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AUTHOR	Linn, Mary McMahon
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ABSTRACT

The purpose of the project was to determine the effects of journal writing on the thinking skills of high school geometry students. The research supports the idea that writing can enhance a student's metacognitive ability. The results show that the journals served effectively in various capacities. Each student became actively involved in his or her own learning process. Writing forced the students to synthesize information and they became aware of what they did and did not know. They recognized their individual learning style and strengths and began to take advantage of those strengths. The journals served as a diagnostic tool for the instructor and they opened lines of communication between teacher and student and personalized the learning environment. The results of the project suggest that this type of journal keeping would be effective in all disciplines but it is especially recommended that it be implemented throughout a mathematics department. (Author)

EFFECTS OF JOURNAL WRITING ON THINKING SKILLS OF HIGH SCHOOL GEOMETRY STUDENTS

by

Mary McMahon Linn

A project submitted to the Division of Curriculum and Instruction in partial fulfillment of the requirements for the degree of Masters of Education

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Dr. Royal Van Horn, Advisor

Dr. Paul Eggen, Committee

Dr. David Jacobsen, Committee

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The purpose of the project was to determine the effects of journal writing on the thinking skills of high school geometry students. The research supports the idea that writing can enhance a student's metacognitive ability. The results show that the journals served effectively in various capacities. Each student became actively involved in his or her own learning process. Writing forced the students to synthesize information and they became aware of what they did and did not know. They recognized their individual learning style and strengths and began to take advantage of those strengths. The journals served as a diagnostic tool for the instructor and they opened lines of communication between teacher and studen and personalized the learning environment The results of the project suggest that this type of jou keeping would be effective in all disciplines but especially recommended that it be implemented throughout a mathematics department.

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Chapter I: Introduction

Problem Statement

How can journal writing be used to improve the thinking skills of students in three levels of high school geometry classes?

Rationale

Geometry is traditionally the subject taught in high schools to teach students to "think" and to become real "problem solvers." Yet how does the teacher determine if the students have indeed developed their thinking skills or if they have just acquired some knowledge about the topics in geometry?

Although individualized instruction is nearly impossible in the typical high school classroom of thirty students who meet for fifty minutes a day, teachers challenged to teach and develop thinking skills must account for individual learning styles if their students' thinking skills are to develop. Teachers must consider the many various type differences of their students when planning classroom instruction (Gordon, 1984). Process must be taught by modeling the behavior of the effective thinker (Newman, 1986), and the teacher must see to it that each student becomes an active learner.

Proponents of "writing-across-the-curriculum" feel that they have a viable solution to provide individualized instruction to a large class of students through the use of learning logs (Pradl, 1985). Hence, each of three groups of geometry classes (each grouped according to ability) were asked to keep individual journals with the hope that each student would (a) master the skills and knowledge in geometry, (b) recognize his or her individual learning style, and (c) use the journal as a forum of thoughts, ideas, and still unanswered questions.

The journals will be used to (a) guide the students toward recognizing their individual learning styles (based on Myers/Briggs types), (b) adjust classroom instruction based on student types and difficulties with material as identified by the students in their journals, and (c) open a personal line of communication with each student.

While many of the anticipated benefits of the learning logs are attitudinal ones--and therefore too subjective to assess--it is hoped that significant increases in scores on classroom tests and quizzes will be realized by individual students regardless of the class level.

Purpose

The purpose of this project is to determine how journal writing can improve the thinking skills of students in three levels of high school geometry classes.

Chapter II: Review of the Literature

The scene is typical: the classroom teacher talking and writing on the blackboard, the students quietly listening or taking notes. A review of the literature, which addresses thinking skills and journal writing, suggests that in a typical classroom, as likely as not, the teacher is not teaching and the students are not learning.

Thinking Skills

The teacher who is to make the teaching of learning skills effective must "specify the cognitive components" used at each level (Beyer, 1984). In this sense, Bloom's taxonomy, although a useful skeleton outline, must be broken down to determine what cognitive steps a person must take to jump to Bloom's next, more complex level. Bloom's taxonomy, Beyer adds, "does not include problem solving, conceptualizing, or decision making," complex operations "that involve the specific operations listed by Bloom--but employed in different sequences to accomplish different goals" (Beyer, 1984).

That the thinking process is far more complex than Bloom would have one believe can be confirmed by an analysis of the factors that lead to learning. J. Barell, in "You Ask the Wrong Questions," uses a litany of terms from the English classroom to define the thinking process. He points to the mind's creativity, which employs "symbols, metaphors, analogies" that link the world "of particulars" to the abstracts "that give them structure" and, in so doing, creates "meaning out of experience" (Barell, 1985). Barell, and others like him, are less concerned with Bloom's level of questions than with the thinking process that one must develop to answer the questions, regardless of the level. In so many words, they are telling teachers to teach thinking skills as a way to teach content.

Teachers, however, face classes of individuals, each of whom "cogitates differently" (Keirsey & Bates, 1984). Not only do their students reside at different "levels of ignorance" (Barrell, 1985), they have unique ways of perceiving the world, a factor which futher affects how they learn. That individuals perceive differently is the premise of type theory, which explores the relationship between the learner and the way he "experiences instruction" (Dutch, 1984). By helping a student first see that he or she will not always respond to a certain method of instruction, and then by helping that student discover the type of instruction that will most likely produce a response, a teacher acts as a guide to self-awareness. A student who understands individual learning types can "discover [his or her] own natural bent"; a teacher, likewise, who knows student types is tempted to search for different methods of instruction so that he offers each type "a learning setting that [gives each his or her] best opportunity to develop" (Lawrence, 1984). In so doing, the teacher also creates different experiences that tempt students to create new, more appropriate images (Barrell, 1985).

To accompany their awareness of perceptual type, students should also be aware that, regardless of their perceptual differences, successful learners share many similarities, the most notable of which is a "well-developed metacognitive ability" (Costa, 1984). Costa defines "metacognition" as follows:

> Metacognition is our ability to know what we know and what we don't know...our ability to plan a strategy for producing what information is needed, to be conscious of our own steps and strategies during the act of problem solving, and to reflect on and evaluate the productivity of our own thinking. (p. 57)

Research suggests, as well, that metacognitive ability is strengthened when a student sees learning as "active, constructive, cumulative and goal oriented" (Shuell, 1986). The stronger the student, the more likely he or she is to "concentrate initially on identifying the correct problem [he or she] is to solve" (Norris, 1985), an ability lacking in the passive student who sees problems as meaningless ends in themselves. Generally, a student with a well-developed metacognitive ability approaches problems in a positive fashion.

Teachers, then, must be equipped with a variety of ways to teach "course[s] in logic and problem solving" (Joyce, 1985) within the context of their respective disciplines. The goal of these courses should be "to bring students to the point where they are willing and able to use thinking skills independently and effectively in a variety of settings" (Beyer, 1984).

The teacher's first step in developing a course in thinking is to "identify the specific skill [he or she] wish[es] to teach" (Beyer, 1984). This requires an analysis of the skill to determine the thought process that a mastery of the skill requires. Beyer points out that the thinking process for any given skill can range from the open-ended (such as problem solving) to the "more discrete and basic...(such as recall, extrapolation, and synthesis)" to "combinations of the two" (Beyer, 1984). Futhermore, a teacher has identified a specific skill only after defining it and "develop[ing] a common language" to describe it so that teachers in other subject areas and grade levels can apply it (Beyer, 1984).

Once a specific skill has been identified, a teacher then must get students "actively engaged in learning activities" (Shuell, 1986). Although the subject of these activities is the course content, the teacher should stress the process by which the content is mastered as much as the content itself. Keeping in mind the cognitive process the students must use "to learn the content" (Shuell, 1986), the teacher engages a student by asking "How did you get the answer?" rather than "What answer did you get?" By asking these types of questions, the teacher serves as a verbal model of the effective problem solver (Costa, 1984). When demonstrating proofs or problems on the board, teachers must not only give the "play-by-play" (which addresses the problem's answer) but the "color commentary" as well. The latter directs students to the process of discovery that leads to the answer, certainly the more enduring and--hence--worthwhile of the two. Once the student recognizes the process, internalizes it, and uses it without coaxing to solve future problems, he or she has become an active rather than a passive learner.

Journal Writing

Individuals working in small groups or seminars make their individual learning styles become apparent. People verbalize their thought processes more readily and are apt to request clarification through a medium that is more suited to their own strengths or learning styles (Olson, 1984). Teachers who work with small groups can take advantage of this type of learning situation and adjust their teaching to address the students' needs as those needs are verbalized.

Teachers in the typical high school classroom, however, are at a disadvantage because class size generally prohibits personal dialogue between the teacher and individual student. The teacher may use a test as a measurement of knowledge attainment, but the real goal--helping to make students problem solvers and independent thinkers--has no mode of delivery or method of assessment. It is unrealistic to expect the teacher to transform each class into the "little red school house" where the teacher coaches each student through the learning process by working at each student's level of thinking in a mode matched to the learning style of each student. Nevertheless, a teacher needs to focus on the needs of each student in the class.

Proponents of "writing-across-the-curriculum" believe that they have a viable method of individualizing instruction for each student and, at the same time, providing a means for the teacher not only to be aware of but also to cater to individual learning styles. Stock (1985) comments, "When James Britton and other members of a research team coined the slogan 'writing-across-the-curriculum,' their purpose was to remind all teachers at all levels of instruction that language--written and spoken--is the most readily and powerful means of learning" (p. 97). The keeping of journals or learning logs was generated from this notion.

Journal writing serves two purposes: to open commmunication between teacher and student and to promote thinking.

When students keep journals, they focus on the subject matter being studied from their own perspectives, which forces them to construct new material with the use of the knowledge they already have. Reading journal entries gives the teacher insight into how the student thinks and allows him to assess the student's mastery of the material (Stock, 1986). Learning logs also afford the teacher feedback as to how students "perceive the class--which techniques work and which do not" (Shaw, 1983). The journals can provide the teacher with an "educational pulse" that the teacher can feel to determine lesson effectiveness, when to modify an approach (Gordon and Mayher, 1985). Students are likely to include in their journals affect comments as varied in subject matter as the material and method being taught to the degree of approval they give the teacher's dress. They also tend to voice anxieties over the subject materials. These types of comments also provide the teacher with useful insights about the learners.

The true value of writing comes from what it forces the writer to do. Murray (1973) states, "Writing is the most disciplined form of thinking"

(p. 22). Olson (1986) gives futher insight to the connection between writing and thinking:

> Thinking and writing are recusive processes; one often has to go back to go forward. Certain stages in the writing process may simultaneously tap two or more thinking levels. Composing involves all of the skills in the taxonomy regardless of the writing task. (p. 32)

Futhermore, processes that students experience while writing mirror those commonly used by successful thinkers and problem solvers: "Different writing tasks require students to deal with the content in a variety of ways--to define, refine, evaluate, integrate and communicate what they have learned at a variety of levels" (Langer and Applebee, 1985). By varying their questions, teachers can demand different levels of thinking that can range from knowledge to the evaluation level (Ruggles, 1985).

Despite the evidence that journal writing would enhance the mathematics curriculum, students are less likely to be asked to write in the mathematics class than in any other. Instead of conducting writing sessions, the mathematics teacher generally spends class time reviewing homework, presenting new concepts, and explaining new material.

All math and no writing leaves little time for the teacher to communicate with each student about his or her thoughts, fears or attitudes regarding the subject. Even less time is spent assessing the problem solving approach taken by students, even though many of the desired outcomes in mathematics are based directly on the ability of the student to communicate. Willoughby (1985) writes that "a characteristic of an effective program for teaching mathematical problem solving is a lot of direct two-way communication between the teacher and student" (p. 90). In order to participate in the class, students must be able to "receive information" that is communicated both orally and in writing and they should be able to present their ideas as well in both mediums (Willoughby, 1985).

D. Schmidt (1985) points out that "mathematics is, after all, communication, but communication in math involves a compact, unambigious symbolism that to many students is cold and rigid. Writing...is a less structured way of expressing ideas" (p. 110). In Schmidt's mathematics classroom writing is also used as a way of "opening lines of communication" between himself and the students who share their feelings about the subject and give him feedback by asking for more information or by reacting to a particular lesson (Ruggles, 1985).

Nahrgang and Peterson (1986), in "Using Writing To Learn Mathematics," found that journal writing provides students with the time to "work informally and personally on mathematical concepts, using their own language and real world experiences" (p. 461). When students are able to connect their experiences with subjects they are studying, they are more likely to internalize the information so that the content becomes "part of their permanent 'intellectual arsenal'" (Gordon & Mayher, 1985).

Mathematics teachers have an excellent opportunity to diagnose students' thought processess when they use the journal to ask students to explain their understanding of a concept. This also provides the teacher with feedback as to the effectiveness of teaching procedures (Shaw, 1983). The use of journals in the geometry classroom is especially revelant since the "understanding process [which] is composed of consolidating, rephrasing, explaining and predicting steps of a solution" (Suydam, 1985) mirrors those steps that students must take while writing.

Since writing promotes thinking it is an excellent tool to teach geometry which, itself, is taught "primarily to develop logical thinking abilities" (Suyham, 1985).

Chapter III: Design of the Procedures

The purpose of this project was to determine the effect writing would have on thinking skills of students in three levels of high shcool geometry classes.

Subjects

Geometry students at St. Joseph Academy in St. Augustine, Florida, were the participants in the journal writing experiment. St. Joseph is a small Catholic high school in a rural community. Approximately 230 students in grades 9 through 12 attend the school. The teaching staff is small (with only two full-time mathematics teachers) and students are likely to have the same instructor two or more times during their four years at the Academy. St. Joseph offers three geometry classes, divided according to general mathematical ability, which is determined by standardized test scores, demonstration of ability by previous achievement in mathematics and teacher recommendations. Each of the levels--Basic, Standard, and Advanced--uses a different text geared to its ability.

The students in the geometry classes are primarily sophomores (15-16 years old) who comprise classes ranging from 20-27 students. The same instructor teaches all three levels. It is noted (without an evaluation of its significance, if any) that the same instructor taught Algebra I to 41 of the 72 geometry students during the previous year.

Method of Procedures

The journal writing did not take place the entire first semester. At the beginning of the second semester, the instructor gave the students notebooks and told them that they would be asked to write in their journals once or twice a week. The instructor did not go into great detail about the purpose of a writing assignment in a mathematics class, but simply told them that it was hoped that their writing would give them an idea of how they were progressing in geometry and that it would be good practice for them to write what they were thinking as quickly and as smoothly as they could. To encourage an easy flow of writing, the instructor told the students that grammar, spelling and punctuation would not be a factor in the evaluation of their journal writing, that they would receive a quiz grade for each writing assignment, and if they wrote for the entire five minutes alloted, they would receive full credit for the assignment.

On the designated writing days, the teacher would pass out the notebooks to the students, write the journal question or questions on the blackboard or overhead projector, and then set a timer for five minutes. The students would then write in their journals while the teacher wrote in the class notebook. At the sound of the timer, the instructor always told them they could take extra time to finish what they were writing. They then passed the journals to the front of each row where the teacher collected them for grading. (This collection method facilitated the return of the notebooks in a similar fashion.)

When formulating response questions for the students, the teacher relied on Gene Galleli's "Activity Mind-set Guide" (1985), which is based on Bloom's taxonomy. Galleli's goal is to "help students perceive the different types of thinking required for different types of questions" (p. 173).

1. Knowledge--list, recite, identify
2. Comprehension--reword, define, outline, calculate, solve
3. Application--relate the problems to a new situation, operate
4. Analysis--take apart, simplify
5. Synthesis--combine, reorder, formualte
6. Evalauation--appraise, referee, justify, criticize, grade

(Ruggles, 1985)

When reviewing student responses, the instructor looked for clues that revealed individual learning styles and pointed them out to each student. The journals became a diagonstic tool, in that students were able to "voice" questions that still remained over various topics, and the teacher was able to point out students' errors and identify misconceptions. The journals were also a direct line of communication between teacher and student as the latter expressed doubts, concerns, ideas and feelings, goals and aspirations.

Evaluation

Each student journal entry received a quiz grade worth two points. At the end of each quarter the teacher totaled the points and entered them as a quiz score as part of each student's quarter grade. The teacher offered this quiz score as an incentive, especially attractive to the reluctant writer. In order to evaluate the effectiveness of the journal writing, the teacher compared the second semester grades with the first semester tests, quizzes and quarter marks. The teacher expected the comparison to show an overall improvement in scores. The teacher also perceived the students' affective comments as important in the evaluation of the results.

It is expected that comments revealing student insights into their individual strengths and learning styles and an overall improvement of scores would show that the journal writing had improved the thinking skills of the students in all three levels of the high school geometry classes.

CHAPTER IV: EVALUATION OF THE RESULTS

Three aspects of the journal writing project were considered in order to evaluate the effectiveness of the project. These considerations took the form of (a) a comparison of grades received by students before and after they began writing in their journals; (b) excerpts from the journals to show their effectiveness as both a teaching and learning tool; and (c) ar evaluation of the affective comments made through the journals in order to judge whether the students themselves deemed it a worthwhile project.

GRADE COMPARISON

Seventy-two students in three levels of geometry classes began the journal writing project at the beginning of the second semester of the 1986-1987 school year. Four of those students withdrew from the school before completing the project; therefore these results compare the first semester grades with those earned by the same students during the second semester of 68 students divided into geometry classes labeled advanced, standard, and basic. The first semester grades were earned by students who were not writing in journals while the second semester grades were earned by those same students who were engaged in the journal writing project.

SEM. 2 : SEM.1	ADVANCED	STANDARD	BASIC	TOTALS
HIGHER	14	17	9	40
LOWER	4	3	9	12
SAME	9	2	1	16
TOTALS	27	22	19	68

The same results shown in percentages are as follows:

SEM. 2 : SEM. 1	ADVANCED	STANDARD	BASIC	TOTALS
HIGHER	52%	77%	47%	59%
LOWER	15%	14%	47%	23%
SAME	33%	9%	6%	18%

Overall, these scores reflect that more students scored higher the second semester while they were writing in the journals than they did before the project began in the first semester. In individual classrooms the most positive effect seemed to occur with the Standard Geometry students while no difference was recorded for the Basic Geometry students.

It must be noted that many variables must be taken into account when comparing scores earned by students from the first semester of a school year to the second. Some of these variables include individual student's histories of semester comparisons and the increasing difficulty of course material over a year. The trend is often toward lower student grades the second semester. While it is hoped that the journals were a very real factor in the overall improvement of the students' scores, it may be more notable to point out that in no class was the mean of the second semester scores earned by the students lower than the first semester mean.

JOURNAL ENTRIES

A preview of various entries made by students in their journals reflect the various levels of understanding of the material presented throughout the second semester. The added communication served as an excellent tool for the instructor to clarify concepts and misconceptions and to share insightful comments made by students. On any given day, the instructor would pose a question or a series of questions for the students to respond to. While the students wrote in their notebooks, the instructor recorded the question in the class journal and recorded personal responses, reflections and expectations (INSTRUCTOR'S JOURNAL ENTRY). Throughout the semester, but not always on the same day, the different class levels received the same or similar questions, so the responses from the various levels of students to those questions are handeled together. It may be of interest to the reader to know the level at which the question was addressed and also the level of the responding student, so the following notation has been used in parenthesis next to each question or response: (A) Advanced, (S) Standard and (B) for the Basic Geometry student. Sample questions and various verbatem examples of student responses are presented here. Occasionally the INSTRUCTOR'S PERSONAL ENTRY is presented as well. During the journal writing project the instructor responded to the students' journal entries by writing answers, comments or questions in their journals, adjusting classroom instruction or addressing individual student needs personally. The INSTRUCTOR'S NOTES reflect the various follow-up steps taken by the instructor. These notes follow the STUDENT RESPONSES.

(A,S) QUESTION: What is wrong with: $\sqrt{81} = 9 = 3$? Why do you think so many people make this mistake?

STUDENT RESPONSES:

(S) Student A: In this problem you reduced 9 and weren't supposed to. 9 is the answer to $\sqrt{81}$ not 3. I think so many people make this mistake because 9 is also a perfect square and people feel the need to simplify it.

(S) Student B: ...because they are so used to looking for numbers with perfect squares when they see one they automatically want to write the perfect square down.

(S) Student C: People probably make this mistake because they really don't think about what the problem really wants.

(S) Student D: ...they are not concentrating on what they are doing or else they need more teaching.

(S) Student E: They think they are dividing.

(S) Student F: ...by going too fast.

(A) Student G: ...they are rushing or just plain careless.

(A) Student H: ...they don't look and think about what they are thinking, they calculate too fast and ahead of problem procedure too much.

(A) Student I: ...9 is the square root of 81 and can't be simplified but many people haven't had good teachers and don't know this stuff.

(S) Student J: It's wrong because 9 can go further. People make this mistake because after you've done the basic part of the problem you don't think of the easy parts like reducing.

(A) Student K: The 9 should be in a square root bar...because many people don't think of 9 as being a perfect square.

(A) Student L: They don't ask for the square root of 9 which is 3 but if you wanted to say that you would need to put a radical sign over the 9. You make this mistake because you're so used to everything being complicated that when there is an easy problem you don't know what to do.

(S) Student M: ...it should be: $\sqrt{81} = (\sqrt{9})(\sqrt{3}) = 3\sqrt{3}$.

INSTRUCTOR'S NOTES:

There is a need to clarify terminology that is used "Fractions are reduced, radicals are simplified." Students J and K seemed to miss the point of the question or are likely to duplicate this mistake themselves. Ask them to simplify $\sqrt{16}$ to make sure they don't write: $\sqrt{16} = 4 = 2$. Student L needs a confidence boost (again). The students have given many insightful answers as to why the mistake is commonly made but have provided no fool-proof method of preventing other students from making the mistake. Perhaps just pointing out the common error clarified the point for most students. Student H points out that "looking" is a big part of solving problems. The process used by student M is correct except that nine times three equals twenty-seven--not eighty-one. Further dialogue is needed with this student.

(A,S) QUESTION: Simplify $\sqrt{16/3}$ and explain each step.

STUDENT RESPONSES:

(S) Student A: $\sqrt{16/3} = \frac{\sqrt{16}}{\cancel{\sqrt{3}}} \frac{\sqrt{4}}{\cancel{\sqrt{3}}} = \frac{2}{1} = 2$

Set up the square root of 16 over the square root of 3 then find the square root of 16 which is 4. Bring the square root of 3 under the 4. Find the square root of 4.

(A) Student B: $\sqrt{16/3} = \frac{\sqrt{16}}{\sqrt{3}} = \frac{4}{\sqrt{3}}$

Multiply the radical sign by both numbers in your fraction. Reduce any perfect squares and leave the other square roots the way they are.

(S) Student C: $\sqrt{16/3}$ First you have to put the numerator and denominator into two parts. $\frac{\sqrt{16}}{\sqrt{3}}$ Then you should see if either part is an even square root. $\frac{4}{\sqrt{3}}$ Since you can't have a radical in the denominator you must multiply both sides by the denominator $\frac{4\sqrt{3}}{3}$.

(S) Student D: a. $\sqrt{16/3}$ b. $\frac{\sqrt{16}}{\sqrt{3}}$ c. $\frac{4}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$ d. $\frac{4\sqrt{3}}{3}$

a. Rewrite the problem.

b. Separate the radical so you can work with them individually.

c. Simplify any that are perfect squares such as $\sqrt{16} = 4$ and then multiply it by one--in this case $\frac{\sqrt{3}}{\sqrt{3}}$ which = 1.

d. Then multiply it out to get your answer.

(A) Student E: $\sqrt{16/3} = \frac{\sqrt{4}}{\sqrt{3}} = \frac{2}{3}$

First you would find the prime number 4, then you would reduce 16, and then find the prime for 4, which is 2. 3 doesn't have a prime so it's carried along. So your answer would be 2/3....I think I need to slow down some and try to get back my confidence with math.

INSTRUCTOR'S NOTES:

Student A seems to have generalized the rule "You can't have a radical sign in the demonimator so rationalize by multiplying by a fraction equal to one" into "You can't have a radical sign in the denominator so move the radical sign up and make the denominator one." Without the additional explanation given by the student, it would seem that the student duplicated the mistake as outlined in the previous question--that is, $\sqrt{16} = 4 = 2$. This revelation points out an entirely new error pattern to look for. Remind student B that it's necessary to rationalize the denominator. Students C and D both solved the problem correctly but student D really communicated complete understanding of rationalizing by multiplying by a fraction equivalent to one. The first clue that student E is having difficulty with this problem is reflected in her inability to use any correct terminology. This student needs individual help. Many students said things such as "Reduce the square root of 16 to 4", when they should have said "Simplify the square." Students also called perfect squares "even square roots." In both cases the correct terminology should be emphasized.

(A,S) QUESTION: Simplifying radicals seems like a backwards process (in my mind). Does it to you? If so, Why?

STUDENT RESPONSES:

(S) Student A: Simplifying radicals is awkward, it is like here is the answer now find the problem.

(A) Student B: ...it seems like the game show "Jeopardy" you get the answer and have to say the question.

(A) Student C: ...well sometimes, but most of the time I treat it as the bacteria in the food chain so simplify radical as bacteria and radical as the dead organism and simplification as decay procedures.

INSTRUCTOR'S NOTES:

Students A and B have reinforced the idea that radicals are difficult for some people because of the "working backward" feeling. Student C suggests an exceptionally unique analogy which may be used as a useful teaching model.

(B) QUESTION: Explain the difference between area and perimeter. When do we use them in our everyday lives?

(Note: In this case it may be helpful to read the instructor's personal entry first.)

INSTRUCTOR'S JOURNAL ENTRY: When teaching the concepts of area and perimeter, I show the students a square with side length 4 inches. When we calculate the area, our answer is 16, when we calculate the perimeter we again get 16 for our answer. I then ask the students "Does this mean that the area and the perimeter are the exact same for this square?" Various students nodd their heads, "Yes, that is so." I then go through the process of putting a 16 inch string agound the square "This 16 inch string represents the perimeter." I go on to put 16 square inches on the square to cover it, "These 16 squares represent the area of the square." I then restate the question: "When we calculated for area and perimeter of this square, we got 16 for both answers- are they the same?" Now that the students have seen the string and squares they appear to understand the conceptual difference between the idea of perimeter and that of area, but I will make sure by asking them to write about it in their journals.

STUDENT RESPONSES:

(B) Student A: Perimeter is the edge of the area around the circumference of a figure.

(B) Student B: Area is the amount covered and perimeter is the line around it.

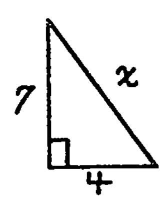
(B) Various student responses: ...we use area when we put our books in our lockers...buying wallpaper for a wall and using area to do so all you would have to do is measure the length and width of your wall and subtract.

...Perimeter is used when you paint a house, you need to know how much paint to buy...it is used to put a new cover on your couch or wrap Christmas presents...when you buy clothes you need to know your waist length to make sure the clothes will fit, your waist is like perimeter.

INSTRUCTOR'S NOTES: Most students did give correct definitions for both area and perimeter but from the responses to the practical use of perimeter and area it is clear that additional concept attainment lessons are in order.

(A,S,B) QUESTION: Solve for x in both problems, then compare the two problems. Are they alike or different? Why?

#1.



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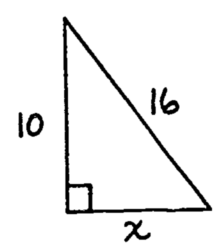


STUDENT RESPONSES:

(B) They are different. I can figure problem #1 out but I can't figure #2 out.

(B) I'm not clear on why the 10 would be the hypotenuse--can the hypotenuse be on a straight line?

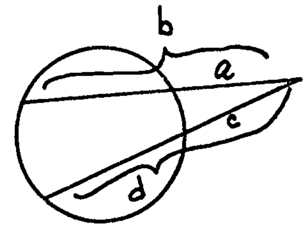
(B) Different because you add one and subtract one.

(S) The problems are different! Because in the first one you add to find the hypotenuse. In the second one you subtract to find the leg.

(A) These two problems are alike in that they both use the same formula, this is the pythagorean theorem: $a^2 + b^2 = c^2$. In the second problem we are not trying to find c but another b. We can rearrange the formula to suffice our needs.

INSTRUCTOR'S NOTES: Most mathematicians would probably have answered that the two problems were basically alike as the last two student comments suggest. However, many students answered that the problems were quite different. One comment shows a student who thought that they were completely different. This student successfully solved problem #1 but stated simply that problem #2 could not be done. The successful math teacher should be aware that these types of student perceptions exist.

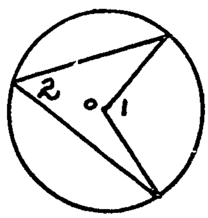
(A) QUESTION: We know that ab = cd. How did we prove this --in other words, what idea was the proof based on?



(A) STUDENT RESPONSE: We proved this by saying that the outside parts (a & c) times the inside parts (b & d) equals the same as the opposite side...In all realness I really have no idea! Am I close though? ...After you showed us on the board I kind of understand now. When you draw the auxiliary lines you form two similar triangles and the crossproducts are equal because all the parts are similar.

INSTRUCTOR'S NOTES: It sometimes becomes clear that the students missed a point before collecting the journals. Usually the students start writing their answers quickly. This time there was very little writing and a lot of perplexed faces. The instructor gave a quick review and the students continued their writing assignment. Interestingly enough, the students seemed to remember the role that similar triangles played in this formula much better after this happened.

(A,S) QUESTION: Angles 1 and 2 intercept the same arc, why aren't they the same size?



STUDENT RESPONSES:

(A) The two angles are not the same size because angle 1 is a central angle and angle 2 is an inscribed angle. A central angle is equal to the measure of the intercepted arc. An inscribed angle is equal to one-half of the intercepted arc.

(S) Angle 2 is smaller because it is an inscribed angle and angle 1 is a central angle therefore you have to pull angle 2 back further bringing the angle sides closer together making the angle smaller.

INSTRUCTOR'S NOTES: More students began to answer the journal questions in a thoughtful manner. The question "Why?" seemed to lose its "test-question-waiting for one correct response" feeling, and more students were willing to speculate. An excellent contrast is illustrated in the two previous student responses.

(S,B) QUESTION: How is finding circumference and area of circles like finding perimeter and area of polygons? How is it different?

STUDENT RESPONSES:

(B)

CIRCLE		POLYGON
Round & you have to use 3.14 & the diameter & the radius to find out the circumference of a circle	circumference & perimeter are both the distance around	a polygon is a square & all you do is add the numbers that you see

(S) Student: Mainly perimeter and circumference are the same thing, just that they work with different figures.

INSTRUCTOR'S NOTES: Throughout the semester many of the questions were the "compare and/or contrast" type. For example, the students were asked to "compare and/ or contrast parallelograms and trapezoids." Another question asked them to "compare and/or contrast congruent and similar polygons." Although the above question was not worded as such, it was exciting to see a Basic Geometry student recognize that the question was--essentially--a compare/contrast type question.

Various examples of journal entries follows. The instructor's comments are included in parenthesis after each student response.

(A) Similar polygons are alike, but are not always identical. (This student illustrates clear and precise language usage.)

(S) I got my answer by doubling the radius and multiplying by 3.14. (In this case, the student correctly solved for the area of a circle by squaring the radius and multiplying by pi. It is a common error to say "double" when we mean "square.")

(B) Parallelograms have four equal sides. (Make sure that the student means that they have two pairs of equal sides, not that all four sides have the same length.)

(A) Trigonometry is the study of three dimensional objects. It is finding the measurements of the angles or the sides. When I heard about trigonometry I thought it would be impossible or very hard. I thought it was big time geometry, but it's not as hard as I thought...it's hard, but not as hard. (Does this student think trigonometry is the study of three dimensional objects because of the practical applications shown in class, or does the student actually have some mis-conceptions about the topic? Sometimes students' preconceived ideas of a topic are so set that the instructor is completely unable to off-set them, so it was refreshing to see students admit their preconceived notions and to change or adjust them.)

(S) I can find the right triangles in pyramids because I usually look for the corners of the figure and I can find the right angle this way. (This is good advice which may help other students.)

(Note: At this time there exists a great deal of interest in right brain/ left brain or picture versus analytical thinking in education. Instructors are concerned with reaching students from either strength in one lesson on a topic. It seems that students have definite preferences in learning style, and it was supported through student responses in their journals. This preference is illustrated in the responses to the next question.)

(S) QUESTION: Explain how to get distance, midpoint, slope and how to graph equations of lines.

We talked about two methods for each in class...plot points and count spaces, draw a right triangle and use the pythagorean theorem, or use algebraic formulas. Which method do you use? Why?

STUDENT RESPONSES:

(S) I get the distance visually by counting how many spaces. This method is faster and easier.

(S) I get the distance mainly by counting the spaces between the points. I don't really understand the formuals and I think it is easier to do it this way.

(S) Usually to get the distance, midpoint or slope, or to graph equations I would use visual methods. Sometimes I will use the formula if I'm stuck on a problem. I would normally use the visual part on a test because I find it easier and faster.

(S) I prefer the formulas because to me they're easier if you just memorize them.

(S) My favorite way is algebraic. It is much more simple to me. $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$, or if you are going to do it the visual way you can use the pythagorean theorem by forming a right triangle on the graph.

(S) I would use the formula. This seems to be an easier way for me instead of drawing the whole thing out. If I needed to check myself I would draw the picture out.

(S) I would use the equation because it is an easier and quicker way.

INSTRUCTOR'S NOTES: These comments indicate that many of the students recognize their individual preferences or learning strengths. The students who are willing to consider using more than one method when solving problems increase their options and are generally more successful problem solvers; essentially they have more tools with which to work. This was recognized by a student who wrote,
I think that I've learned that you can use different

methods you already know to figure a problem."

Throughout the journal writing project, the instructor specifically looked for clues that would indicate learning preferences and would attempt to point them out to the students by underlining key words, writing follow-up questions in the students' journals, or by making direct comments in their journals about the students' procedures used in solving problems. In the example that follows, the instructor simply underlined key words.

(S) I liked the visual review the most. It helped to trigger the information that had been stored in my mind.

The following are journal entries made throughout the semester by a student in the Standard Geomety class. It can be seen that the student became more aware of his learning style and better able to articulate his thoughts and procedures.

(S) This doesn't seem backward to me because this is mostly the way I think. It's a little hard to explain on paper.

On this problem you can't have a fraction as a radicand and you cannot have a radical in the denominator. First you make each one a radical then multiply by one and then simplify further if needed.
$\sqrt{16/3} = \frac{\sqrt{16}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{48}}{3}$.

I don't know maybe I think in three dimensions. Angle 2 is smaller than angle 1 because when you move an angle back away from the center, the angle becomes smaller to accommodate for the largeness of the opening near the arc. The further you have to move it, the smaller it gets, until you reach the other side of the circle.

My way of thinking is different from yours, but your way of teaching is helpful still. This did bring out into the open certain things: I now know how I really think and that helps and I know more about the way you think (or at least realize it) In a lot of ways my thinking is careless, but I am the impatient type and don't like the same thing over and over. I do like challenges though.

Writing in this journal was pretty helpful in that it gave each of us a chance to see what we're thinking, but sometimes I find it hard to find the words.

Near the end of the writing project the students in all three levels were given a check list that reflected the effects of preferences in work situations. The students were asked to consider which preferences, which mirrored the Meyers-Briggs learning types, matched their own, and how these preferences compared with the work situation demanded by the geometry course and the instructor. They were also asked if thinking about their individual learning styles gave them a clue regarding their ability to do well in geometry class or why they were --or perhaps were not-- comfortable in the classroom. Some insightful comments follow.

(S) I think in liking math you have to be intuitive and I guess I am.

(S) I think my thinking and my way of doing things has changed since I have been in this class, for many reasons. I now do my homework alot more easily than I did awhile back. My way of thinking for tests has changed too. Geometry has also showed me new ways of step by step methodical ways of reaching conclusions.

(A) I knew I was impatient when details were complicated but I didn't realize I could work in different types of situations. It helps me see why I like the class but get impatient and bored with the subject. For me there are too many details to remember. The only way I can remember anything is to write it down.

(A) I can usually understand something better when someone is showing me in a picture or relating it to something I already know.

(S) I think that being in your class for two years has got me thinking like you. When I do my work and don't leave it unfinished or do it sloppy. I do like how this class is ran because it is never anything but a learning environment. I gotta admit it--I learned alot in this class for two years.

(B) Like your class, I like routine details and step by step things. I work better with peace and quiet although I do have a ding bat sitting behind me but that's another point.

(B) Making the checks helped me to notice a few different ways of learning that I didn't think I would enjoy.

(B) I like organization and when I'm doing my work I like it quiet and hate interruption and in this class I do not have to worry about interruption and I know always to be prepared because you are.

(A) I think in this class we're sometimes a little of each. It just depends if we're discussing something and working as a class or if we're working by ourselves. This class and the way we do things to me are that we are as a class, introverts, intuitive, and the thinking and judging types. This may be one of the reasons I don't have an easy time in this class. (Comments: This student's type was extroverted, sensing, feeling and judging. The way she described the class matched the instructor's style exactly.)

AFFECTIVE COMMENTS

Teachers seldom receive feedback from students--in terms of reaction to particular lessons, classroom rules that govern the atmosphere of the class or how the students feel about their progress. Journal entries were filled with those types of affective comments. Examples follow.

(S) It is always quiet which makes it easier to work and no one laughs at you when you mess up which makes you more comfortable with asking questions and that is what I call a good class.

(B) I like organization.

(B) I enjoy this class because we do think for ourselves and also if I do not do very well on a test or quiz, I don't feel like I only let myself down--it feels like I've let you down too which makes me want to do better.

(B) I think geometry helped me learn to think more logically.

(B) I know much more than I ever would have thought I would. I'm glad that geometry is a required class.

(B) Being organized helps me to learn and think easy. This gave me an opportunity to learn new things about myself and others as well.

(A) Now I know how to combine geometry and algebra to solve geometry problems.

(S) After I saw that you cared enough to help me out I figured I should care even more so I started studying and paying more attention in class and I just want to tell you thank you.

(S) This is the way I see how to do it. I think if I had to teach it, everyone would fail.

(S) I have made a lot of progress these past three quarters because I started out thinking I would never be smart in math and now I know I can be.

(A) I like the class and how it is run, I am not too fond of the subject, I guess Spanish best suits my learning style. I don't know why, I think because it interests me more than other subjects.

(A) This year I think all of it was nothing but pure learning. I don't think I knew anything we have learned this year and I also think you're the best teacher for this job and you're doing a great job.

Sometimes the instructor's perception of the lesson presents a dramatic contrast to that of the students' perceptions. This is clearly illustrated when comparing the instructor's journal entry regarding the class with that of the students'. An example from the Advanced Geometry class follows.

(A) QUESTION: What was yesterday's lesson like? Was it different or the same? Did you like or dislike it?

INSTRUCTOR'S JOURNAL ENTRY: Yesterday's lesson was very different from the norm. I gave a concept attainment lesson on similar polygons. I had numerous posters of examples of two similar polygons as well as non-examples. The students had to discover that similar polygons have congruent angles and proportional sides and that if they have only one of the two properties they would not be similar. Although I had to start class in a stern manner (the students were not seated and ready to begin class when the bell rang and I had to wait on them to begin the lesson, so they received a lecture on excellence: time is critical--accept the challenge to become the best--people have the right to waste their own time but never anyone else's time...etc.), but I felt that once the lesson got started they got involved and understood the concepts.

STUDENT RESPONSES:

...The class was generally the same except when you yelled at us. Most of the lessons are usually the same. I would like some variety.

...Yesterday's lesson was the same. I kind of disliked it because it was kind of boring. I think it's because this is the last class of the day and by this time everyone is tired. That's why I find this class boring sometimes. I wish this class was in the morning when I'm awake.

...Yesterday's assignment was like your normal everyday assignment. Nothing spectacular, but much easier to learn geometry. The cardboard sheets made it better to understand which is why I liked it more than your normal assignments.

...It was the same and I liked it.

...Yesterday's lesson was on similar triangles, squares polygons and so on. It was a little better than other lessons. I liked it because it was funner.

...Yesterday's lesson was different because we had the pictures out showing similarities. This helps me learn it easier although it is not real hard without the pictures. I liked the idea, it was beneficial.

...Yesterday's lesson was different than usual and I liked it because it was different.

COMMENTS: Sometimes it's not all that wonderful to get feedback from the students. One might wonder if each of the above comments are about the same lesson, which took place just the previous day.

The final journal question of the project asked the students directly "Was journal writing helpful? Did you like writing in your journal?" 95% of the students responded that journal writing was helpful and 94% did enjoy the assignment. The survey results and student comments follow.

QUESTION	YES	NO
1. Was journal writing helpful?	63	3
2. Did you like writing in your journal?	62	4

RESPONSES FROM BASIC GEOMETRY STUDENTS:

...Yes, it helped me sort out out problems and things I didn't understand.

...Yes, you asked us to write what we learned and that gave me a chance to really see what I learned and I liked it.

...Yes, we could let you know what was wrong--express questions that we didn't understand so we didn't have to ask them in class again.

...It gave us a time to relax and write our thoughts but still be thinking of our class and what is going on in it.

...I like writing in the journal, being able to share your thoughts and what you think of a class and it's very interesting to look back and see what you wrote and what you thought. It really made you think about the work you were doing, not just working it out and that's it but think about the many different steps it took to find the answer. I love to express what I have to say.

...I liked it and thought it was helpful because I knew that if I didn't understand something I could write it down for you personally and you could answer fully without confusion.

...I love journal writing because it gives us a chance to let you know how we feel about geometry. It's easier to write it on paper than say it. It also gives you an idea on how we are and pretty much what we are capable of.

...It gave a way to tell your teacher something without saying it to her straight and be embarrassed.

...I didn't really see the purpose, I don't think it even really helped me but I liked it because it was an easy grade.

RESPONSE FROM STANDARD GEOMETRY STUDENTS:

...I think writing makes me realize how much I really know and understand, I like it!

...On some of the problems the journal helped but others it was hard to put it down on paper and describe it. I did like the journal because it allowed us to write about how I feel about this class and what we were studying.

...Journal writing made me think about what I was doing and learning in class which helped me alot and I liked writing in them. It was a way to express things.

...Journal writing didn't really help me because I have a good mind for math and I understood it before I wrote about it in the journal. I liked writing in the journal because it is something different in math class.

...The journal was kind of helpful because in the beginning I had to look through the book for what I wanted to say and now I just know what to say and I'm not afraid about what to say. I really liked the journals it was something different and fun.

...The journal writing was helpful to me because it showed me or not if I was picking the material up. I didn't mind doing it at all. If anything it helped me.

...The journal writing was helpful and I liked it because it showed me some of my weak points in certain chapters and some things you said when you responded to my answers it boosted up my confidence and made me realize I could do alot better than I was.

RESPONSES FROM ADVANCED GEOMETRY STUDENTS:

...I don't like writing in journals, I never have. It did help me study though.

...The journal was helpful, it was a kind of review.

...writing in the journal was real effective, it helped me discover what I did and didn't need to work on.

...The journal was helpful. I enjoyed it and it was a nice change of pace.

...I enjoyed writing in my journal. It has helped me to be more open and understanding within myself.

...I liked writing in the journal because I can write down and think about problems I didn't know too much about. I think it helped me alot in this class.

...I do think the journal was helpful. You've answered some strange questions I've had.

CONCLUDING REMARKS

It was hoped that the journals would reveal student insights into their individual strengths and learning styles, and that they would act as a classroom barometer, of sorts, to aid the instructor in assessing students' progress as well as their needs for additional help or instruction. It can be seen from the journal entries that these goals were, indeed, achieved and, in fact, surpassed. The students' metacognitive ability was definitely enhanced through their journal writing experience. The reason for this may be that the students became active participants in the class while writing in their journals. The typical highschool classroom full of passive learners was transformed--at least while they were writing--into an active learning and therefore positive environment.

It was also hoped that an overall improvement of scores would show that journal writing had improved the thinking skills of the students. While the overall scores did improve, the strongest argument that they improved as a direct result of journal writing is that the students believed, and made statements to the effect, that writing in their journals improved their understanding of the material and resulted in higher grades. An overwhelming majority (95%) of the students said that journal writing was helpful. That in itself is a strong argument for the project's worth.

An additional, unforseen benefit for the classroom instructor was realized through the project. Many students' affective comments voiced their appreciation for the instructor's efforts. Some literally said "Thanks." In a profession where the burnout rate is high, a word of thanks is deeply appreciated.

The only negative aspect of the project was that keeping up with the writing assignments was an added burden on the instructor. Reading the journal entries and responding to them did take a good deal of time, and they were not easy to keep up with. The project would be enhanced if this element of time could be resolved.

The overwhelming evidence suggests that journal writing in the mathematics classroom does, indeed enhance learning and is a worthwhile endeavor for the instructor as well as the students.

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