $8.3 = \frac{z}{4}$
f. $\frac{w}{.05} = 4.1$

2. John gave his entire stamp collection to 3 friends. When they divided the stamps equally, each received 123 stamps. How many stamps were in the collection? (Use s to represent the number of stamps.)

SUMMARY:

1. What is the inverse operation for division? For multiplication?

2. What property of equality is used to solve each of the following and how is that property used?

a) $\frac{x}{7} = 9$ (Multiplication property of equality. Multiply both sides of the equation by 7.)

b) $16 = y - 17$

c) $5p = 123$

d) $x + 7 = -3$

CLIFF-HANGER:

Solve and check:

$\frac{x}{-3} = -9$

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LESSON 6

AIM: To multiply and divide signed numbers

PERFORMANCE OBJECTIVE: Students will be able to:

- state and apply the techniques for multiplication and division of signed numbers to obtain the appropriate products and quotients.

VOCABULARY:

Review: signed numbers, positive, negative, product, quotient, factor

CHALLENGE PROBLEM:

What operation is used to solve the equation

$$\frac{x}{-3} = -9$$

DEVELOPMENT:

1. Elicit from the class that, in order to solve the above equation, the multiplication property of equality is used and both sides of the equation must be multiplied by -3.

   The solution will look like this:

   $$-3\left(\frac{x}{-3}\right) = -3\ (-9)$$

   $$x = -3\ (-9)$$

   Alert class that they do not yet know how to find the product of two negative numbers.

   The problem will be solved after the class has developed a procedure for multiplying signed numbers.

2. Write the following set of multiplication examples on the board:

   (1) $\begin{array}{r} 4 \\ \underline{\times\ 3} \end{array}$ (2) $\begin{array}{r} 4 \\ \underline{\times\ 2} \end{array}$ (3) $\begin{array}{r} 4 \\ \underline{\times\ 1} \end{array}$ (4) $\begin{array}{r} 4 \\ \underline{\times\ 0} \end{array}$ (5) $\begin{array}{r} 4 \\ \underline{\times\ \quad} \end{array}$

   a. Have class supply missing factor in 5th example by observing the pattern of previous factors (-1).

   b Have students evaluate first four products (12, 8, 4, 0).

   Based on this sequence, what must be the next product? (Since the numbers are decreasing by 4, the next product is - 4. It may be useful to refer to the number line.)

   c. Recall that "+4" is another way to write 4.

d. Have students determine the next set of factors:

$$\begin{array}{r} +\ 4 \\ \underline{\times\ -\ 2} \end{array}$$ and the product (- 8)

e. Therefore, state the following:

1) The product of 2 positive numbers is a positive number (from the first 3 examples).

2) The product of a positive number and a negative number is a negative number (from examples 5 and 6).

3. Do Application A.

4. Use the following set of examples to elicit the technique for finding the sign of the product of two negative numbers:

(1)	(2)	(3)	(4)	(5)
+ 3	+ 2	+ 1	0	- 1
x - 4	x - 4	x - 4	x - 4	x - 4
(- 12)	(- 8)	(- 4)	(0)	(+ 4)

Have students state:

The product of two negative numbers is a positive number.

5. Do Application B.

6. Return to challenge problem.

The solution is x = + 27.

Have students attempt to check the solution.

They cannot because they do not as yet know the techniques for division of signed numbers. (Use this as a motivation for developing these techniques.)

$$\frac{x}{-\ 3} = -\ 9$$

Does $\frac{+\ 27}{-\ 3} = -\ 9$?

7. Develop techniques for division of signed numbers using the fact that multiplication and division are inverse operations.

For example: $\frac{12}{4} = 3$ because 12 = 3 x 4

Pose the question $\frac{+\ 12}{-\ 4} = ?$ This is equivalent to 12 = ? x (- 4).

Have class determine the missing number (- 3).

8. Have students recognize that the quotient of a positive and a negative number is a negative number.

9. Use a similar technique to develop a way of finding the sign of the quotient of two negative numbers.

   Have students state: The quotient of two negative numbers is a positive number.

10. Complete the check of the challenge problem:

$$\frac{x}{-3} = -9$$

Does $\frac{+27}{-3} = -9$?

$$-9 = -9 \checkmark$$

11. Do Applications C and D.

APPLICATIONS:

A. Evaluate the following:

1) $\begin{array}{r} (+7) \\ \underline{\times(-7)} \end{array}$ 2) $\begin{array}{r} -5 \\ \underline{\times +9} \end{array}$ 3) $\begin{array}{r} 8 \\ \underline{\times 6} \end{array}$

4) $(+.5)\ (-.2)$ 5) $(-7.3)\ (+4)$ 6) $(.5)\ (+.08)$

B. Find each of the following products:

1) $(-12)\ (-7)$
2) $(+55)\ (-18)$
3) $(+2.5)\ (-.3)$
4) $(-6)\ (36.5)$
5) $(+\frac{1}{2})\ (-\frac{2}{3})$
6) $(-\frac{5}{8})\ (-\frac{2}{15})$

C. Find the following quotients:

1) $10 \div (-2)$
2) $(-12) \div (-3)$
3) $\frac{+125}{-5}$
4) $\frac{-420}{35}$
5) $\frac{-100}{-10}$

D. Evaluate each of the following:

1) $(-4)\ (-2)\ (+3)$
2) $(+4)\ (-2)\ (+3)$
3) $(-6.4)\ (-3.2)\ (20)$
4) $\frac{(-20)\ (-10)}{-5}$
5) $\frac{-33}{-11}\ (-17)$

SUMMARY:

Elicit from the class the following simplified statements:

> A. The product or quotient of two numbers with the same sign is a positive number.
>
> B. The product or quotient of two numbers with different signs is a negative number.

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LESSON 7

AIM: To solve equations of the form $\pm a x = \pm b$ and $\pm \frac{x}{a} = \pm b$

PERFORMANCE OBJECTIVES: Students will be able to:

- apply the techniques for multiplication and division of signed numbers.
- apply the techniques for equation solving to solve equations of the form $\pm ax = \pm b$ and $\pm \frac{x}{a} = \pm b$.

VOCABULARY:

Review: inverse operation

CHALLENGE PROBLEM:

Solve and check: $-2.8x = 5.6$

DEVELOPMENT:

1. Use challenge problem to review both the division property of equality as applied to solving equations and the technique for division of signed numbers.

   a. Use division property of equality and divide both sides of the equation by -2.8

   $$\frac{-2.8x}{-2.8} = \frac{5.6}{-2.8}$$
   $$x = \frac{5.6}{-2.8}$$

   b. Use the technique for division of signed numbers to obtain the sign of the quotient. Perform the calculation.

   $$x = -\frac{5.6}{2.8}$$
   $$x = -2$$

   c. Have students check the solution, thus reviewing the techniques for multiplication of signed numbers.

2. Solve and check:

   $$\frac{y}{3.8} = -7$$

   a. Use the multiplication property of equality and the technique for multiplication of signed numbersto obtain the solution.

   $$\frac{y}{3.8} = -7$$
   $$(3.8)\left(\frac{y}{3.8}\right) = 3.8\,(-7)$$
   $$y = -26.6$$

   b. Have students check the solution to review the techniques for division of signed numbers.

3. Do applications.

APPLICATIONS:

1. Find the solution for each equation and check.

   a. $3m = 18$ b. $9t = 45$ c. $14x = -42$

   d. $21z = -63$ e. $-55 = 11k$ f. $-5x = -9$

   g. $-5 = 3y$ h. $-9a = -6$ i. $\frac{y}{4} = -2$

   j. $\frac{x}{-3} = 12$ k. $\frac{c}{2} = -7$ l. $\frac{x}{-6} = -1.8$

   m. $\frac{x}{7} = -2\frac{1}{2}$ n. $\frac{m}{-4} = -4\frac{1}{4}$ o. $-8k = 8.8$

SUMMARY:

How do you determine whether to use the multiplication property of equality or the division property of equality to solve a particular equation?

LESSON 8

AIM: To reinforce the solution of simple equations

PERFORMANCE OBJECTIVES: Students will be able to:

- select which property of equality to use to solve a one-step equation
- solve any linear equation where only one property of equality must be used.

CHALLENGE PROBLEM:

Name the property of equality that was used for each equation given in Column I, to find its solution, given in Column II.

	Column I	Column II
a.	$20m = -60$	$m = -3$
b.	$x + 7 = 12$	$x = 5$
c.	$\frac{q}{3} = -9$	$q = -27$
d.	$t - 4 = 3.9$	$t = 7.9$

DEVELOPMENT:

1. a. Review the four properties of equality previously developed.
   b. Have students state the meaning of each property of equality.
   c. Review with students that the inverse operation is used to determine the choice of property needed to solve the equation
2. Do applications.

APPLICATIONS:

Solve and check each equation. Show all steps.

1. $4x = 32$
2. $w - 1.6 = .3$
3. $-3a = -21$
4. $x - 18 = -30$
5. $y + 5 = -22$
6. $s + 18 = 7$
7. $\frac{7}{8} = x - \frac{15}{8}$
8. $34 = -8x$
9. $-29 = x - 47$
10. $14 = \frac{x}{-3}$
11. $\frac{x}{-4.3} = -12$
12. $6y = -18$
13. $-2.6r = 12.74$
14. $10\frac{1}{2} = x + 4\frac{3}{8}$
15. $x - 5\frac{1}{4} = 8\frac{1}{2}$

SUMMARY:

Prepare a short quiz on problems of the same type as those shown above.

LESSON 9

AIM: To solve equations of the form $ax \pm b = c$ by trial and error

PERFORMANCE OBJECTIVE: Students will be able to:

- use substitution to determine whether or not a specific value of a variable is a solution to an equation of the form $ax \pm b = c$ or $\frac{x}{a} \pm b = c$.

VOCABULARY:

Review: substitution, order of operations, statement

CHALLENGE PROBLEM:

Seven less than 4 times a number is 17. Find the number.

DEVELOPMENT:

1. Discuss the challenge problem. Translate this problem into an tion equation using n to represent "the number."

   $$4n - 7 = 17$$

   Elicit from the class that one method of solving this problem is to substitute different values for the variable until we find the value of n which makes the statement true.

   For example: If $n = 1$, $4n - 7 = 17$ becomes
   $4 (1) - 7 = 17$

   (It is important to review the order of operations for evaluating expressions. Remind students that multiplications and divisions are performed before additions and subtractions.)

   $4 - 7 = 17$ is a false statement.
   Therefore, 1 is not the solution.

   Continue substituting values for n.(The students may suggest these values until the correct solution has been found.) If

   $n = 6$ then $4n - 7 = 17$
   $4 (6) - 7 = 17$
   $24 - 7 = 17$ is a true statement. Therefore, 6 is the number that solves the challenge problem.

2. Have students solve the equation below by substituting different values for x.

   $$6 + 3x = 18$$

   (If the student thinks "2" is the solution, an error has been made in the order of operations.) The correct solution is

   $x = 4$ because: $6 + 3x = 18$

   $6 + 3 (4) = 18$

   $6 + 12 = 18$ (a true statement)

3. Do applications.

APPLICATIONS:

A. Determine whether or not the number in parentheses is a solution to the open sentence:

1. $6n + 17 = 47$ $(n = 5)$
2. $55 - 2t = -32$ $(t = -6)$
3. $27 - 2y = 50$ $(y = 2)$
4. $\frac{x}{-6} + 2 = 0$ $(x = 12)$
5. $+ 2 = 24 - 20y$ $(y = \frac{1}{2})$

B. Using the integers from -2 to +2, find solutions for the following equations. (If there are no solutions, write "none".)

1. $3x + 3 = 6$
2. $5x - 9 = 1$
3. $4y + 2 = 10$
4. $7x - 9 = -9$
5. $\frac{c}{2} + 6 = 5$

C. Using trial and error to choose a value for the variable, find the solution for each equation.

1. $2c - 4 = 6$
2. $\frac{b}{2} + 6 = 12$
3. $4a + 2 = -34$

SUMMARY:

1. Review the method of substitution to determine whether or not a specific value for a variable is a solution to an equation.
2. Review-the order of operations.

CLIFF-HANGER:

Solve the equation:

$3x - 5 = 88$

LESSON 10

AIM: To solve equations of the form $ax \pm b = c$ using properties of equality

PERFORMANCE OBJECTIVES: Students will be able to:

- state that the addition or subtraction properties must be used before the multiplication or division properties in the solution of equations of the form $ax \pm b = c$.
- solve and check equations of the form $ax \pm b = c$.

VOCABULARY:

Review: inverse operations, order of operations

CHALLENGE PROBLEM:

Solve the equation: $5x + 9 = 79$

DEVELOPMENT:

1. Discuss the challenge problem. Students should be led to determine that trial and error substitution is difficult if solutions to equations are not obvious. A technique for solving equations whose solutions are not obvious must be developed.
2. Remind students that solving an equation means that we must "undo" everything that was done to the variable. The equation above indicates that the variable x was multiplied by 5. 9 was then added to the product. What steps are necessary to "undo" these operations?
3. Present the students with a real-life situation in which several operations are performed. Determine the order in which the inverse must be performed to undo all the operations. For example: when a man dresses wearing a 3-piece suit, he puts on a shirt, then a vest, and then the jacket. To "undo" this, or get undressed, the inverse operations are performed in the reverse order.
4. Return to the challenge problem. Since addition was the last operation performed, it must be "undone" first. Then undo the multiplication. Review the inverse operation as the method of "undoing" operations.

$$5x + 9 = 79$$
$$\underline{\quad - 9} = \underline{-9} \quad \text{Undo addition}$$
$$\frac{5x}{5} = \frac{70}{5} \quad \text{Undo multiplication}$$
$$x = 14$$

Check: $5x + 9 = 79$

Does $5(14) + 9 = 79$?

$70 + 9 = 79$ ✓

APPLICATIONS:

A. Solve and check numbers 1-10 below:

1. $4n - 6 = 10$	6. $\frac{x}{-5} - 19 = -24$
2. $18 = 2a + 6$	7. $-3a - 1.2 = 7.2$
3. $3y - 5 = 8$	8. $8.3 = .5x + 6$
4. $5t - 10 = -25$	9. $\frac{z}{-2} + 6 = -15$
5. $-9x + 1 = 10$	10. $\frac{y}{5} - 3 = -4$

B. Write an equation for each problem using n as "the number." Solve and check each equation.

1. If ten is added to 3 times a number, the result is 28. Find the number.
2. Three times an unknown number decreased by 12 equals negative ninety-six. Find the number.
3. If seven times a number is increased by fifty, the result is 15. Find the number.
4. If ten is subtracted from a number divided by 4, the difference is 14. Find the number.

SUMMARY:

Have students state the order in which the properties of equality ar- applied to the solution of equations of the form $ax \pm b = c$ or $\frac{x}{a} \pm b = c$.

(Elicit that properties must be used in reverse of the order of operations.)

CLIFF-HANGER:

If two-thirds of a number is 4, find 'he number

30

LESSON 11

AIM: To solve equations of the form $\frac{ax}{b} = c$ using properties of equality

PERFORMANCE OBJECTIVES: Students will be able to:

- write the reciprocal of any number.
- state that dividing by any number is equivalent to multiplying by its reciprocal.
- solve and check equations of the form $\frac{ax}{b} = c$ using properties of equality.

VOCABULARY:

New terms: reciprocal

Review: multiplication property of equality, division property of equality

CHALLENGE PROBLEM:

Solve the equation: $\frac{2}{3}x = 4$

DEVELOPMENT:

1. Discuss the challenge problem. Some students may solve the problem by trial and error, while others will treat it as requiring two steps (first multiplying by 3 and then dividing by 2). Still others may attempt to divide both sides by $\frac{2}{3}$.

2. Consider $\frac{2}{3}x = 4$ as a 2-step equation:

<u>Multiplying by 3</u>) $\frac{2}{3}x = 4$

$$3\left(\frac{2}{3}x\right) = 3\ (4)$$

<u>Dividing by 2</u>) $\frac{2x}{2} = \frac{12}{2}$

$$x = 6$$

<u>Check</u>: $\frac{2}{3}x = 4$

Does $\frac{2}{3}(6) = 4$?

$$\frac{2}{\cancel{3}_1} \times \frac{\cancel{6}^{2}}{1} = 4$$

$$\frac{4}{1} = 4$$

This procedure can be simplified into a one-step process using <u>only</u> the multiplication property of equality with the number $\frac{3}{2}$. Thus:

$$\frac{2}{3}x = 4$$

$$\frac{\cancel{3}^{1}}{\cancel{2}_1}\left(\frac{\cancel{2}^{1}}{\cancel{3}_1}x\right) = \frac{3}{\cancel{2}_1}\left(\frac{\cancel{4}^{2}}{1}\right)$$

$$1\ x = 6$$

$\frac{3}{2}$ is called the reciprocal of $\frac{2}{3}$.

Define: Reciprocals are two numbers whose product is 1. Each number is the reciprocal of the other.

3. Do Applications A and B.

4. Consider $\frac{2}{3}x = 4$ as an equation of the form $ax = b$. In previous lessons students learned to "undo" multiplication by using the division property of equality. Therefore, we may solve the problem in the following way:

$$\frac{\frac{2}{3}x}{\frac{2}{3}} = \frac{4}{\frac{2}{3}}$$

The right side of the equation may be re-written as $4 \div \frac{2}{3}$. Therefore,

$$x = 4 \div \frac{2}{3}$$

The solution is equivalent to evaluating the arithmetic expression

$$4 \div \frac{2}{3}.$$

Write: $\frac{4}{1} \div \frac{2}{3}$.

Since we know that the solution is 6 and that $\frac{4}{1} \times \frac{3}{2} = 6$, students should be led to write the statement:

$$\frac{4}{1} \div \frac{2}{3} = \frac{4}{1} \times \frac{3}{2}.$$

Elicit the statement: Division by a number is equivalent to multiplication by the reciprocal of that number.

5. Do Applications C, D, and E.

APPLICATIONS:

A. Write the reciprocal for each of the following:

1) $\frac{3}{4}$	6) $-\frac{1}{7}$
2) $\frac{1}{3}$	7) -8
3) 5	8) $-2\frac{1}{3}$
4) $1\frac{1}{4}$	9) $\frac{5}{8}$
5) $-\frac{2}{5}$	10) .3

B. Solve each of the following equations using the multiplication property of equality and reciprocals.

1) $\frac{3}{4}y = 12$	6) $-\frac{1}{7}r = 4$
2) $\frac{1}{3}m = -2$	7) $-8q = 2$
3) $17 = 5x$	8) $-2\frac{1}{3}n = \frac{5}{8}$
4) $1\frac{1}{4}y = \frac{1}{4}$	9) $-2\frac{1}{3} = \frac{5}{8}y$
5) $-40 = -\frac{2}{5}t$	10) $.3x = 15$

C. Perform the indicated operations and express the results in lowest terms.

1) $\frac{2}{3} \div \frac{8}{9}$	4) $33\frac{3}{4} \div 11\frac{1}{4}$
2) $14 \div \frac{2}{7}$	5) $30 \div \frac{3}{10}$
3) $\frac{3}{4} \div 4$	6) $\frac{2}{3} \div \frac{3}{16}$

D. Solve each of the following equations using the division property of equality.

1) $-\frac{3}{5}x = 12$

2) $\frac{1}{5}n = -40$

3) $-\frac{2}{3}x = -20$

4) $2\frac{3}{4}y = 33$

5) $-5\frac{1}{4} = 3\frac{1}{2}x$

E. Solve each of the following:

1 If a board $10\frac{1}{4}$ yards long is cut into 4 equal pieces, how long will each piece be?

2 A 50 pound bag of peanuts is to be placed into small bags holding $\frac{5}{16}$ of a pound each. How many smaller bags can be filled?

SUMMARY:

1 Review the meaning of the terms "reciprocal" and "finding the reciprocal of a number."

2 Review the method used for division of fractions.

3 Review the methods for solving equations of the form $\frac{ax}{b} = c$.

CLIFF-HANGER:

If $\frac{3}{4}$ of a number is increased by 7, the result is 16. Find the number.

LESSON 12

AIM: To solve equations of tne form $\frac{ax}{b} \pm c = d$

PERFORMANCE OBJECTIVES: Students will be able to:

- state the sequence of steps required for the solution of equations of the form $\frac{ax}{b} \pm c = d$ using inverse operations.
- solve the equations and check the solutions.

CHALLENGE PROBLEM:

Solve the equation: $\frac{3}{4}y + 7 = 16$

DEVELOPMENT:

1. Review the procedure for solving equations where several operations are performed. Recall the development from Lesson 10 in which operations must be "undone" in exactly the opposite order from that in which they were originally performed.

Subtract 7 from both sides.

$$\begin{array}{r} \frac{3}{4}y + 7 = 16 \\ \underline{\phantom{\frac{3}{4}y +} - 7 = -7} \\ \frac{3}{4}y = 9 \end{array}$$

Choose one of the methods developed in the previous lesson to solve $\frac{3}{4}y = 9$.

Method 1 or Method 2

Method 1:

$$\frac{3}{4}y = 9$$

$$\frac{4}{3} \times \frac{3}{4}y = \frac{4}{\cancel{3}_1} \times \frac{\cancel{9}^3}{1}$$

$$y = 12$$

Method 2:

$$\frac{3}{4}y = 9$$

$$\frac{\frac{3}{4}y}{\frac{3}{4}} = \frac{9}{\frac{3}{4}}$$

$$y = 9 \div \frac{3}{4}$$

$$y = \frac{{}^3\cancel{9}}{1} \times \frac{4}{\cancel{3}_1}$$

$$y = 12$$

Check the result.

2. Do applications

APPLICATIONS:

A. Solve and check the following equations:

1) $\frac{1}{2}m + 6 = 15$

2) $9 = \frac{1}{3}z + 11$

3) $\frac{2}{5}y - 3 = 9$

4) $\frac{2}{3}a + 7 = 29$

5) $\frac{3}{4}b + 14 = 8$

6) $-25 = \frac{7}{3}s - 11$

7) $\frac{5d}{6} + 1\frac{1}{2} = 8\frac{1}{2}$

8) $8\frac{2}{3} = \frac{5b}{6} + 3\frac{5}{6}$

B. Solve each of the following:

1) A number is multiplied by $\frac{2}{5}$. When this product is decreased by 13, the result is 7. Find the number.

2) $\frac{7}{8}$ of a number decreased by $\frac{3}{4}$ is $4\frac{1}{2}$. Find the number.

3) If 38 is added to $\frac{5}{9}$ of a number, the result is 128. Find the number.

SUMMARY:

Review the procedure used to solve equations of the form $\frac{ax}{b} \pm c = d$.

LESSON 13

AIM: To review the solution of linear equations using one or more properties of equality

PERFORMANCE OBJECTIVES: Students will be able to:

- use formulas to solve problems by substituting given values and solving the resulting equations.
- determine if the solution satisfies the conditions of the problem.

VOCABULARY:

Review: formulas, and all terms related to solution of equations.

CHALLENGE PROBLEM:

$F = \frac{9}{5}C + 32$ is a formula relating Fahrenheit and Celsius temperatures. Find the Celsius temperature when the Fahrenheit temperature is 85°.

DEVELOPMENT:

1. Have students recall the procedure for substituting values in a formula.

   Elicit from the class that once the substitution is performed, an equation with one variable is obtained.

   $$F = \frac{9}{5}C + 32$$

   $$85 = \frac{9}{5}C + 32$$

   Using substitution, review the procedure for solving this equation.

   Subtract 32 from both sides.

   $$\begin{aligned} 85 &= \frac{9}{5}C + 32 \\ -32 &= \quad -32 \\ \hline 53 &= \frac{9}{5}C \end{aligned}$$

   Multiply both sides by $\frac{5}{9}$

   $$\begin{aligned} \frac{5}{9}(53) &= \frac{5}{9}\left(\frac{9}{5}C\right) \\ \frac{265}{9} &= C \\ 29\frac{4}{9} &= C \end{aligned}$$

   The temperature is $29\frac{40}{9}$ c.

   Check the result.

2. Provide students with a variety of equations and verbal problems reviewing all techniques presented in this unit. Some interesting applications follow:

APPLICATIONS:

Use the given formula to solve each problem. Check your results.

1. A car was driven a distance (d) of 51 kilometers in $\frac{3}{4}$ of an hour (t). Find its rate of speed (r). $(d = r\ t)$
2. The retail price of an item is the sum of the wholesale price and the mark-up $(r = w + m)$. A jacket has a retail price of \$37. A store's mark-up is \$17.25. What is the wholesale price?
3. The sale price of an item(s) is found by subtracting the discount (d) from the regular price (r). What was the regular price of an item that was on sale for \$12.74 if the discount was \$2.25?
4. The formula relating a person's blood pressure and age is $p = \frac{a}{2} + 110$. Sandra's blood pressure is 134. How old should she be?
5. A person's height in inches and weight in pounds are approximately related by the formula $w = \frac{11}{2}h - 220$. According to this formula, what is the height of a person weighing 220 pounds?

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UNIT VIII.
Tables, Graphs, and Coordinate Geometry

LESSON 1

AIM: To read and interpret data found in tables

PERFORMANCE OBJECTIVES: Students will be able to:

- provide examples in which presentation of data in tables is useful.
- read tables of data and answer specific questions relating to these data.
- describe ways of presenting data: lists, tables, charts, maps, graphs.

VOCABULARY:

New terms: tables, charts, graph, horizontal, vertical

Review: data

CHALLENGE PROBLEM:

List at least five examples of information commonly presented in tables, charts or graphs.

DEVELOPMENT:

1. Students will probably suggest examples such as:

   sales tax, train or bus schedules, hospital charts, T.V. schedules, postal rates, weather charts, ideal weight chart, sports statistics, stock market summary, travel maps, etc.

2. Elicit from students that tables and graphs are used to present data in a convenient, easy-to-read way. The form used for the presentation depends upon the information given and the intended use.

3. Consider information that is often presented in table or chart form. One example is a mileage chart.

Distances Between Cities in New York State

	Albany	Buffalo	New York City	Niagara Falls	Poughkeepsie	Syracuse	Watertown
Albany	---	291	154	300	71	142	173
Buffalo	291	---	397	22	320	146	200
New York City	154	397	---	419	72	251	323
Niagara Falls	300	22	419	---	372	165	220
Poughkeepsie	71	320	72	372	---	45	298
Syracuse	142	146	251	165	45	---	71
Watertown	173	200	323	220	298	71	---

Distances Given in Miles

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a) What is the distance between Niagara Falls and Poughkeepsie? Discuss the two ways of finding this answer. (Note horizontal and vertical labels.) Allow students to discover the symmetry of this table and compare that with symmetry in addition and/or multiplication tables.

b) Consider other questions relating to this table such as:

   i. Which two cities are the closest together? Furthest apart?

   ii. If the speed limit in New York State is 55 miles per hour, what is the minimum travel time between Syracuse and Niagara Falls? etc.

   iii. On a business trip a company provides a travel allowance of 21¢ a mile. What is the allowance for a trip from New York City to Albany? A round trip between Watertown and Poughkeepsie?

4. Do applications using the samples presented or more current tables and charts found in newspapers, magazines, etc.

APPLICATIONS:

A. Use the telephone rate table shown to answer the questions below. (Note the difference between the presentation of the table her and the one used in the development. Why is only "half" of a table necessary?)

Rate Chart

All prices are for 20 minute out-of-state calls dialed direct, without operator assistance, any time Saturday, Sunday 'til 5 p.m., or any night after 11 p.m. 'til 8 a.m. Rates do not apply to Alaska and Hawaii. To find the rate between two cities, trace down the vertical column of one city to its intersection with the horizontal column of the other city, as illustrated.

	SAN FRANCISCO	PHILADELPHIA	OMAHA	NEW YORK	NEW HAVEN	NEWARK	MILWAUKEE	MEMPHIS	INDIANAPOLIS	DETROIT	DENVER	DALLAS	CLEVELAND	CHICAGO	BOSTON	ATLANTA
BOSTON																3 16
CHICAGO															3 00	3 00
CLEVELAND														2 99	3 00	3 00
DALLAS													3 16	3 00	3 16	3 00
DENVER												3 00	3 16	3 16	3 16	3 16
DETROIT											3 16	3 16	2 50	2 83	3 00	3 00
INDIANAPOLIS										2 83	3 16	3 00	2 83	2 66	3.00	2.99
MEMPHIS									2 99	3 00	3 00	2 .9	3 00	3 00	3 16	2 99
MILWAUKEE								3 00	2 83	2 83	3 00	3 00	2 99	2 50	3 00	3 00
NEWARK							3 00	3 16	3 00	3 00	3 16	3 16	2 99	3 00	2 83	3 00
NEW HAVEN						2 50	3 00	3 16	3 00	[illegible]	3 1(	3 16	3 00	3 00	2 50	3 00
NEW YORK					2 34	0 78	3 00	3 16	3 00	3 00	3 16	3 16	2 99	3 00	2 66	3 00
OMAHA				3 16	3 16	3 16	2 99	3 00	3 00	3 00	3 00	3 00	3 00	2 99	3.16	3 00
PHILADELPHIA			3 16	2 50	2 66	2 50	3 00	3.00	3 00	3.00	3 16	3 16	2 99	3 00	2 83	3 00
SAN FRANCISCO		3 33	3 16	3 33	3 33	3 33	3 16	3 16	3 33	3 33	3 16	3 16	3 33	3 16	3 33	3 33
SEATTLE	3 00	3 33	3 16	3 33	3 33	3 33	3 16	3 16	3 16	3 33	3 16	3 16	3 33	3 16	3 33	3 33

1. What is the cost of a 20-minute call between Dallas and Philadelphia?

2. What is the total cost of three 20-minute calls between New York City and Boston?

B. Use the sales tax table shown to compute the total cost of the purchases of taxable items in a local candy store.

SALES TAX

$0.01	to	$0.10 -	0¢	3.07	to	3.18 -	25¢
.11	to	.17 -	1¢	3.19	to	3.31 -	26¢
.18	to	.29 -	2¢	3.32	to	3.43 -	27¢
.30	to	.42 -	3¢	3.44	to	3.56 -	28¢
.43	to	.54 -	4¢	3.57	to	3.68 -	29¢
.55	to	.67 -	5¢	3.69	to	3.81 -	30¢
.68	to	.79 -	6¢	3.82	to	3.93 -	31¢
.80	to	.92 -	7¢	3.94	to	4.06 -	32¢
.93	to	1.06 -	8¢	4.07	to	4.18 -	33¢
1.07	to	1.18 -	9¢	4.19	to	4.31 -	34¢
1.19	to	1.31 -	10¢	4.32	to	4.43 -	35¢
1.32	to	1.43 -	11¢	4.44	to	4.56 -	36¢
1.44	to	1.56 -	12¢	4.57	to	4.68 -	37¢
1.57	to	1.68 -	13¢	4.69	to	4.81 -	38¢
1.69	to.	1.81 -	14¢	4.82	to	4.93 -	39¢
1.82	to	1.93 -	15¢	4.94	to	5.06 -	40¢
1.94	to	2.06 -	16¢	5.07	to	5.18 -	41¢
2.07	to	2.18 -	17¢	5.19	to	5.31 -	42¢
2.19	to	2.31 -	18¢	5.32	to	5.43 -	43¢
2.32	to	2.43 -	19¢	5.44	to	5.56 -	44¢
2.44	to	2.56 -	20¢	5.57	to	5.68 -	45¢
2.57	to	2.68 -	21¢	5.69	to	5.81 -	46¢
2.69	to	2.81 -	22¢	5.82	to	5.93 -	47¢
2.82	to	2.93 -	23¢	5.94	to	6.06 -	48¢
2.94	to	3.06 -	24¢				

1. A game for $5.99
2. Three birthday cards at 60¢ each
3. The following schools supplies:

    1 pack loose-leaf paper - 99¢

    2 ball-point pens @ 39¢ each

    1 spiral notebook - $1.19

    6 pencils at 10¢ each

Long distance rates within New York

FROM MANHATTAN TO:	DIRECT DISTANCE DIALED (Paid By Calling Party) DAY 8 AM to 5 PM Mon.-Fri. Init. 2 Mins.	DAY Ea. Add. Min.	EVENING 5 PM to 11 PM Mon.-Fri. Weekends 8 AM to 11 PM Init. 2 Mins.	EVENING Ea. Add. Min.	NIGHT 11 PM to 8 AM All Days Init. 1 Min.	NIGHT Ea. Add. Min.
Albany	$ .90	$.40	$.58	$.26	$ 20	$.16
Binghamton	.90	.40	.58	.26	.20	.16
Huntington	.52	.22	.33	.14	.12	.08
Monticello	.80	.35	.52	.22	.18	.14
Montauk Point	.90	.40	.58	.26	.20	.16
Mt. Kisco	.52	.22	.33	.14	.12	.08
Niagara Falls	1.02	.45	.66	.29	.22	.18
Poughkeepsie	.73	.31	.47	.20	.16	.12
Riverhead	.80	.35	.52	.22	.18	.14
Rochester	1.02	.45	.66	.29	.22	18
Syracuse	1.02	.45	66	.29	.22	.18
Utica	1.02	.45	66	.29	.22	.18

C. Use the table of Long Distance Telephone Rates within New York To compute the cost of telephone calls from Manhattan.

1. A 2-minute telephone call to Huntington at 9:00 A.M. on Monday? at 9:00 P.M.? at 7:45 A.M.?
2. How much more does it cost to place a 5-minute call between New York City and Mt. Kisco on a Friday afternoon (before 5:00 P.M.) than on a Saturday afternoon?

D. Use the Guide to Street Numbers in Manhattan to answer the fc owing:

GUIDE TO STREET NUMBERS IN MANHATTAN

To find nearest street, drop last digit of house number divide by 2 and further calculate as shown below

Ave A B C, D	Add 3	Audubon Ave	Add 165
1st & 2nd Ave	Add 3	Broadway	Up to 754
3rd Ave	Add 10	is below 8th St	
4th Ave		754 to 858	Subt 29
(Park Ave S)	Add 8	858 to 958	Subt 25
5th Ave		Above 958	Subt 31
Up to 200	Add 13	Columbus Ave	Add 60
Up to 400	Add 16	Convent Ave	Add 127
Up to 600	Add 18	Ft Washington	
Up to 775	Add 20	Ave	Add 158
776 to 1286		Lenox Ave	Add 110
Cancel last figure		Lexington Ave	Add 22
and subt	18	Madison Ave	Add 26
Up to 1500	Add 45	Manhattan Ave	Add 100
Above 2000	Add 24	Park Ave	Add 35
Ave of the		St Nicholas	
Americas	Subt 12	Ave	Add 110
7th Ave		West End Ave	Add 60
Up to 1800	Add 12	Central Park West	
Above 1800	Add 20	Divide house number by	
8th Ave	Add 10	10 and add 60	
9th Ave	Add 13	Riverside Drive	
10th Ave	Add 14	Divide house number by	
11th Ave	Add 15	10 and add 72	
Amsterdam Ave	Add 60	up to 165th St	

1. What is the closest street to 147 Fifth Avenue? 666 Fifth Avenue?

2. Between which two streets is 878 Broadway?

SUMMARY:

Have students describe situations in which tables are used to present information.

LESSON 2

AIM: To read and interpret pictographs (picture graphs)

PERFORMANCE OBJECTIVES: Students will be able to:

- read the scale on a pictograph.
- translate the symbols on a pictograph into numerical quantities.
- extrapolate new information or form opinions using the data on a pictograph.

VOCABULARY:

New terms: pictograph, symbol, legend, scale

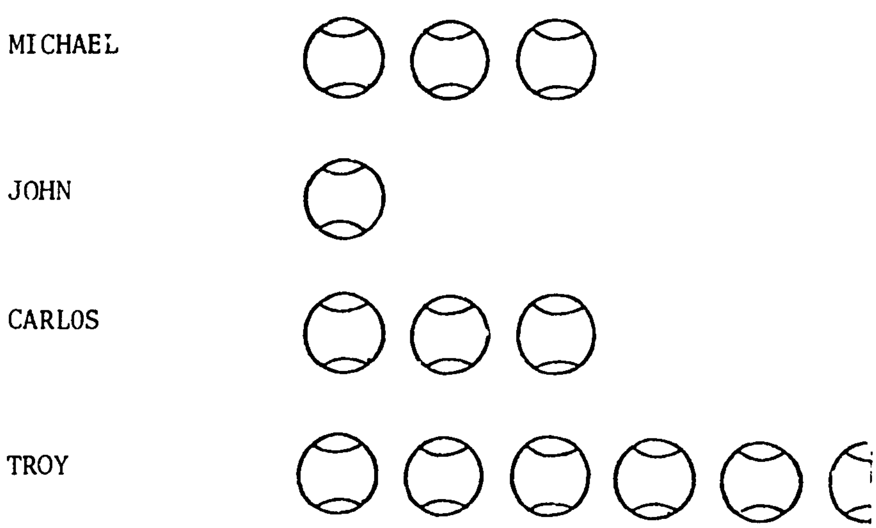
CHALLENGE PROBLEM:

If the symbol  represents 10 hits for a baseball player, draw the representation for 55 hits.

DEVELOPMENT:

1. Elicit the response to the challenge and discuss how the answer is obtained.
2. Have students recall that data may be presented in many forms. One form that is usually easy to read and visually attractive is a <u>pictograph</u>. The pictures in a pictograph often convey the subject matter of the graph.

   (<u>Note</u>: Appropriate graphs should be prepared for overhead projector or rexographs, etc.)
3. Present a pictograph similar to the one shown below and discuss the components of a pictograph. (Note: This graph is incomplete; items below explain how the graph should be completed.)



4. a) What information can be obtained from this graph? Elicit that each row of symbols is preceded by a label. This label can be compared to the vertical labels on some tables.

   b) What does this graph show? Elicit from students that a title is necessary in order to answer this question. Provide the title: Leading Hitters at Eastern High School, and repeat the question.

5. Which player on the team had the most hits? How many hits did this player have? Elicit that further information is necessary. Every pictograph must have a <u>scale</u> or <u>legend</u> which indicates the value of each <u>symbol</u>. Provide the information = 10 hits. Repeat the question and provide additional questions relating to this graph such as:

   a) How many more hits did Troy have than Carlos?

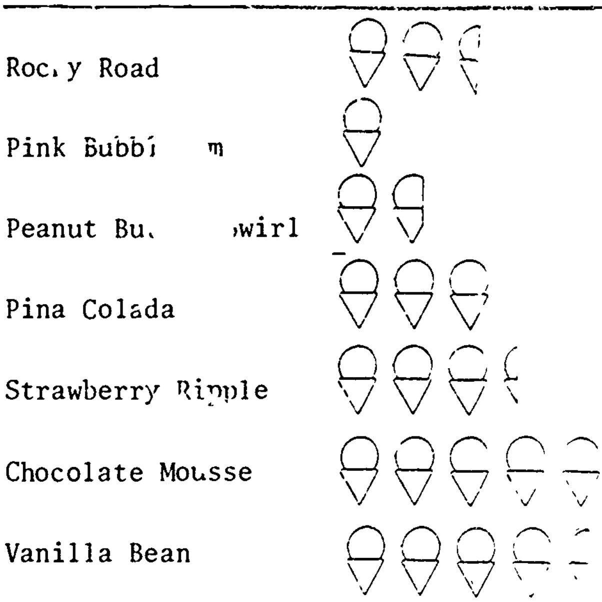
   b) How many more hits does the leader have compared to the 2nd place hitter? 3rd place hitter?

   c) If this chart represents hits for the first half of the year, how many hits would you expect Michael to have at the end of the year if he continued hitting at his present rate?

6. Do applications.

APPLICATIONS:

A. <u>Ice-cream cones sold at Steve's 7 Flavors on July 4.</u>




= 20 cones

1. Which flavor was the most popular? least popular?
2. How many cones of Pink Bubblegum ice-cream were sold? Pina Colada? Rocky Road?
3. How many cones were sold on July 4?
4. If this pattern continues, how can Steve use the information to plan his production?

B. <u>Points Scored in the Play-Off Series</u>
<u>By the Starting-Five Basketball Players</u>

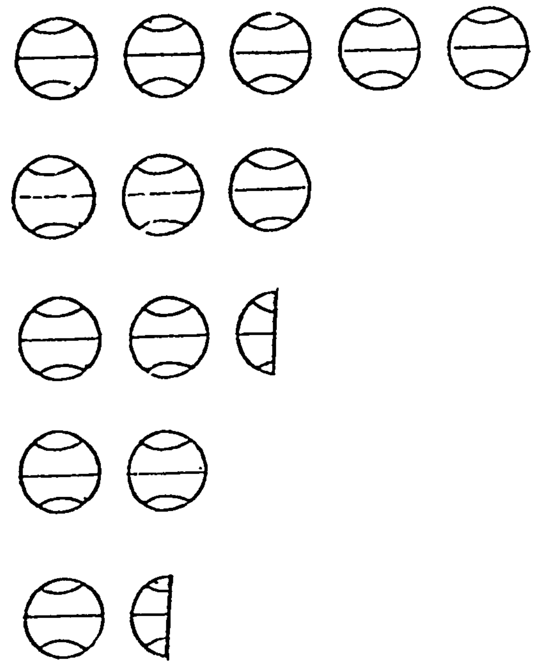
FRANKLIN

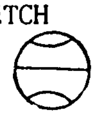
DOUGLAS

MONROE

DEREK

STRETCH

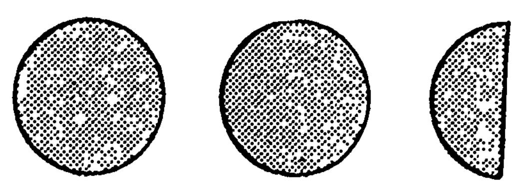


Legend:  = 16 points

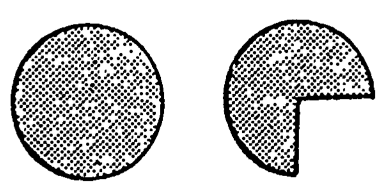
How many points did each player score?

C. If each symbol represents 10,000 records sold, how many records are represented by each row?

a)



b)



SUMMARY:

1. What information is needed in all pictographs?
2. If one symbol represents 4,000 cars sold, how many symbols are needed to show 300,000 cars?
3. Would you consider this method of showing data very accurate?

LESSON 3

AIM: To read and interpret bar graphs

PERFORMANCE OBJECTIVES: Students will be able to:

- identify bar graphs as a type of graph that can illustrate data more accurately than pictographs.
- translate the scale units used on a bar graph into numerical quantities.
- read and interpret bar graphs.
- read and interpret bar graphs with double sets of data.

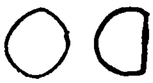
VOCABULARY:

New terms: horizontal and vertical scale (axes), bar graph

Review: legend, pictograph, symbol, scale

CHALLENGE PROBLEM:

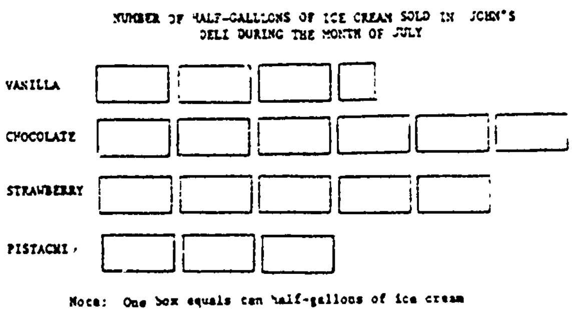
If each  represents 15 points, how many points are represented by

 ?

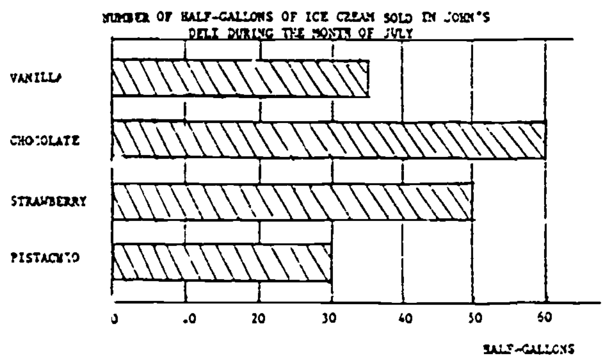
DEVELOPMENT:

1. Elicit from the class that the solution to the challenge problem is difficult because it is hard to estimate the fractional part represented by the figure. Similar problems arose in the previous lesson. Have the class determine that a more accurate means of graphically presenting data is desirable.
2. One such graph, closely related to pictographs, is called a bar graph. Present the following:

A.



B.



Compare the two graphs and elicit the following:

a) Both graphs have titles and an indication on the vertical axis telling what each row of symbols or bars represents.

b) The scale for a pictograph is symbol, but the scale for a bar graph is a length on a line

c) To find the number of items in a pictograph you must count the number (or parts) of symbols and multiply by the value of each symbol. To find the number of items represented by a bar in a bar graph, estimate where the end of the bar falls in relation to the scale.

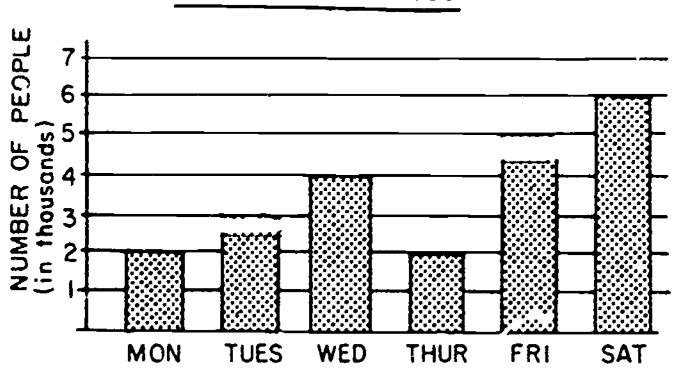
3. Elicit that the differences make the bar graph easier to read. (However, a pictograph can be visually more attractive.)

4. Do Application A.

5. Elicit that in the bar graph presented, the bars were drawn horizontally and a number scale was on the horizontal axis. The vertical axis contained the labels. It is also possible to draw the bars vertically. In this case, where must the number scale be shown? What information must be shown on the other axis?

6. Present the following vertical bar graph. Discuss its parts and answer the following questions:

ATTENDANCE AT A CONCERT
ONE WEEK IN AUGUST



a) On which day did the most people attend? Fewest?

b) Approximately how many people attended on Wednesday? Thursday?

c) What is the difference between the largest crowd and the smallest crowd?

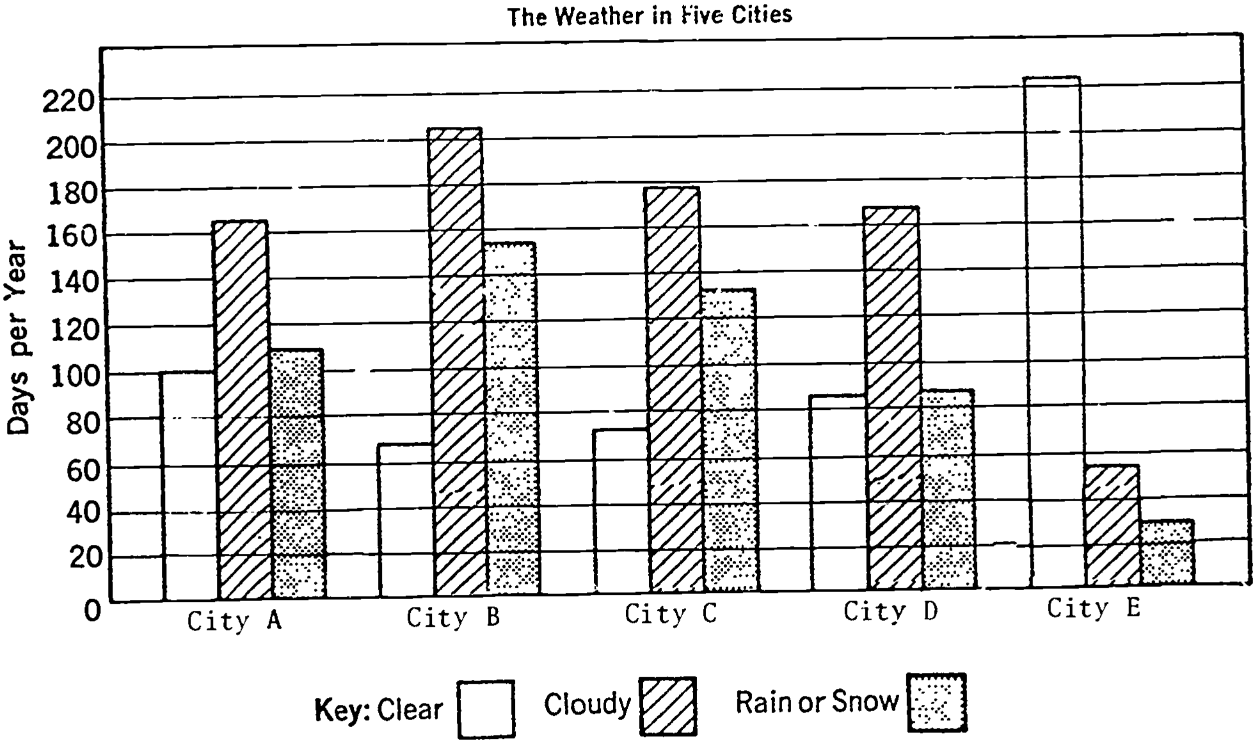
d) What was the approximate total attendance for the week? The average daily attendance?

7. Do Application B.

8. Present the table and multiple bar graph that follows. Compare how the same information can be presented in two different ways. From which presentation is it easier to extrapolate information?

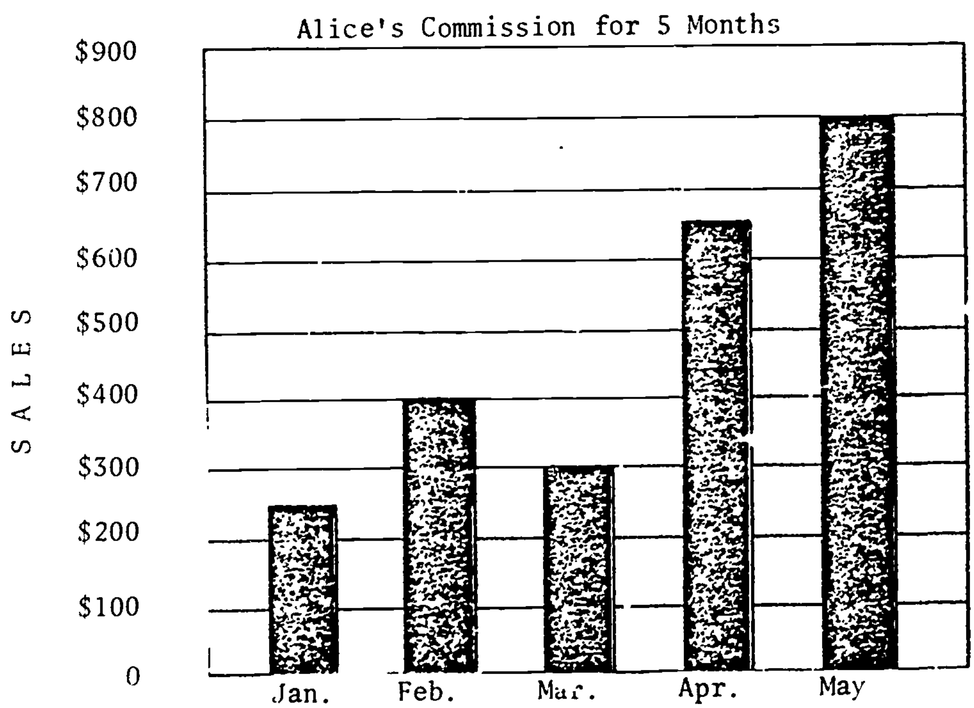
The Weather in Five Cities

	City A	City B	City C	City D	City E
Clear	101 days	68 days	72 days	84 days	223 days
Cloudy	166 days	204 days	178 days	167 days	54 days
Rain or Snow	109 days	154 days	132 days	87 days	28 days



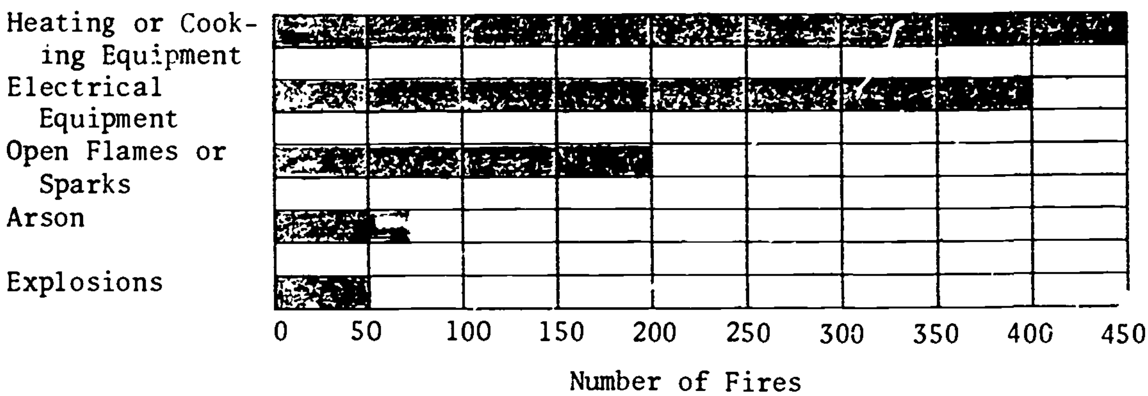
APPLICATIONS:

A. Answer the following questions based on the given bar graph:



1. What was Alice's total commission for January? For February?
2. For which month was Alice's commission highest? Lowest?
3. For which two months was it the same?

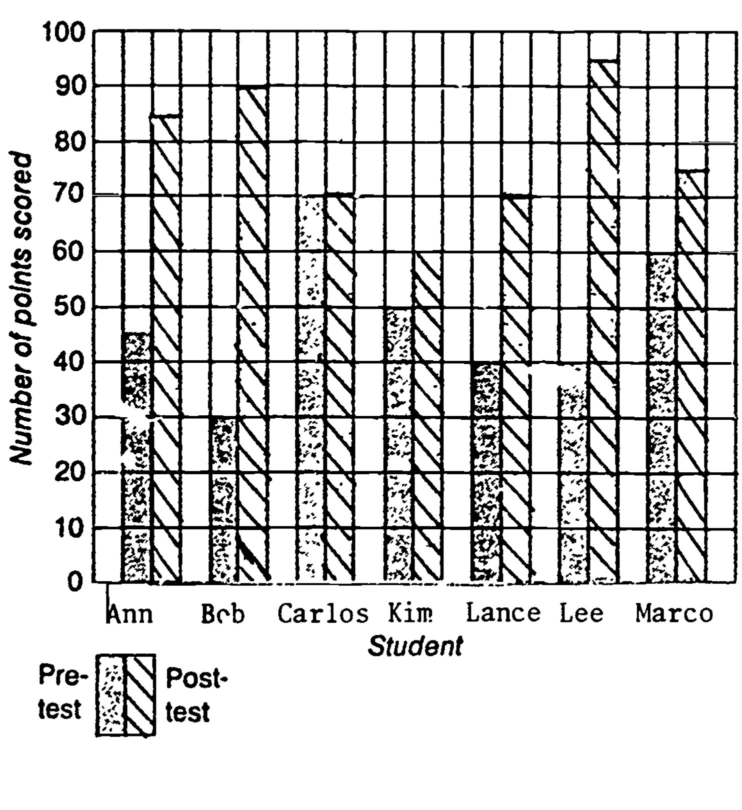
B. Major Causes of Fire in New York City



1. What are the two major causes of fires?
2. About how many fires did electrical equipment and heating or cooking equipment cause?
3. About how many times more fires are caused by explosions than are caused by heating or cooking equipment?
4. Approximately how manv fires were caused by arson?

C. Use the double bar graph to answer the following questions:

Student Scores on Math Tests



1. Who scored the most points on the pretest? The fewest points?
2. Who scored the most points on the post-test? The fewest points?
3. Who scored 30 points higher on the post-test than on the pretest? 15 points higher?
4. Who scored the same on the pretest and post-test?
5. Which students had the same scores on their post-tests?
6. Which student showed the most improvement from pretest to post-test?

SUMMARY:

List the major components of a bar graph (title, horizontal and vertical scales or labels).

ERIC

LESSON 4

AIM: To read and interpret line graphs

PERFORMANCE OBJECTIVES: Students will be able to:

identify line graphs as types of graphs that illustrate trends more effectively than bar graphs.

- read and interpret line graphs.
- state that line graphs tend to show a continuous series, process, or tendency (optional).

VOCABULARY:

New terms: line graph, trend, (interval)

Review: horizontal and vertical scales (axes)

CHALLENGE PROBLEM:

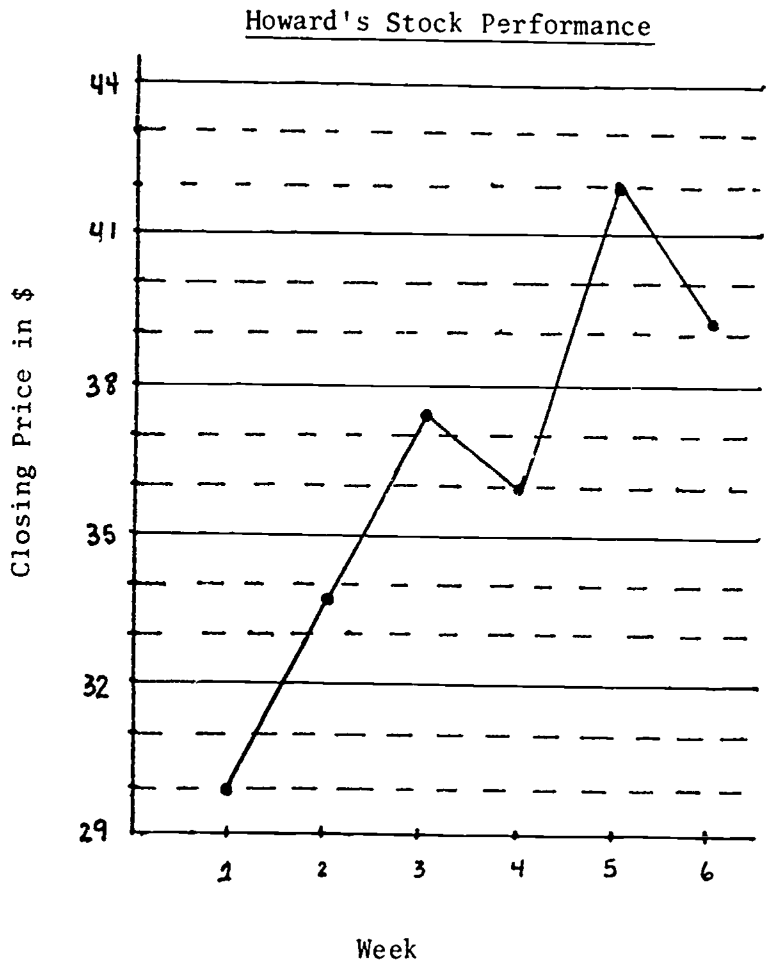
Howard kept track of the weekly closing of his stock for six weeks and found the following data:

Week	Closing Price (in dollars)
1	30
2	$33\frac{3}{4}$
3	$37\frac{1}{2}$
4	36
5	42
6	$39\frac{1}{4}$

During which weeks did his stock rise in price?

DEVELOPMENT:

1. Discuss the challenge problem. Elicit that stock rose from week 1 through week 3 and week 4 to week 5.
2. In what way might this data be presented so that it may be interpreted more easily? (a graph) Data of this type is usually presented using a line graph. (Teacher may wish to present line graphs from newspapers.) Present the following line graph:



3. Discuss the major components of a line graph.

   a) Title

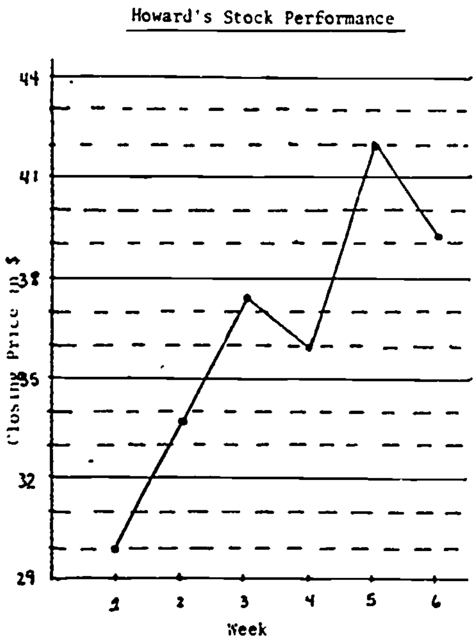
   b) Horizontal and vertical axes each having a scale.

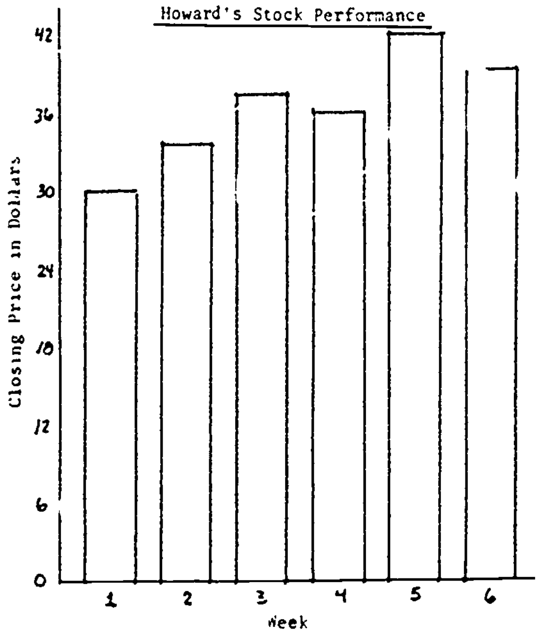
   c) Each point (dot) represents 2 pieces of information - the week number and its corresponding closing price.

   d) A line segment is drawn between each pair of consecutive dots. Each line segment shows the <u>trend</u> for the indicated period.

4. Do applications.

5. Compare bar graphs and line graphs, by presenting a bar graph drawn from the data in the challenge problem.





Elicit the following comparisons:

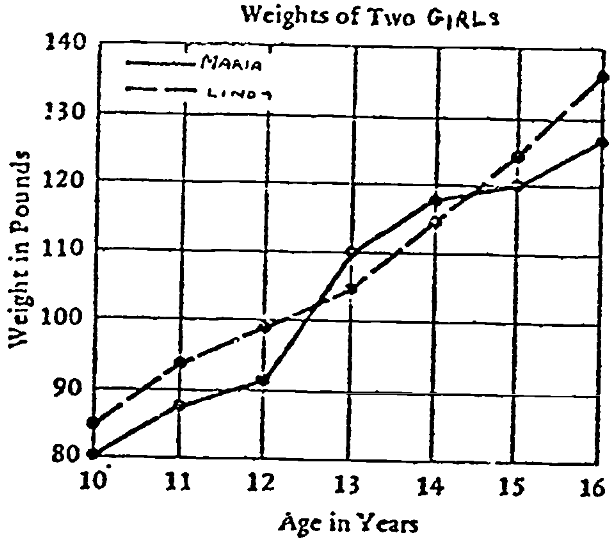
a) Both graphs have the same title and horizontal axis.

b) On the bar graph, the vertical scale begins at zero whereas on the line graph, we chose to begin at 29. Why? Could 31 be used as the starting point?

c) Show that by placing a dot in the center of each bar and connecting the dots consecutively, a line graph is obtained. Why do the two line graphs appear different? (Different scales.) On which graph are the trends more easily seen? (line graph) Which graph shows a truer picture of the stocks' performance in relation to its overall cost?

(Teacher may wish to compare this stock's performance to the performance of another stock whose prices range from 230 to 239¼.)

6. Present and discuss multiple line graph shown in Figure 3:



Study the graph in Fig. 3 and answer the following questions:

a) What type of graph is pictured?

b) What is the subject of the graph?

c) What is shown on the horizontal axis? On the vertical axis?

d) Whose weight is represented by the dotted line graph?

e) At age 15 what was Maria's weight? What was Linda's weight?

f) At which ages did Maria weigh more than Linda?

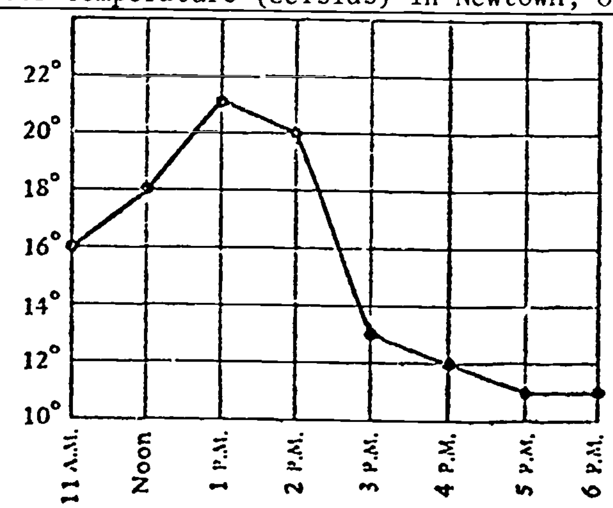
g) Between which two ages did Maria's weight increase most?

h) At which age was the difference between Linda's weight and Maria's weight greatest?

7. (Optional) Using graph in Application A, pose the following question: Approximately what was the temperature at 2:30 P.M.? Discuss the fact that the lines in a line graph may represent uniform change in an interval.

APPLICATIONS: Outdoor Temperature (Celsius) in Newtown, Oct. 30

Figure 4

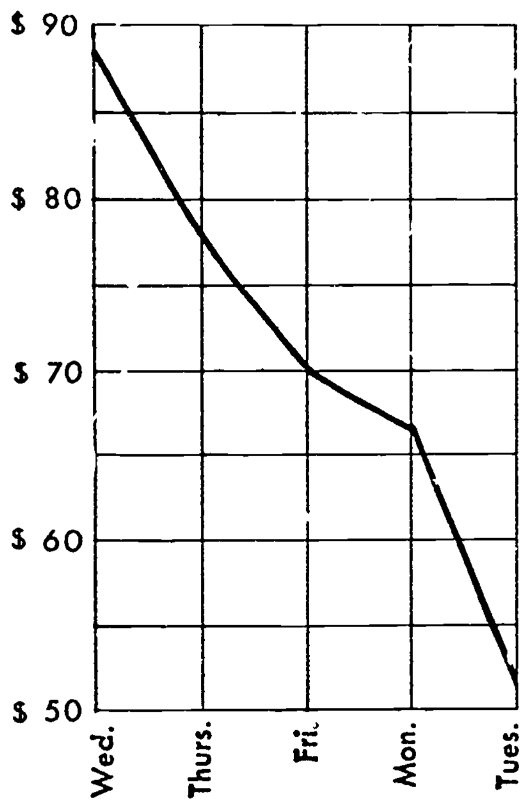


A. Study the line graph in Fig. 4 and answer the following questions:

1. What is the title of the graph?
2. What does the horizontal axis show?
3. What does the vertical axis show?
4. At what temperature does the scale start?
5. On a line graph, must the scale start at zero?
6. What was the temperature at noon? At 3:00 P.M.?
7. At what hours was the temperature highest? Lowest?
8. Between which hours was the temperature rising? Falling? Unchanged?
9. How much did the temperature rise between 11:00 A.M. and noon? Between noon and 1:00 P.M.?
10. Between which two successive hours did the temperature fall most rapidly?

Performance of JCN Company Stock

Figure 5



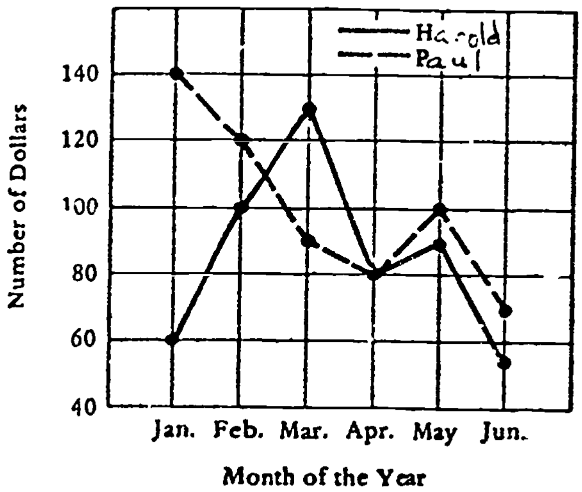
B. Study the line graph in Figure 5 and answer the following questions:

1. What is the decrease in price of the stock between Wednesday and the following Tuesday?
2. What was the stock's price on Thursday?

3. Between which two days did the stock fall most in price?
4. What was the stock's price at the close of business on the last day?
5. Between which two days did the stock fall least in price?
6. What is the trend in the stock price?

Earnings of Harold and Paul

Figure 6



C. Study the line graph in Figure 6 and answer the following questions:
   1. What is the title of the graph?
   2. How is the horizontal axis labeled?
   3. What scale is used on the vertical axis?
   4. Whose earnings are represented by the solid line graph?
   5. In which month were the earnings greatest for Harold? For Paul?
   6. In which month were the earnings least for Harold? For Paul?
   7. What was the greatest amount earned by either of the boys?
   8. What was the least amount earned by either of the boys?
   9. During how many months were Paul's earnings greater than Harold's?
   10. In which month did Harold earn the same amount as Paul?
   11. In which month was the difference between Paul's earnings and Harold's earnings greatest?

SUMMARY:

Review the purpose and interpretation of line graphs. Emphasize that the choice of scales can distort or exaggerate the facts being shown. One major purpose of a line graph is to indicate trends.

ERIC

LESSON 5

AIM: To read and interpret circle graphs

PERFORMANCE OBJECTIVES: Students will be able to:

- identify circle graphs as good graphs to use when illustrating the subdivisions of a whole quantity.
- read and interpret circle graphs where the subdivisions of a whole are given as numerical quantities.
- compare the relative sizes of sectors of a circle by inspection

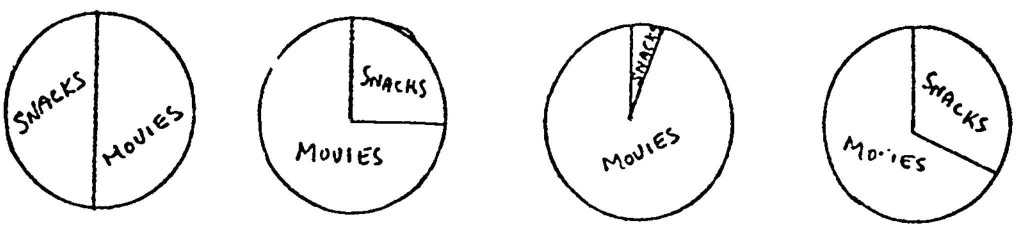
VOCABULARY:

New terms: circle graph

Review: sector

CHALLANGE PROBLEM:

John received $4.00 for his weekly allowance. He spent $1.00 for school snacks and $3.00 for movies. Which diagram correctly represents this information?



DEVELOPMENT:

1. Using the challenge problem, elicit the following:

   The circle represents John's total allowance. The divisions of the circle represents the approximate portions of the allowance allotted to each activity. What part of his allowance was spent on snacks? (¼) Which diagram show snacks as ¼ of the whole circle?

2. Graphs using the diagrams presented above are called circle graphs.

   a) In a circle graph the whole circle represents the entire quantity discussed (allowance). [The teacher may present other circle graphs obtained from newspapers, income tax instruction booklets, magazines, etc.] The title of the graph indicates what the whole circle represents. What might be an appropriate title for a graph of the information in the challenge problem?

   b) Each sector of the circle represents a subdivision of the whole quantity and is labeled appropriately. The size of each sector represents the part of the whole taken up by a subdivision. In the

the challerge problem, what are the subdivisions? How much of his all›wance is spent on each activity? Enter this information on the diagram. Elicit that the sum of these quantities is equal to ~:.00 (the whole allowance). The graph is now complete because:

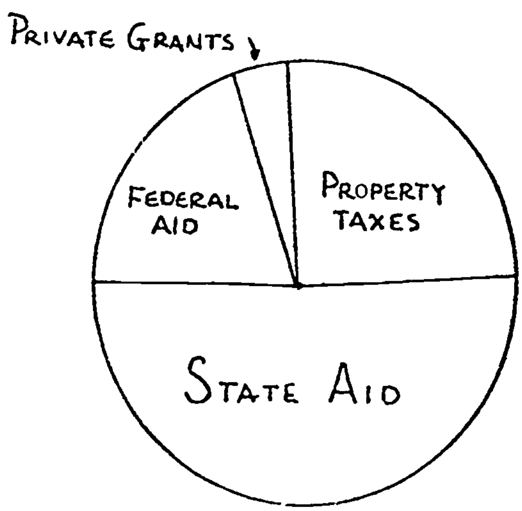
(1) it has a title, and

(2) each sector has a label and an indicated size.

3. Dr applications.

APPLICATIONS:

A. Sources of income for a local school district:



List from which sources the money is received in the order of largest to smallest.

B. Health Education Recreational Activity Choices:

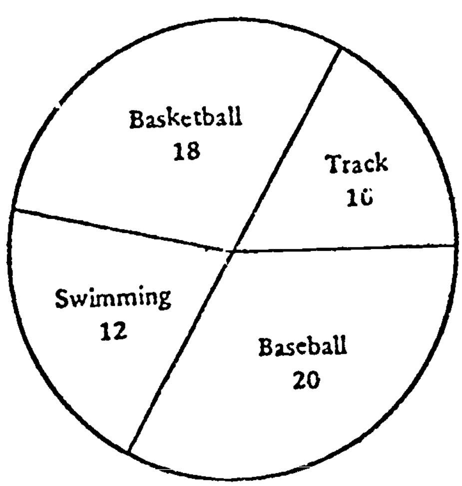
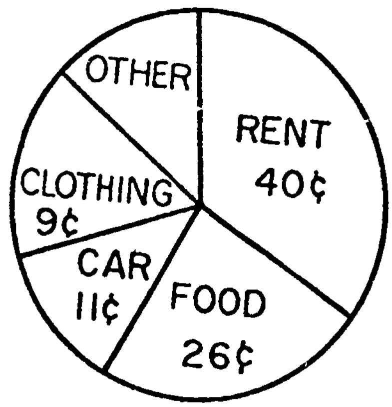


Fig. 3

58

1. How many pupils were in the health education class?
2. What fractional part of the number of pupils in the class selected basketball? Track? Baseball? Swimming?
3. Which sport was selected by the greatest number of pupils? The smallest ..umber of pupils?
4. How many more students selected baseball than track? Selected basketball than swimming?
5. What ıs the ratio of the number of pupils who selected baseball to the number of pupils who selected track? Who selected basketball to the number of pupils who selected swimming?

C. <u>How Wai-Keung Spends Each Dollar He Earns</u>



1. How much of each dollar is spent on rent? Food? Car? Clothing? Other?
2. On which item is the most money spent? Least money spent?
3. How much more is spent on rent than on clothing?
4. What part of each dollar is spent on rent?

SUMMARY:

What kind of information is represented by circle graphs?

How does the size of the sector relate to the size of the whole?

CLIFF-HANGER:

In Application C, if Wei-Keung earns $600 a month, how much does he spend monthly on car expenses?

LESSON 6

AIM: To define percent and find percent of a number

PERFORMANCE OBJECTIVES: Students will be able to:

- express percent as the ratio of a number to 100.
- find the percent of a number by expressing the percent as a fraction and multiplying.
- state that the sum of all the parts of a quantity is 100%.

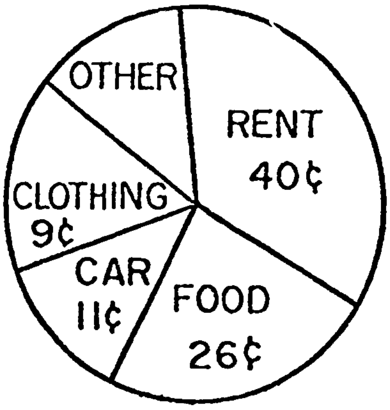
VOCABULARY:

New terms: percent

Review: circle graph, ratio

CHALLENGE PROBLEM:

How Carl Spends Each Dollar He Earns



If Carl earns \$600 a month, how much does he spend on rent?

DEVELOPMENT:

1. Discuss the challenge problem. Recall that each sector in a pictograph relates the item shown to the whole quantity. In this case 40¢ of each \$1.00 is spent on rent. This can be expressed as $\frac{40}{100}$ of the money spent. Therefore this problem can be solved by finding $\frac{40}{100}$ of \$600.

$$\frac{40}{100} \times \frac{600}{1} =$$

$$\frac{2}{\cancel{5}_1} \times \frac{\overset{120}{\cancel{600}}}{1} \quad \text{or} \quad \frac{40}{\cancel{100}_1} \times \frac{\cancel{600}^{6}}{1}$$

$$= 240 \qquad\qquad = 240$$

2. Consider the fraction $\frac{40}{100}$. Fractions with denominators of 100 are often written in the form 40%. Have students observe the similarity between the symbol % and its meaning "$\frac{}{100}$" (division by 100).

   Define a percent as the ratio of a number to 100.

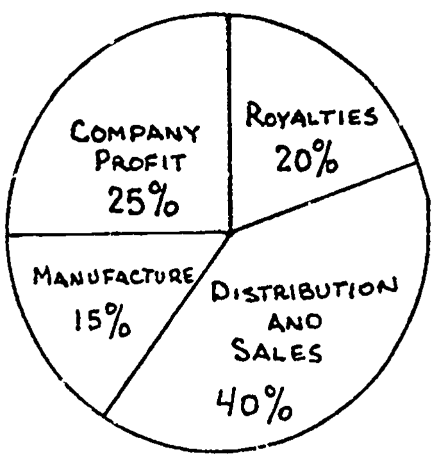
   For example: $\frac{40}{100}$ = 40%   65% = $\frac{65}{100}$

3. Do Application A.

4. Return to the graph in the challenge . Express each sector label as a fractional part of a dollar. That is: Car, $\frac{11}{100}$; Food, $\frac{23}{100}$; etc. What is another way in which each of these fractions could have been presented? (As percents, i.e., 11%, 23%, etc.)

5. Explain that many circle graphs present information using percents. Provide students with examples, such as:

   Where Money From Record Sales Goes



   a) Where does the smallest part of the money go? Largest part?
   b) What percent is paid in royalties to the song writer, singer and/or musicians?
   c) What fractional part of the money from sales is used for manufacturing?

6. Suppose a record album costs \$8. How much of this cost is returned to the company as profit? Have students express this problem as:

   25% of \$8

   Since 25% can be written as a fraction, this problem can be expressed in the same form as the challenge problem, that is $\frac{25}{100}$ of \$8. Solve the problem.

7. Generalize a procedure for finding the percent of a number:
   a) Express the percent as a fraction.
   b) Multiply the fraction by the given amount and express the answer in simplest form.

8. Do Applications B and C.

APPLICATIONS:

A. 1. Express each of the following as a fraction in lowest terms.

a)	25%	e)	100%
b)	30%	f)	15%
c)	20%	g)	16%
d)	7%	h)	72%

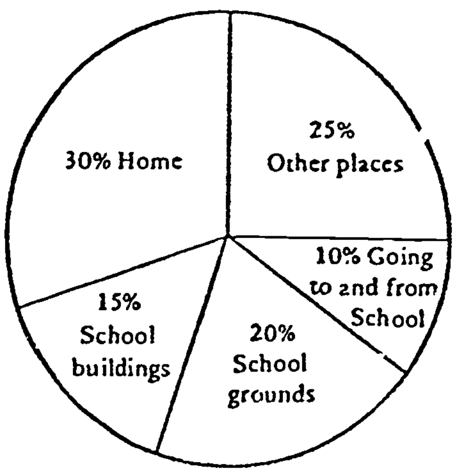
2. Express each of the following as a percent.

a)	$\frac{3}{100}$	e)	$\frac{4}{5}$
b)	1 to 100	f)	$\frac{3}{4}$
c)	$\frac{8}{100}$	g)	$\frac{3}{15}$ (note: Reduce to $\frac{1}{5}$ first)
d)	$\frac{3}{10}$	h)	1

B. Find the indicated percent of the given numbers.

1.	68% of 100	5.	50% of 362
2.	10% of 750	6.	75% of 36
3.	24% of 75	7.	100% of 721
4.	7% of 600	8.	1% of 2000

C. Accidents to Children in Viewridge



1. What percent of the total number of accidents occur going to and from school? On school grounds?
2. Where do the greatest number of accidents to school children happen?
3. How many times more accidents happen in the home than happen in school buildings?

4. What fractional part of the accidents to school children happen at home? In School buildings? On school grounds? Going to and from school?

5. If there were 480 accidents to school children in one year, how many accidents happened at home? In school buildings? On school grounds? Going to and from school?

SUMMARY:

1. In a circle graph, what percent is represented by the entire circle?
2. Review the procedure for finding the percent of a number.

CLIFF-HANGER:

Using the "Record Sales" circle graph, find the cost of manufacturing a record album that retails for $8.90.

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LESSON 7

AIM: To express percents as decimals and to find a percent of a number

PERFORMANCE OBJECTIVES: Students will be able to:

- express whole number percents as decimals.
- find a percent of a number when the percent is expressed as a decimal.

VOCABULARY:

Review: decimal, percent

CHALLENGE PROBLEM:

Find 15% of $8.90

DEVELOPMENT:

1. Discuss challenge problem and compare it to the cliff-hanger.

   $$\frac{15}{100} \times \frac{\$8.90}{1}$$

   Why is this problem more difficult than the ones done previously? (This problem is not easily solved using fractions.)

2. Elicit that another way of expressing 15% is .15 (read 15 hundredths).

3. Do Application A.

4. Return to challenge problem. Solve as follows: 15% of $8.90.

   $$.15 \times \$8.90 \quad \text{or} \quad \begin{array}{r} \$8.90 \\ \underline{\times\ .15} \\ 4450 \\ \underline{890} \\ \$1.3350 \end{array} \longrightarrow \$1.34.$$

   Review multiplication of decimals, placing decimal point, and rounding off to nearest cent.

5. Do Applications B and C.

APPLICATIONS:

A. 1. Express each of the following as a decimal:

a) 42%	e) 100%
b) 13%	f) $\frac{7}{10}$
c) 5%	g) $\frac{2}{5}$
d) 1%	h) $\frac{1}{4}$

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2. Write each of the following as a percent:

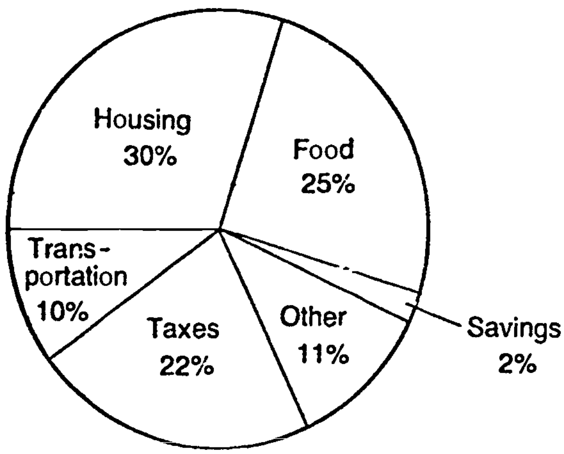
a) .47
b) .86
c) .30
d) .3
e) .06
f) .6
g) 1
h) 1.35

B. Compute the following:

a) 39% of 671
b) 2% of 36
c) 6% of 150
d) 98% of 79
e) 65% of $12.50
f) 82% of 9.9

C. Answer the following questions based on the given circle graph. Express each result to the nearest cent. (Change percents to either fractions or decimals.)

How the Allen Family Budgets its $18,750 Annual Income



1. Which budget item is $\frac{1}{3}$ of the cost of housing?
2. Taxes are twice as much as which item?
3. What is the total percent spent on food and housing?
4. Find the dollar amount budgeted for each item.
5. How much more is spent than is saved?

SUMMARY:

What are the two methods for finding a percent of a number? How do you decide which to use? Why?

LESSON 8

AIM: To locate points on a number line, to find the distance between two points and to locate a point midway between two other points

PERFORMANCE OBJECTIVES: Students will be able to:

- locate any signed number on the number line.
- state that the number associated with a point on the number line is called its coordinate.
- locate a point midway between two other points and determine its coordinate.
- find the distance between two given points the number line.
- state that the midpoint of a line segment is the average of the coordinates of the end points.

VOCABULARY:

New terms: coordinate, midpoint

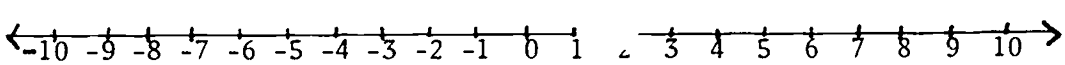
Review: average, positive, negative, distance, number line, opposites, additive inverses

CHALLENGE PROBLEM:

In a movie theater, there are two sections of seats, one to the left of the center aisle and one to the right. The theater is already dark because the show has begun. The usher tells you that there is an empty seat 5 seats from the center in a certain row. Does this information tell you exactly where to look for the empty seat?

DEVELOPMENT:

1. Elicit from the pupils the fact that, since the usher did not say whether the seat was to the right or to the left of the center aisle, the exact location of the seat is uncertain. Pupils should note that the seat could be in either of two places. Translate these thoughts into a geometric model, a graph in one dimension (a number line). Draw a number line on the board, as shown:



Using the center aisle to separate seat numbers considered positive from those considered negative, elicit the following conventions from the class: the center aisle location may be represented as the "0" point on the number line, all seats to the right of the center are designated with positive integers, and all seats to the left of center are designated with negative integers.

2. Ask the class how the two seats that are possibly empty might be described in terms of the number line. The pupils should readily respond with the answers "+5" (or "5") and "-5". Ask the class to locate on the number line a variety of seats to the left (negative)

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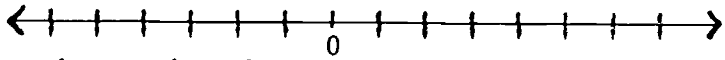
and to the right (positive) of the center aisle. Also mark several points on the number line and ask pupils to read their coordinates. For this exercise, use a number line marked only with the zero point:



Define the number naming the location of a point on a line as the coordinate of that point.

3. Ask the class how far apart are the two seats discussed. They may observe that the answer is 10, since each seat is 5 spaces to the right or left of the center aisle. This question is equivalent to asking for the distance between -5 and +5. Refer to the number line on which +5 and -5 have been indicated. Lead pupils to determine this answer by counting the number of spaces between -5 and +5.

4. Do Application A.

5. Call to the attention of the class the fact that the "center aisle" in the theater is given that name because is is midway between the seats on the right and the corresponding seats on the left. The geometric equivalent of this observation is the fact that the zero point on the number line is the midpoint of the line segment joining any two points whose coordinates are +a and -a. Illustrate this concept with such points as those with coordinates +5 and -5, +7 and -7, +3 and -3, etc. (These number pairs are also called opposites or additive inverses.)

6. Generalize to points other than those with coordinates of +a and -a in the following manner: Have pupils locate points on the number line midway between any two other points on the line with integral coordinates. Permit them to do this by counting. For example, to find the point midway between +5 and -3, the pupils should note that these points are 8 units apart on the number line and that the point midway between them is 4 units distant from each of the points. Recall that movement to the right on a number line is positive and movement to the left is negative. To get to the midpoint of the line segment joining the two points, it is necessary to move 4 units to the right of -3 [- 3+ (+4) = + 1] or move 4 units to the left of +5 [+5 + (-4)= + 1]. Therefore + 1 is the midpoint of the line segment joining + 5 and - 3, or +1 is the number halfway between + 5 and - 3.

7. Elicit that finding the midpoint of a line segment is equivalent to finding the average of the coordinates of the endpoints. Recall that the average of two numbers is found by adding the numbers and dividing by 2. Thus the average of +5 and -3 is:

$$\frac{(+5) + (-3)}{2} = \frac{+2}{2} = +1$$

8. Do Applications B and C.

APPLICATIONS:

A. For each of the following pairs of points locate the points on a number line and find the distance between them:

1) + 1 and + 9	5) - 7 and -2
2) 0 and +3	6) -1 and + 4
3) 4 and 10	7) - 2 and + 10
4) - 3 and - 9	8) 6 and - 10

B. For each of the coordinates listed in Application A, find the coordinate of the midpoint of the line segment joining the points, by counting. Check your results by averaging each pair of numbers.

C. 1. In a theater, you are sitting in the third seat to the right of the center aisle and your friend is sitting in the ninth seat to the right of the center aisle in the same row.

a) Show these facts on a number line.

b) How many seats apart are you and your friend?

c) What is the number of the seat which is halfway between your friend's seat and your seat?

2. In the auditorium, two pupils are sitting in the same row. One pupil is sitting seven seats to the right of the center aisle while the other is sitting three seats to the left of the center aisle.

a) Show these facts on a number line.

b) How many seats apart are these two seats?

c) Which seat is midway between these two seats?

SUMMARY:

1. Review the fact that points on the number line have coordinates which are positive, zero, or negative, according to their location on the line.

2. Also review the method of finding the coordinate of a point midway between any two other points on the number line and the method of finding the distance between any two points on the number line.

CLIFF-HANGER:

If there were only one empty seat in an entire dark movie theater, what information would you need from the usher in order to find that seat?

LESSON 9

AIM: To name points on the coordinate plane using ordered number pairs

PERFORMANCE OBJECTIVES. Students will be able to:

- describe a location in a coordinate plane using two numbers (coordinates).
- distinguish between location (3, 2) and location (2, 3), and define an ordered number pair.
- identify coordinates of a point to the left of 0 on the horizontal number line as negative, and similarly, coordinates below 0 on the vertical number line as negative.
- label the horizontal number line with x, and call it the x-axis (horizontal axis).
- label the vertical number line with y, and call it the y-axis (vertical axis).
- locate the origin and state its coordinates.
- read and locate the coordinates of given points on the coordinate plane, including points on the axes.

VOCABULARY:

New terms: x-axis, y-axis, coordinate axes, origin, quadrant, ordered pair, coordinates of a point

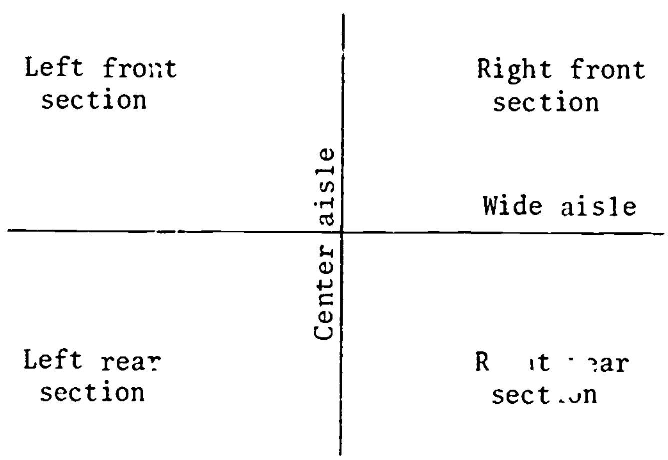
Review: coordinate

CHALLENGE PROBLEM:

A theater has a wide aisle dividing the front and rear sections as well as a center aisle dividing the <u>right</u> and <u>left</u> sections of the theater. How many sections does this theater have?

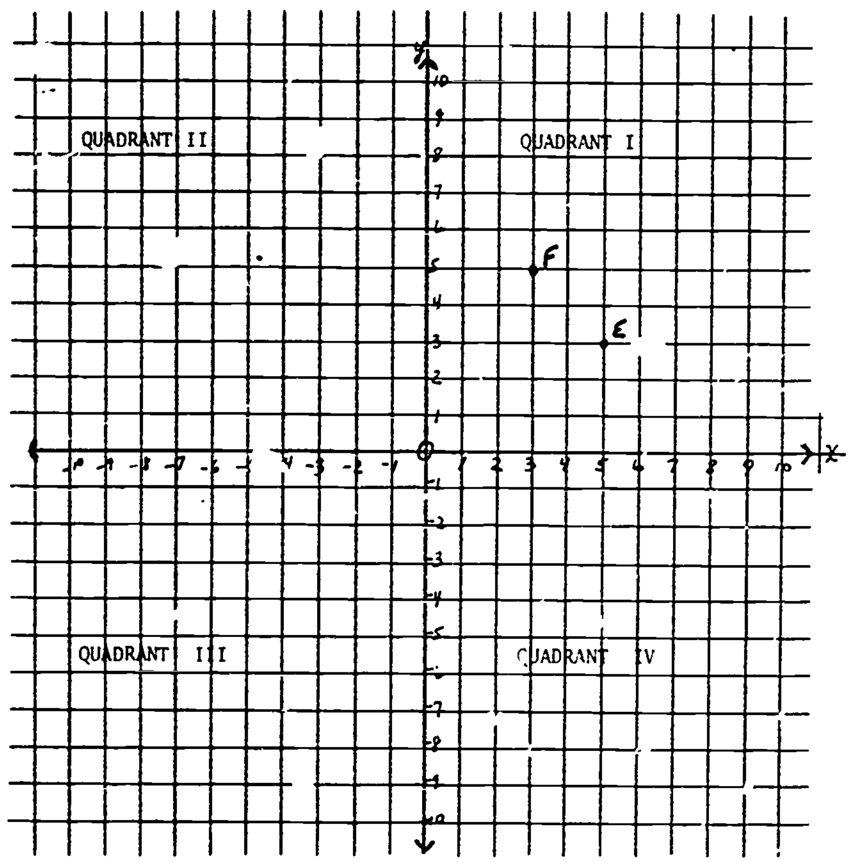
DEVELOPMENT:

1. Using the challenge problem, elicit from the students that the theater is divided into four sections and can be represented in a diagram as follows:



Using the aisles as reference lines, ask students what informat they would need to find a particular location in the theater. Elicit that two pieces of information are all that is needed to find any location (in this theater). The pieces of information needed are the row number and the seat number.

2. Develop the analogy between aisles and coordinate axes. Label them x and y and refer to them as the "x axis" and the "y axis". The sections of seats correspond to the four quadrants in the plane. Mark integral values along both axes, in a manner similar to that used on the number line in the previous lesson. Explain that, just as points to the right of zero were designated as positive and those to the left of zero were designated as negative in the previous lesson, so we now also designate points above zero (on the vertical axis) as positive and those below zero as negative. Elicit the fact that the y axis is another number line. The zero point on the plane may be referred to as the "origin" (the "original" reference point from which measurements are made in the horizontal and vertical directions). The following diagram should now be on the board: Compare this diagram to the diagram of the theater.



Indicate that the axes form right angles at the origin.

In comparing the two diagrams consider such questions as:

a) What quadrant corresponds to the left front section?

b) Which aisle corresponds to the x-axis?

c) Where in the theater is the origin located?

3. Using the diagram of the coordinate plane, (figure 2) choose a location labeled E in the diagram. Have students state how they would describe the location of point E if the origin is used as the starting point. Students may respond: go 5 units to the right and 3 units up, or 3 units up and 5 units to the right. Inform the students that the usual order in which this information is given is 5 units to the right and 3 units up. This can be simplified by writing the ordered pair of coordinates (5, 3).

   Definition: In an ordered pair of coordinates, movement to the right or left of the vertical axis is indicated first. The second number indicates movement up or down from the horizontal axis. These coordinates are written within parentheses and separated by a comma.

   What are the coordinates of point F in figure 2? (3, 5) Have students observe that the coordinates (5, 3) name a different point than the coordinates (3, 5). Emphasize the fact that movement right or left of the y-axis is always named first in an ordered pair.

4. Do Applications.

APPLICATIONS:

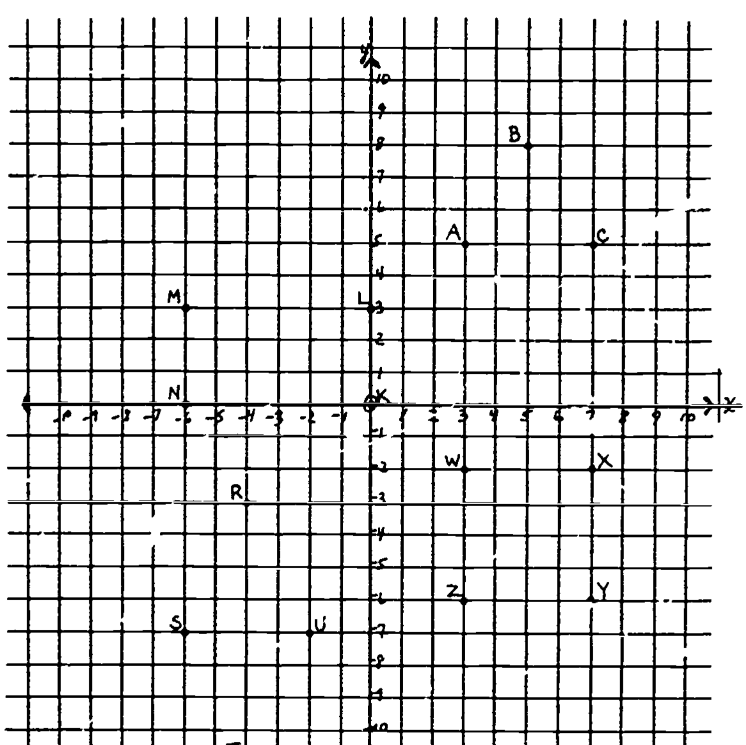


Figure 3

Use Figure 3 to answer all of Part A.

A. 1. Give the coordinates for each of the points shown (in Figure 3).

2. In what quadrant is each of the following points located?

(a) B (d) Y

(b) M (e) R

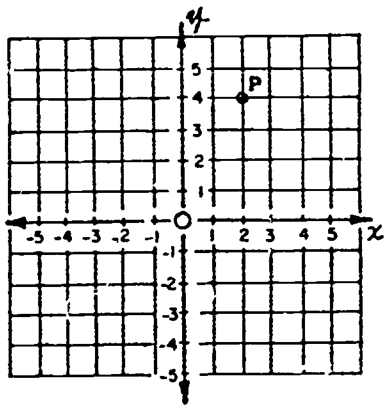
(c) S (f) C

3. Connect each set of alphabetically lettered points. What figures do you get?

Set ABC: Set RSTU:

Set KLMN: Set WXYZ:

B.



1. How many units is P from the vertical axis?

2. How many units is P from the horizontal axis?

3. Give the coordinates of the following points:

(a) The point 4 units below P.

(b) The point 4 units above P.

(c) The point 4 units to the right of P?

(d) The point 4 units to the left of P?

(e) The point 3 units to the left of P and 2 units below it.

SUMMARY:

Review the manner in which the coordinate plane is divided into four quadrants by the coordinate axes and how the locations of points in the plane are described by ordered number pairs. Identify coordinates of the origin. Review the new vocabulary introduced in this lesson.

LESSON 10

AIM: To locate points on the coordinate plane

PERFORMANCE OBJECTIVES: Students will be able to:

- plot points on the coordinate plane, given their coordinates.
- state that the first number in an ordered number pair is called the x-coordinate and the second is called the y-coordinate.

VOCABULARY:

New terms: x-coordinate, y-coordinate, plot

Review: coordinate plane, ordered number pair, coordinate axes, quadrant, coordinates of a point, origin

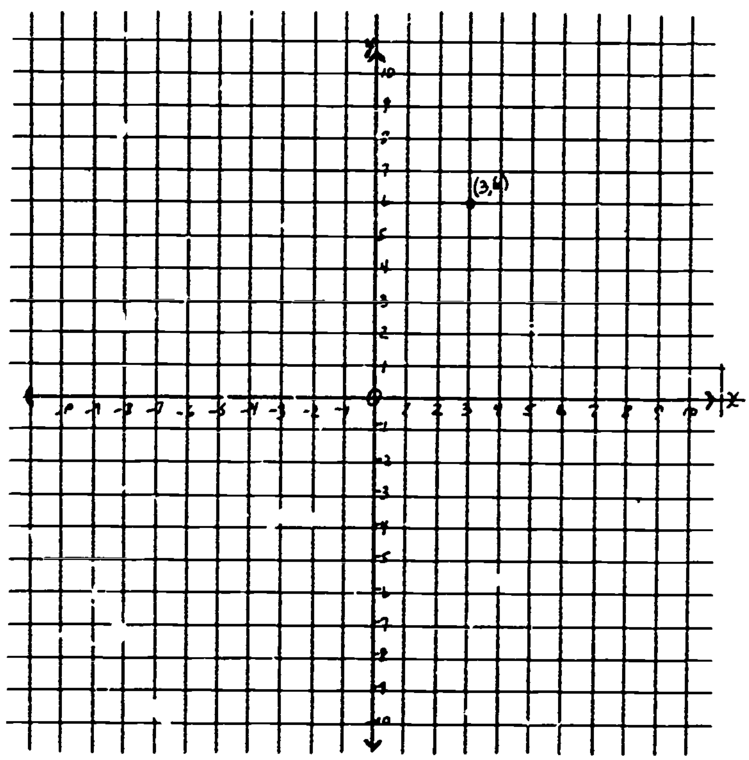
CHALLENGE PROBLEM:

Lorraine leaves her house at (3,6) and heads for the drugstore which is located at (-2, -8). However she meets Rochelle at (-1, -5) and they then decide to go to the candy store at (-3, -2). Will they pass the drugstore on their way to the candy store?

DEVELOPMENT:

1. Refer to the challenge problem. Have students determine that, in order to answer this question, a diagram of the coordinate plane must be drawn. Elicit the following:

   (a) A set of coordinate axes must be drawn and labeled in the appropriate manner. (See diagram below.) Emphasize that the scale on each axis is uniform.



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(b) Review the meaning of ordered pair. Introduce the phrase "plot a point" as locating an ordered pair in the coordinate plane. For example (3,6) indicates a point 3 units to the right of the vertical axis and 6 units above the horizontal axis. 3 is called the <u>x-coordinate</u> of the point (3,6) and 6 is called the <u>y-coordinate</u> of the point (3,6).

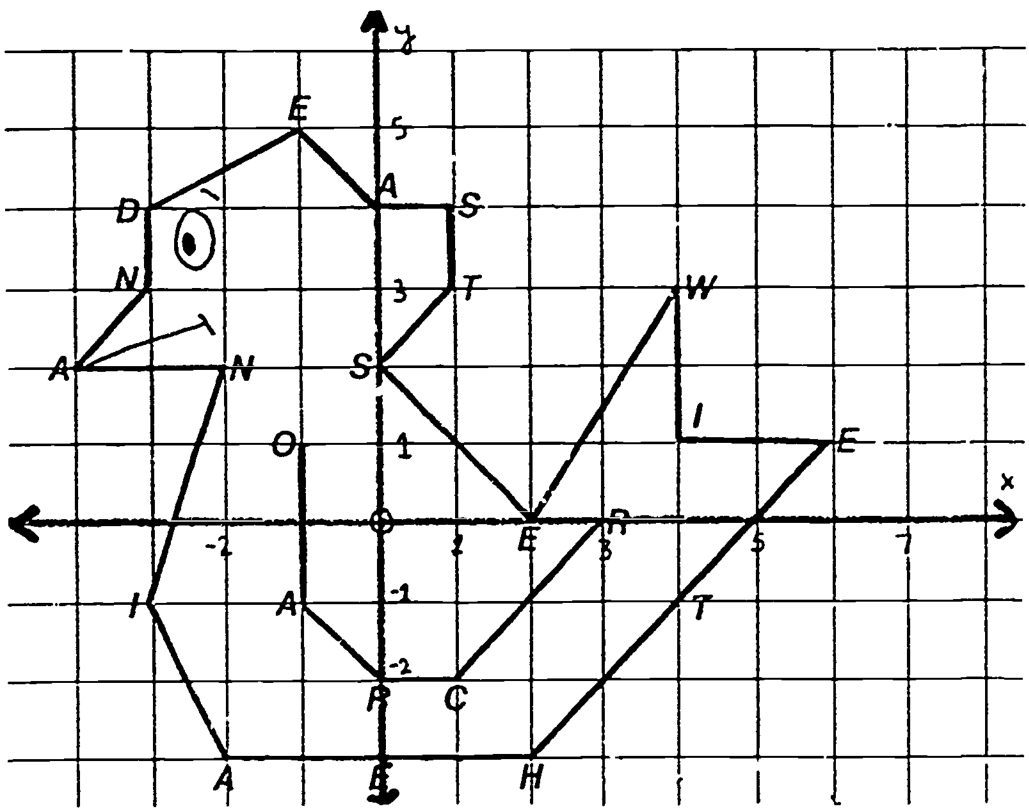
What information is given by the x-coordinate of an ordered pair? (distance right or left of the vertical axis)

Complete the solution to the challenge problem.

2. Do applications.

APPLICATIONS: Note to teacher: One set of coordinate axes should be used for Application B (all parts) and one set for Application D (all parts).

A.



Have students find the answer to the riddle: "Why did the student plant birdseed?" by following the directions below. Each point on the graph is lettered. To answer the riddle, write each letter above the matching ordered pair.

(2, -3) (2,0) (4, 3) (0, 4) (-2, 2) (1, 3) (-1, 5) (-3, 4)

(4, -1) (-1, 1) (3, 0) (-1, -1) (-3,-1) (1, 4) (6, 1)

(1, -2) (-4, 2) (-3, 3) (-2, -3) (0, -2) (4, 1) (0, -3) (0, 2)

B. 1. Connect the following points in alphabetical order. Then connect point M to point A:

A (10, 10)	F (-8, 0)	J (-3, -4)
B (8, 4)	G (-6, -6)	K (4, -6)
C (-10, 8)	H (-8, -8)	L (3, -8)
D (-16, 2)	I (-8, -5)	M (6, -8)
E (-12, -2)		

2. Shade the quadrilateral whose vertices are: (or whose corners are the points:)

   (-10, 8), (-10, 3), (-8, 3), (-5, 7)

3. Shade the triangle bounded by:

   (-16, 2), (-15, 1), (-15, 3)

4. Make a large dot at (-12, 4)

5. Name your figure.

C. Connect the following points in order and name the figure formed:

A (1, 2)	G (-4, 0)	M (1, -11)
B (6, 15)	H (-4, -3)	N (2, -8)
C (8, 18)	I (-7, -6)	O (2, -4)
D (6, 18)	J (-7, -9)	P (3, -2)
E (0, 2)	K (-6, -11)	Q (3, 1)
F (-2, 2)	L (-2, -12)	R (1, 2)

D. 1. Connect the following points in alphabetical order. Then connect U to A.

A (6, 14)	I $(-4, 2\frac{1}{2})$	Q (3, -6)
B (2, 10)	J $(-6\frac{1}{2}, \frac{1}{2})$	R (4, -3)
C (2, 3)	K (-7, -1)	S (4, 1)
D (-5, 10)	L (-6, -3)	T (3, 3)
E (-9, 13)	M (-3, -5)	U (6, 6)
F $(-8, 9\frac{1}{2})$	N (-3, -7)	
G (-6, 6)	O (-1, -6)	
H $(-1, 3\frac{1}{2})$	P (-1, -7)	

2. In each set of coordinates connect the points in order:

   a. (-5, -7), $(-5, -9\frac{1}{2})$, (-2, -7), (2, -10), (-5, -7)

   b. (-1, -8), (3, -7), (3, -8), (-1, -9), (-1, -8)

   c. $(-2\frac{1}{2}, 0)$, $(-2\frac{1}{2}, 1\frac{1}{2})$, $(-4\frac{1}{2}, 1\frac{1}{2})$, $(-4\frac{1}{2}, 0)$, $(-2\frac{1}{2}, 0)$

SUMMARY:

Review the technique of locating points on a plane given their coordinates and determining the ordered number pairs associated with given points.

CLIFF-HANGER:

Plot R(-2, -2), S(-2, 4) and T(6, -2)

Find the perimeter of the figure formed.

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LESSON 11

AIM: To find the distance between 2 points on the coordinate plane

PERFORMANCE OBJECTIVES: Students will be able to:

- find the length of a line segment that is parallel to one of the coordinate axes.
- apply the Pythagorean Formula to find the length of a line segment that is not parallel to either coordinate axis.
- find the distance between 2 points on the coordinate plane.

VOCABULARY:

Review: Pythagorean Formula, coordinate axes, perimeter, right triangle

CHALLENGE PROBLEM:

Given: C (0, 0), B (0, 4), A (3, 0)

Plot these points and connect these points to form a triangle.

What is the length of $\overline{AC}$? of $\overline{BC}$?

DEVELOPMENT:

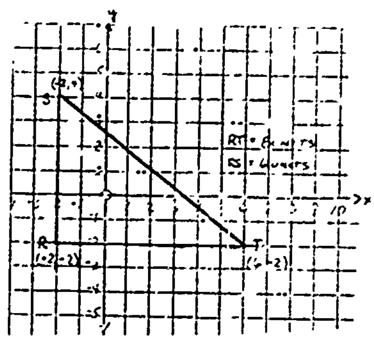
1. Discuss the challenge problem. Recall that both the x-axis and the y-axis are number lines. Review the procedure for finding the distance between 2 points on a number line by counting (counting the number of unit spaces between them).

   Elicit BC = 4 units and AC = 3 units.

2. Refer to the cliff-hanger problem from the previous lesson. Develop technique for finding the lengths of line segments parallel to either AXIS. Elicit that the method of counting the spaces between the end points may be used to find the length of a line segment which is parallel to either axis.

   Therefore:

   Fig. 1



   RT = 8 units

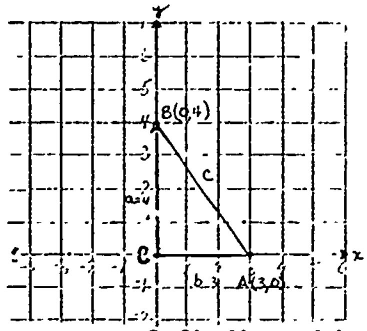
   RS = 6 units

3. Do Application A.

4. Return to the challenge problem. Ask students to find the length of $\overline{AB}$. Why can't the answer be 4?

Elicit that this problem can't be solved by the method of counting since there are no longer unit spaces along the line. One method of obtaining the length of $\overline{AB}$ is to use another piece of graph paper as a ruler. Place this piece of graph paper as shown below:

Fig. 2



Determine that the length of AB = 5 units

5. A more direct method of finding this result should be developed as follows:

   a) What type of triangle is $\triangle$ ABC? (right triangle)

   b) What name is given to side $\overline{BC}$ and $\overline{AC}$? (legs)

   c) What name is given to $\overline{AB}$? (hypotenuse)

   d) What formula can be used to find the length of the hypotenuse of a right triangle when its legs are known? (Pythagorean Formula) Show how the Pythagorean Formula may be used to find the length of $\overline{AB}$.

$$(\text{leg})^2 + (\text{leg})^2 = (\text{hypotenuse})^2$$

or

$$a^2 + b^2 = c^2$$

Therefore

$$4^2 + 3^2 = c^2$$
$$16 + 9 = c^2$$
$$25 = c^2$$
$$5 = c$$

Apply the Phythagorean Formula to find the length of $\overline{ST}$ in the cliff-hanger problem.

Since leg RT = 8 and leg RS = 6,

$$8^2 + 6^2 = (ST)^2$$
$$64 + 36 = (ST)^2$$
$$100 = (ST)^2$$
$$10 = ST$$

Complete the solution to the cliff-hanger problem reviewing the definition of perimeter.

6. Apply the Pythagorean Formula to find the distance between any 2 points in the coordinate plane.

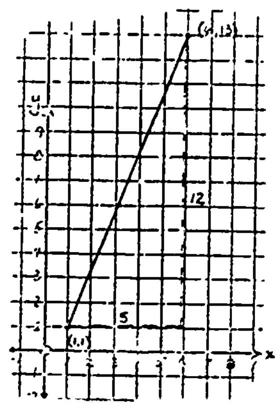
   Pose the problem: What is the distance between (1, 1) and (6, 13)?

   Elicit the following procedure:

   a) plot the points and draw the line segment between them.

   b) construct a right triangle with this segment as the hypotenuse and the legs parallel to the axes. See Figure 3.

Fig. 3



   c) find the length of each leg.

   d) use the Pythagorean Formula to find the length of the hypotenuse (distance between the 2 given points.

$$d^2 = 12^2 + 5^2$$
$$d^2 = 144 + 25$$
$$d^2 = 169$$
$$d = 13$$

APPLICATIONS:

A. Draw each of the following line segments and find its length

   1. (2, 1) and (9, 1)
   2. (2, -2) and (2, -7)
   3. (6, -2) and (-1, -2)
   4. (-1, -3) and (-1, 5)

B. Find the distance between each pair of points.

   1. (2, 1) and (5, 5) [5]
   2. (-8, 1) and (-3, 13) [13]
   3. (2, -3) and (-4, -11) [10]
   4. (-2, 3) and (4, -5) [10]
   5. (1, 4) and (6, -1) [$\sqrt{50}$]

C. Find the perimeter of each of the following quadrilaterals:

   1. A (4, 0), B (20, 0), C (8, 5), D (4, 5) Trapezoid
   2. A (0, 2), B (4.5), C (4, -4), D (0, -7) Parallelogram

SUMMARY:

Review the procedures for finding the distance between 2 points in the plane.

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LESSON 12

AIM: To determine the coordinates of the midpoint of a line segment

PERFORMANCE OBJECTIVES: Students will be able to:

- state that the x-coordinate of the midpoint of a line segment is the average of the x-coordinates of the endpoints.
- state that the y-coordinate of the midpoint of a line segment is the average of the y-coordinates of the endpoints.
- determine the coordinates of the midpoint of a line segment.

VOCABULARY:

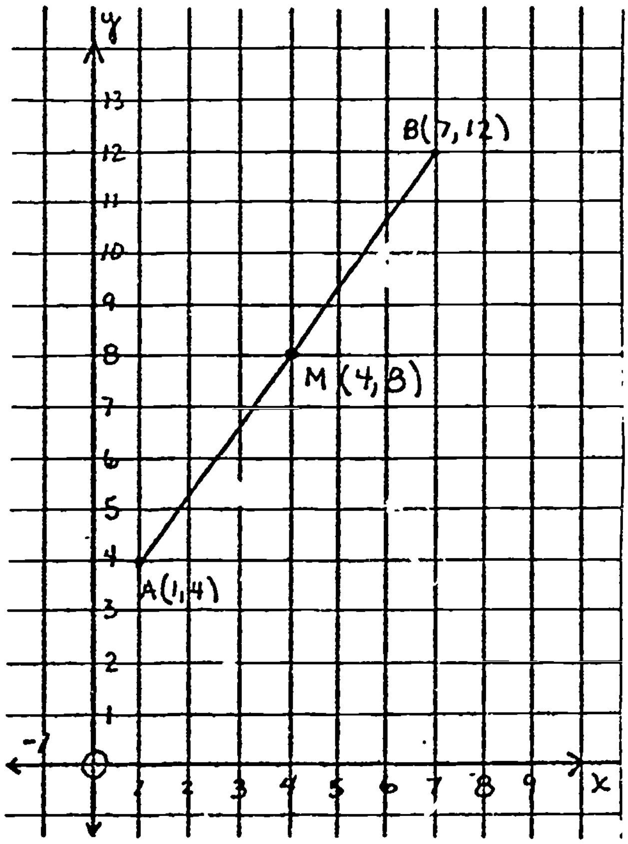
Review: x-coordinate, y-coordinate, midpoint, average

CHALLENGE PROBLEM:

When plotted on the same set of coordinate axes, Ship A is located at (1, 4) and Ship B is located at (7, 12). If the ships travel towards each other at the same rate of speed, at what point will they meet?

DEVELOPMENT:

Figure 1



1. Discuss the challenge problem using a graph of the given information (see figure 1). Elicit from students that the solution to the problem involves finding the midpoint of the line segment joining points A and B. Recall the procedure for finding the midpoint of a line segment on a number line (a coordinate axis). Recall that

to find the midpoint we averaged the endpoints. Since each point in the coordinate plane is named by a pair of numbers (x-coordinate and y-coordinate), it seems logical to try to find the coordinates of the midpoint by averaging the x-coordinates and by averaging the y coordinates as shown:

$$x \text{ at the midpoint} = \frac{x_{\text{at A}} + x_{\text{at B}}}{2}$$

$$= \frac{1 + 7}{2}$$

$$= \frac{8}{2}$$

$$= 4$$

$$y \text{ at the midpoint} = \frac{y_{\text{at A}} + y_{\text{at B}}}{2}$$

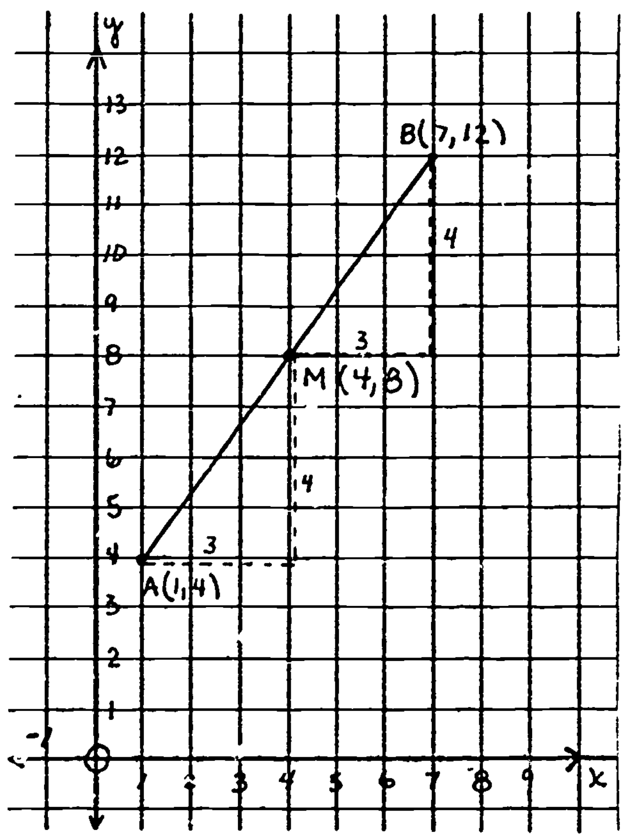
$$= \frac{4 + 12}{2}$$

$$= \frac{16}{2}$$

$$= 8$$

Therefore the coordinates of the midpoint are (4, 8).

2. How can it be verified that (4,8) is the midpoint of $\overline{AB}$? Rec 11 that a midpoint is located the same distance from each endpoint. Have students find the distance from the midpoint to each endpoint (by using the Pythagorean Formula). See Figure 2.



$(AM)^2 = 3^2 + 4^2$ $(MB)^2 = 3^2 + 4^2$

$(AM)^2 = 9 + 16$ $(MB)^2 = 9 + 16$

$(AM)^2 = 25$ $(MB)^2 = 25$

$AM = 5$ $MB = 5$

Therefore, AM = MB which shows that M is the midpoint of $\overline{AB}$.

3. Do applications.

APPLICATIONS:

1. Find the coordinates of the midpoint of each of the line segments whose endpoints have the coordinates shown:

    a) (3, 5) and (9, 13)

    b) (-2, -6) and (-10, -8)

    c) (1, 4) and (7, -4)

    d) (5, 3) and (-9, -1)

    e) (0, 0) and (6, 6)

    f) (-9, 0) and (12, 0)

    g) (1, 3) and (3, 1)

    h) (-6, 0) and (0, 10)

2. Have students plot each line segment used in example 1 and show that the midpoint is the same distance from each endpoint.

SUMMARY:

Review the method for finding the coordinates of the midpoint of a line segment having students state that the x-coordinate of the midpoint is the average of the x-coordinates and that the y-coordinate of the midpoint is the average of the y-coordinates.

UNIT IX. Banking and Taxes

LESSON 1

AIM: To compute simple interesc

PERFORMANCE OBJECTIVES: Students will be able to:

- use the words deposit, withdrawal, principal, interest, interest rate, and balance to describe banking procedures.
- explain why a bank pays interest.
- state that interest rates are always expressed as an annual rate.
- identify the principal, rate, and time period in a bank interest problem.
- compute interest using the simple interest formula I = Prt, where t is an integral number of years.

VOCABULARY:

New terms: principal, interest, interest rate, balance, annual rate

Review: deposit, withdrawal

CHALLENGE PROBLEM:

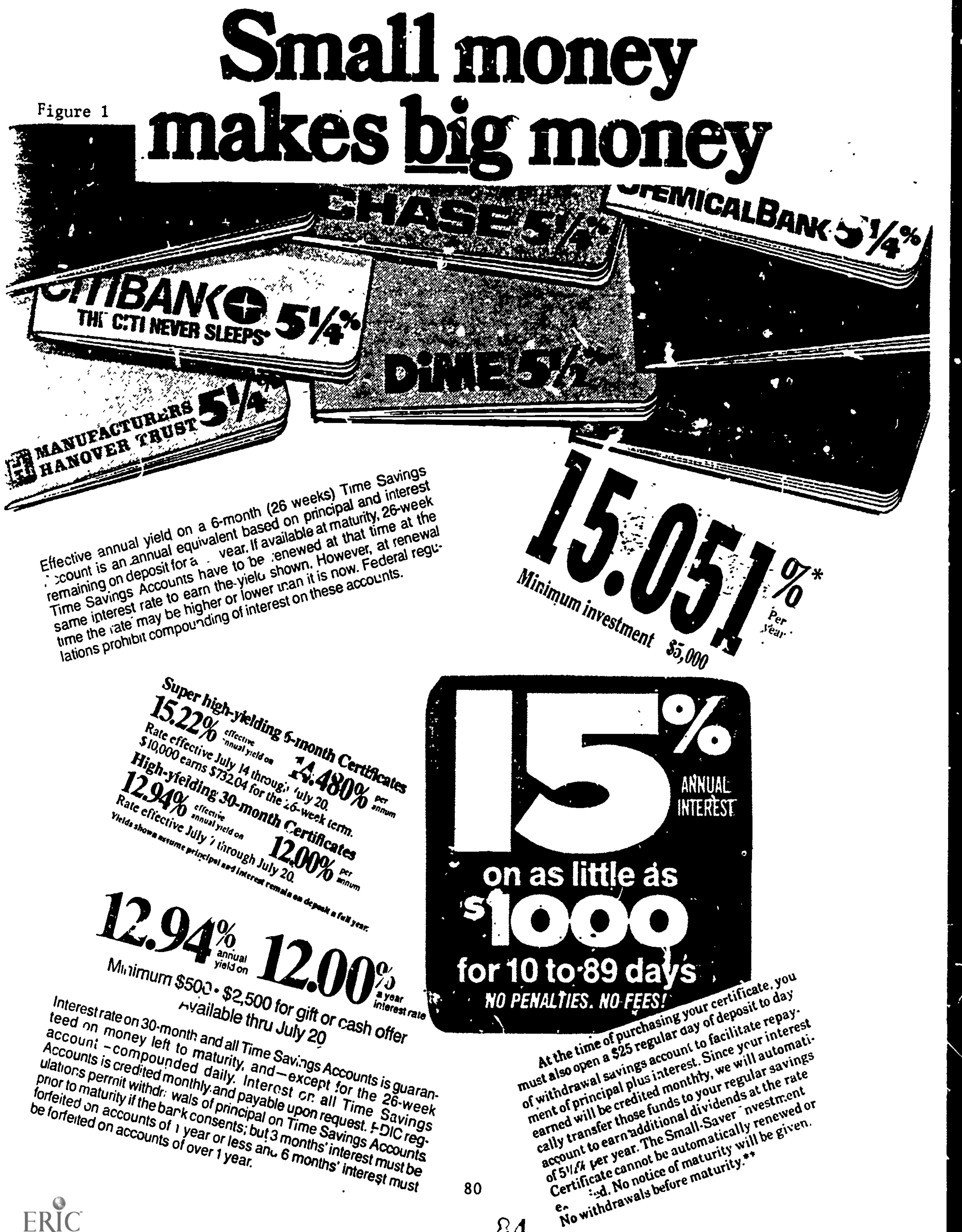
Mr. Rodriguez saved $10 each week in his new savings account. By the end of one year he had made 52 deposits and his bankbook showed a balance of $539.80.

1. How much money did he deposit in 52 weeks?
2. Where did the extra money come from?

DEVELOPMENT:

1. Discuss the challenge problem. Elicit that the extra money is called <u>interest</u>. Define interest as a rental paid for the use of money. When money is deposited in a savings account, the money is used by the bank. The bank then pays the depositor interest (a rental fee). Similarly, when someone borrows money (takes a loan), they must pay interest (a rental fee) to the lender.
2. Examine bank advertisements and elicit the following points: (see figure 1)
    a) <u>Interest rate</u> is expressed as a percent.
    b) Interest rates are given per year or per <u>annum</u>.
    c) <u>Principal</u> is the amount on which interest is figured (deposited or borrowed).
    d) <u>Deposit</u> is money added to an account.
    e) <u>Withdrawal</u> is money taken out of an account.
    f) <u>Balance</u> is the amount of money in an account at any given time.

Figure 1



Small money makes big money
CHASE 5¼%
CHEMICALBANK 5¼%
CITIBANK THE CITI NEVER SLEEPS 5¼%
DIME 5½
MANUFACTURERS HANOVER TRUST 5¼%
Effective annual yield on a 6-month (26 weeks) Time Savings Account is an annual equivalent based on principal and interest remaining on deposit for a year. If available at maturity, 26-week Time Savings Accounts have to be renewed at that time at the same interest rate to earn the yield shown. However, at renewal time the rate may be higher or lower than it is now. Federal regulations prohibit compounding of interest on these accounts.
15.051%* Per Year
Minimum investment $5,000
Super high-yielding 6-month Certificates
15.22% effective annual yield on 14.480% per annum
Rate effective July 14 through July 20.
$10,000 earns $732.04 for the 26-week term.
High-yielding 30-month Certificates
12.94% effective annual yield on 12.00% per annum
Rate effective July 7 through July 20.
Yields shown assume principal and interest remain on deposit a full year.
15% ANNUAL INTEREST
on as little as $1000
for 10 to 89 days
NO PENALTIES. NO FEES!
12.94% annual yield on 12.00% a year interest rate
Minimum $500 • $2,500 for gift or cash offer
Available thru July 20
Interest rate on 30-month and all Time Savings Accounts is guaranteed on money left to maturity, and—except for the 26-week account—compounded daily. Interest on all Time Savings Accounts is credited monthly and payable upon request. FDIC regulations permit withdrawals of principal on Time Savings Accounts prior to maturity if the bank consents; but 3 months' interest must be forfeited on accounts of 1 year or less and 6 months' interest must be forfeited on accounts of over 1 year.
At the time of purchasing your certificate, you must also open a $25 regular day of deposit to day of withdrawal savings account to facilitate repayment of principal plus interest. Since your interest earned will be credited monthly, we will automatically transfer those funds to your regular savings account to earn additional dividends at the rate of 5¼% per year. The Small-Saver Investment Certificate cannot be automatically renewed or extended. No notice of maturity will be given.
No withdrawals before maturity.**

3. Pose the problem: If an interest rate is 9% per year, find the amount of interest on a principal of $500 fcr o ~ year.

   a) What information is given? (interest rate, principal, length of time)

   b) What information must be found?

   c) Elicit that the solution means 9% of $500 must be found.

   d) Obtain the solution by the following calcuation:

Interest = 9% of $500	Calculation
Interest = .09 x 500	$ 500
Interest · $45.00	x .09
	$45.00

   e) If the interest rate remained the same, how much interest would be paid in three years?

   Elicit that the solution is found by multiplying the yearly interest by 3. In cases where the interes is computed on the same principal each time, the interest is called simple interest.

4. Have students state in words the procedure used to calculate the simple interest. (Interest is found by multiplying the principal by the rate of interest by the number of years.)

   Elicit the formula $\boxed{I = Prt}$ where I = interest, P = principal, r = rate of interest, t = number of years.

5. Do applications.

APPLICATIONS:

1. Find the interest on a $900 deposit for one year at 7%.
2. How much interest will Mr. Sargent receive if he deposits $1500 for two years at 10% interest each year?
3. Mr. Lee makes an investment of $5000 that yields 15% interest. Hc nuch will the investment earn in one year?
4. A & B Auto Repairs borrowed $4000 to purchase new tools. If the loan was repaid at the end of 2 years and the rate of interest was 21%, find the interest due.

SUMMARY:

Use the following problem to review the major concepts in this lesson.

$12,000 was borrowed for a period of 3 years at an interest rate of 19%.

a) What is the principal?

b) What is the annual rate of interest?

c) What is the interest charged?

d) How much money must be repaid at the end of three years?

CLIFF-HANGER:

A savings bank pays $7\frac{3}{4}$ % effective annual yield on a deposit of \$500. How much interest is earned in one year?

LESSON 2

AIM: To compute simple interest

PERFORMANCE OBJECTIVES: Students will be able to:

- express non-integral percents as decimals.
- use the simple interest formula to calculate interest where the rate is a non-integral percent.

VOCABULARY:

Review: percent, interest, interest formula, interest rate

CHALLENGE PROBLEM:

A bank advertises 12.94% annual yield on deposits of $500 or more. How much interest is earned in one year on a deposit of $600?

DEVELOPMENT:

1. a) What information is given in the Challenge Problem? (interest rate - 12.94%, principal - $600 time - 1 year)

   b) What formula can be used to solve the problem? (I = Prt)

   c) Substitute the values in the formula to obtain the statement:

      I = $600 x 12.94% x 1

   How must 12.94% be written so that the calculation can be performed?

2. Develop a procedure for changing a percent containing a decimal into a decimal number as follows:

   a) Recall the meaning of percent as the ratio of a number to 100.

   b) Express 12.94% as $\frac{12.94}{100}$

   c) Divide

$$\begin{array}{r} .1294 \\ 100\,\overline{)12.9400} \\ -10\,0 \\ \hline 2\,94 \\ -2\,00 \\ \hline 940 \\ -900 \\ \hline 400 \\ -400 \\ \hline \end{array}$$

   Therefore 12.94% = .1294.

   d) Recall the shortcut method for division by a power of 10. Since $100 = 10^2$, the result of division by 100 can be found by placing the decimal point 2 places to the left of its original position in the given number. Demonstrate: 12.94% = .12 94.

   e) Complete the solution to the challenge problem.

3. Do Application A.

4. a) Consider the cliff-hanger problem from the preceding lesson. In order to solve this problem, the student must substitute the values \$500, $7\frac{3}{4}\%$ and one year into the simple interest formula obtaining:

   $$I = \$500 \times 7\frac{3}{4}\% \times 1$$

   b) Using a procedure similar to the one outlined in item #1, elicit a need to express $7\frac{3}{4}\%$ as a decimal

5. Develop a procedure for changing a percent containing a fraction into a decimal number as follows:

   a) What is the difference between the form of the annual rate in the challenge and cliff-hanger problems? (One contains a fraction, the other contains a decimal.)

   b) How can $7\frac{3}{4}\%$ be expressed so that the procedure learned previously can be used? Elicit that $7\frac{3}{4}$ must be expressed as a decimal.

   c) Review changing a fraction to a decimal by division:

   $$\frac{3}{4} \text{ means } \begin{array}{r} .75 \\ 4\overline{)3.00} \\ \underline{-2\ 8} \\ 20 \\ \underline{-20} \end{array}$$

   Since $\frac{3}{4} = .75$, $7\frac{3}{4} = 7.75$.

   d) Therefore $7\frac{3}{4}\% = 7.75\%$. Now the student can change 7.75% to .0775 using the procedure previously learned.

6. Complete the solution to the cliff-hanger.

7. Do Application B.

APPLICATIONS:

A. 1. Express each of the following percents as a decimal:

a) 14.48%	e) 8.03%
b) 17.5%	f) 7.25%
c) 15.301%	g) 5.75%
d) 7.5%	h) 9.051%

2. The annual yield on a minimum deposit of \$10,000 is 5.22%. Find the interest received for two years on a deposit of \$10,000.

B. 1. Express each of the following percents as a decimal:

a) $12\frac{1}{2}\%$ e) $5\frac{1}{2}\%$

b) $18\frac{1}{4}\%$ f) $8\frac{3}{4}\%$

c) $17\frac{3}{4}\%$ g) $6\frac{3}{8}\%$

d) $12\frac{5}{8}\%$ h) $\frac{1}{2}\%$

2. A bank charges $17\frac{1}{2}\%$ interest on a credit card cash advance. If a cash advance for \$800 is paid off in a lump sum after one year, how much interest is charged?

SUMMARY:

Review the procedures for changing non-integral percents to decimals.

CLIFF-HANGER:

What would be the interest charged on an 18% loan of \$3000 for a period of 15 months?

LESSON 3

AIM: To compute simple interest involving fractional parts of the year

PERFORMANCE OBJECTIVES: Students will be able to:

- express any number of months as a fraction of a year.
- express a non-integral percent as a fraction in lowest terms.
- use the simple interest formula to calculate interest where the rate is a non-integral percent and the time is a non-integral number of years.

VOCABULARY:

Review: annual, percent interest

CHALLENGE PROBLEM:

If $120 interest is earned in one year, at the same rate how much interest is earned after 6 months? after 3 months?

DEVELOPMENT:

1. Discuss the challenge problem. Have students recall that the rate of interest on a deposit or loan is always an <u>annual</u> rate. Elicit that 6 months or 3 months is a part of a year. Have students determine the fractional part as follows:

   There are 12 months, so each month is $\frac{1}{12}$ of a year. Therefore,

   6 months = $\frac{6}{12}$ of a year; 3 months = $\frac{3}{12}$ of a year.

   Reducing each fraction, 6 months = $\frac{1}{2}$ of a year and 3 months = $\frac{1}{4}$ of a year.

   Have students generalize that when a loan is made for a period other than a whole number of years, the time must be expressed as a multiple or part of a year.

   Solve challenge problems as follows:

   interest earned after 6 months = $\frac{1}{2}$ of $120

   interest earned after 6 months = $\frac{1}{2}$ (120)

   interest earned after 6 months = $60

   interest earned after 3 months = $\frac{1}{4}$ of $120

   interest earned after 3 months = $\frac{1}{4}$ (120)

   interest earned after 3 months = $30

2. Do Application A.

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3. Consider the cliff-hanger problem. Elicit the simple interest formula (I = prt)

Elicit 15 months = $\frac{15}{12}$ or $\frac{5}{4}$ years.

Guide students to discover that the calculation would be simpler if the interest rate were written as a fraction rather than as a decimal. Therefore: $18\% = \frac{18}{100}$

$$I = Prt$$

$$I = \overset{30}{\cancel{\$3000}} \times \frac{18}{\cancel{100}_1} \times \frac{5}{4}$$

$$I = \frac{\overset{15}{\cancel{30}}}{1} \times \frac{18}{1} \times \frac{5}{\cancel{4}_2}$$

$$I = \frac{15}{1} \times \frac{\overset{9}{\cancel{18}}}{1} \times \frac{5}{\cancel{2}_1}$$

$$I = \$675$$

4. Do Application B.

5. Present the problem:

At $7\frac{1}{2}\%$, what is the interest on an \$800 loan for 1 year 9 months?

Elicit the steps needed to solve this problem:

$$I = Prt$$

$$t = 1 \text{ year } 9 \text{ months} = 1\frac{9}{12} \text{ years}$$

$$t = 1\frac{3}{4} \text{ years}$$

$$t = \frac{7}{4} \text{ years}$$

Guide students to discover that the calculation would be simpler if $7\frac{1}{2}\%$ were expressed as a fraction as follows:

$$7\tfrac{1}{2}\% = \frac{7\frac{1}{2}}{100} \text{ or } 7\tfrac{1}{2} \div 100 \text{ or } \frac{15}{2} \div 100$$

$$7\tfrac{1}{2}\% = \frac{15}{2} \div \frac{100}{1}$$

$$7\tfrac{1}{2} = \frac{\overset{3}{\cancel{15}}}{2} \times \frac{1}{\cancel{100}_{20}}$$

$$7\tfrac{1}{2}\% = \frac{3}{40}$$

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Complete the solution:

$$I = \$800 \times \frac{3}{40} \times \frac{7}{4}$$

$$I = \frac{420}{4}$$

$$I = \$105$$

6. Do Application C.

APPLICATIONS:

A. Express in lowest terms each of the following, using fractional parts of a year:

1. 5 months	5. 1 year 4 months
2. 2 months	6. 5 years 6 months
3. 4 months	7. 4 years 8 months
4. 13 months	

B. 1. Martha made a \$400 loan for 1 year 6 months at 15%. Find the interest she owed.

2. Mr. Greg owed \$900 in taxes for 20 months. The penalty was a 6% interest charge. How much interest was he charged?

C. 1. Express each of the following percents as a fraction:

a. $5\frac{3}{4}\%$	d. $12\frac{5}{8}\%$
b. $12\frac{1}{2}\%$	e. $33\frac{1}{3}\%$
c. $18\frac{1}{4}\%$	f. $6\frac{3}{8}\%$

2. A credit card account charges $17\frac{1}{2}\%$ interest on unpaid balances. Sandra owed \$200 on her credit card. What was the interest charge for one month?

SUMMARY:

Review changing percent to fractions and use of the simple interest formula.

CLIFF-HANGER:

A bank advertises 6% interest compounded daily. What does this mean?

LESSON 4

AIM: To understand the meaning of compound interest

PERFORMANCE OBJECTIVES: Students will be able to:

- differentiate between simple and compound interest by stating:
  1. that when calculating simple interest:
     a. the interest is figured all at once for the entire period,
     b. the principal does not change during the period.
  2. that when calculating compound interest:
     a. the interest is figured separately for each unit of time (e.g., per year for annual compounding),
     b. the principal increases after each unit of the time period (e.g., this year's principal equals last year's principal plus last year's interest).
- calculate interest compounded annually.

VOCABULARY:

New terms: compound interest, interest periods

CHALLANGE PROBLEM:

How much money will you have in the bank at the end of 1 year if you deposit $500 at 6% interest?

DEVELOPMENT:

1. Discuss the challenge problem. Recall that I = Prt is used to compute simple interest. What happens to the $30 interest that the bank pays? (It is credited to the account.) Therefore, the solution to the challenge problem is $530.
2. Now pose the problem: Suppose the money is left in the account for another year. On what amount of money (principal) will the bank be paying interest? Elicit the response $530. Using this as the principal, compute the interest paid for the second year at the same rate of interest. Then compute the balance:

   I = Prt

   I = $530 x .06 x 1

   I = $31.80

   Balance = previous balance + interest.

   Balance = $530 + $31.80

   Balance = $561.80
3. Have students compute the simple interest for $500 at 6% for 2 years, and the final balance.

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$I = Prt$

$I = \$500 \times .06 \times 2$

$I = \$60$

Balance = $500 + $60

Balance = $560

4. Compare the results of both computations. Have students speculate why the balances are different. Discuss with the class the fact that the procedure follow d in item 2 is called <u>compounding</u> interest. In compounding interest, as interest is earned, it is added to the principal. This creates a new principal, on which the new interest is calculated.

5. Discuss the fact that many banks compound interest quarterly. What does this mean? (Interest is paid every 3 months; in other words: $\frac{3}{12} = \frac{1}{4}$)
   Have students recognize that $\frac{1}{4}$ year represents the time, t, used in computing the nterest for the 3 month period.

6. Recall the cliff-hanger problem. What value of t is used in computing the daily interest in a normal year? ($\frac{1}{365}$) Computations involving compounding are generally done using a calculator or tables.

7. Do applications.

APPLICATIONS:

1. What is the total interest earned if $2000 is deposited for 2 years at 15% compounded annually?

2. Mr. Strong deposited $4,000 at 8% compounded annually.
   a. How much will he have on deposit at the end of two years?
   b. How much will he have on deposit at the end of three years?

3. Have pupils find examples of sums of money deposited in banks which have grown to large amounts due to compounded interest.

SUMMARY:

Have pupils state that:

1. when calculating simple interest--
   a. the interest is figured all at once for the entire period,
   b. the principal does not change during the period.

2. when calculating compound interest--
   a. the interest is figured separately for each unit of time (e.g., per year for annual compounding),
   b. the principal increases after each unit of the time period (e.g., this year's principal equals last year's principal plus last year's interest).

LESSON 5

AIM: To develop the concept of equal repayments for a loan

PERFORMANCE OBJECTIVES: Students will be able to:

- calculate the amount of interest charged on a loan.
- calculate the total amount to be repaid (interest plus principal).
- state that each installment is the quotient of the total amount borrowed divided by the number of equal installments.
- compute each installment on a loan.

VOCABULARY:

Review: interest, principal, installment, mortgage

CHALLENGE PROBLEM:

Mr. Williams borrowed $1000 for 2 years at 10%. How much did he have to repay?

DEVELOPMENT:

1. Have students recall that a borrower must repay the full amount borrowed, called the principal, plus the interest owed. Let the class calculate the amount of interest due.

   $$I = Prt$$

   $$I = 1000 \times \frac{10}{100} \times 2$$

   $$I = 200$$

   Mr. Williams must repay $1,000 plus $200 interest, for a total o $1200.

2. Mr. Williams plans to repay this loan in equal monthly <u>installments</u> over the 2-year period. How much will he repay each month?

   Guide students to an understanding that there will be a total of 24 monthly payments in a two-year period. Since the payments are to be equal, $1200 ÷ 24 = $50, Mr. Williams will have to repay $50 per month.

3. Do Application 3 (use this opportunity to reinforce calculations involving a variety of percents).

APPLICATIONS:

1. Mrs. Quinn borrowed $1800 for 2 years at 12%. What was the amount of each equal monthly repayment?

2. To purchase a house, the Hernandez family obtained a 10 year mortgage for $40,000 to be repaid in equal monthly payments at 9% interest. What would their monthly mortgage payment be?

3. To pay for his daughter's wedding, Mr. Weber borrowed $900 to be repaid in 18 equal monthly installments. At 14% interest, what was his monthly payment?

4. In order to buy a sports car, Fred borrowed $9000. He agreed to make equal monthly payments over a five year period, at 11.5% annual interest. How much did he have to repay each month?

5. The B & G Sportwear Corp. borrowed $18,000 at 14½% to be repaid in equal quarterly (four times a year) payments over three years. Find the amount due each quarter.

SUMMARY:

Review the idea that loans are often repaid in equal installments and that these payments include both the amount borrowed (principal) and the interest owed.

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LESSON 6

AIM: To introduce the idea of "bank discount"

PERFORMANCE OBJECTIVES: Students will be able to:

- compute the amount of interest charged on a loan.
- compute the actual amount of money available to the borrower (net proceeds).
- determine whether a given amount yields sufficient net proceeds.

VOCABULARY:

New terms: net proceeds, bank discount

CHALLENGE PROBLEM:

The Second National Bank agreed to lend Mr. Freeze \$500 for 6 months at 8%. When Mr. Freeze went to get the money, he was given only \$480. What happened to the other \$20?

DEVELOPMENT:

1. Ask pupils to try to solve the challenge problem. If they fail to do so, suggest that we consider one of the most important aspects of any loan, the interest.

   $$I = Prt$$
   $$I = 500 \times \frac{8}{100} \times \frac{6}{12}$$
   $$I = 20$$

   The interest on Mr. Freeze's loan will be \$20.

2. Explain that a bank often "discounts" or subtracts the amount of interest on the loan in advance. Thus, Mr. Freeze will have to repay \$500 which includes the \$480 he received, called the <u>net proceeds</u>, plus the \$20 interest. (Net proceeds = P - I)

3. Have the class consider this situation: Marlene wishes to purchase a new car that will cost \$4200. Her bank agrees to give her a three year loan, discounted at 9%.

   a. If she borrows \$4200, will she receive enough money to buy the car?

   b. If she borrows \$5000, will she have enough?

   c. If she borrows \$6000, how much more than the price of the car will she receive?

      Solution to a.

      $$I = Prt$$
      $$I = \cancel{4200}^{42} \times \frac{9}{\cancel{100}_{1}} \times \frac{3}{1}$$
      $$I = 1134$$

97

Net proceeds = P · I

Net proceeds = 4200   1134

Net proceeds - $3066

The answer is that she will not have enough money if she borrows $4000.

Similarly for b.

I = $1350

Net proceeds = $3650.

The answer is that she will not have enough money if she borrows $5000.

Similarly for c.

I = $1620

Net proceeds = $4380

The answer is that $4380 is $180 more than the $4200 she needs.

4. Do applications.

APPLICATIONS:

1. Mr. Cohen borrows $1000 for 1 year discounted at 10%. How much will he actually receive?
2. Martha borrowed $750 from her bank. If the loan was discounted at 8% for a period of six months, how much did she receive?
3. In order to purchase a car costing $4500, Mr. Walters borrowed $5000 for 2 years. The 10% interest was discounted in advance.
   a. How much did he receive?
   b. How much more (or less) than he needed was this?
4. The Mulligan Family went to two banks to discuss a loan. The Northwest Bank offered them a $700 loan for 3 years at 12% simple interest. The Citywide Bank offered them a $1000 loan for 3 years at 10%, discounted in advance.
   a. Find the interest charge for each loan.
   b. Which would have been the best choice? Why?

SUMMARY:

Review the idea that interest is often deducted in advance ("discounted") on a loan, so that the borrower does not receive the full amount of the loan.

CLIFF-HANGER:

In what situations other than banking is the term "discount" used? (Note to teacher: Have students bring in newspaper advertisements showing discounts and discounted prices.)

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LESSON 7

AIM: To compute the selling price when discounting

PERFORMANCE OBJECTIVES: Students will be able to:

- compute the amount of a discount if the discount rate is a unit fraction.
- compute the selling price (net price) if the discount rate is a unit fraction.

VOCABULARY:

New terms: discount, markdown, rate of discount, original price, sale price, discount price, net price

CHALLENGE PROBLEM:

During an "end-of-season" sale, a store gave a discount of $\frac{1}{3}$ off every ski jacket. Mark purchased a jacket with an original price of $42. What was the discount? How much did he have to pay?

DEVELOPMENT:

1. Use the challenge problem to list and discuss the meanings of the words commonly associated with "sales". What do each of the numbers stated in this problem represent? Elicit:

   a. $42 represents the original price (also called the retail price, regular price, ticket price, marked price or list price).

   b. $\frac{1}{3}$ represents the rate of discount.

2. a. Solve the challenge problem. Define the term discount (or markdown) as the reduction in the original price of the article. Since the discount was given as a rate, the dollar value of the discount m..t be computed as follows:

   $$\text{Discount} = \frac{1}{3} \text{ of original price}$$

   $$\text{Discount} = \frac{1}{3} \times \frac{\cancel{42}^{14}}{1}$$

   $$\text{Discount} = \$14$$

   Emphasize that the discount is the actual dollar amount of the reduction ($14) as compared to the rate of discount ($\frac{1}{3}$).

   b. Have students compute the cost of the ski jacket as follows:

   Sale price = ORIGINAL PRICE - DISCOUNT

   Sale price = $42 - $14

   Sale price = $28

   Discuss other terms meaning "sale price" (discounted price, net price)

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3. Do Application A.

4. Refer to newspaper ads:

   Use the following example (or similar ones brought in by students) to determine the validity of ads.

   RECORD ALBUMS - Regularly \$9.99

   Now $\frac{1}{3}$ off - only \$6.89

   Can we assume that the sale price in the ad is correct?

   Have the students compute the discount and the sale price. Have them compare their sale price to the advertised sale price.

5. Do Applications B and C.

APPLICATIONS:

A. 1. At $\frac{1}{4}$ off, what is the discount you will receive when purchasing a \$64 coat?

2. Arlene purchases a \$27 calculator during a $\frac{1}{3}$ off sale. What is the markdown? What is the sale price?

3. Original Price: \$12.99

   Now: $\frac{1}{3}$ off

   Net price: ?

4. A newspaper advertisement read: Take $\frac{1}{4}$ off the price of all summer clothes. Carl saw a \$25 jacket he wished to buy. If he had \$20, was he able to make the purchase? (Disregard sales tax.)

5. Two department stores advertised the same TV game this way:

THE BARGAIN STORE	GAMELAND
Our Regular Price: \$72	Original Price: \$81
This Week Only: $\frac{1}{4}$ off	Now: $\frac{1}{3}$ off

   Which store had the lower selling price?

B. The following items were placed on sale in a local store:

	(a) B & W TV	(b) Radio	(c) Tape Recorder	(d) Color TV
List Price	\$128	\$57	\$89	\$499
Discount	$\frac{1}{4}$ off	$\frac{1}{3}$ off	$\frac{1}{4}$ to $\frac{1}{3}$ off	$\frac{1}{4}$ to $\frac{1}{3}$ off
Sale	\$96	\$38	\$62	\$399

Is the store correct in its advertisements?

C. Mrs. Cruz purchased a $90 coat for $60. How much of a discount had she received? What fraction of the original price is the discount (rate of discount)?

Note: Teacher may wish to determine the rate of discount for each item in Application B using the given list price and sale price.

SUMMARY:

Recall that a discount is an amount deducted from the original price of an article, and the rate of discount is often expressed as a fraction of the original price.

Compare a store discount with a bank discount. Compare net price of a purchase with net proceeds from a loan.

CLIFF-HANGER:

Employees of the M & V Stationery Co. receive a 15% discount on all purchases. What will an employee pay for a $39 file cabinet?

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LESSON 8

AIM: To find the selling price when the rate of discount is expressed as a percent

PERFORMANCE OBJECTIVES: Students will be able to:

- convert several common percents to their fractional equivalents (10%, 20%, 25%, 33 1/3%, etc.)
- compute the net price where the discount rate is given as a percent by expressing the rate as a decimal or common fraction.

VOCABULARY:

Review: rate of discount, discount, sale price, etc.

CHALLENGE PROBLEM:

Employees at Marsh's Shoe Emporium receive a 20% discount on all purchases. How much of a discount will Maria, an employee, receive on a $50 pair of shoes?

DEVELOPMENT:

1. Discuss the challenge and cliff-hanger problems. In what way do these problems differ from the problems in the previous lesson? (The rate of discount is expressed as a percent.) Recall various newspaper ads.

2. Review the procedure for finding a discounted price when the rate of discount is a fraction. Recall that every percent can be expressed as either a fraction or a decimal.

   For example: $20\% = \frac{20}{100} = \frac{1}{5}$ or $20\% = .20$

3. Elicit that the challenge problem can be solved in either of two ways:

   Discount = 20% of $50

   a. $20\% = \frac{1}{5}$

   Discount = $\frac{1}{5}$ of $50

   Discount = $\frac{1}{5} \times \frac{50}{1}$

   Discount = $10

   b. 20% = .20

   Discount = .20 x $50

   Discount = $10.00

4. Elicit the two methods of solving the cliff-hanger problem.

   The discount = 15% of $39

   a. $15\% = \frac{15}{100}$

   b. 15% = .15

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Discount = $\frac{15}{100}$ of \$39

Discount = $\frac{15}{100} \times \frac{39}{1}$

Discount = $\frac{585}{100}$

Discount - \$5.85

Discount = .15 x \$39

Calculation

$$\begin{array}{r} \$39 \\ \times\ .15 \\ \hline 1.95 \\ 3.9 \\ \hline \$5.85 \end{array}$$

Discount - \$5.85

Compute Net (employee's) Price to be \$33.15.

Note: Allow students to choose either method of solution. You may wish to discuss with students their reasons for favoring either method.

5. Do applications.

APPLICATIONS:

1. Find the discount on a \$350 TV set being sold at 20% off.
2. A \$40 pair of shoes is priced at 30% off.
   a. What is the markdown?
   b. What is the sale price of the shoes?
3. Regular Price: \$60. Now: $33\frac{1}{3}\%$ off. Find the discount price. (Note: Since the decimal form of $33\frac{1}{3}\%$ is not convenient for use, express $33\frac{1}{3}\%$ in fractional form.)
4. Members of the Art Guild receive a 5% discount on all purchases. How much will Rosa save on a purchase of \$26?
5. G.O. members can often attend 'pro' games at 50% off the normal admission price. If the normal price is \$4.50 per ticket, what will it cost for a G.O. member?
6. For a Valentine's Day gift, Hector had to decide between a \$30 necklace at 20% off and a \$40 bracelet at 40% off. Which would have been the least expensive? (Have pupils realize that the larger percent discount or the smaller original price do not, by themselves, guarantee the cheapest selling price.)
7. At 20% off, find the cost of a \$36 skate board.
8. For their 23rd Anniversary Sale, the Globe Sporting Goods store gave a discount of 23% off every item. If Bill buys an \$18 basketball during this sale, what will the sale price be?

SUMMARY:

1. Have students state that when the rate of discount is expressed as a percent, the percent must be given as a fraction or decimal before computing the discount.

2. Express some frequently used percents as fractions and decimals (50%, 25%, 20%, $33\frac{1}{3}\%$).

CLIFF-HANGER:

Carolyn purchased a cassette recorder marked \$48.95. An 8% sales tax was then added to this price. What was Carolyn's total cost?

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LESSON 9

AIM: To find the total cost of a purchase

PERFORMANCE OBJECTIVES: Students will be able to:

- calculate the sales tax (or tip) by finding the product of the expediture and the rate of sales ıx (or tip).
- express monetary values to the nearest cent.
- compute the total cost of a purchase including tax, tip, or both.

VOCABULARY:

New terms: Sales tax, tax rate, tip

Review: rounding off

CHALLENGE PROBLEM:

Paulette had dinner in a restaurant. The cost of the food was \$12.50. She told Linda that dinner cost her \$15.50. How do you account for this difference?

DEVELOPMENT:

1. Discuss the challenge problem. Elicit from the class that sales tax and a tip account for the \$3 difference.
2. Elicit from students that sales tax is generally charged on most items purchased. In some cases the sales tax is already included in the cost listed (for example - gasoline, taxi rates, admission tickets). In most cases the sales tax is a percent of the purchase price and must be added on to find the total cost. The sales tax rate varies in different places and for different items.

   Sometimes the amount of tax is determined by using a sales tax table. When a table is not available or when the cost of the purchase exceeds the scope of the table, the tax is computed by using the sales tax rate (usually expressed as a percent).
3. In the challenge problem have students compute the sales tax if the sales tax rate is 8%.

   Solution: 8% = .08

   Sales tax is 8% of \$12.50

   Sales tax = .08 x \$12.50

   Sales tax = \$1.0000

   Recall that cents is expressed as a two plac' decimal. Therefore \$1.000 should be expressed as \$1.00. Elicit from the class that the tip Paulette left was \$2.00.
4. Recall the challenge problem. The solution is as follows: Find 8% of \$48.95

$$\begin{array}{r} \$48.95 \\ \underline{\times\ .08} \\ \text{Sales tax} = \$3.9160 \end{array}$$

Round off $3.9160 to $3.92. Since the total cost means the cost including the sales tax, add $3.92 to $48.95. ($52.87 is the total cost.)

5. Have students state in words how the total cost of a purchase is obtained.

6. Do Application A.

7. Return to discussion of the challenge problem. Elicit examples of situations where tips are generally given (services rendered). Explain that tips usually range from 15% to 20% of the cost of service before tax. These amounts are usually rounded off to the nearest 25¢. Compute 15% of $12.50 and determine whether the tip was adequate. Tip = 15% of $12.50:

$$\begin{array}{r} \$12.50 \\ \times\ .15 \\ \hline 62\ 50 \\ 125\ 0 \\ \hline \$1.87\ 50 \end{array} \longrightarrow \$1.88$$

Since $1.88 is an awkward amount to leave as a tip, Paulette rounded the amount of the tip up to $2.00. (Discuss why she left $2.00 instead of $1.75.)

8. Do Application B.

APPLICATIONS:

A. 1. Arthur's new coat was priced at $77.50. At 8%, how much tax must be paid?

2. Carmen purchased a new bicycle, priced at $129. If the sales tax is $8\frac{1}{4}\%$, how much tax must she pay? (Recall $8\frac{1}{4}\% = 8.25\% = .0825$)

3. Eileen's family saved $300 to purchase a new television set. If the list price of the set is $289.50 and the tax is 8%, do they have enough money to pay for the set? If not, how much more do they need?

4. Fred visited his uncle in Albany, where the sales tax was 5%. If he bought two records at $4.75 each, how much did he have to pay? How much change did he receive if he paid with a $20 bill?

5. The tax on parking a car is 6%. After parking for 3 hours, Mr. Gordon owed $3.77 plus tax. What was the total he owed?

6. Bob saw a sports car priced at only $4550. How much sales tax, at $7\frac{1}{4}\%$ must he add to the price?

B. Find a 15% tip for each of the following bills:

1. A haircut for $12.00.

2. A taxi ride costing $8.10.

3. A meal for $14.65.

SUMMARY:

Have pupils review the components of the total cost of a purchase.

CLIFF-HANGER:

Louis saw that a $25 calculator was on sale at 25% off. He knew that an 8% sales tax would be added. If he had exactly $20, could he purchase the calculator? How much more - or less - than the total cost did he have?

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LESSON 10

AIM: To calculate the total cost of a purchase involving both discount and sales tax

PERFORMANCE OBJECTIVES: Students will be able to:

- state that sales tax is added after the sale price is calculated.
- calculate the purchase price for goods advertised in a newspaper sale including discounting and sales tax.

VOCABULARY:

Review terms: discount, sale price, total cost

CHALLENGE PROBLEM:

An item which lists for \$25, sells for \$20. If the tax rate is 8%, how much tax must be paid on this item?

DEVELOPMENT:

1. Elicit from pupils that sales tax is charged on the <u>actual</u> cost of an item. If a discount is given, the sales tax is charged on the reduced price of the article. Therefore in the challenge problem the sales tax is 8% of \$20 or \$1.60.

2. Have pupils work out the cliff-hanger problem:

a)	\$25	original price	b)	\$25.00	original price
	x.25	rate of discount		- 6.25	discount
	\$6.25	discount		\$18.75	reduced price
c)	\$18.75	reduced price	d)	\$18.75	reduced price
	x .08	rate of tax		+1.50	sales price
	\$ 1.50	sales tax		\$20.25	total cost

Louis could not purchase the calculator since he needed 25¢ more.

3. Pupils may determine the total cost of actual items advertised in the local newspapers. The applications will be more meaningful to the students if they relate to specific purchases of interest to the students.

APPLICATIONS:

(<u>Note</u>: In all the following problems, the teacher may specify that the effective local tax rate be used.)

1. Nina bought a bathing suit originally marked \$25, for 40% off. Including the sales tax, what was her total cost?
2. During a "$\frac{1}{3}$ off" sale, Bill purchased two shirts originally priced \$7.50 each. Find his total cost when the sales tax was added.
3. The Aquarius Specialty Shop advertised: "25% off the first sweater you buy - 50% off the second..." Edith purchased two sweaters, each with a list price of \$18. Find her total cost.

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4. In Chuck's town, a \$60 record player is on sale at 30% off plus 4% tax. In the big city nearby, he can purchase the same record player at $\frac{1}{3}$ off, plus 8% tax. Where would he obtain the lower price? How much lower?

SUMMARY:

Have the pupils state that the discount or reduction must be subtracted from the original price before the sales tax is computed.

AIM: To solve business and consumer problems involving rates

PERFORMANCE OBJECTIVES: Students will be able to:

- state the meaning of common business terms involving the rate expressed as a percent (commission, markup, successive discount).
- compute commissions, markups, and successive discounts when the rates are given.

VOCABULARY:

New terms: commission, markup, successive discount, across the board, base sticker price, surcharge, successive discount

CHALLENGE PROBLEM:

A salesperson receives a commission of 36% of total sales. How much does the salesperson earn on sales totaling $642?

DEVELOPMENT:

1. Discuss meaning of commission. Commission is the money paid by an employer, based on the value of the goods sold or the services rendered to an employee. The rate of commission is generally expressed as a percent. As with all other problems where a rate is given, the commission is found by multiplying the rate of commission by the total value of goods, sales or services. Solve the challenge problem as follows:

   Commission = 36% of sales

   Commission = .36 x 642

   Commission = $231.12

2. There are many business situations involving rates expressed as percents. Some of these are covered in the following applications. The teacher should supplement these with a variety of problems emphasizing business terms and computation with percents.

APPLICATIONS:

1. In order to keep up with rising costs, an automobile manufacturer announced a 9% across the board increase on the base sticker price of a car. If the price of a car was listed as $6390,
   a. What is the increase in cost?
   b. What is the new base sticker price?
   c. If the automobile salesman makes a 12% commission, how much more will he receive after the price increase on this car?
2. Mr. and Mrs. Beale dine at their favorite restaurant. Mr. Beale orders a steak dinner for $12.95. Mrs. Beale orders a lobster dinner for $14.95. The menu states that there is a 10% surcharge on lobster.
   a. Find the actual cost of the lobster.

b. Find the total bill for the 2 dinners if the sales tax is 8%.

c. If Mr. Beale leaves a 15% tip, how much change will he keep from a $50 bill?

3. By paying his bill within ten days, Mr. Fernandez received successive discounts of 25% and 5%. If the items he purchased had a list price of $480, how much did he have to pay? (Explain that to calculate successive discounts, use the first discount rate to find the first reduced price, and then use the second discount rate on this reduced price to find the final reduced price.)

4. Pat's father can purchase $15 footballs from a company that offers double discounts of 20% and 20%, or from a dealer who offers a single discount of 35%. Which would be the least expensive?

5. A video cassette recorder costs a dealer $590. In order to cover his expenses and make a profit, he must use a 35% markup. At what price will he sell the recorder?

6. Ronnie's sister is paid a weekly salary of $150 and a 3% commission on all sales over the first $1000 in sales each week. During the week when her sales were $4297, what were her total earnings?

SUMMARY:

Review vocabulary of business and consumer problems involving rates.

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UNIT X. Areas

LESSON 1

AIM: To find the areas of rectangles

PERFORMANCE OBJECTIVES: Students will be able to:

- provide practical examples where area is used (e.g., floor tiles, carpeting, wallpaper).
- state that a surface (area) must be measured with a square unit, and is different from perimeter which is measured with linear units.
- find areas of rectangles by counting the number of squares they enclose on a grid.

VOCABULARY:

New terms: area, square unit, surface

Review: perimeter, linear unit, rectangle

CHALLENGE PROBLEM:

On a pair of coordinate axes plot the following points and draw the quadrilateral ABCD:

A(2,1), B(6,1), C(1,4), D(2,4),

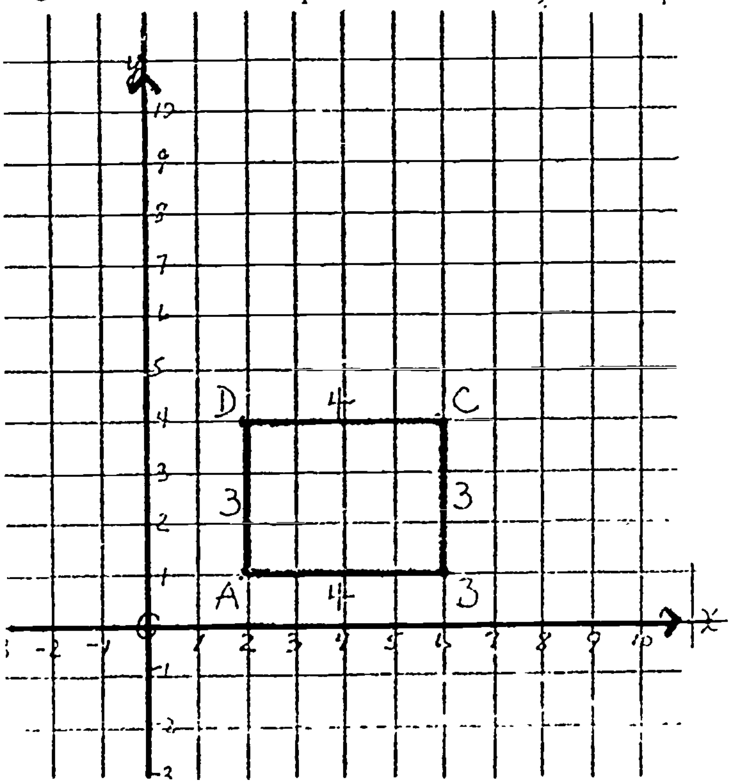
a. What is the perimeter of quadrilateral ABCD?

b. What type of quatrilateral is formed?

DEVELOPMENT:

1. Using the challenge problem, have students recall the meaning of perimeter (the number of linear units of measure around a figure) and the method for finding the length of a line segment on a coordinate plane (counting the number of spaces on a line). The perimeter is 14 units.

Figure 1



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2. Elicit that the quadrilateral formed is a <u>rectangle</u> because it has 2 pairs of opposite sides equal in measure and the angles are right angles.

3. Have students find the number of squares within the rectangle ABCD. The number of squares within ABCD is 12. Define <u>area</u> as the number of square units within a figure.

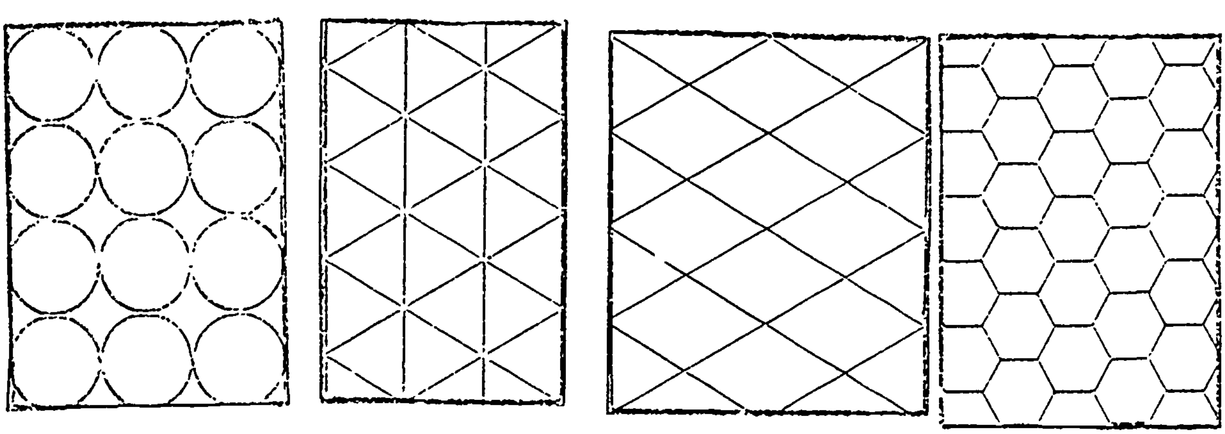
4. What situations require the computation of area? What situations require the computation of perimeter? List responses and make a table similar to the following:

<u>AREA</u>	<u>PERIMETER</u>
carpeting a room	installing tackless trips for carpeting
seeding a law	fencing around a lawn
buying fabric for a table cloth	baseboard molding around a floor
painting a mirror's surface	buying trim for the edge of a table cloth
mirror a wall	frame for a mirror

Generalize that area involves covering a <u>surface</u> completely, while perimeter involves measurement <u>around</u> a figure.

5. Discuss the type of units used to measure area and compare these to the units used to measure perimeter. Elicit that square units are used to measure areas because they are a convenient shape completely covering any surface. Demonstrate this by showing several figures as follows and discuss why each of these would be harder to work with than squares.

Figure 2



6. Do Applications A, B, and C.

APPLICATIONS:

A. Determine which of the following are examples of linear measure (perimeter) and which are surface measure (area):

1. the amount of thread on a spool
2. a boy's height
3. a plank, a slat, or a wire fence to be whitewashed
4. a floor to be sanded
5. a fringe to be put on a bedspread
6. a rug to be cleaned
7. a painting on a wall
8. a frame to be put on a painting

B. Find the area and perimeter of each rectangle:

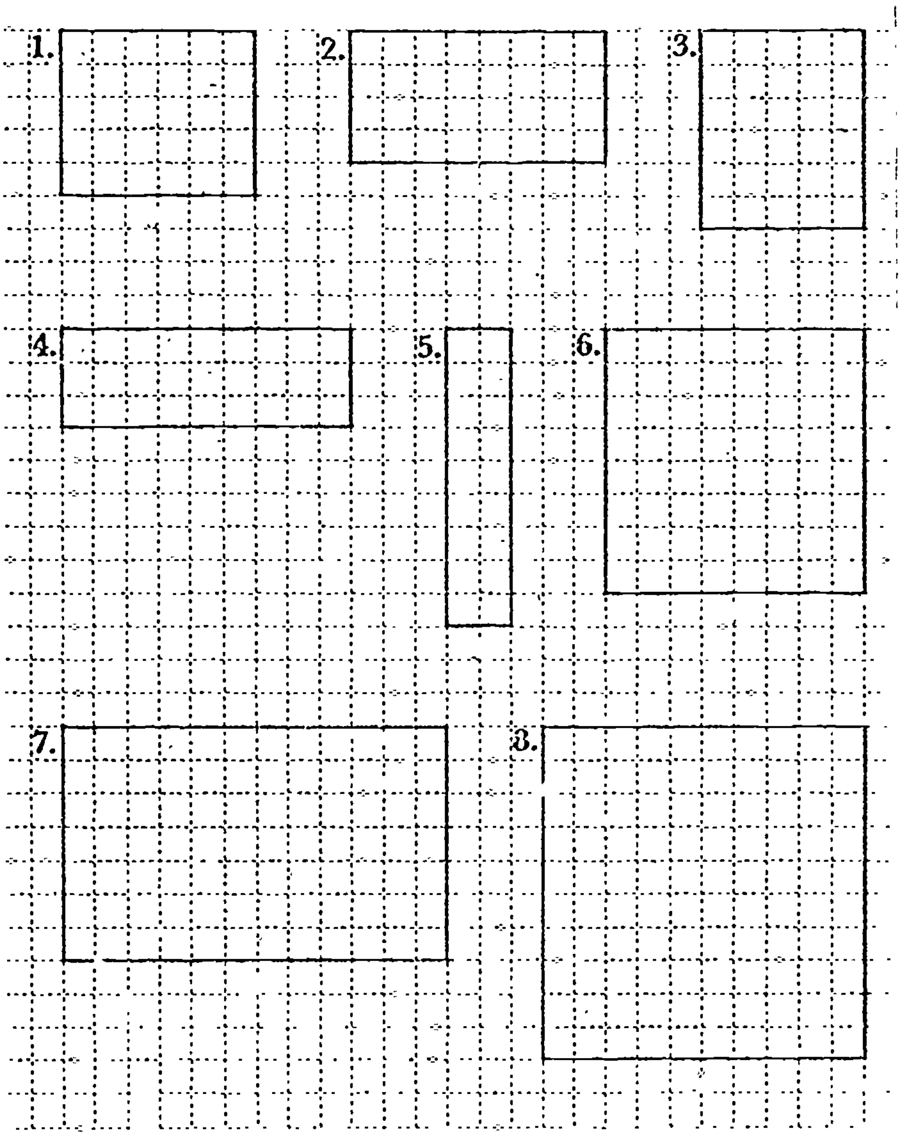


Figure 3

1.4

C. Draw rectangles having the following dimensions. Draw in the square units and find the area of each rectangle.

   1. 6 by 4

   2. 5 by 3

   3. 10 by 2

SUMMARY:

Review that a surface or area must be measured with a square unit, whereas a perimeter is measured with a linear unit.

CLIFF-HANGER:

Find the area of a rectangle which measures $3\frac{1}{2}$ units by $4\frac{1}{2}$ units.

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LESSON 2

AIM: To calculate the areas of rectangles and squares

PERFORMANCE OBJECTIVES: Students will be able to:

- state that the area of a rectangle can be found using the formula $A = lw$ (or $A = bh$).
- state that the area of square can be found using the formula $A = s^2$.
- use the formulas $A = lw$ and $A = s^2$
  - (a) to calculate areas and,
  - (b) to find a missing dimension when the area and one dimension are given.

VOCABULARY:

New terms: length, width, base, height, side

Review: area, formula

CHALLENGE PROBLEM:

Find the area of a rectangle which measures 15 units by 18 units

DEVELOPMENT:

1. Elicit the response to be 270 square units. Discuss the techniques used by the students to obtain this result. It should be clear to students that, while counting squares will yield the solution, this procedure is not convenient. The same result can be obtained by recognizing that a rectangle has rows of squares, each containing the same number of squares. The total number of squares can be found by multiplying the number of squares in each row by the number of rows.

18 rows

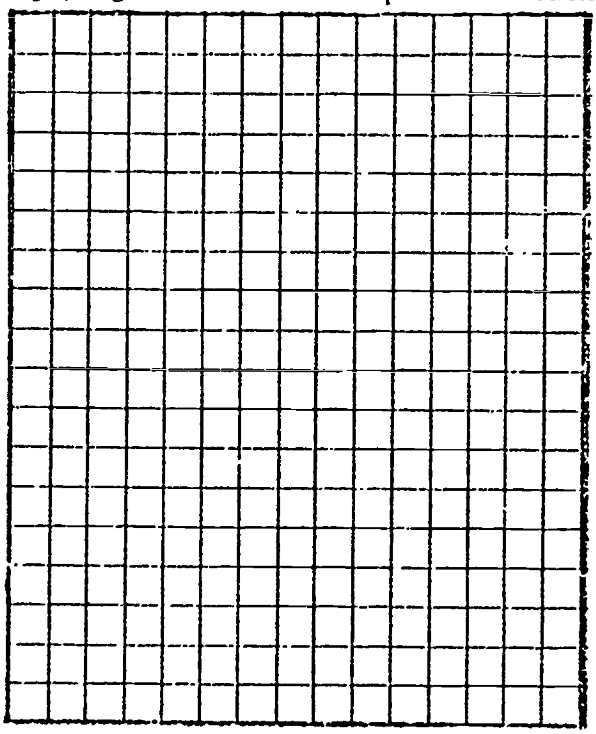


Figure 1

15 squares in each row

2. If the number of units in each row is called the <u>length</u> (l) or <u>base</u> (b) of the rectangle, and the number of rows is called the <u>width</u> (w) or <u>height</u> (h) of the rectangle, then the area of the rectangle may be found by multiplying the length by the width or the base by the height. The formula which expresses this relationship is:

$$\boxed{A = lw} \quad \text{or} \quad \boxed{A = bh}$$

3 Use this formula to check the result for the challenge problem as follows:

$$A = lw$$
$$A = 15 \times 18$$
$$A = 270$$

This area is 270 square units.

4. Use the formula $A = lw$ to solve the cliff-hanger problem as follows:

$$A = lw$$
$$A = 3\frac{1}{2} \times \frac{9}{2}$$
$$A = \frac{7}{2} \times \frac{9}{2}$$
$$A = \frac{63}{4}$$
$$A = 15\frac{3}{4}$$

This area is $15\frac{3}{4}$ square units.

5. Do Application A.

6. If the side of a square is 15 cm, what is the area of this square? Elicit from the class that a square is a type of rectangle where the length and width are the same. If each dimension of the square is called a side (s), then the area of the square can be found by multiplying a side by a side.

   The formula which expresses this relationship is: $A = s \times s$ or $A = s^2$. (Recall the use of exponents and how to read s^2 as s squared or s to the second power.)

7. Use the formula $A = s^2$ to solve the problem in item 6.

$$A = s^2$$
$$A = 15^2$$
$$A = 15 \times 15$$
$$A = 225 \text{ cm}^2$$

8. Do Application B.

9. Given that the area of a rectangle is 72 sq. inches and the length of this rectangle is 6 inches, find the width. Elicit from the students that this problem can be solved in the following way:

   Given: $A = 72$, $l = 6$

   Using the formula $A = lw$, substitute
   $72 = 6w$

Solve this equation recalling the division property of equality:

$$\frac{72}{6} = \frac{6w}{6}$$

$12 = w$: The width is 12 inches

Check the problem: If $l = 6$ inches and $w = 12$ inches

$A = lw$

$A = (6)\ (12)$

$A = 72$ sq in. which was the given area

10. In a similar manner have students find a side of a square whose area is 49 m^2.

$A = s^2$

$49 = s^2$

$\sqrt{49} = s$

$7 = s$ This side is 7 meters

(Recall the meaning of the square root of a number and the symbol "$\sqrt{}$")

11. Do Application C.

APPLICATIONS:

A. Find the area of the rectangles whose dimensions are given:

1. $l = 8$ m, $w = 22$ m
2. $b = 36$ mm, $h = 17$ mm
3. $l = 3.2$ cm, $w = 6.3$ cm
4. $b = 7\frac{1}{4}$ in., $h = 2\frac{1}{2}$ in.
5. $l = 1\frac{1}{2}$ yd, $w = 3\frac{3}{4}$ yd

B. Find the area of each square having the given side:

1. 28 m
2. $2\frac{1}{2}$ ft
3. 1.8 mm
4. 2.25 m

C. Find the missing dimension in each quadrilateral.

1. A rectangle whose area is 325 cm^2 and width is 13 cm
2. A rectangle whose area is 3.45 km^2 and length is 2.3 km
3. A rectangle whose area is $18\frac{1}{8}$ sq. in. and width is $2\frac{1}{2}$ in.
4. A square whose area is 400 m^2.
5. A square whose area is 50 cm^2 (answer: $\sqrt{50}$ cm)
6. A square whose area is $1\frac{9}{16}$ sq in. (answer: $\sqrt{\frac{25}{16}} = \frac{5}{4}$ in.)

ERIC

SUMMARY:

Review the formulas for finding the area of a rectangle and a square.

CLIFF-HANGER:

Raphael's back yard is in the shape of a square. He used 196 sq ft of sod to cover the ground. How many feet of fencing will he need to enclose this area?

ERIC

LESSON 3

AIM: To solve verbal problems involving area and perimeter

PERFORMANCE OBJECTIVES: Students will be able to:

- distinguish between area and perimeter in verbal problems.
- solve verbal problems involving area and perimeter of rectangles and squares.

VOCABULARY:

Review: area, perimeter, square, rectangle, enclose, cover

CHALLENGE PROBLEM:

Floor tile costs $15 per square meter. What is the cost of covering a rectangular floor 5 meters (5m) long and 4 meters (4m) wide?

DEVELOPMENT:

1. Use challenge problem to elicit answers to the following questions:
   a. What information is given? (cost per square meter and dimensions of a rectangular floor)
   b. What information is asked for? (Cost of tiling the entire floor.)
   c. What information must be found in order to find this total cost? (number of square meters in the area of this rectangular floor)
   d. What words in the problem indicate the need for finding area? (covering a floor, or cost per square meter)
2. Solve the problem:

   Area of rectangular floor = $l \times w$

   Area of rectangular floor = $5\ m \times 4\ m$

   Area of rectangular floor = $20\ m^2$

   20 square meters of tile are needed to cover the floor.

   Total cost = Cost per Unit x Number of Units

   Total cost = $15 x 20

   Total cost = $300
3. Solve the cliff-hanger problem given in the previous lesson by asking questions similar to those in item 1.

   Emphasize that the terms "enclose" and "feet" indicate the need to find perimeter and that "square feet" indicates that the area is given.

ERIC

Given: $A = 196$ sq ft

Since $A = s^2$

$$196 = s^2$$
$$\sqrt{196} = s$$
$$14 = s$$

Each side of the square is 14 ft. long.

Since $P = 4s$

$$P = 4 \times 14$$
$$P = 56 \text{ ft}$$

Raphael needs 56 feet of fencing to enclose his yard.

4. Do applications.

APPLICATIONS:

Before obtaining the solutions for the following problems, the teacher should have students underline in each problem the key words or phrases which indicate the need for finding area or perimeter.

1. Nancy is making a holiday tablecloth and wishes to trim it with fringe. The dimensions of the finished tablecloth are 1½ yards by 2 yards. The cost of the fabric is \$3.99 a square yard, and the cost of the fringe is \$2.49 a yard.

   a. What is the cost of the fabric?

   b. What is the cost of the fringe?

   c. What is the total cost?

2. Gerald intends to wallpaper one wall in his hallway. The wall is 8 ft high and 15 ft long. Each roll of wallpaper will cover 30 sq ft How many rolls will he need to cover the wall? If each roll costs \$12.95, what is the cost of wallpapering this wall?

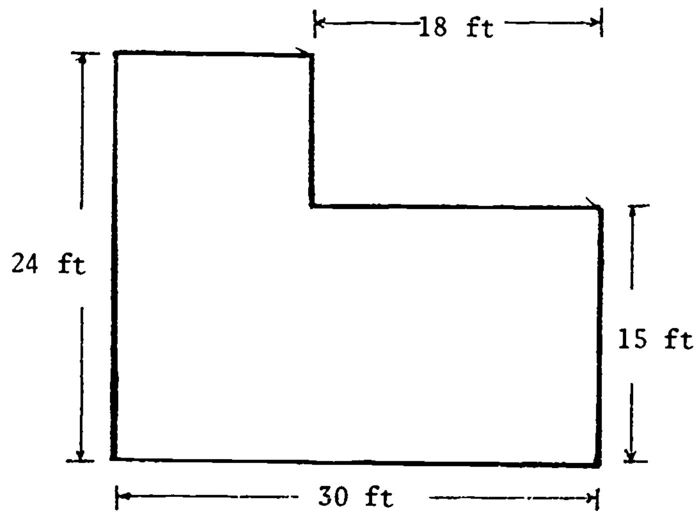
3. If grass seed costs \$5 a pound, and one pound of grass seeds covers 2500 square feet, how much will it cost to seed a rectangular lawn 20 ft by 50 ft ?

SUMMARY:

Review key phrases or terms which indicate whether area or perimeter is required.

CLIFF-HANGER:

An L-shaped room has the dimensions shown:



If carpet costs $12.99 a square yard, how much will it cost to carpet this L-shaped room?

LESSON 4

AIM: To solve problems involving areas of complex figures

PERFORMANCE OBJECTIVES: Students will be able to:

- calculate areas of complex figures involving rectangles by dividing the figures into component rectangles and adding.
- solve problems involving figures of this type.

VOCABULARY:

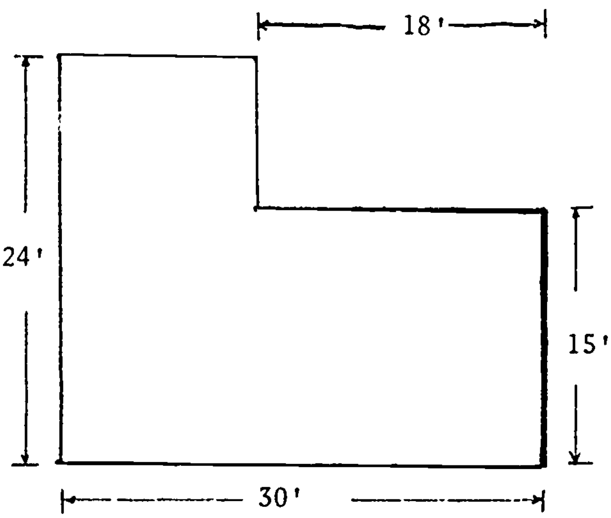
New terms: complex

Review: dimensions, rectangle

CHALLENGE PROBLEM:

Find the area of Figure 1.

Figure 1



DEVELOPMENT:

1. Elicit that the figure in the challenge problem can be divided into 2 rectangles in either of the following ways:

Figure 2

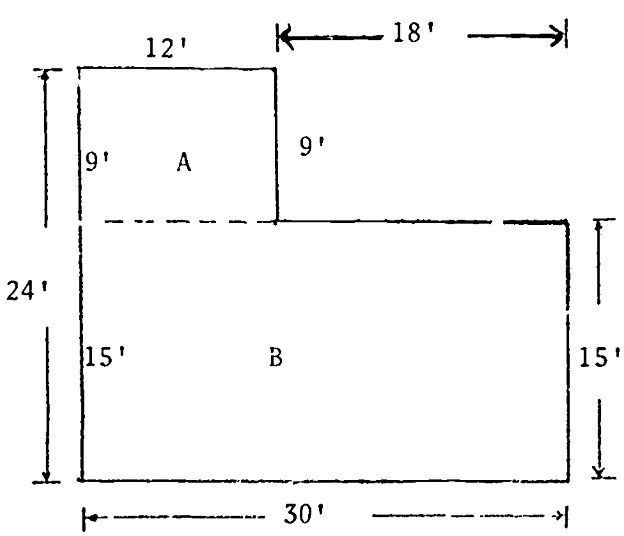
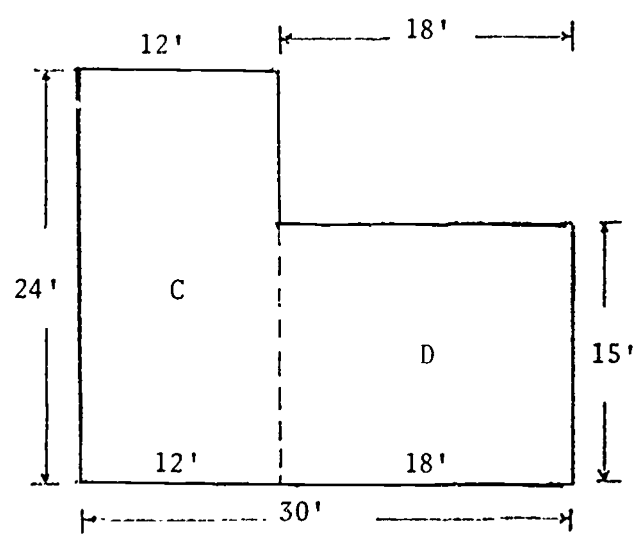


Figure 3



2. Determine the dimensions of each of the smaller rectangles.

In figure 2 rectangle B is 30' x 15' and rectangle A is 9' x 12'. Have students explain how the dimensions of rectangle A are obtained.

Have students calculate the areas of rectangles A and B.

Area of A = 9 ft x 12 ft	Area of B = 30 ft x 15 ft
Area of A = 108 sq ft	Area of B = 450 sq ft

Elicit that the total area is the sum of A and B or 558 sq ft

3. Similarly, calculate the areas of C and D in Figure 3 to be 288 sq ft and 270 sq ft respectively, whose sum is also 558 sq ft

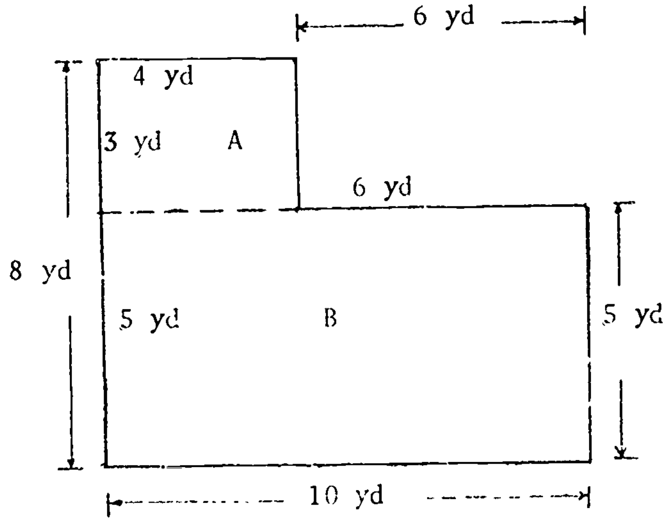
4. Have students state that in order to find the area of a complex figure made up of rectangles, it is necessary to divide the figure into its component rectangles. The dimensions of each rectangle must be found and the areas calculated. The total area of the figure is the sum of the areas of the component rectangles.

5. Do Application A.

6. Return to the cliff-hanger problem.

Elicit that to find the cost of carpeting the L-shaped room, the area of the room must first be found by using the method described in item 2 or 3. (Since the price is given for square yards, the dimensions of the room should be changed from feet to yards before any calculations are done). Once the area of the room has been calculated, the number of square yards is multiplied by the cost per yard.

Figure 4



Total area = 12 sq yd + 50 sq yd

= 62 sq yd

Cost of carpeting room = 62 x $12.99.

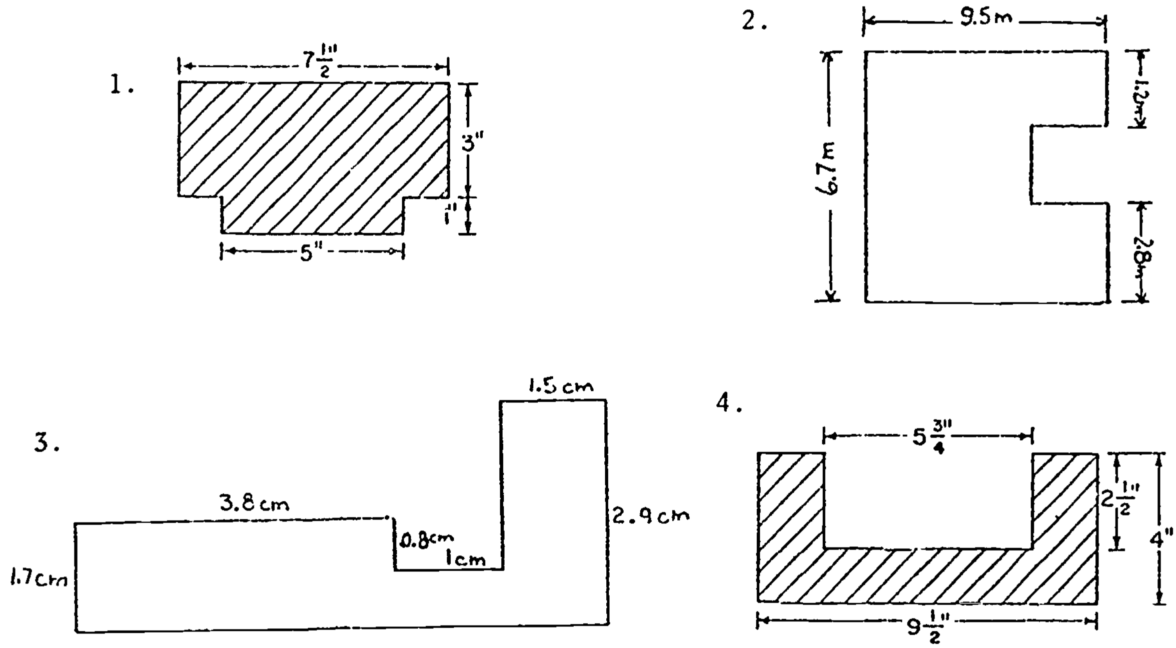
Cost of carpeting room = $805.38.

7. Do Application B.

APPLICATIONS:

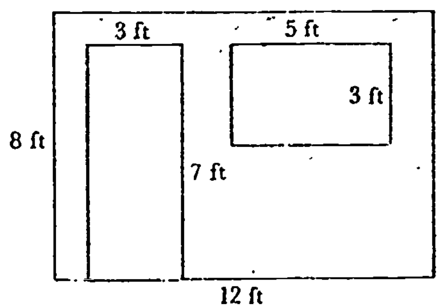
A. Find the areas of each of the given complex figures whose dimensions are shown:

Figure 5



B. Figure 6 shows a wall with a door and a window. What is the area of the window? What is the area of the door? If the wall is to be painted, how many square feet of surface need to be covered? (The door and window won't be painted.) If the 2 side walls of the room are 10 ft long and there are no other windows or doors in the room, what is the total surface that needs to be covered with paint?

Figure 6



(Note to Teacher: Find other irregular figures of this type and compute areas. Use the Real Estate section of a Sunday paper.)

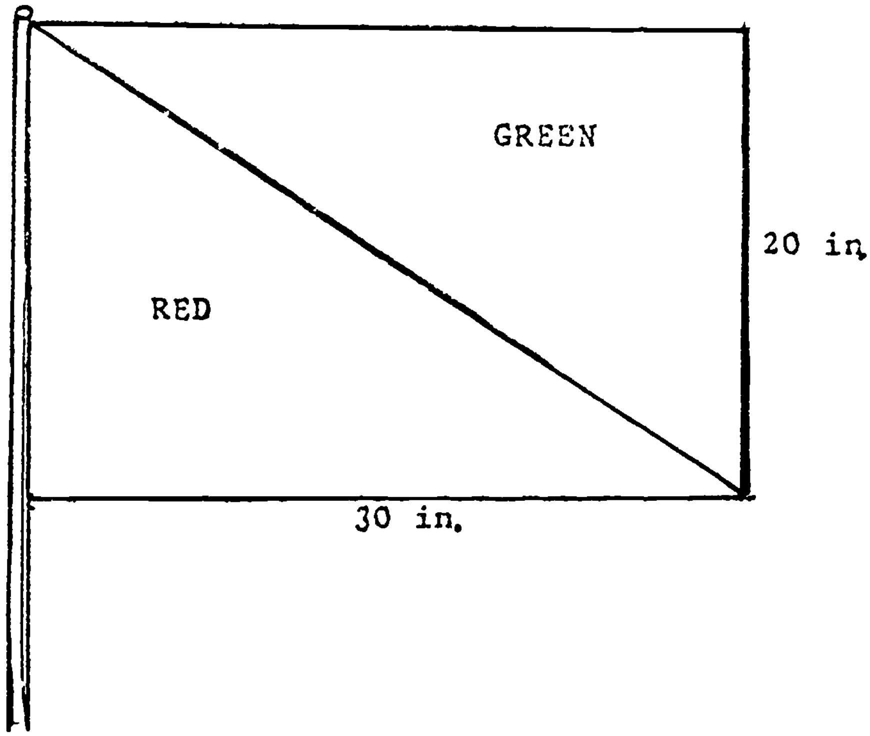
SUMMARY:

Review the methods used to compute the areas of complex rectangular shapes.

ERIC

CLIFF-HANGER:

As shown in the diagram, a flag is to be made in two colors, red and green. If the dimensions of the flag are 30 in. in length and 20 in. in width, what is the area of the red cloth needed?



LESSON 5

AIM: To find the area of a triangle

PERFORMANCE OBJECTIVES: Students will be able to:

- apply the formula $A = \frac{1}{2}bh$ to calculate the area of right triangles and express the area in square units.
- identify the base and height of any triangle where the bottom side is the base.
- use the formula $A = \frac{1}{2}bh$ to calculate the area of any triangle.
- use the formula $A = \frac{1}{2}bh$ to calculate the base or height of a triangle if the other dimensions are given.

VOCABULARY:

Review triangle, right triangle, diagonal, rectangle, base, height

CHALLENGE PROBLEM:

Find the area of the figure shown.

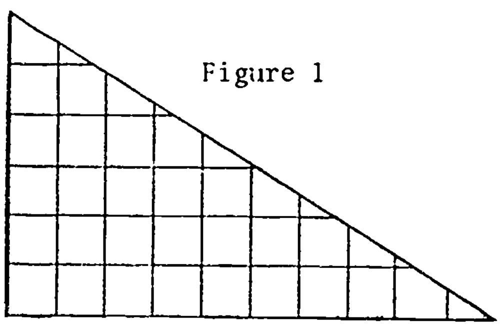


Figure 1

DEVELOPMENT:

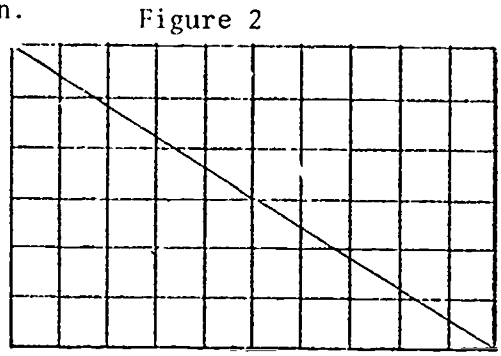
1. a. Have students suggest ways of finding the area in the challenge problem. (counting)

   b. Students should realize from their work with rectangles that counting is not always practical.

   c. Elicit that if this right triangle were seen as half of a rectangle, we could multiply $\frac{1}{2} \times 10 \times 6$ and the area would be 30 square units.

   d. Have pupils build up a rectangle on the original right triangle as shown.

Figure 2



(Note: This construction should be demonstrated at the board or on the overhead projector.)

e. Elicit that the area of the right triangle is equal to ½ the area of the rectangle. (Pupils may cut along the diagonal of a rectangle and place the triangles on top of each other showing that the figures are equal in area.)

f. Elicit that since a formula for the area of a rectangle is $A = bh$ and the area of a right triangle is ½ the area of a rectangle, a formula for the area of a right triangle is:

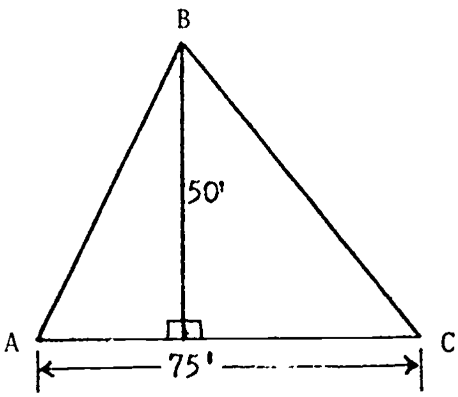
$$A = \frac{1}{2}bh$$

2. Use the formula for the area of a right triangle to answer the cliff-hanger problem and arrive at the result 300 square inches of red cloth.

3. Do Application A.

4. a. In Figure 3, △ABC has a <u>base</u> of 75 feet and a <u>height</u> of 50 feet. Elicit the meaning of <u>height</u> (as in tree, building, etc.) as the perpendicular distance from the highest point to the bottom (base).

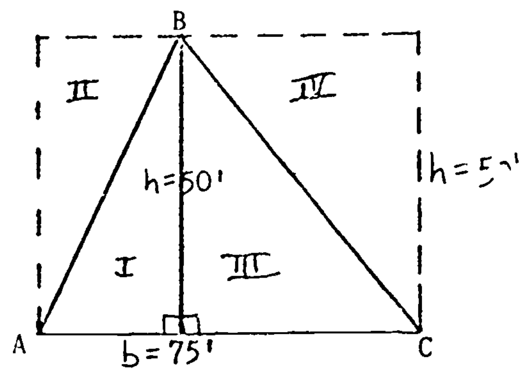
Figure 3



Since △ABC is not a right triangle, ask the pupils if the formula $A = \frac{1}{2}bh$ can still be used to find the area of △ABC? Why?

b. Develop the concept that the formula $A = \frac{1}{2}bh$ may be used to find the area of any triangle by constructing a rectangle around triangle ABC as in Figure 4 and proceeding as follows:

Figure 4



(1) The area of ΔI = the area of ΔII and the area of ΔIII = the area of ΔIV. (Refer to Figure 2 in order to explain this.)

(2) Since Δ I = ΔII and

Δ III = Δ IV

then Δ I + ΔIII = ΔII + ΔIV using the addition property of equality.

(3) Since ΔI + ΔIII = ΔII + ΔIV and all four triangles add up to the entire rectangle, each sum (ΔI + ΔIII) and (ΔII + ΔIV) is one half of the rectangle.

(4) Therefore the area of ΔI + ΔIII = ½ the area of the rectangle

or

the area of ΔABC = ½bh

5. Use the formula A = ½bh to compute the area of Δ ABC.

$$A = \tfrac{1}{2}bh$$

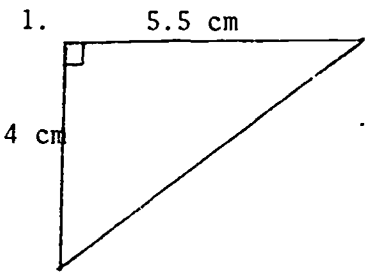
$$A = \tfrac{1}{2} \times \frac{75}{1} \times \frac{50}{1}$$

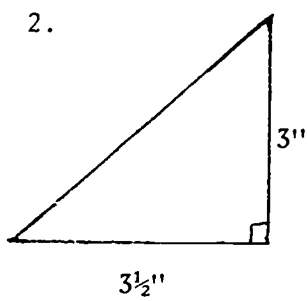
$$A = 1875 \text{ sq ft}$$

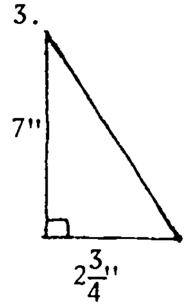
6. Do Applications A, B, C, and D.

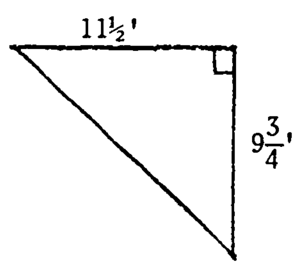
APPLICATIONS:

A. Find the area of each of the right triangles pictured:

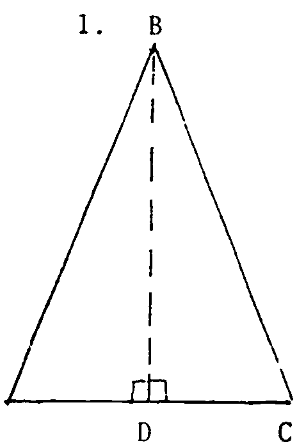
1.


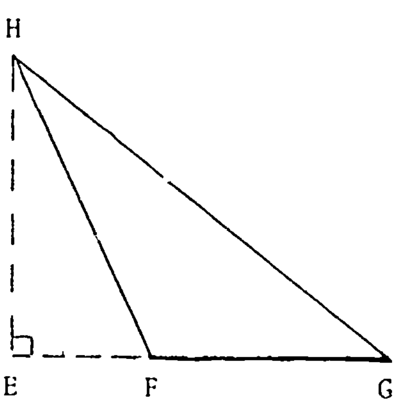
2.


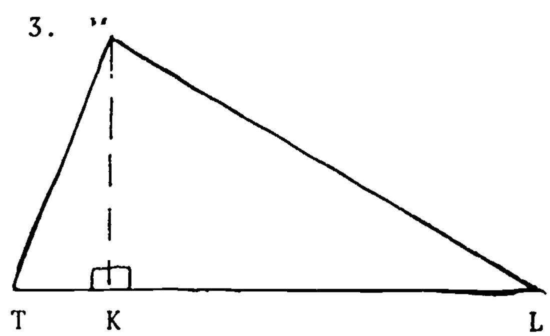
3.


4.


B. Name the base and the height of each of the triangles drawn:

1.


2.


3.


C. Find the area of each of the triangles in Application B (Figure 6) if the lengths of the line segments are given.

1. BD = 12.6 m, AC = 9.5 m

2. FG = 5 in., HE = 4½ in.

   (Point out that for this type of triangle, the length of the base does not include its extension outside the triangle.)

3. TL = 18 ft, MK = 6¼ ft

D. Complete the chart below by finding the missing dimension in each triangle.

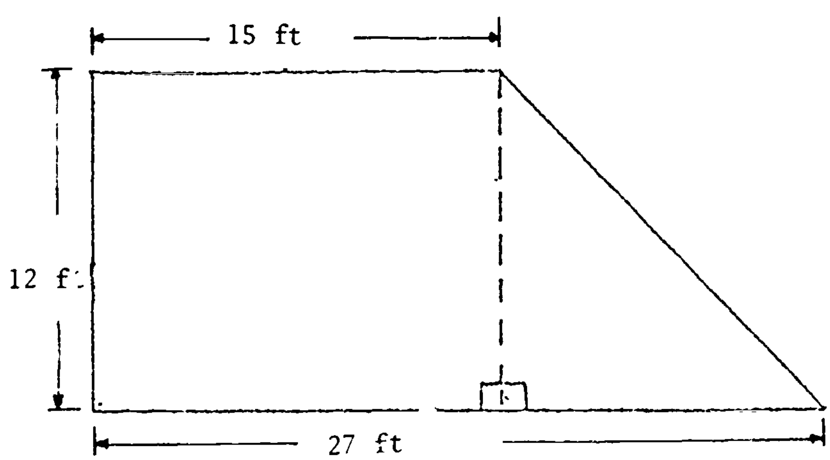
	Base	Height	Area
a.	28 cm	?	140 cm^2
b.	?	60 m	720 m^2
c.	21 in.	?	42 sq in.
d.	?	10 cm	2.5 cm^2

SUMMARY:

Review the meaning of the terms base and height of a triangle and the formula for finding the area of a triangle.

CLIFF-HANGER:

One room at the corner of a large building has a floor shaped as shown. If tile costs $3.00 a square foot, find the cost of tiling this room.



LESSON 6

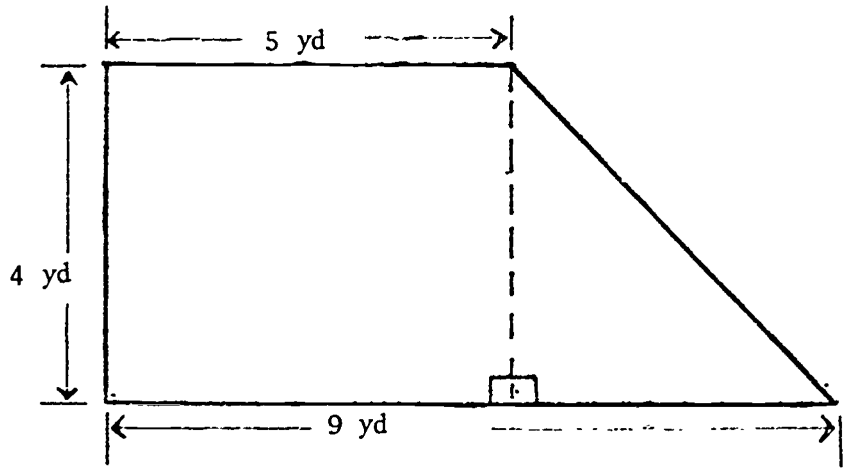
AIM: To find the areas of complex regions involving rectangles and triangles

PERFORMANCE OBJECTIVES: Students will be able to:

- divide complex regions given in a diagram into rectangles and triangles
- calculate the areas of complex regions involving rectangles and triangles.

CHALLENGE PROBLEM:

One room at the corner of a large building has a floor shaped as shown. Find the area of this floor.



DEVELOPMENT:

1. Using the challenge problem, elicit that the area of the floor can be found by finding the sum of the area of the rectangle and the area of the triangle. The students should obtain a result of 20 square yards for the area of the rectangle plus 8 square yards for the area of the triangle. Therefore the area of the floor is 28 square yards.

2. Solve the cliff-hanger problem by eliciting that the area of the floor can be found in the same way as was shown in the challenge problem. The students should obtain a result of 180 sq ft for the area of the rectangle and 72 sq ft for the area of the triangle for a total of 250 sq ft.

   Therefore: The cost of tiling the floor = 250 x $3.00.

   Cost of tiling the floor = $750

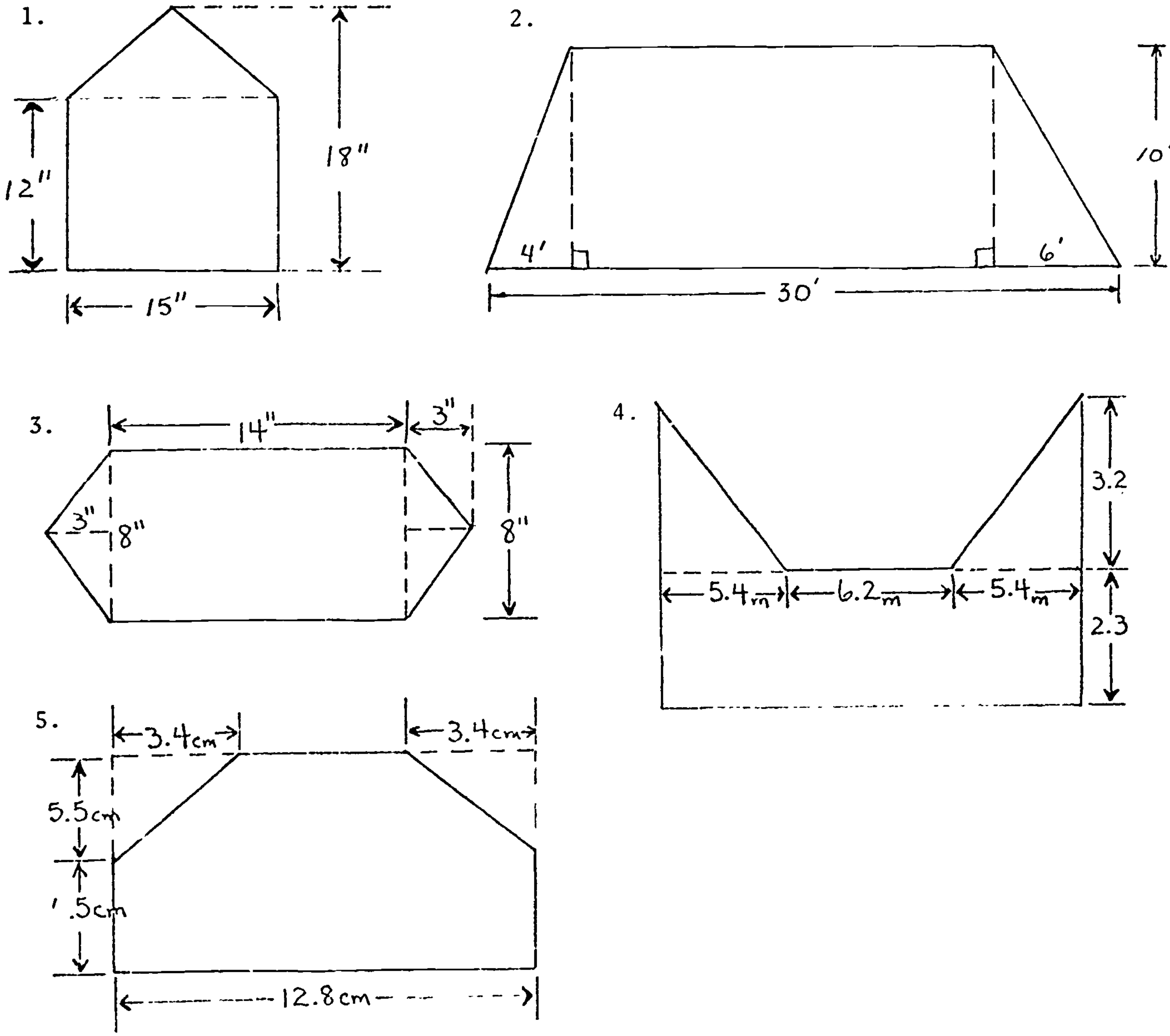
3. Do applications.

APPLICATIONS:

A. Using the given information in each of the following diagrams, find the total area of each figure:

ERIC

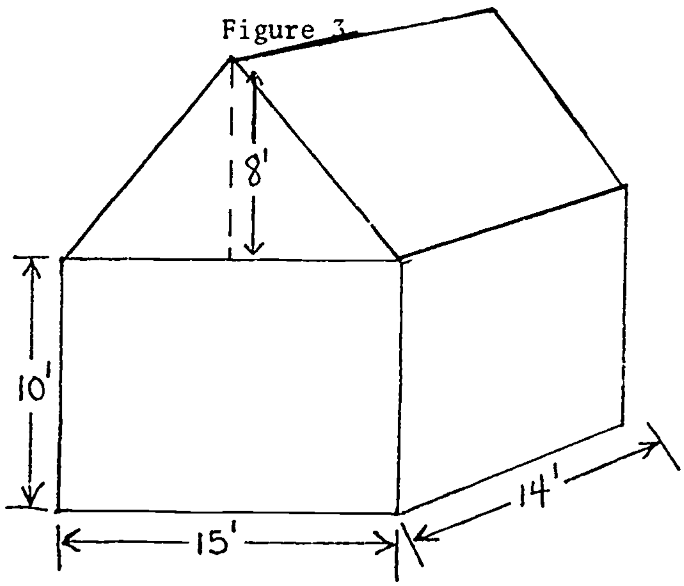
Figure 2


1.
18"
12"
15"
2.
10'
4'
6'
30'
3.
14"
3"
3"
8"
8"
4.
3.2
5.4m
6.2m
5.4m
2.3
5.
3.4cm
3.4cm
5.5cm
.5cm
12.8cm

B. A barn has a shape as in Figure 3. Using the information given in the diagram, find the cost of painting the front, back and walls of the barn if a gallon of paint costs $15.95 and covers an area of 20 sq ft.

(Note to Teacher: Discuss the need for purchasing a full gallon even though only a part of it will be used.)

Figure 3



SUMMARY:

Review the method of finding the area of complex regions involving rectangles and triangles.

CLIFF-HANGER:

A revolving sprinkler sprays a lawn for a distance of 7 m. How many square meters does the sprinkler water in 1 revolution?

LESSON 7

AIM: To find the area of a circle

PERFORMANCE OBJECTIVES: Students will be able to:

- state the formula used to find the area of a circle: $A = \pi r^2$.
- apply the formula to find the areas of circles using $\pi = \frac{22}{7}$

VOCABULARY:

Review: circle, radius, pi, diameter, circumference

CHALLENGE PROBLEM:

Find the circumference of a circle whose radius is $3\frac{1}{2}$ m [use $= \frac{22}{7}$].

DEVELOPMENT:

1. Use challenge problem to review diameter, radius, and circumference of a circle. Recall meaning of circumference and the formulas used to find circumference of a circle:

$$C = 2\pi r$$

$$C = \pi d$$

Review that π is the ratio of the circumference to the diameter in any circle and approximately equals $\frac{22}{7}$ or 3.14.

2. Solve the challenge problem as follows:

$$C = 2\pi r$$

$$C = 2 \times \frac{22}{7} \times 3\frac{1}{2}$$

$$C = \frac{{}^{1}\cancel{2}}{1} \times \frac{22}{\cancel{7}_1} \times \frac{\cancel{7}^{1}}{\cancel{2}_1}$$

$$C = 22 \text{ m}$$

3. Recall the cliff-hanger problem. Elicit that the students must find the area of the circle in order to solve the problem; however, they have not yet learned a method for doing so.

4. Develop the formula for finding the area of a circle in the following way:

   a. Draw a circle and divide it into 16 equal parts as in Figure 1.

   b. Demonstrate that the circle may be cut apart and put back together as in Figure 2. This can be done effectively on the overhead projector.

   c. Figure 2 is similar in appearance to a rectangle. However a closer approximation can be obtained by cutting the sector on the far right in half and placing this half on the left side as in Figure 3.

Figure 1

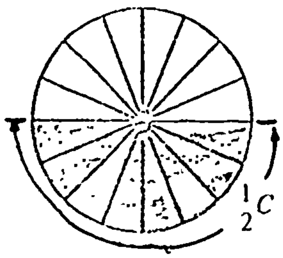


Figure 2

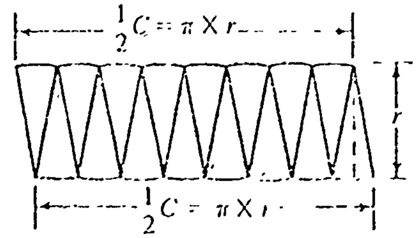
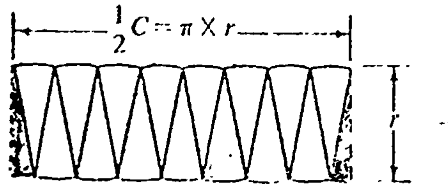


Figure 3



d. Elicit that the dimensions of the rectangle formed in Figure 3 are approximately ½C (the base) and r (the height); therefore, the area of the rectangle is approximately given by:

$$A = bh$$

$$A = \frac{1}{2} C \text{ x } r$$

e. Recall that $C = 2 \pi r$, so by substitution

$$A = \frac{1}{2} \ (2 \pi r) \text{ x } -$$

or

$$A = \frac{1}{\cancel{2}} \text{x} \frac{\cancel{2}}{1} \text{ x } \pi \text{ x } r \text{ x } r$$

This can be written in simplest form as:

$$A = \pi r^2$$

This is the formula for the area of a circle in terms of its radius.

5. Solve the cliff-hanger problem as follows:

$$A = \pi r^2 \quad \text{where } r \text{ is 7 meters}$$

$$A = \frac{22}{7} \text{ x } 7^2$$

$$A = \frac{22}{\cancel{7}_1} \text{ x } \frac{\cancel{7}^1}{1} \text{ x } \frac{7}{1}$$

$$A = 154 \text{ m}^2$$

6. Do applications.

APPLICATIONS:

A. Find the areas of circles having the following radii. (Use $\pi = \frac{22}{7}$)

1. 14m
2. 20cm
3. $4\frac{2}{3}$ yd
4. ½ foot
5. $3\frac{1}{4}$

B. Find the areas of circles having the following diameters (Note: Review finding the radius when a diameter is given.)

Use $\pi = \frac{22}{7}$

1. 84m
2. 7mm
3. 2½ ft

C. Solve each of the following problems (use $\pi = \frac{22}{7}$)

1. If an FM radio station can broadcast within a radius of 91 km, over how large an area can the signal be received?
2. From his tower a forest ranger can see a distance of 28 km in any direction. How many square kilometers can he patrol from his tower?

SUMMARY:

Review the meaning of pi and how to find the area of a circle.

LESSON 8

AIM: To apply the formula for the area of circle

PERFORMANCE OBJECTIVE: Students will be able to:

- apply the formula $A = \pi r^2$ to find the areas of circles, using $\pi = 3.14$.

CHALLENGE PROBLEM:

A circular swimming pool is 18 feet in diameter. A plastic cover is to be purchased to fit this pool. If the cost of plastic is $.35 per square foot, what will be the cost of the cover to the nearest dollar?

DEVELOPMENT:

1. Discuss the challenge problem. Elicit that the area of the cover must be found in order to find the cost. Have the students recognize that $\frac{22}{7}$ is an inconvenient approximation for π in this problem. Elicit that another approximation for π is 3.14, by having students convert the fraction $\frac{22}{7}$ to a decimal.
2. a. Using the value $\pi = 3.14$ find the area of the pool cover.

   Since the diameter = 18 feet, r = 9 feet.

   $A = \pi r^2$

   $A = 3.14 \times 9 \times 9$

   $A = 254.34$ square feet

   b. The cost of the cover = cost per sq. ft. x number of sq. ft.

   Cost = \$.35 x 254.34 sq. ft.

   Cost = \$89.019

   The cost to the nearest dollar is \$89.
3. Do applications.

APPLICATIONS:

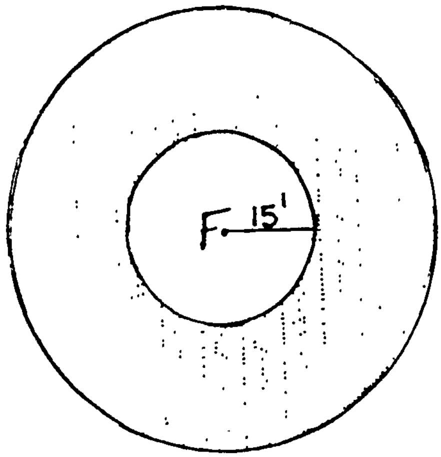
A. Find the area of the circle described using $\pi = 3.14$.

1. r = 6 cm
2. r = 8 cm
3. r = 2.2 m
4. r = 0.7 mm

B. 1. Would you get more pizza if you bought two 10-inch pizzas or one 16-inch pizza?

2. Which is larger: the area of a circle 6 cm in diameter or the area of a square whose side is 6m? How much larger?
3. A circular walk surrounds a fountain in the park as shown:

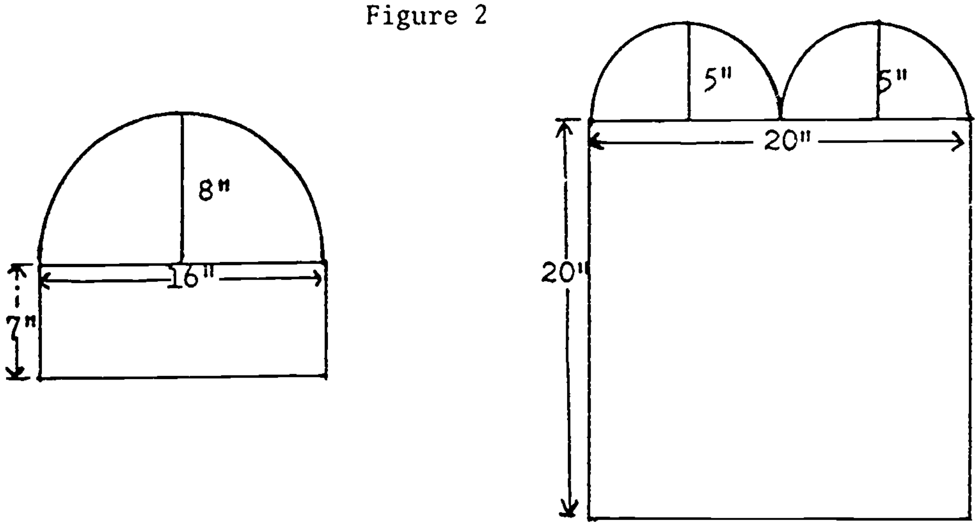
Figure 1



F represents the fountain. The walk, which is 6 ft wide, is represented by the shaded region. If the cost of concrete is $2.34 a square foot, what is the approximate cost of constructing this walk?

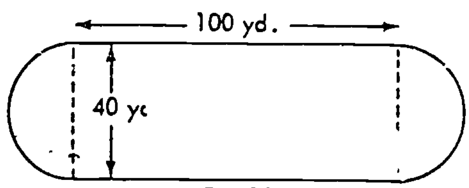
4. Find the total area of each of the following figures:

Figure 2



5. A football field consists of a rectangle with semicircles at each end, as shown. Using the dimensions given, find, to the nearest square yard, the number of square yards of sod needed to cover the field. (Use $\pi = 3.14$.)

Figure 3



Ex. 13

SUMMARY:

Review the meaning of area, the formulas used for finding the areas of rectangles, squares, triangles, circles, and the method of finding the area of a composite figure. Compare the units used to measure area with those used to asure perimeter.

UNIT XI. Indirect Measure and Scaling

LESSON 1

AIM: To distinguish between direct and indirect measurement

PERFORMANCE OBJECTIVES: Students will be able to:

- describe situations in which a measurement of length or distance cannot be made directly.
- apply formulas to find measurements of quantities indirectly by using measurements that have been made directly.

VOCABULARY:

New terms: direct measurement, indirect measurement

Review: formula

CHALLENGE PROBLEM:

How far away is a flash of lightning which is seen 5 seconds before the thunder is heard?

DEVELOPMENT:

1. Elicit from class that the challenge problem requires finding a distance or measure of length. Suggest the possibility of obtaining the result directly by measuring the distance from the point where the thunder is heard to the point where the lightning actually struck.

2. Discuss the meaning of direct measurement (where the unit of measure is applied directly to the quantity being measured). List objects whose measure can be found directly such as dimensions of a room, height of a person, length of a piece of fabric or length of a piece of wood.

3. Elicit that to measure the distance described in the challenge problem directly is possible but very difficult. Another method of solving this problem is by indirect measurement. One type of indirect measurement is through the use of a formula. One formula for finding distance is d = rt. Since the rate or speed of sound (in air) is approximately 1100 feet per second and it took 5 seconds until the thunder was heard:

   $d = rt$

   $d = 1100 \times 5$

   $d = 5500$ ft

   Note: Since the speed of light is 186,000 mi/sec, it is disregarded in this problem.

   Since 5280 feet = 1 mile, the lightning was a little more than one mile away.

4. Elicit situations in which direct measurement would be impossible or impractical; for example: the height of a mountain, or a building, or a flagpole; the distance across a body of water, or between the earth and the other planets, or between cities.

5. Discuss the fact that most measurement is indirect. For example, when you weigh an object, you are usually finding the stretch of a spring; when temperature is measured, you are measuring the amount of expansion or contraction of metals; when time is measured, the amount of movement of gears is being measured. One of the few measurements that can be made directly is that of length.

6. Discuss situations in which there is a choice as to the type of measurement used. For example, the area of a rectangle can be found directly by counting the number of square units it contains or indirectly by multiplying its length by its width. Elicit that indirect measurement is often more convenient - easier, faster, and more efficient.

7. Do applications.

APPLICATIONS:

1. Determine which of the following measurements can be done directly and which must be done indirectly:

   a. finding the height of Mount Everest

   b. finding the length of a corridor in school

   c. finding the diameter of a dime

   d. finding the height of the teacher's desk

   3. finding the depth of the Pacific Ocean at the deepest part

   f. finding the distance from the earth to the sun

   g. finding the height of the school flagpole

   h. finding the temperature of a blast furnace

   finding the weight of an elephant

   some cases, there may be a choice between direct and indirect urement. If so, ask the pupils which would appear to be preferable.)

2. What direct measurements do we need to know to find each of the following indirectly?

   a. the area of the floor of a classroom

   b. the perimeter of a triangle

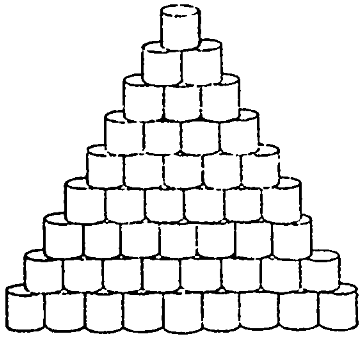
   c. the circumference of a circle

   d. the area of a triangle

3. Radar travels at 186,000 miles per second. If it takes 2.8 seconds for a radar signal to bounce off the moon and back to earth, what is the distance from the earth to the moon? (Use d = rt. Note it takes 1.4 seconds for the signal to go from the earth to the moon.

4. A merchant wishes to set up a display of canned goods. How many cans will he need for the display if he wishes to place 9 cans in the bottom row and 1 can less in each row until there is only

1 can in the top row? Use the formula $S = \frac{n}{2}(a + l)$ where S = the sum, n = the number of rows, a = the number in the bottom row, and l = the number in the top row.



Check the result of the indicrect measurement by directly counting the number of cans.

5. Henry borrowed a book from a library. The library charges 15¢ for the first 3 days and 2¢ a day for each additional day. If Henry kept the book for 19 days, how much did he have to pay? (Use the formula: $c = 15 + 2(n-3)$ where c = the charge in cents and n = the number of days he kept the book.)

6. How high is a building if a rubber ball which is dropped from the roof takes 2 seconds to hit the ground? (Use the formula $s = 16t^2$ where s = the distance (height) in feet and t = the time in seconds.)

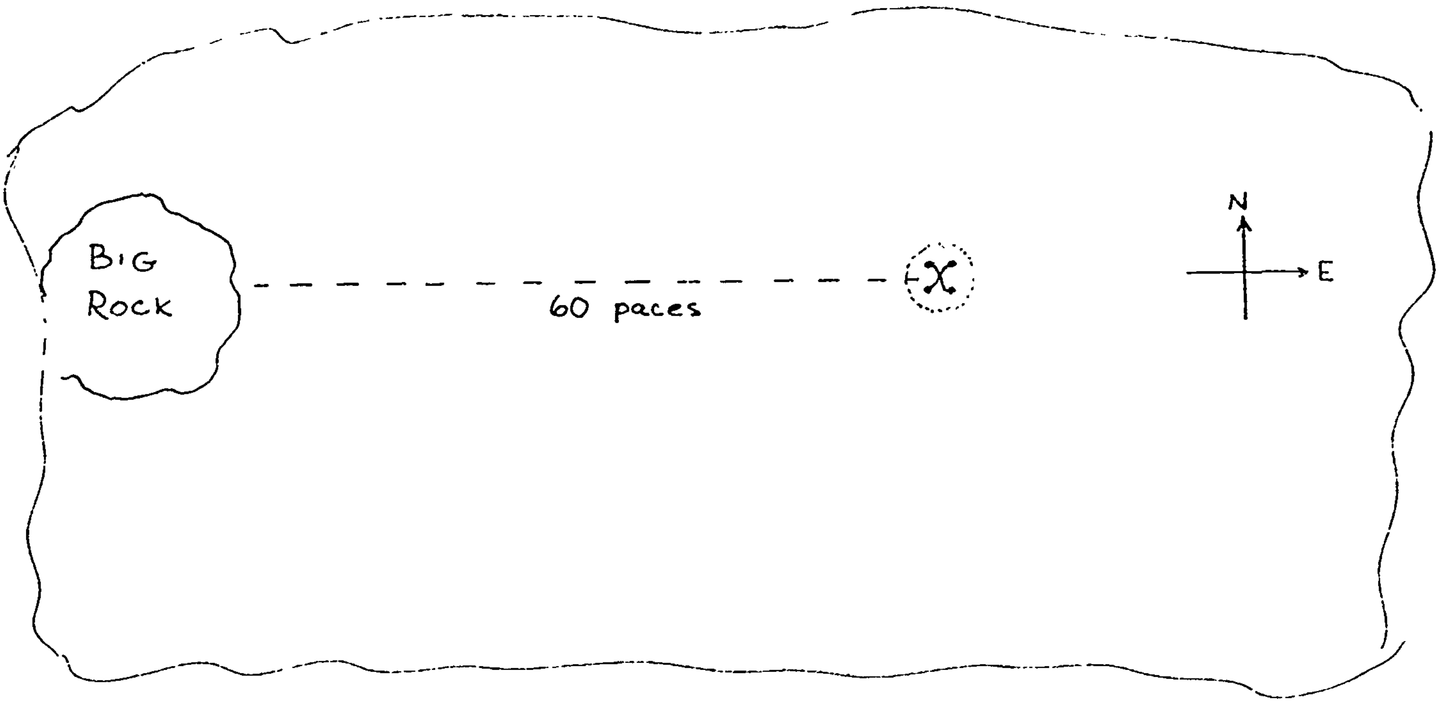
SUMMARY:

Review the difference between direct and indirect measurement and the need for the latter in some measurement problems. Pupils should understand that indirect measurement requires the measurement of <u>some</u> quantity or quantities <u>directly</u> and the performing of calculations using these direct measurements to obtain the desired results.

CLIFF-HANGER: See next page.

CLIFF-HANGER:

A treasure map shows that a treasure is buried 60 paces due east of the BIG ROCK. If Stan is 6'4" tall and Oliver is 5'6" tall and each looks for the treasure, will they dig in the same place? Who will find the treasure?



'SON 2

AIM: To understand the meaning anւ use of scale drawing

PERFORMANCE OBJECTIVES: Students will be able to:

- state a need for standard units of measure.
- use a scale to determine the actual number of unit· represented in a drawing.
- measure the length of line segments using a ruler
- determine an actual distance given a unit scale and a scaled length.

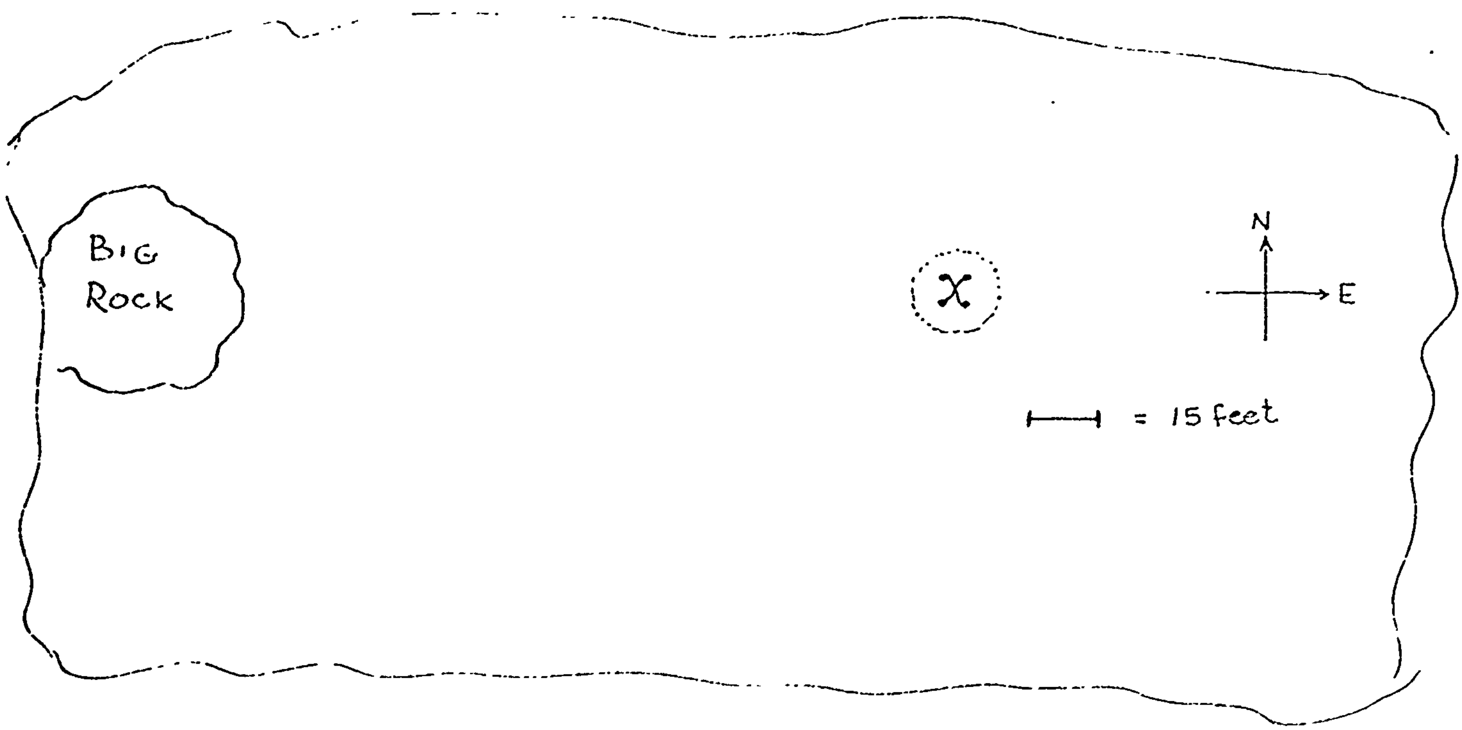
VOCABULARY:

New terms: scale, scale drawing

Review: unit, indirect measurement, direct measurement

CHALLENGE PROBLEM:

Tom and Jerry found the following treasure map:



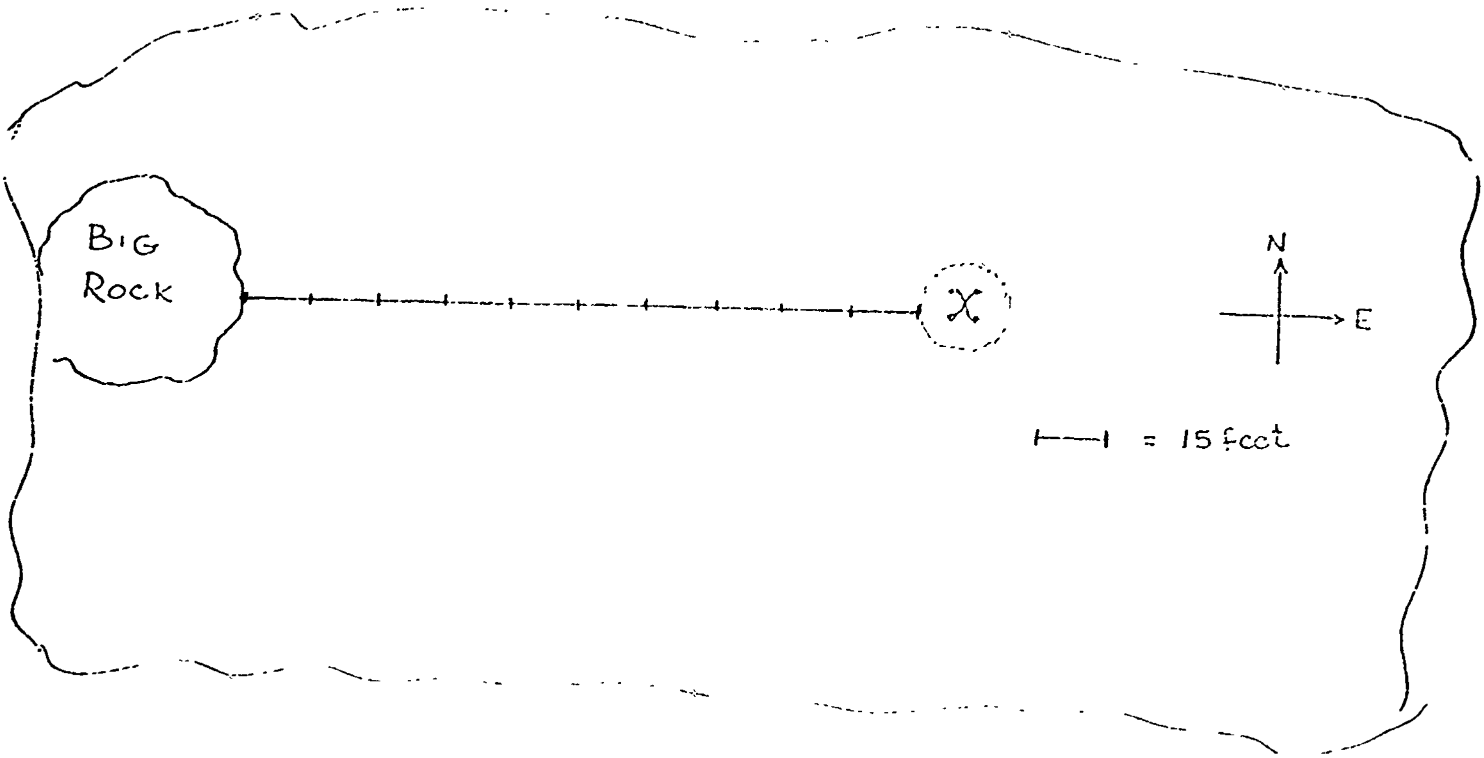
How far from the rock must Tom and Jerry dig?

DEVELOPMENT:

1. Discuss the challenge problem. Compare it with cliff-hanger problem. How do the two maps differ? Elicit: the scale on the challenge problem gives a standard unit of measure whereas the map in the

cliff-hanger problem uses paces (a non-standard unit of measurement which differs from person to person).

2. Relate the scale (⊢—⊣ = 15 ft) to the scale on a pictograph. How can the distance from Big Rock to the treasure be found? Elicit the need for finding the number of units in the length of the line joining the Big Rock to the treasure on the map.



Have students determine that the line contains 10 of the given unit lengths. Therefore, since each unit length represents 15 feet, the line is 15 (10) or 150 feet long.

3. A map of this type is called a <u>scale drawing</u>. Have students list other examples which involve <u>scale drawings</u> or scaling. Elicit such examples of scale drawings as: road maps, blueprints, patterns in dress-making, model making (airplanes, cars), enlargements (blow-ups). Elicit that actual measures can be found indirectly by measuring a scaled figure directly and knowing the value of the scale.

4. Do applications.

APPLICATIONS:

A. Measure each line segment and write the distance using the scale of miles or kilometers given. (See Figure 3)

APPLICATIONS:

A. Measure each line segment and write the distance using the scale of miles or kilometers given.

Figure 3

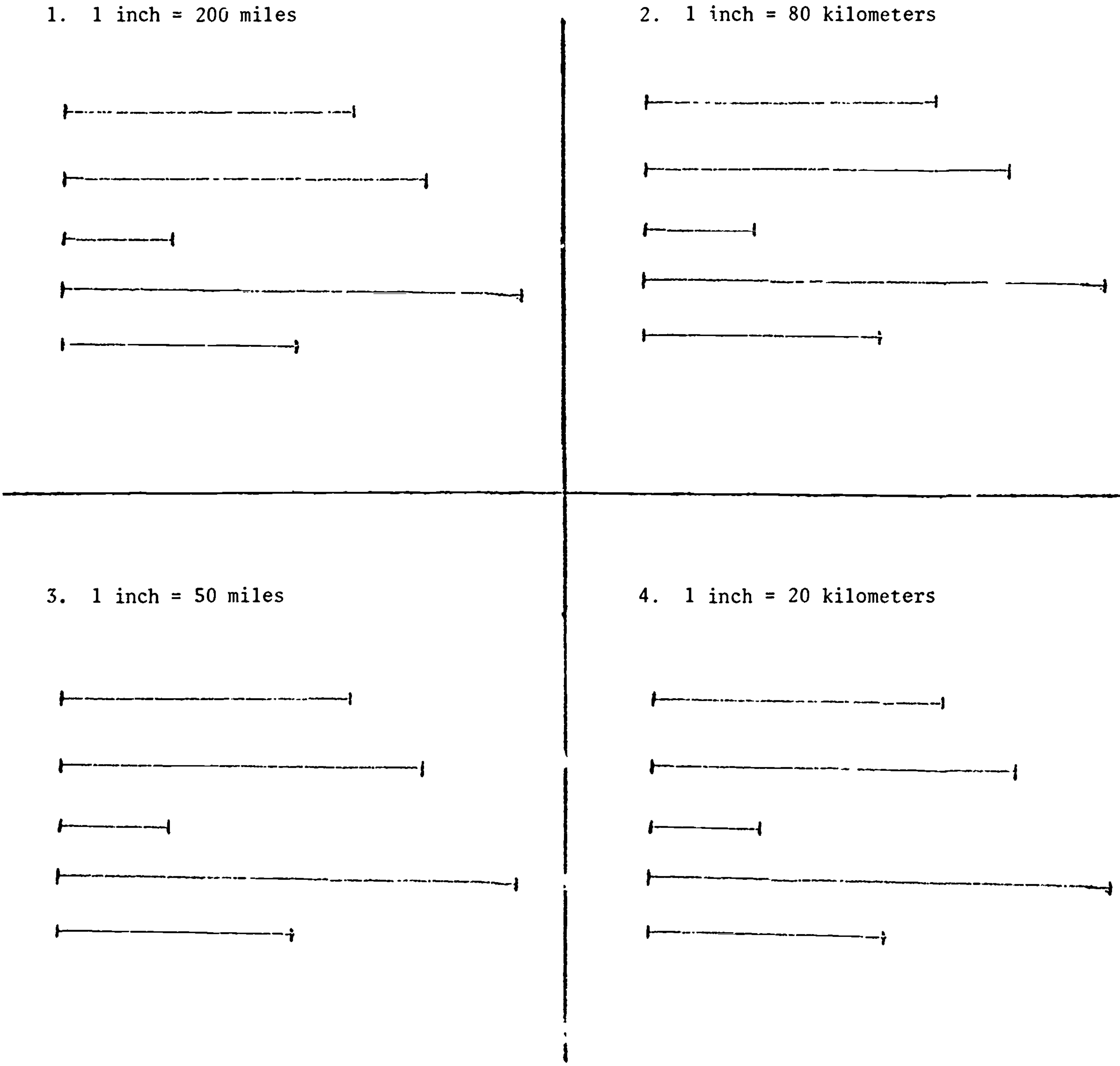
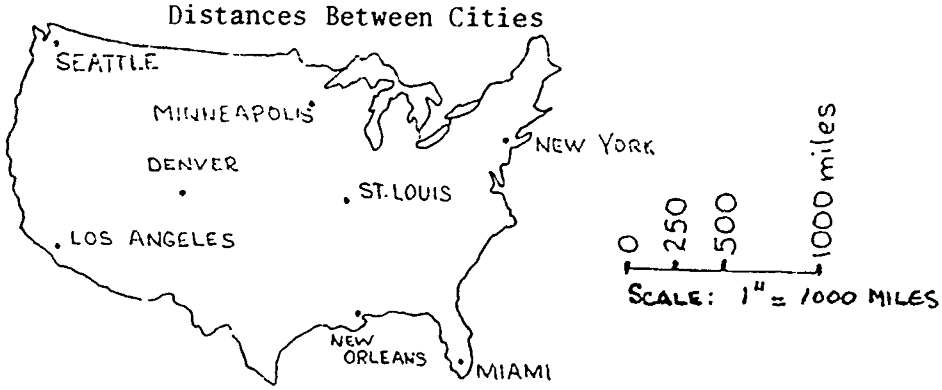


Figure 4

Distances Between Cities

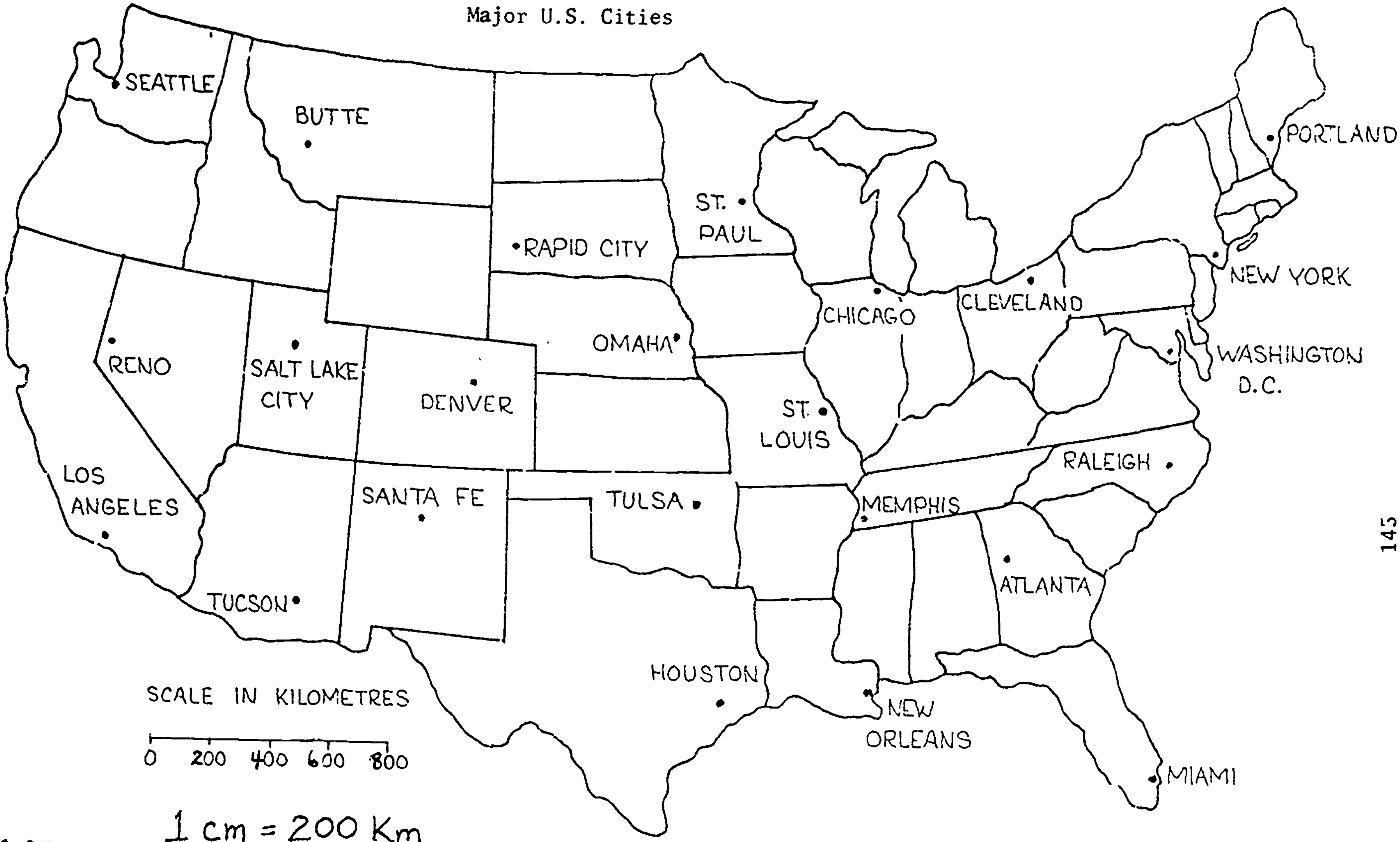


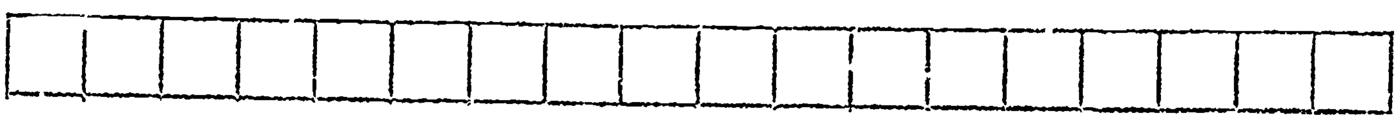
B. Using the map in Figure 4 record the distan e between cities in the chart below. (Measure to the nearest ¼ in.)

	DENVER	LOS ANGELES	MIAMI	MINNEAPOLIS	NEW ORLEANS	NEW YORK	SEATTLE	ST. LOUIS
DENVER								
LOS ANGELES								
MIAMI								
MINNEAPOLIS								
NEW ORLEANS								
NEW YORK								
SEATTLE	1000							
ST. LOUIS								

Figure 5

Major U.S. Cities





C. Figure 5. (Note to Teacher: Students may use metric ruler below map.)

1. Using the given scale, find the approximate distance between each pair of cities listed.

   a. Salt Lake City to Denver

   b. Tucson to Tulsa

   c. Chicago to Cleveland

   d. St. Paul to Houston

2. Which of the following trips is longer? How much longer?

   Los Angeles ⟶ Chicago ⟶ Washington

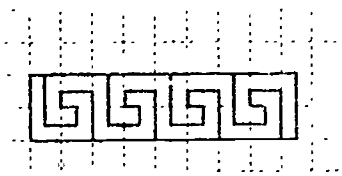
   Los Angeles ⟶ Atlanta ⟶ Washington

SUMMARY:

Review how to use a scale and a scale drawing to determine actual measurements.

CLIFF-HANGER:

Wendy saw the following design pictured in a book. She decided to use this design for a belt that would be 1 inch wide. How would she accomplish this?



LESSON 3

AIM: To identify similar figures

PERFORMANCE OBJECTIVES: Students will be able to:

- state that the enlargement or the reduction ratio is called a scale.
- compare any two quantities with a ratio.
- state that enlargements or reductions create figures similar to the original.
- state that when figures are similar, the corresponding parts (lines) are in proportion.
- identify a true proportion by:
  1. comparing fractions.
  2. using the proportion property.

VOCABULARY:

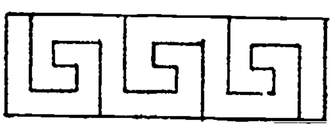
New terms: proportion, means, extremes, similar, terms, corresponding

Review: scale, ratio

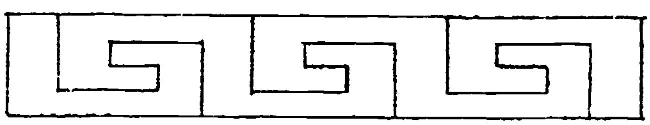
CHALLENGE PROBLEM:

Which of the following pictures represents an enlargement of Figure A?

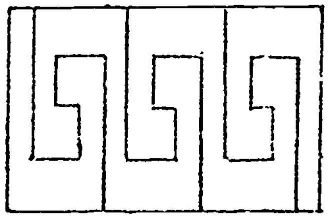
A.



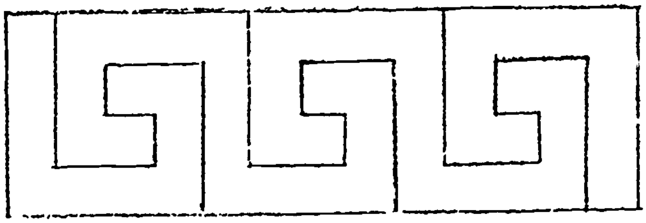
1.



2.



3.



DEVELOPMENT:

1. Discuss the challenge problem. Elicit from students that in Picture 1 the length was increased (doubled) but the width was kept the same. This resulted in a distortion of the original diagram. Similarly, in Picture 2, the width was increased (doubled) but the length was left the same. This resulte in another distortion of the original diagram. However, in Figure 3 both the length and the width were increased (doubled) maintaining the same shape as the original design.

   Explain that figures having the same shape are called <u>similar figures</u>. Therefore, Figure 3 is similar to Figure A.

2. Have students measure the length and width of Figure A and the length and width of Picture 3. Have students write the ratio of the width of Figure 3 to the width of Figure A (2 to 1 or 2:1 or $\frac{2}{1}$). Ask students to hypothesize the ratio of the <u>length</u> of Figure 3 to the <u>length</u> of Figure A. Verify the hypotheses by measuring. Explain that the ratio of corresponding lines is called the <u>enlargement scale</u>.

3. When figures are simil the enlargement or reduction scale is the same for all corresp ing lines. Verify this fact by measuring a pair of corresponding lines (other than length or width) from the patterns, and writing the ratio of the enlargement part to the original part. Elicit that in this challenge problem each ratio could be simplified to 2:1

4. Define a <u>proportion</u> as a statement of 2 equal ratios. Have stud write proportions such as:

$$\frac{\text{Length of Figure 3}}{\text{Length of Figure A}} = \frac{\text{Width of Figure 3}}{\text{Width of Figure A}}$$

or

$$\frac{\text{Width of Figure 3}}{\text{Width of Figure A}} = \frac{\text{Any part of Figure 3}}{\text{The corresponding part in Figure A}}$$

   Therefore, when two figures are similar, their corresponding parts are in proportion.

5. Do Application A.

6. Do the ratios $\frac{9}{12}$ and $\frac{15}{20}$ form a proportion? Have students recall the definition of a proportion (a statement that two ratios are equal) and use this definition to write the following: Does $\frac{9}{12} = \frac{15}{20}$? Lead students to answer this question by the two methods described below:

   a. Does $\frac{9}{12} = \frac{15}{20}$? Method A: Rewrite each ratio lowest terms:

$\frac{9}{12}$	$\frac{15}{20}$
Dividing numerator and denominator by 3 results in the ratio $\frac{3}{4}$	Dividing numerator and denominator by 5 results in a ratio $\frac{3}{4}$

Therefore $\frac{9}{12} = \frac{15}{20}$ is a true proportion.

b. Does $\frac{9}{12} = \frac{15}{20}$?

Method B involves first defining the terms of a proportion. The four numbers that form a proportion are called the terms. In this proportion 9 is the first term, 12 is the second term, 15 is the third term, and 20 is the fourth term. In any proportion the second and third terms are called the means and the first and fourth terms are called the extremes. The following property is used:

> In a true proportion, the product of the means is equal to the product of the extremes.

Illustrate the proportion property as follows:

$$\frac{9}{12} = \frac{15}{20} \text{ if } 12 \times 15 = 9 \times 20.$$

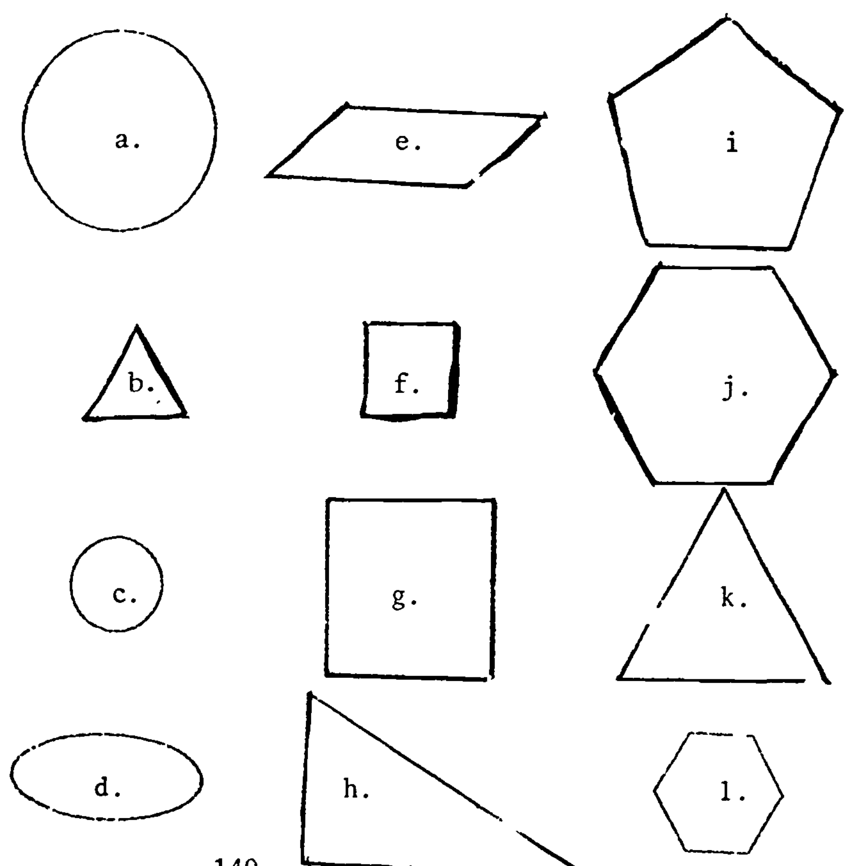
Since $12 \times 15 = 180$ and $9 \times 20 = 180$, the proportion $\frac{9}{12} = \frac{15}{20}$ is a true proportion.

7. Do Applications B and C.

APPLICATIONS:

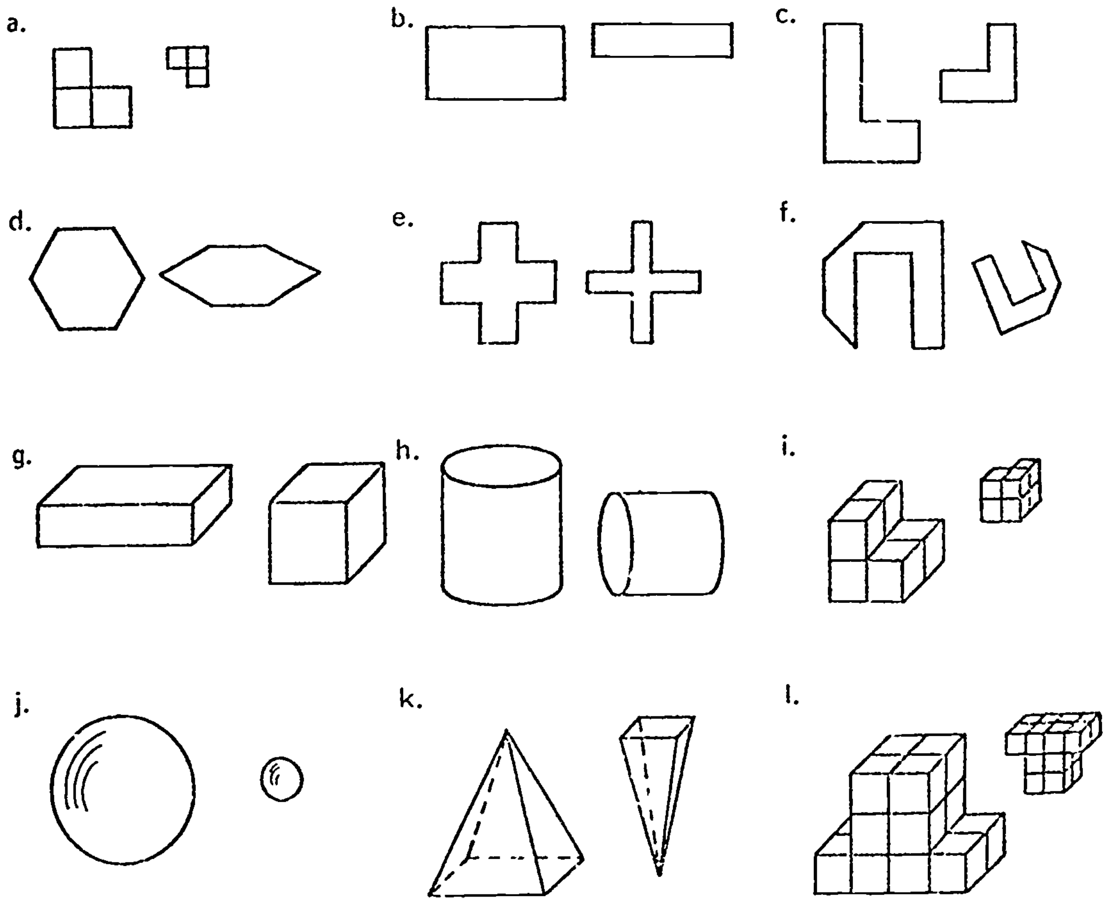
A. 1. Name the pairs of figures which appear to be similar.

Figure 2.



2. Tell which pairs of figures appear to be similar in figure 3.

Figure 3



B. Which of the following are true proportions?

1. $\frac{3}{4} = \frac{9}{12}$ 3. $\frac{19}{3} = \frac{57}{9}$ 5. $\frac{10}{15} = \frac{4}{6}$

2. $\frac{2}{7} = \frac{5}{16}$ $\frac{5}{9} = \frac{11}{20}$ $\frac{2}{7} = \frac{10}{35}$

C. For each pair of similar figures, write a proportion which relates two pair of corresponding sides. Show that each proportion is a true proportion.

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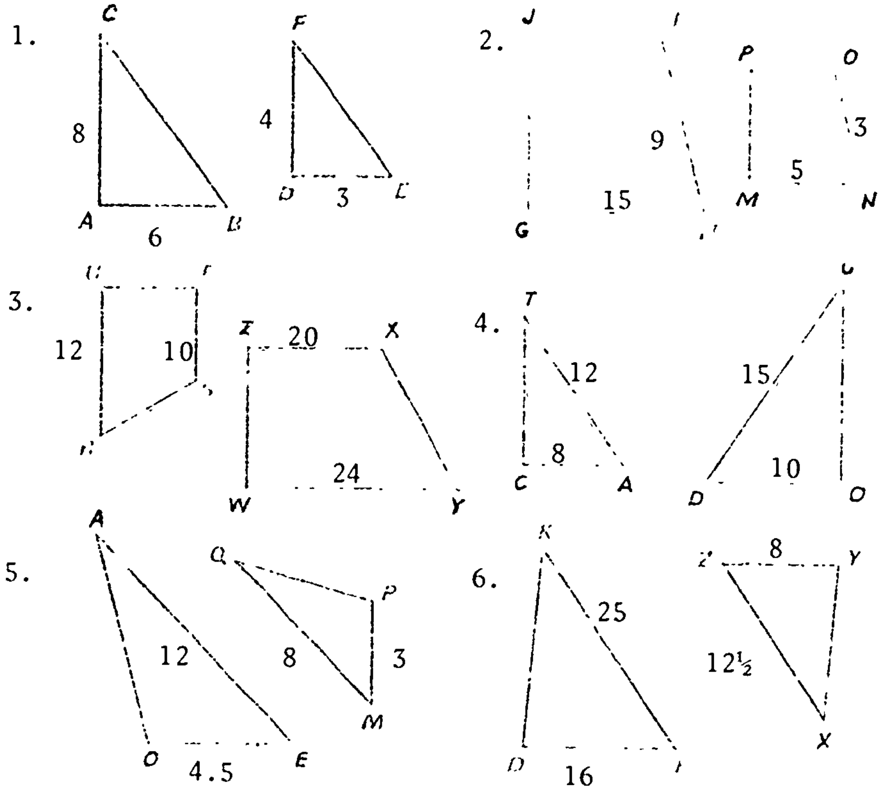
Figure 4



SUMMARY:

Review the meaning of ratio and scale. Review the definitions of similar figures, proportions, and the terms of a proportion. Have students state the two methods used to show whether a proportion is true.

CLIFF-HANGER:

The shadow of a tree is 16 feet long at the same time that the shadow of a boy is 4 feet long. If the boy is 5 feet tall, how tall is the tree?

LESSON 4

AIM: To use proportions to find unknown sides of similar triangles

PERFORMANCE OBJECTIVES: Students will be able to:

- use the proportion property to solve proportions (e.g., $\frac{a}{b} = \frac{x}{c}$).
- write and solve proportions to find an unknown side in similar triangles.

VOCABULARY:

New terms: perpendicular

Review: similar, proportion, means, extremes, division property of equality, indirect measurement

CHALLENGE PROBLEM:

Find the value of x that makes the proportion true.

$$\frac{x}{5} = \frac{16}{4}$$

DEVELOPMENT:

1. Students may obtain a solution to the challenge problem in several ways. Elicit the following :

   a. Recall the cross-product property of true proportions. Name the means (5 and 16) and the extremes (x and 4) and write the products $5 \cdot 16$ and $4x$ and the equation relating these products:

   $$80 = 4x$$

   b. Recall the use of the division property of equality to solve this equation:

   $$\frac{80}{4} \quad \frac{4x}{4}$$

   $$20 = x$$

   c. Check that 20 is the solution.

	By reducing	By proportion
Does	$\frac{20}{5} = \frac{16}{4}$?	$\frac{20}{5} = \frac{16}{4}$?
	$4 = 4$	$5 \cdot 16 = 4 \cdot 20$
		$80 = 80$

2. Do Application A.

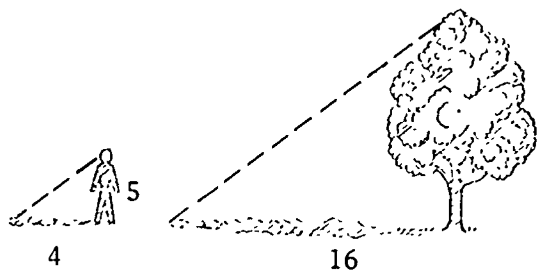
3. Discuss the cliff-hanger problem.

   a. Guide the pupils to realize that on a sunny day a tree, as well as a building, people, and all kinds of objects in the street, cast shadows. Since these shadows are on the ground, it is possible to measure them quite easily with the usual instruments for measuring length (rulers, tapes, etc.). The lengths of shadows change as the sun rises and sinks in the

sky, so comparative measures of shadows must always be made at the same time of day.

b. Have students draw a picture representing the situation described in the cliff-hanger as in Figure 1. The dotted lines indicate the path of the sun's rays. Since all measurements are being made at the same time of day, the rays strike the ground at the same angle in both objects pictured.

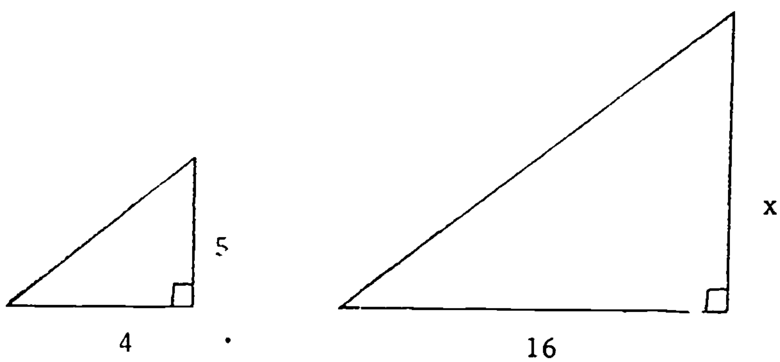
Figure 1



NOTE: Diagrams are not drawn to scale.

c. Elicit that the physical situation can be represented by a geometric model as in Figure 2. The students will note that both triangles are right triangles since the tree and the pupil standing next to the tree make right angles with (are <u>perpendicular</u> to) the ground. Because of the nature of the way the sun's rays strike, students should note that the triangles formed are similar.

Figure 2



NOTE: Diagrams are not drawn to scale.

d. Have students recognize that the cliff-hanger problem actually requires them to find the missing leg in the larger of the two similar triangles. Since the triangles are similar, the corresponding sides are in proportion.

e. If the unknown leg of the triangle is represented by x, have students write a proportion relating the corresponding sides of the triangles.

$$\frac{x}{5} = \frac{16}{4}$$

f. Students should recognize this proportion as the same one solved in the challenge problem. The solution to this equation is   = 20. Therefore, the height of the tree is 20 feet. Elicit that the height of the tree was formed <u>indirectly</u> by solving a proportion resulting from similar triangles.

4. Do Applications B and C.

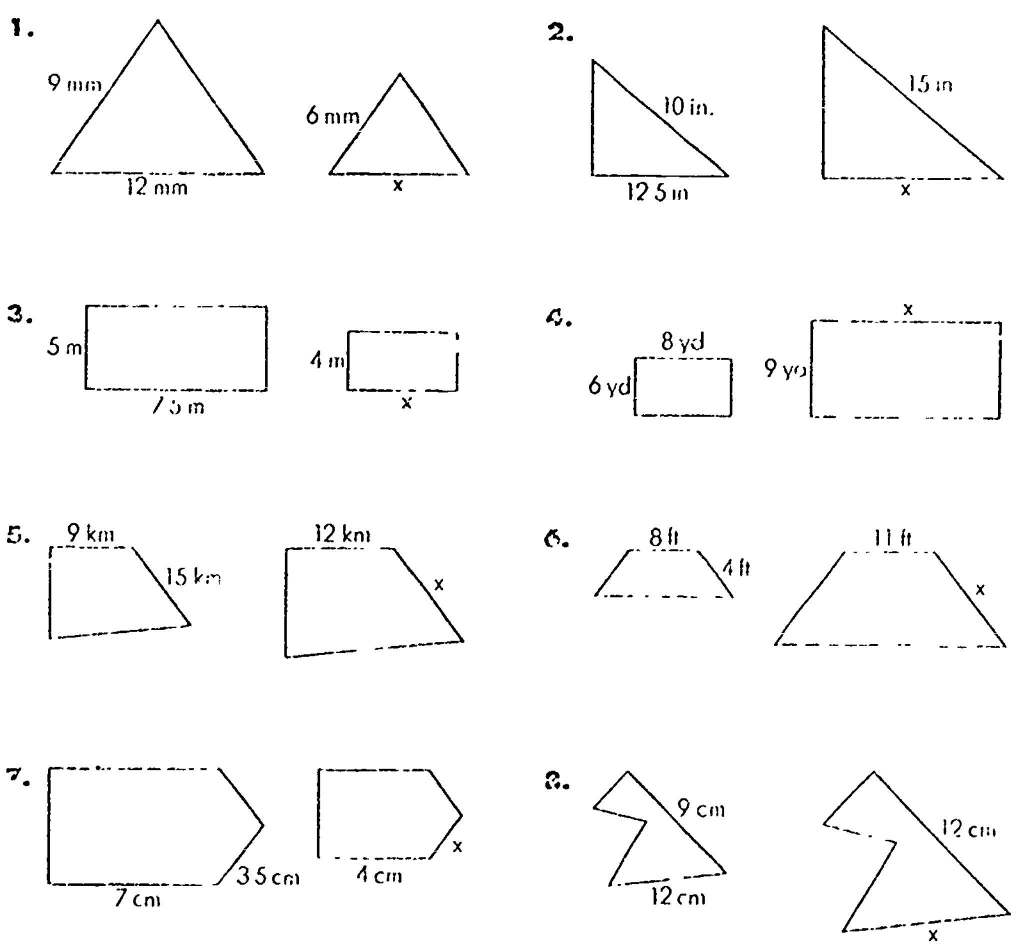
APPLICATIONS:

A. Find the missing term in each of the following proportions.

1. $\frac{8}{16} = \frac{x}{64}$
2. $\frac{14}{x} = \frac{42}{21}$
3. $\frac{x}{36} = \frac{9}{4}$
4. $\frac{9}{36} = \frac{x}{4}$
5. $\frac{\frac{1}{2}}{3} = \frac{x}{6}$
6. $\frac{x}{8} = \frac{2.4}{2}$
7. $\frac{3}{1.4} = \frac{6}{x}$
8. $\frac{.9}{3.6} = \frac{x}{1.5}$

B. Each of the following pairs of figures are similar. Find the indicated side.

Figure 3



C. 1. A tree casts a shadow 120 feet long at the same time as a man standing next to the tree casts a shadow 8 feet long. If the man is 6 feet tall, what is the height of the tree?

2. An apartment building casts a shadow 75 feet long at the same time as a telephone pole next to it casts a shadow 15 feet long. If the pole is 20 feet high, what is the height of the building?

SUMMARY:

Review the principles upon which indirect measurement using shadow is based: the triangles created by the shadows are similar to each other and the corresponding sides of similar triangles are in proportion.

CLIFF-HANGER:

On a typing test Alicia typed 80 words in 4 minutes. At the same rate, how many words would she type in 10 minutes?

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LESSON 5

AIM: To use proportions to solve problems involving ratios and rates

PERFORMANCE OBJECTIVES: Students will be able to:

- represent relationships in verbal problems as proportions involving rates or ratios.
- solve proportions using the proportion property.

VOCABULARY:

Review: rate, ratio, proportion

CHALLENGE PROBLEM:

In a school the ratio of the number of girls to the number of boys is 2 to 3. If 630 boys attend the school, what is the number of girls attending the school?

DEVELOPMENT:

1. Elicit that many situations besides similar figures lead to proportions. Since this problem involves ratio, its solution can also be found by solving a proportion. Translate the given information as follows:

$$\frac{\text{number of girls}}{\text{number of boys}} = \frac{2}{3}$$

Using the additional information given in the problem, obtain the following :

$$\frac{\text{number of girls}}{630} = \frac{2}{3}$$

Use a variable to represent the number of girls to simplify the expression:

$$\frac{g}{630} = \frac{2}{3}$$

Solve the proportion using the proportion property.

$$3g = 2 \times 630$$

$$\frac{3g}{3} = \frac{1260}{3}$$

$$g = 420$$

Therefore, the number of girls attending this school is 420

2. Discuss the cliff-hanger problem. What ratio is implied by the statement "Alicia typed 80 words in 4 minutes"? (words per minute or $\frac{\text{words}}{\text{minutes}}$). Guide students to write the following proportion:

$$\frac{\text{number of words}}{\text{number of minutes}} = \frac{80}{4}$$

Using the additional information from the problem and "w" to represent the number of words, obtain the proportion:

$$\frac{W}{10} = \frac{80}{4}$$

obtain the result w = 200. Therefore, Alicia can type 200 words in 10 minutes.

3. Pose the problem: If 10 oranges cost $1.29, what is the cost of 8 oranges at the same rate? Elicit the following:

$$\frac{\text{number of oranges}}{\text{cost}} = \frac{10}{1.29}$$

Using the additional information and "c" to represent the cost, obtain the proportion:

$$\frac{8}{c} = \frac{10}{1.29}$$

Obtain the result c = 1.032 round to $1.04. Therefore, the cost of 8 oranges is $1.04.

4. Do applications.

APPLICATION:

(Note to the teacher: Have students solve each problem using the same procedure outlined in the development.)

1. A car travels 165 miles in 3 hours. At the same rate, how long does the car take to travel 275 miles?

2. Two candidates in a school election split the vote in a 3:2 ratio. The loser received 182 votes. How many votes did the winner receive? (Note: 3:2 is the same as $\frac{3}{2}$.)

3. A picture 11 in. x 14 in. is to be reduced to a wallet size picture whose longer side is $3\frac{1}{2}$ inches. Find the shorter side of the wallet size picture.

4. A recipe to make 4 dozen cookies requires $\frac{1}{2}$ cup of butter. How much butter is needed to make 10 dozen cookies?

5. If three cans of soup cost 35 cents, what is the cost of two dozen cans at the same rate?

6. In East High School, the ratio of girls to boys in a class is 4:5. If there are 20 boys in this class, how many students are in this class? (Hint: How many girls are in the class?)

SUMMARY:

Review the steps needed to solve a rate or ratio problem using proportions.

CLIFF-HANGER:

On an airline map, the scale is $\frac{1}{4}$ "= 100 miles. If the distance from Los Angeles to Denver measures $2\frac{1}{8}$", what is the distance in miles between these cities?

LESSON 6

AIM: To use proportions to solve problems involving scaling

PERFORMANCE OBJECTIVES: Students will be able to:

- represent the relationships in problems by using proportions.
- solve problems involving scaling by using proportions.

VOCABULARY:

Review: scale, scale drawing

CHALLENGE PROBLEM:

In drawing plans an architect uses the scale $\frac{1}{8}$ = 1 foot. How long a line must he draw to represent a 40 foot wall?

DEVELOPMENT:

1. Use the challenge problem to elicit that a scale drawing is similar in shape to the object it represents. The scale or legend establishes the ratio between the scale length and the actual length. Therefore, the following proportion may be used to solve the challenge problem:

$$\frac{\frac{1}{8}}{1 \text{ foot}} = \frac{\text{scale length}}{\text{actual length}}$$

2. Using the remaining information, the challenge problem can be solved as follows: (use s to represent scale length)

$$\frac{\frac{1}{8}}{1} = \frac{s}{40}$$

$$1s = \frac{1}{8} \times \frac{40}{1}$$

$$s = 5$$

Therefore a 5 in. line must be drawn to represent a 40 ft wall.

3. Recall the cliff-hanger and solve it in the same manner as the challenge problem:

$$\frac{\frac{1}{4} \text{ in.}}{100 \text{ mi}} = \frac{\text{scale length}}{\text{actual distance}}$$

Using d to represent actual distance:

1. $\frac{\frac{1}{4}}{100} = \frac{2\frac{1}{8}}{d}$

2. $\frac{1}{4}d = 2\frac{1}{8} \times 100$

3. $\frac{4}{1} \times \frac{1}{4}d = \frac{17}{\cancel{8}_{\cancel{2}\,1}} \times \frac{\cancel{100}^{25}}{1} \times \frac{\cancel{4}^{2}}{1}$

4. $d = 850$

Therefore the distance between Los Angeles and Denver is 850 miles.

4. Do Applications A and B.

5. Pose the problem: Jeff has to make a scale drawing on standard construction paper, 8½" x 11", of his 30-foot long by 25-foot wide classroom. How can he obtain an appropriate scale for this project?

   a. Recall that a scale is the ratio of the scale length to the actual length.

   b. Write a ratio so that the length of the scale drawing will just fit the length of the paper $\frac{11 \text{ in.}}{30 \text{ ft}}$. Elicit from pupils that such a scale would be impractical because it does not allow for drawing the lines and also because $\frac{11}{30}$ is an inconvenient fraction. Elicit that $\frac{10 \text{ in.}}{30 \text{ ft}}$ would be a better choice (this is not the only choice) because the scale could be reduced to $\frac{1 \text{ in.}}{3 \text{ ft}}$ or 1 in. = 3 ft.

   c. Using this scale determine the scale width of the room. Will the entire drawing fit on the paper?

   $\frac{1}{3} = \frac{x}{25}$

   $3x = 25$

   $x = 8\frac{1}{3}$

   The scale width is $8\frac{1}{3}$ in. which is less than the given $8\frac{1}{2}$ in., therefore the picture will fit.

6. Do Application C.

APPLICATIONS:

A. 1. If a distance of 80 miles is represented on a map by 2½ inches, how many miles are represented by 2 inches?

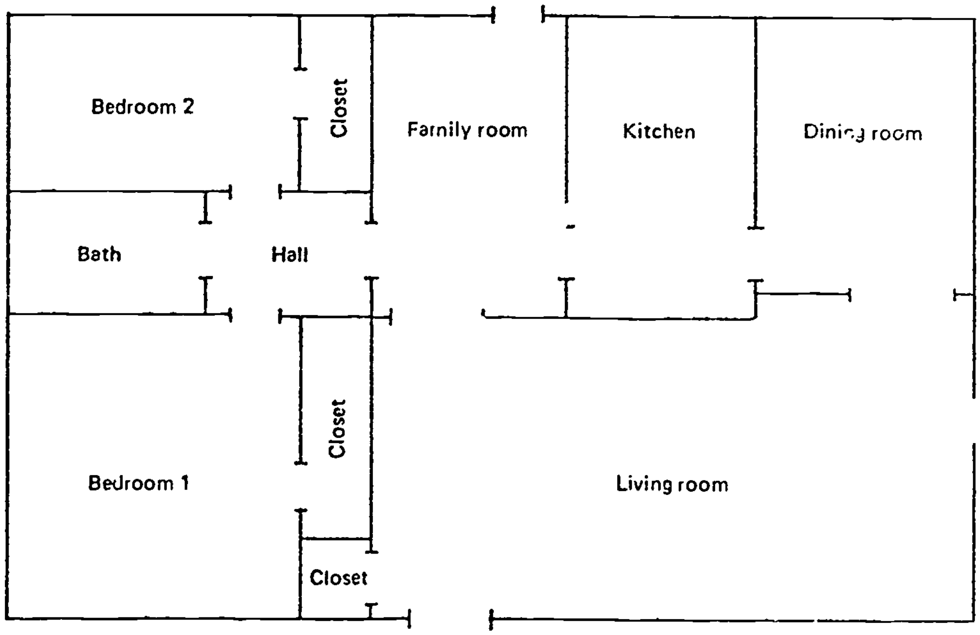
   2. If on a blueprint of a house a $\frac{3}{8}$ inches long represents 6 feet, what is the length of a line which represents a distance of 32 feet?

   3. If a scale is ¼ in. = 2 ft find the area of the floor of a room whose dimensions are 2½ in. by 2¼ in.

   4. In a diagram of a computer component, a square silicon chip is 8 cm wide. If the diagram is 10 times actual size, how wide is the chip?

B. In the given floor plan the scale is $\frac{1}{8}$ in. = 1 ft. (See figure 1)

Figure 1



1. Find the actual dimensions of the dining room.
2. What is the number of feet across the front of the house?
3. How many square feet of closet space does the house contain?
4. What is the cost of tiling the hall with 1-foot square tiles which cost \$3.50 per tile?

C. 1. If 80 miles is represented by $2\frac{1}{2}$ in., how many miles are represented by 1 in.?

2. Find the scale if the actual length is 350 miles and the scale length is 7 in.

SUMMARY:

How can proportions be used to measure indirectly?

UNIT XII. Solid Geometry

LESSON 1

Note to Teacher: All lessons in this unit will be more meaningful if models of the solids as well as common objects having these forms are brought to class.

AIM: To introduce solid geometric figures and the concept of volume

PERFORMANCE OBJECTIVES: Students will be able to:

- identify plane and solid objects as rectangular, square, circular, and triangular.
- distinguish between drawings or objects which are plane or solid figures.
- state that a prism is named by the shape of its base.
- state that the space displaced by a solid object is different from length or area, and requires a cubic unit of measure.
- use cubes (sugar, plastic, etc.) to measure volume.

VOCABULARY:

New terms: plane, solid, prism, cube, volume, displacement, space, face, edge

Review: rectangle, square, circle, parallelogram, parallel, dimension, triangle, polygon

CHALLENGE PROBLEM:

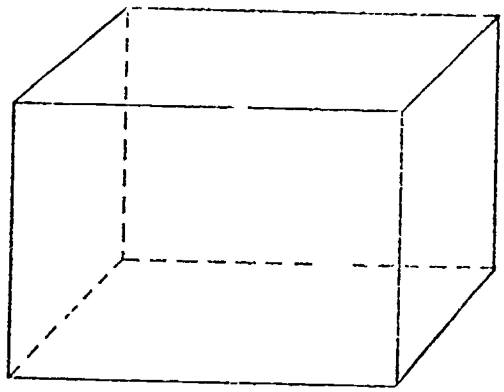
What is the difference between a map of the world and a globe?

DEVELOPMENT:

1. Use the challenge problem to discuss the difference between flat or <u>plane</u> figures and <u>solid</u> figures. Have students recognize that solid figures are all around us. Although objects are solid figures, their faces maybe familiar plane figures. Have students list the names of plane figures they have studied such as squares, rectangles, triangles, circles. Elicit that plane figures have only 2 dimensions but solid figures have 3 dimensions. Plane figures have surface area but solid figures occupy <u>space</u>.

2. Have students list examples from their experience of common solid objects such as boxes, cans, balls, books, candles, buildings, dice, etc. One of the most common shapes is the <u>rectangular solid</u>. Which of the objects listed are in the shape of a <u>rectangular solid</u> (boxes, books, buildings)? Elicit the following description of a rectangular solid: 6 rectangular surfaces called <u>faces</u>, each side of a rectangle is an <u>edge</u> of the solid, the opposite faces are parallel and have the same size and shape. (See Figure 1 on next page.)

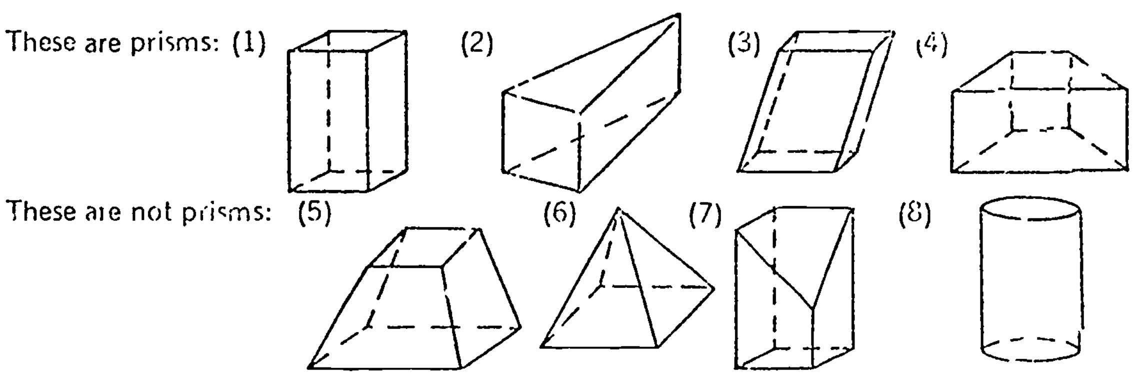
Figure 1



Have students make a picture of a rectangular solid and label the parts described. (Note: Dashed lines indicate edges that cannot be seen.) Stress that this is a 2-dimensional representation of a 3-dimensional object.

3. A rectangular solid is one type of prism. A prism is a solid figure with faces that are polygons. One pair of these faces must be parallel and have the same size and shape. These are called the bases. The other faces must be parallelograms. (See Figure 2)

Figure 2



Use Figure 2 to identify (1), (2), (3), and (4) as prisms and (5), (6), (7), and (8) as solids which are not prisms. When the bases of a prism are rectangles, the solid is called a rectangular solid or rectangular prism (1). When the bases of a prism are triangles the solid is called a triangular prism (2). Recall that

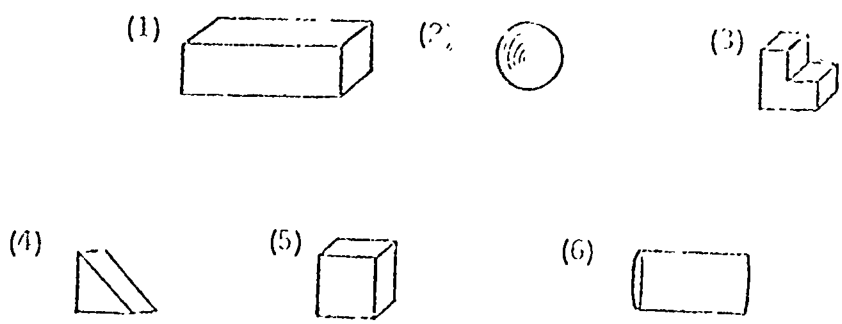
a triangular prism is used to break light into its component colors (the sprectrum). Elicit the reasons that (5), (6), (7), and (8) are not prisms. (For example in (7) the bases are not the same size and the other faces are not parallelograms; in (8) the bases are not polygons.)

4. Recall that a plane figure has a surface area, but a solid figure occupies space. How is the amount of space occupied by a solid figure determined?

   a. Review that to measure surface area, a unit which had a surface was chosen. This was called a square unit. Recall the reasons for this choice. To find the area was to determine the number of square units in a surface.

   b. Elicit that to measure space or <u>volume</u>, a unit must be chosen which has a volume. Which of the objects shown in Figure 3 could be used as a standard unit of volume? Why are the others unsuitable?

Figure 3

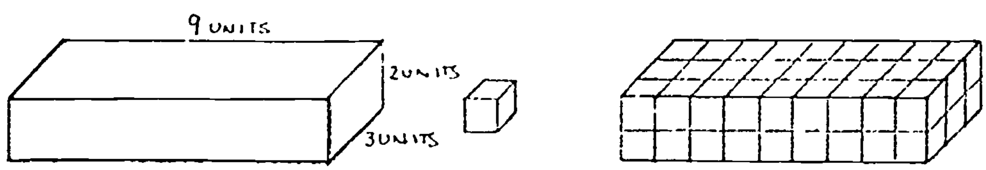


   Name (5) as the best choice. Elicit that it is a special <u>rectangular prism</u> called a cube. Describe what is special about a cube. (Each face is a square and all edges are equal.

   c. Elicit that to find the volume of an object means to determine the number of cubes (cubic units, cu in., cu ft, m^3) contained in the object.

   d. Find the volume of the rectangular prism pictured in Figure 4, or use a model of a rectangular prism and fill it with unit cubes (if available).

Figure 4

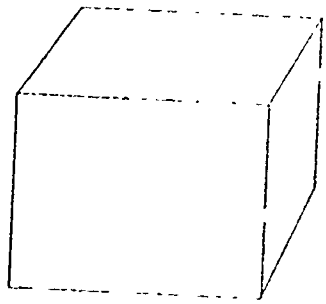


5. Do applications.

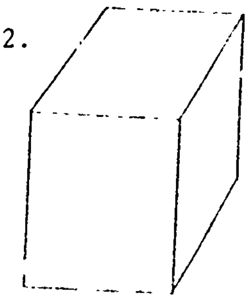
APPLICATIONS:

A. Draw dashed lines in each of the solids below to show the edges which cannot be seen.

1.

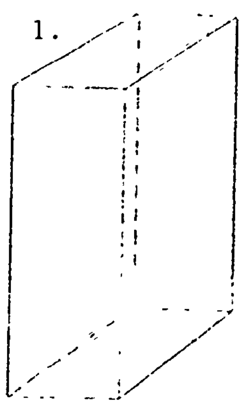


2.

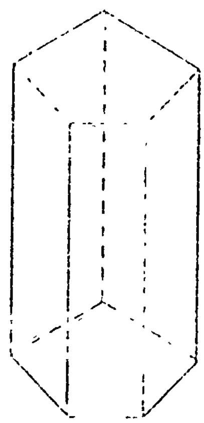


B. For each given prism name the shape of its base.

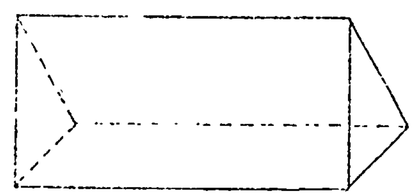
1.



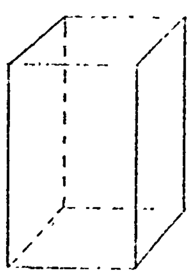
2.



3.



4.



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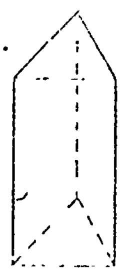

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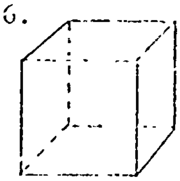

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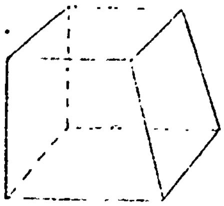
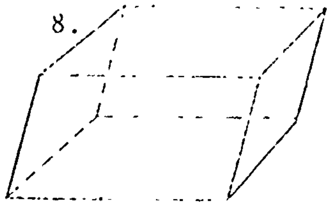
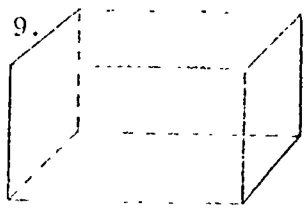


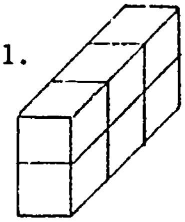
8.

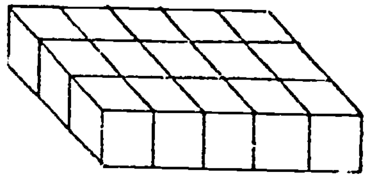


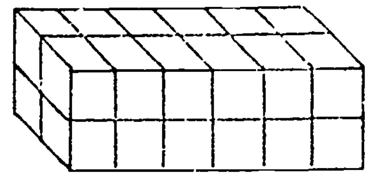
9.

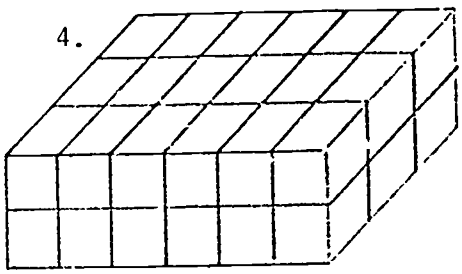


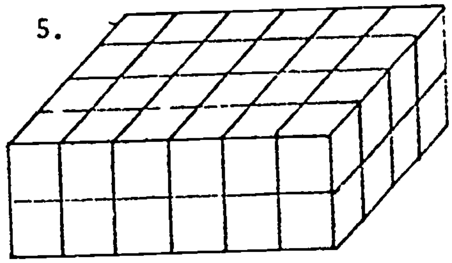
C. Find the volume of each figure by counting the number of cubic units.

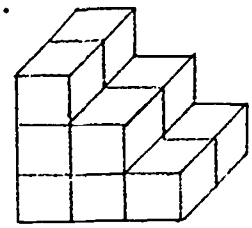
1. 

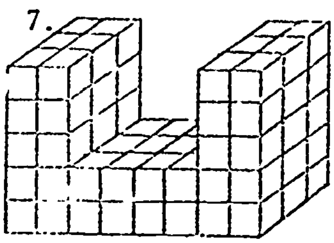
2. 

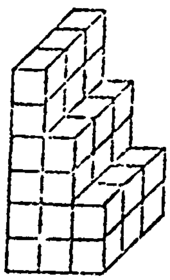
3. 

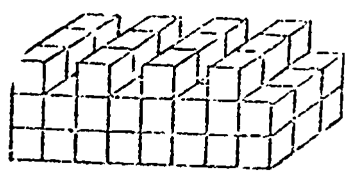
4. 

5. 

6. 

7. 

8. 

9. 

SUMMARY:

1. How many faces does a rectangular solid have? How many edges?
2. A rectangular solid in which each face is a square is called a ________________.
3. What does volume measure?

CLIFF-HANGER:

What is the volume of a rectangular prism whose dimensions are: length, $4\frac{1}{2}$ in.; width, $3\frac{1}{2}$ in.; height, 2 in.?

LESSON 2

AIM: To find volumes of rectangular solids

PERFORMANCE OBJECTIVES: Students will be able to:

- Use the formula v = lwh to calculate the volume of a rectangular prism, where l, w, and h are expressed as whole numbers, decimals and fractions.
- find a dimension of a rectangular prism if its volume and 2 dimensions are given.
- apply the formula v = lwh to the solution of verbal problems.

VOCABULARY:

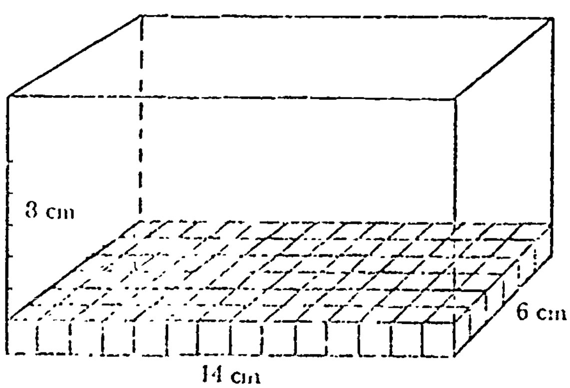
Review: length, width, height, rectangular prism, volume

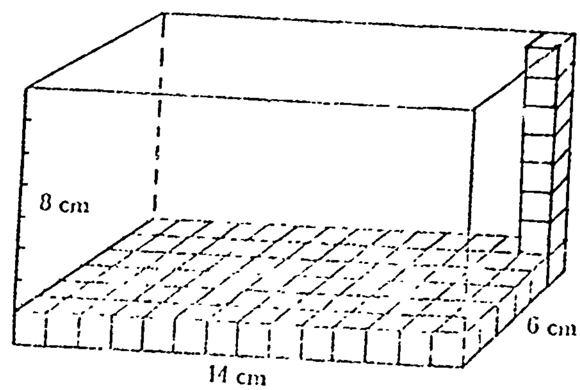
CHALLENGE PROBLEM:

Find the volume of a rectangular prism whose length equals 14 cm, width equals 6 cm and height equals 8 cm.

DEVELOPMENT:

1. Discuss the challenge problem. Recall the procedure used to find the volume of a rectangular prism (by counting the number of cubes contained in the prism).





a. Demonstrate that one method of filling the solid with cubes is to first fill the entire bottom with a single layer of cm cubes. This bottom layer is called the base layer. How can the number of cubes in this base layer be found? Elicit that the product of the number of rows and the number of cm cubes in each row gives the number of cubes in the base layer. Elicit that the number of cubes in the base layer can be found by obtaining the product of the length and width of the base layer. Therefore, the number of cubes in the base layer equals l x w.

b. How many such layers will be contained in this rectangular

prism? Elicit that the number of layers is equal to the height of the rectangular prism. Therefore, the number of layers = h.

c. Elicit that the total number of cm cubes contained in this rectangular solid can be found by multiplying the number of cubes in the base layer of the number of layers. Therefore the total number of cubes in the rectangular prism(volume) is equal to its length times its width times its height:

$$\text{or} \quad V = lwh$$

2. Use the formula V = lwh to solve the challenge problem:

$$V = 14 \times 6 \times 8$$

$$V = 672$$

The volume is 672 cubic centimeters or 672 cm^3.

3. Similarly, the solution to the cliff-hanger problem can be found by using:

$$V = lwh$$

$$V = 4\tfrac{1}{2} \times 3\tfrac{1}{2} \times 2$$

$$V = 31\tfrac{1}{2}$$

The volume is $31\frac{1}{2}$ cubic inches.

4. Do applications.

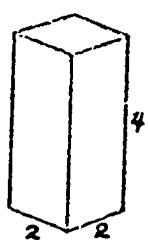
APPLICATIONS:

A. Complete the following table:

	Volume	Length	Width	Height
1.		7 ft	9 ft	8 ft
2.		8 in.	6 in.	$4\frac{1}{2}$ in.
3.		$7\frac{3}{4}$ yd	5 yd	$6\frac{1}{2}$ yd
4.		6.1 m	2.3 m	4.1 m
5.	720 cm^3		10 cm	6 cm
6.	216 cu ft	6 ft		8 ft
7.		5 yd	5 yd	5 yd

B. Find the volume of a cube whose
   1. edge is 5.2 cm.
   2. edge is $\frac{1}{2}$ in.

C. Find the volume for each figure:

1.


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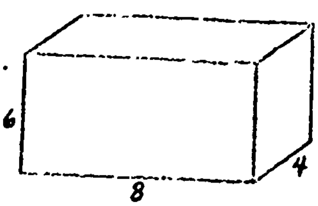

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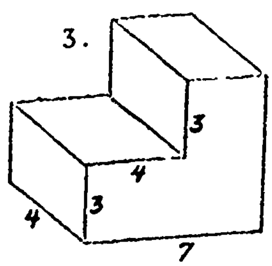


D. Solve each of the following problems:

1. A walk 40 ft long and 5 ft wide is to be paved. It is dug to a depth of 6 in. and filled with crushed rock. (Note: 6 in. = $\frac{1}{2}$ft)

   a. How many cu ft of crushed rock are needed?

   b. Find the cost of the crushed rock at $2.35 per cu ft.

2. An aquarium is 44 in. long, 28 in. wide, and 15 in. high. How many gallons of water will the tank hold? (1 gallon = 231 cu in.)

3. The building code requires 200 cu ft of air per person in a classroom. What is the maximum number of people who can occupy a classroom 27 ft by 25 ft by 12 ft?

4. A liter is 1000 cm^3. How many liters of water will fill a tank 60 cm long, 30 cm wide and 20 cm deep?

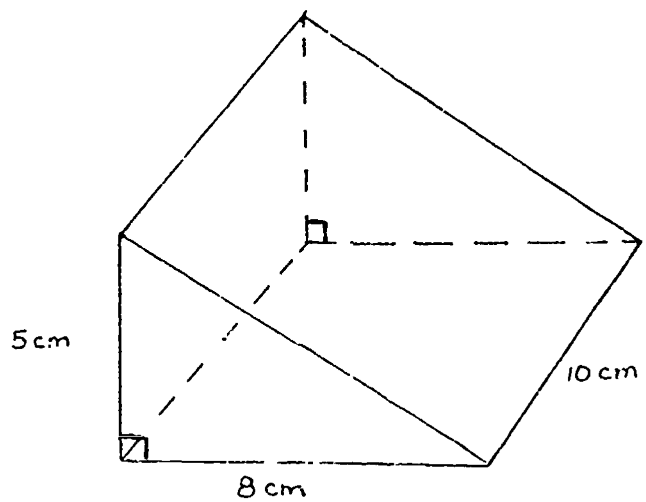
5. $4\frac{1}{2}$ cu ft of sand is poured into a sandbox and leveled to a uniform depth. If the sandbox is 3 ft on each side, what is the depth of the sand?

SUMMARY:

Review the formula for finding the volume of a rectangular prism.

CLIFF-HANGER:

Find the volume of the figure shown.



LESSON 3

AIM: To find the volumes of prisms and cylinders

PERFORMANCE OBJECTIVES: Students will be able to:

- state that $l \times w$ is the area of the base of a rectangular prism, so that its volume is the product of the area of its base and its height.
- calculate the volume of any prism using the relationship that the volume is equal to the area of the base times the height of the prism.
- distinguish between prisms and cylinders.
- find the volume of a (right-circular) cylinder.

VOCABULARY:

New terms: cylinder

Review: prism, volume

CHALLENGE PROBLEM:

Find the volume of a rectangular prism if its base has an area of $84\ cm^2$ and its height is 8 cm.

DEVELOPMENT:

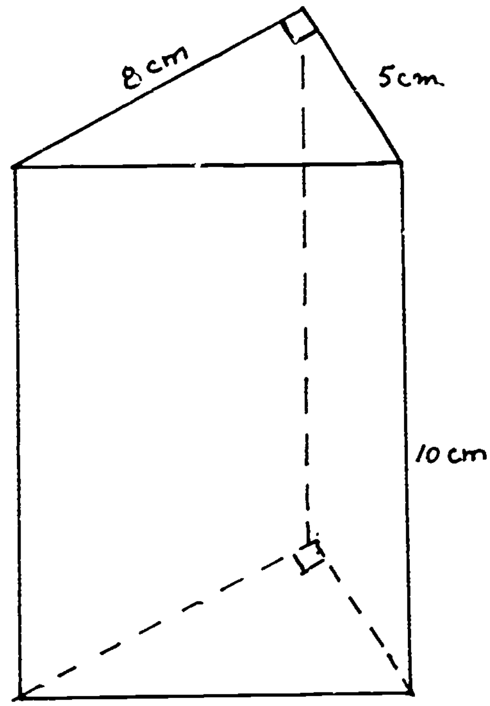
1. Discuss the challenge problem. Elicit that $84\ cm^2$ represents the number of cubes in the base layer (84) (see development in Lesson 2) and the height represents the number of layers (8). Therefore the volume of a rectangular prism can be found by multiplying the number of cubes in each layer (area of base layer) by the number of layers (height). Using B to represent the area of the base (layer) write the formula for volume of a rectangular prism: $V = Bh$ (The volume is $672\ cm^3$.)

2. Elicit that the formula $V = Bh$ is equivalent to the formula $V = lwh$ when the base of a rectangular prism is a rectangle. The area of the base (B) is the length times the width (lw). Substituting B for lw, $V = (lw)h$ becomes $V = Bh$.

   (Note: In the formula $V = Bh$, h is the length of the edge between the bases. The problems presented in this unit will deal only with right-circular cylinders and prisms, i.e., where the edge is perpendicular to the bases.)

3. Recall the cliff-hanger problem. Name the figure (triangular prism). Describe its base (a right triangle).

Figure on next page.



a. Since it has been established that the volume of a solid can be found by multiplying the number of unit cubes in the base layer (B) by the number of layers (h), the volume of this prism may be found by V = Bh. In order to use this formula, it is necessary to calculate B (the area of the base).

b. Using the formula for the area of a right triangle, compute the area of this triangle to be:

$$A = \tfrac{1}{2} \times 8 \times 5$$

$$A = 20 \text{ cm}^2$$

Therefore, the area of the base (B) = 20 cm^2 and the volume of the triangular prism is:

$$V = 20 \times 10$$

$$V = 200 \text{ cm}^3$$

4. Do Applications A and B.

5. Review the definition of prism as a solid whose bases are polygons. What is the name of a solid whose bases are circles? Name some common objects whose shape is a cylinder. Have students suggest a technique which could be used to find the volume of a cylinder (V = Bh). Use this formula to find the volume of a can 3 in. across and 6 in. high. Since the base of the cylinder is a circle, its diameter is 3 in. The area of a circle is found using $A = \pi r^2$. Therefore, since r = 1.5 in.

$$A = 3.14 \times (1.5)^2$$

$$A = 7.065 \text{ sq in.}$$

The area of the base is 7.065 sq in.

The volume of the cylinder is found using V = Bh, therefore

$$V = 7.065 \times 6$$

$$V = 42.39 \text{ cu in.}$$

The volume of the can is 42.39 cu in.

6. Do Application C.

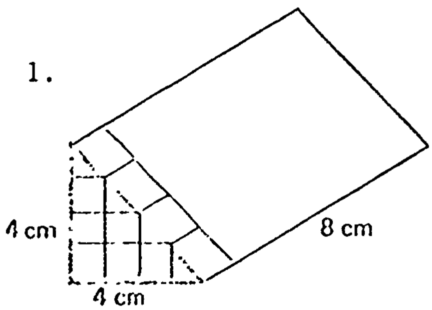
APPLICATIONS:

A. Find the volume of each prism where B is the area of the base and h is the height.

1. B = 9 sq ft, h = $6\frac{1}{3}$ ft
2. B = 14 cm^2, h = 3.5 cm
3. B = 8.6 sq in. h = 1.4 in.
4. B = $3\frac{1}{7}$ sq in. h = $2\frac{1}{3}$ in.

B. Find the volume of each triangular prism.

1.



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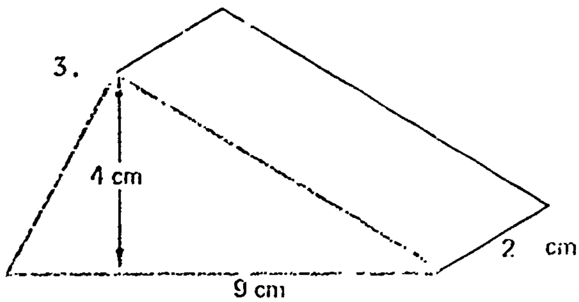

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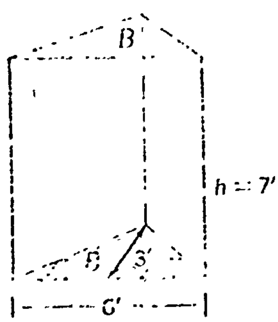

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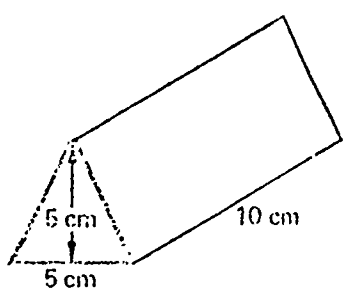
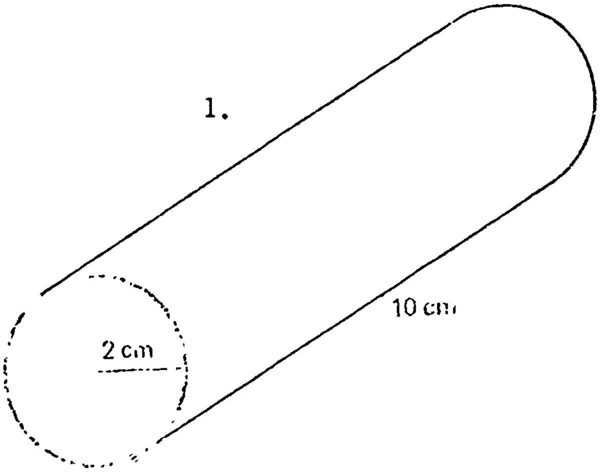
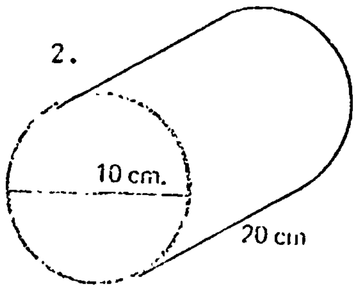


C. Find the volume of each circular cylinder.

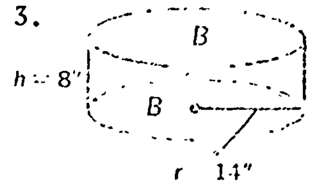
1.



2.

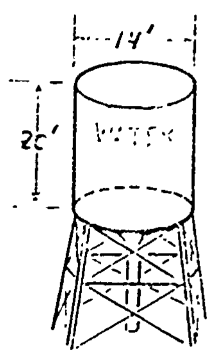


3.



D. 1. Which container holds more, one that is 3 cm in diameter and 4 cm high or one that is 4 cm in diameter and 3 cm high? How many cubic centimers more can be held by the container with the greater volume?

2. The water tower on top of the Hillcrest Apartments is a cylinder 14 feet across and 20 feet high. Find the volume of water in the water tower when it is full. If there are 7.5 gallons of water per cubic foot, find the number of gallons in the tank.



3. A cylindrical eyedropper is .8 cm in diameter and 3.2 cm long. How many milliliters of liquid will the eyedropper hold ($1\ cm^3 = 1$ ml)?

4. A water tank truck is 10 feet in diameter and 42 feet long. It is brought to a park to fill a wading pool 30 feet by 50 feet to a

depth of 2 feet. Will there be enough water? How much more water is needed or how much extra water is left over?

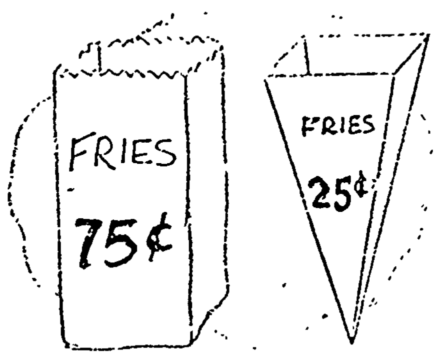
SUMMARY:

Elicit that the formula V = Bh may be used to find the volume of a cube, a rectangular prism, a triangular prism, and a cylinder.

CLIFF-HANGER:

If portions of french fries come in the two sizes indicated (see Figure 5), which do you think is the better buy? Explain your answer.

Figure 5



LESSON 4

AIM: To find the volumes of pyramids and cones

PERFORMANCE OBJECTIVES: Students will be able to:

- distinguish between pyramids and prisms by recognizing that pyramids have triangular sides and prisms have sides that are parallelograms.
- use $V = lwh$, or $V = \frac{1}{3}$ the area of the base x height, to calculate the volume of a pyramid.
- distinguish between cones (circular base) and pyramids (polygon base).
- use $V = \frac{1}{3}$ area of base x height to calculate the volume of a cone.
- identify a given geometric solid as a prism, a cylinder, a pyramid, or a cone.

VOCABULARY:

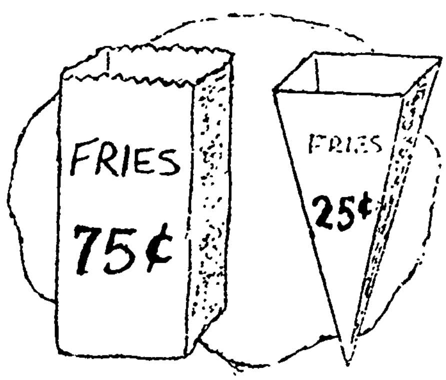
New terms: pyramid, cone

Review: prism, base, height, volume, cylinder, perpendicular

CHALLENGE PROBLEM:

If french fries come in the two serving sizes indicated (see Fig. 1), describe an experiment to show which is the better buy.

Figure 1



DEVELOPMENT:

1. Discuss the challenge problem. Elicit from students that they could count or weigh the fries in each bag to show which is the better buy. This problem is equivalent to finding which container has the greater volume and comparing these volumes. Have students observe that the heights and bases of both containers are the same; yet the volumes differ.

2. Elicit that the shape of the bag of french fries is a rectangular prism. Recall the characteristics of a rectangular prism. Have

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students identify the shape of the other container as a pyramid. Where have they seen this shape before? List the characteristics of a pyramid: only one base, the base is a polygon, the faces are triangles. (Note: A pyramid with a square or rectangular base is the most common, but pyramids may have any polygon as a base.)

3. The following experiment can be used to show the relationship between the volume of a pyramid and that of a rectangular prism having the same base and height. (Models of these shapes are commercially available for this purpose.) Fill the pyramid with sand or water and empty it into the prism. Have students observe the height reached on the first filling. Repeat this procedure until the prism is full. This should take three fillings. Have students state the following in words: The volume of the rectangular prism is 3 times the volume of the pyramid or the volume of a pyramid is one-third the volume of a rectangular prism if both have the same base area and height. Thus in the challenge problem neither is the better buy since the unit price is the same.

4. Use the relationship just established to write the following formula for the volume of a pyramid:

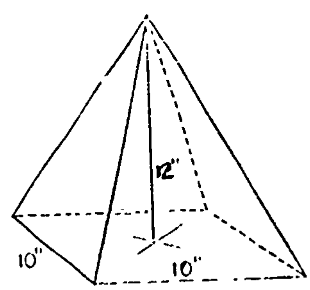
$$V = \frac{1}{3} Bh$$

Students should recognize that <u>B</u> is the area of the base and <u>h</u> is the length of the perpendicular drawn from the highest point to the base.

5. Have students find the volume of the pyramid shown in Figure 2.

$$V = \frac{1}{3} Bh$$

Figure 2



The area of the base is 10 x 10 or 100 sq in.; therefore using the formula:

$$V = \frac{1}{3} Bh$$

$$V = \frac{1}{\cancel{3}_1} \times \frac{100}{1} \times \frac{\cancel{12}^4}{1}$$

$$V = 400 \text{ cu in.}$$

6. Do Application A.

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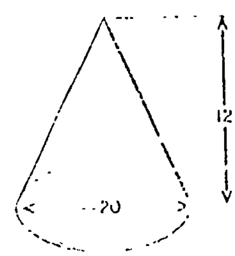
7. Have students imagine the shape of a pyramid if its base were circular. Elicit that this is called a cone. List the characteristics of a cone: only one base, the base is a circle. What are some objects which have the shape of a cone? What solid is closely related to the cone? (Cylinder.) Have students hypothesize the relationship between the volume of a cylinder and the volume of a cone if both have the same base radius (or diameter) and height. Perform an experiment similar to the one described in Item 3 to show that the volume of a cone is one-third the volume of a cylinder if both have the same base radius and height, or $V = \frac{1}{3}Bh$.

8. Have students find the volume of the cone shown in Figure 3.

$$V = \frac{1}{3}Bh$$

Figure 3



The area of the base is $A = \pi r^2$ or

$$A = 3.14 \times 10 \times 10$$

$$A = 314 \text{ sq in.}$$

Therefore using the formula $V = \frac{1}{3}Bh$

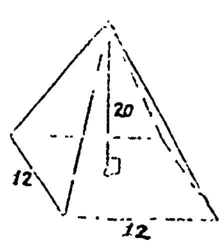
$$V = \frac{1}{\cancel{3}_1} \times \frac{314}{1} \times \frac{\cancel{12}^4}{1}$$

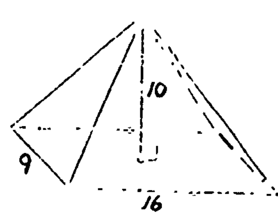
$$V = 1256 \text{ cu in.}$$

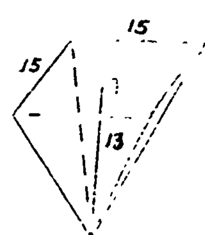
9. Do Applications B and C.

APPLICATIONS:

A. Find the volume of each figure.

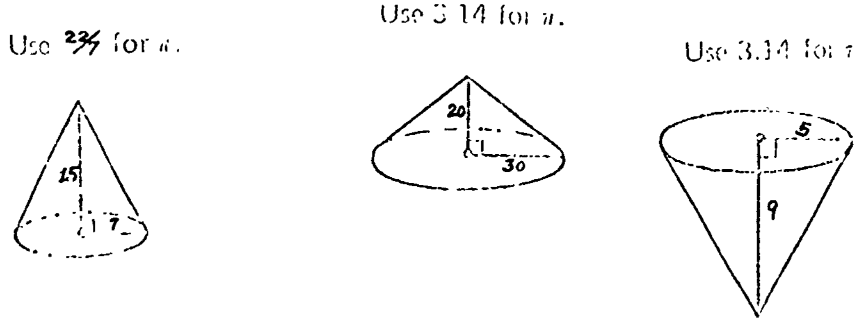






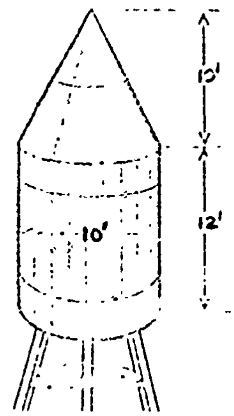
B. Find the volume of each figure. (See Figure 5)

Figure 5



C. 1. A water tower is in the shape of a cylinder surmounted by a cone (see Figure 6).

Figure 6

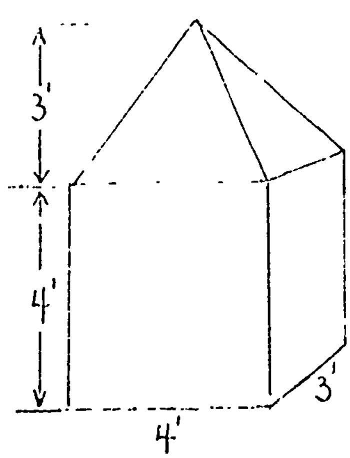


a. Find the capacity of the water tower in cubic feet.

b. If there are 7.5 gallons of water in a cubic foot, find the number of gallons of water in the tank when the tank is 58% full.

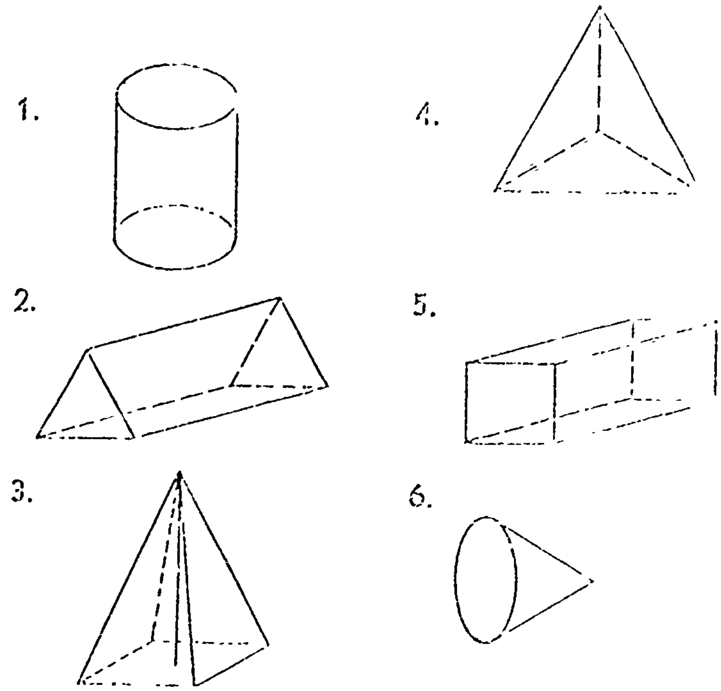
2. A solid stone monument is in the shape of a rectangular prism capped by a pyramid (see Fig. 7). If the stone weighs 173 pounds per cubic foot, find the weight of the monument.

Figure 7



SUMMARY:

Have students identify each solid pictured and write the formula for its volume ($V = Bh$ or $V = \frac{1}{3} Bh$).



CLIFF-HANGER:

A crystal ball is being shipped in a box. If the crystal ball is 10 inches in diameter and just fits inside the box, how many cubic inches of styrofoam are needed to fill the space left inside the box?

LESSON 5

AIM: To find volumes of spheres

PERFORMANCE OBJECTIVES: Students will be able to:

- distinguish the sphere from other solid figures.
- use the formula $V = \frac{4}{3}\pi r^3$ to find volumes of spheres.

VOCABULARY:

New terms: sphere, hemisphere

Review: volume, cylinder, cube

CHALLENGE PROBLEM:

If a crystal ball 10 inches in diameter just fits inside a box, find the volume of the box.

DEVELOPMENT:

1. Elicit from the class that the shape of the box must be a cube because of the properties of the ball. Since the ball is 10 inches wide, the length, width and height of the cube must be 10 inches. Therefore using V = Bh, obtain V = 10 x 10 x 10 or 1000 cu in. for the volume of the cube.

2. Elicit that the geometric name for the ball is a <u>sphere</u>. Have students distinguish the sphere from other solid figures (sphere has no plane surfaces). Name common objects shaped like a sphere.

3. Have students recognize that the volume of a sphere with a diameter = 10 is less than the volume of the cube whose length is 10 and can be found by the following formula: $V = \frac{4}{3}\pi r^3$

4. Use the formula to find the volume of the crystal ball from the challenge problem.

$$V = \frac{4}{3}\pi r^3$$

$$V = \frac{4}{3}(3.14)(5^3)$$

$$V = \frac{4}{3} \times 3.14 \times 5 \times 5 \times 5$$

$$V = \frac{1570.00}{3} = \frac{1570}{3}$$

$$V = 523\frac{1}{3} \text{ cu in.}$$

5. Solve the cliff-hanger problem. Since the volume of the cube is 1000 cu in. and the volume of the ball is $523\frac{1}{3}$ cu in., then the number of cubic inches left for the styrofoam is $1000 - 523\frac{1}{3} = 476\frac{2}{3}$ cu in.

6. Do applications.

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APPLICATIONS:

A. Find the volume of each sphere described.

1. $r = 7$ in. (use $\pi = \frac{22}{7}$)

2. $r = 1.3$ m (use $\pi = 3.14$)

3. $d = 2$ cm (use $\pi = 3.14$)

B. 1. A gas storage tank is in the shape of a sphere. The tank is 30 ft in diameter. How many cubic feet of gas can be stored in this tank?

2. A laser light show is presented in a hemisphere (hemisphere is ½ of a sphere) 42 ft in diameter. If 100 cu ft of space is required for each person occupying the room, what is the maximum occupancy for this room?

3. Tennis balls are vacuum packed in cylindrical cans, 3 balls to the can. If each tennis ball is approximately 2 inches in diameter, how much air must be removed from the can before it is sealed at the factory?

SUMMARY:

1. Name each of the geometric figures represented by the following objects.

2. State the formula used to find the volume of each object.

   a. a base all

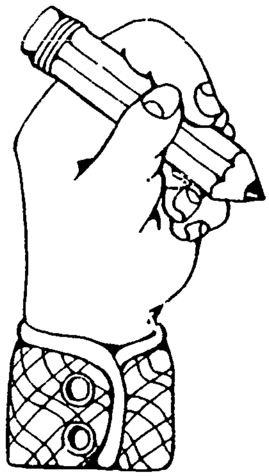
   b. a teepee

   c. a crate of apples

   d. a water pipe

   e. the top of the Washington Monument

Teacher's notes



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