Advocates and teachers of critical thinking tend to deny that intuition and justification are logical, even though they assume that both processes are rational. However, it can be demonstrated that the relation between intuition and inference, between justification and explanation, is dialectical and complementary, so that there is no mystery as to (1) how informal reasoning supplies the content of the knowledge articulated in formal reasoning or (2) how formal reasoning explicates the form of the knowledge acquired in informal reasoning. Although inference and insight are contraries in the strong sense, in a weaker sense inference can include the kind of informal inference implicit in a rational assertion, which makes intuition and inference not only compatible but complementary. In addition, formal inference can be used to make the structure of informal inference explicit, thereby creating further compatibility between informal inference and formal reasoning. Still, discovering a solution to a problem also requires the alternation between justification and explanation of theory; thus inference must be interpreted informally as a dialectic between justification and explanation in the process of achieving equilibrium in the assessment of an argument. (Examples of formal argumentation supporting the claims of this paper are included.) (SKC)
FORMALITY AND INFORMALITY IN REASONING

William E. Murnion
Ramapo College of New Jersey

January 27, 1987

Paper presented at the Conference on Critical Thinking
Christopher Newport College
Newport News, VA
April 9–12, 1987

"PERMISSION TO REPRODUCE THIS MATERIAL HAS BEEN GRANTED BY

William E. Murnion

TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)."
Formality and Informality in Reasoning

The distinction between informal and formal reasoning can be explained as either structural or functional. At this point, the structural explanation seems to be dominant, among the advocates of critical thinking as well as among formal logicians. Intuition is taken to be the contrary of inference, justification the opposite of explanation. Hence, logicians regard intuition and justification as irrational, although they admit these operations are the only source of the knowledge explicable through inference. And critical thinkers (Is there a name for the advocates and teachers of critical thinking?), for their part, tend to deny intuition and justification are logical, even though they assume both of these processes are rational. There is something suspect about a position that leads to such contradictory and counterintuitive conclusions.

What if, instead, the distinction between informal and formal reasoning is functional? What if, that is, the relation between intuition and inference, between justification and explanation, is dialectical and complementary, so that there is no mystery how informal reasoning supplies the content of the knowledge articulated in fo. =1 reasoning or how formal reasoning explicates the form of the knowledge acquired in informal reasoning? That is the intuitively appealing and irenic thesis I wish to demonstrate. It supposes that the rationality manifest in the communication of knowledge is already operative in the attainment of knowledge, making discovery
as well as proof, learning as well as teaching, critical thinking, and proof as well as discovery, teaching as well as learning, creative thinking.

1 The Dialectic between Intuition and Inference

In the strong sense, intuition and inference are contraries by definition. For intuition in the strong sense is unjustifiable immediate apprehension (a convention or a tautology or else a hunch or a feeling),\(^1\) and inference in the strong sense is the mediately justified knowledge of a formal argument. But in a weak sense, intuition can be a justifiable immediate apprehension of the meaning evident in the pattern of the data for an event or an object. Intuition, in this sense, is what both Peirce and Lonergan labeled insight.\(^2\) And inference in the weak sense can include the kind of informal inference implicit in a rational assertion. This is the kind of assertion for which questioning can elicit the premisses, even if initially only in an enthymeme. In the weak sense, therefore, intuition and inference can be not only compatible but complementary. For an insight is justifiable, while a rational assertion is immediate, and without inference the justifiability of an insight would remain inarticulate, while an assertion would be irrational without an insight to supply the meaning.

An illustration of the dialectic between insight and inference is to be found in what might at first seem to be an unlikely spot. It is the locus classicus in the Meno for the demonstration of the necessity of anamnesis.\(^3\) Remember, what prompted this demonstration was the dilemma Meno presented Socrates that learning seemed to be either unnecessary or impossible, for either we already knew something
and did not need to inquire into it, or else we did not know it and could not recognize if we discovered it. Socrates' response was to claim that knowledge was not a matter of learning but of recollection: a recall of the knowledge gained by our souls in a previous existence. The proof he offered was to show how a slave boy, who showed no sign of innate knowledge, nor had ever been instructed in geometry, could nevertheless come to understand the Pythagorean theorem without a formal proof. By process of elimination, Socrates argued, the only explanation for the boy's apprehension of the theorem was anamnesis. This was tantamount to saying that intuition was the contrary of inference, that discovery was irrational.

What the proof actually shows, however, is that discovery is a function of a dialectic between intuition and inference, as rational a process as can be. Socrates demonstrated the theorem to the boy by posing the problem of how to find the root of a square double the size of a square of four (that is, one with a root of two). He led the boy to discover the solution by sketching for him a series of squares, each with a root closer to the right one, until, finally, he drew for him a square whose root was the diagonal of the original square (figure 1).

(Figure 1 here)

Since this square is obviously twice the size of the original square of four and half the size of a circumscripive square of sixteen, it is clearly the square of eight, for the root of which the boy had been looking. By looking at the figure, the boy could see that the root of this square, even though it had no determinate number, was the solution to his problem. At this point, Plato cuts off the story, without allowing
Socrates to help the boy infer the significance of his insight — that the square of a diagonal of a triangle is equal to the sum of the squares of the other two sides.

This example illustrates how the discovery of the solution to a problem depends upon using the imagination to draw from the data of the problem the lineaments of a model in which we can gain an insight into the solution. The criteria for the model are the requirements for the solution. Even so, we must ordinarily analyze the specifications and vary the features of the model before the solution becomes manifest in it. Indeed, unless we confirm these intuitive strategies with the statistical strategies operative in formal inference, we are likely to confuse a necessary with a sufficient solution. In any case, the concomitant insight is the recognition of a solution in a pattern emergent from the data of the problem. The insight becomes the basis for an assertion whose presuppositions and consequences we can articulate through inference. As Norwood Hanson has commented, "This is what philosophers and natural philosophers were groping for when they spoke of discerning the nature of a phenomenon, its essence; this will always be the trigger of physical inquiry." Therefore an insight is an intuition for which inference can elaborate the rational justification.

How inference articulates the rationale implicit in insight is something Bernard Lonergan has explained. His argument is that formal inference amounts to an articulation of the simple hypothetical argument implicit in the informal inference behind a rational assertion.

It appears a fact that spontaneous thinking sees at once the conclusion, \(B\), in apprehending the antecedents, \(A\). Most frequently the expression of this
inference will be simply the assertion of B. Only when questioned do men add that the "reason for B" is A; and only when a debate ensues does there emerge a distinction between the two elements in the "reason for B," namely, the antecedent fact or facts, A, and the implication of B in A (if A, then B).7

This structure, implicit in informal inference, becomes explicit in formal inference. The statement in informal inference of the reason for the assertion becomes in formal inference the minor premise of an argument. And the formulation in informal inference of the condition for the reason becomes in formal inference the major premise of the argument.

Thus the transition from informal to formal inference is a process of analysis; it makes explicit, at once in consciousness and in language, the different elements of thought that were present from the first moment. For when B simply is asserted, it is asserted not as an experience but as a conclusion; else a question would not elicit the answer, B because of A. Again, when this answer is given, there would be no meaning to the "because" if all that was meant was a further assertion, A. On the contrary, the causal sentence (because A, therefore B) compresses into one the three sentences of the formal analysis (if A, then B; A; .:. B).25

The syntax of formal inference, therefore, simply explicates the argument concealed in the more primitive assertions and enthymemes of informal inference.

The logical form cognate to the syntax of formal inference, Lonergan concludes, is the simple hypothetical argument (table 1).

7
This form is flexible enough for the units, A or B, to stand for any number or any type of propositions and yet so simple every inference demonstrates the implication of the conclusion in the premises.26

On this analysis, the function of inference is to articulate the rationale for an insight. Just as insight supplies the content for the form of inference, inference gives form to the content of insight. The distinction between the two is functional rather than structural.

2. The Dialectic between Justification and Explanation

The dialectic of learning is not absolved in the interaction between insight and inference, however. It also includes an alternation between justification and explanation. For we cannot be sure we have discovered the solution to a problem simply by inferring whatever hypothesis is suggested by an insight. We have to assess the hypothesis by evaluating the relation between its presuppositions and its consequences until we reach a reflective equilibrium between theory and fact.8 What we propose as a solution to a problem must fit within conventional theory unless we are prepared to revise the theory, just as it must correlate with conventional explanations for the facts unless we are prepared to defend it as an exception to the rule. To reach the point of reflective equilibrium, therefore, the form of inference must be a dialectic between justification and explanation.9
7.

To understand the nature of this dialectic, it is necessary to elaborate a more complete model of the form of inference. In this model, the premisses comprise a biconditional proposition in the major premise and the position or negation in the minor premise of the condition postulated in the antecedent of the major. The conclusion is represented as a disjunction between the complementary logical implications of the positive and negative insights represented in the minor premise. In addition to the premisses and the conclusion, the model includes the alternative possibilities, positive and negative, of the existential import of the conclusion. This is a model of inference as a series of dichotomies in the articulation of the implications, logical and existential, of a biconditional proposition. It supposes that as inference actually functions in the learning process, it represents a selection of the abstract possibilities allowed by formal logic for the structure of argument (table 2).

(Table 2 here)

The major premise is the terminus of justification and the foundation of explanation. It must be a biconditional because as the articulation of an insight into the nature of an object, it is supposed to represent the grasp of a necessary condition and to entail strict implication. A complete and precise statement of the form for the major would be, therefore, If and only if \( A \), then \( B \) (or, If \( A \), then \( B \), and if not \( A \), then not \( B \)). \( A \) is postulated to be the independent variable necessary for the occurrence of \( B \). Therefore, coming at the turning-point between justification and explanation, the major represents an articulation of the conditions necessary for the minor to imply the conclusion.
The minor itself has both a logical and an existential function. In justification, it has the logical function of representing the reasons for either alternative in the conclusion, while in explanation it has the existential function of positing or negating the (bi)condition in the antecedent of the major. For in justification the minor represents the significance of an insight into a pattern emergent in a model and articulated in an assertion that, when inference is complete, will become the conclusion of an argument. In this case, the disjunction in the minor represents the process of determining the appropriate features to be included or excluded as independent variables in the formation of a prototype of the event or object under investigation. In explanation, by contrast, with the major already specifying the conditions (the independent variables) for the relevant insight, the minor represents the determination of whether or not the conditions are fulfilled; that is, whether or not the independent variables represented in the prototype occur in actuality. In the alternation between justification and explanation necessary for achieving a reflective equilibrium between hypothesis and empirical data, the disjunction in the minor represents the alternative possibilities of accommodating the premisses to the data or of assimilating the data to the premisses. Hence, the minor is the axis of the interaction between justification and explanation, the point for assigning a truth-value, hypothetical or actual, to the antecedent of the biconditional in the major.

With the minor a disjunction, the conclusion must be one also. This is the terminus of explanation, but only because it is originally the initiation of justification. In justification, the disjunction in the conclusion represents the assertion or denial of a logically possible solution to a problem. If we gain an insight into the solution implicit in this disjunction, articulate it in the premisses of an argument, and affirm the fulfillment of the condition necessary for a solution, the conclusion will be
positive, an explanation of the soundness of the original insight. And if explanation leads to a denial in the minor of any actual fulfillment of the rationale for the insight, the conclusion will be negative — once again, a reassertion of the original insight, but now in the form of a denial that the object in question can occur in the absence of the conditions predicated as necessary for it. Thus the immediate function of the conclusion in demonstration is to represent the logical consequences of affirming or denying in the minor the actual fulfillment of the conditions postulated in the major as necessary for the occurrence of the event or object in question.

But the conclusion has an existential function as well. For it also represents a statement about whether or not empirical data verify or falsify either the positive or the negative formulation of a putative solution to a problem. Before there can be a rational assertion to be justified there must an assessment of the import of the available data for a problem, and after an explanation has been offered, it must be confirmed by the data. In the conclusion, therefore, besides the disjunction between the positive and the negative formulation of the logical consequences of an argument, there is also a distinction between the logical consequences themselves and the actual data. And within the data, another distinction has to be made between the positive and negative eventualities of either the positive or the negative formulation of the logical conclusion.

This leads, therefore, to the possibility of four simple sets of existential conclusions — Identity, Negation, Correlation, and Reciprocation — the state descriptions of the world implied in the logical conclusion. They correspond both to the four possibilities in the truth table for the conditional and to the quarternary group Jean Piaget devised to interpret formal operations. Yet it is one thing to state the alternative
possible existential conclusions of an inference, another to determine whether any of the possibilities has been fulfilled, and yet another to assess the significance of various kinds of fulfillment. Within each possibility, a distinction must be made between a judgment about the actual occurrence of a certain set of data and a decision about whether it instantiates the possibility in question. And since evidence may be discovered for none, any, some, or all four of the sets of alternative possibilities, there are sixteen possible combinations of existential conclusions that may be drawn. What is more, this formal complexity prescinds from the substantive complications to be expected from assigning diverse weights to the sets of data in the various possibilities. The consequence is that the projectibility of a justification as well as the confirmability of an explanation is formally undecidable. Therefore, inference must be interpreted informally, as a dialectic between justification and explanation in the process of achieving reflective equilibrium in the assessment of an argument.

Summary

Attention to the origin as well as to the communication of knowledge reveals a dialectic between insight and inference, between justification and explanation, in the process of reasoning. This compound dialectic explains how the distinction between informal and formal reasoning can be interpreted as functional rather than structural. Thus, informal reasoning can be regarded as rational not only teleologically but archeologically as well, while the justification of formal reasoning can be by a construction of its function in argumentation as well as by a reduction of its structure to the prerequisites of an axiomatic system.
Notes


3. Plato, Meno, 82a-86b. This passage so impressed Aristotle he treated it twice, once in the Posterior Analytics (II, 94b 20-35) and again in the Metaphysics (IX [1051a 22-34), but he argued that it demonstrated, not as Plato thought the influence of anamnesis, but the capacity of intelligence to reach understanding through appropriate images. Aquinas believed this analysis was one of Aristotle's most important contributions to cognitional theory, and he adopted for his own theory: see In II Post. Anal. lect. 9, §495 (Spiazzi, p. 360) and In IX Meta., lect. 10, §1888 (M.-R. Cathala and R. M. Spiazzi, eds., S. Thomae Aquinatis, In Duodecim Libros Metaphysicorum Aristotelis Expositio [Turin-Rome: Marietti, 1964], pp. 453-54). Modern Thomists have defended the importance of this analysis: see Karl Rahner, Geist in Welt: Zur Metaphysik der endlichen Erkenntnis bei Thomas von Aquin, 2d ed. (Munich: Kösel-Verlag, 1957); Bernard Lonergan, Verbum, Word and Idea in Aquinas, ed. D. Burrell (Notre Dame, IN: University of Notre Dame Press, 1967), pp. 25-33; William Murnion, "St. Thomas Aquinas's Theory of the Act of Understanding," The Thomist, 37 (1973), 115-16.

4. Plato, 80 d5-e5.

5. See Ralph Johnson, "On Reasoning: The Relevance of Some Recent Work in
Cognitive Psychology," Conference '86 on Critical Thinking (Christopher Newport College, Newport News, VA, April 10-12, 1986).

6. Norwood Russell Hanson, Patterns of Discovery: An Inquiry into the Conceptual Foundations of Science (Cambridge: Cambridge University, 1958), p. 87. Taking a cue from C. S. Peirce, Hanson made an elegant and compelling analysis of how Kepler finally discovered the curvature of the orbit of Mars by converting the data of Brahe's observations into a series of mathematical models until he finally saw that it must be an ellipse (pp. 77-85). Lonergan, in Insight, made insight into images of empirical data the keystone for a philosophy of science and a comprehensive theory or knowledge, not to speak of a methodology for metaphysics, ethics, and theology.


11. This set of outcomes is modelled on the combinatorial lattice Piaget devised to give a concrete interpretation to the logical alternatives represented in his quarternary group: see Inhelder and Piaget, pp. 114-21.
Table 1
The Form of Inference: Simple Hypothetical Argument

If A, then B
But A
∴ B
Table 2

The General Form of Inference

<table>
<thead>
<tr>
<th>If and only if A, then B</th>
<th>And A</th>
<th>OR</th>
<th>And not A</th>
</tr>
</thead>
<tbody>
<tr>
<td>(If A, then B, and if not A, then not B)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>And A</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>B (logically)</td>
<td>Not B (logically)</td>
<td></td>
</tr>
<tr>
<td>Verification</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Positive: Identity</td>
<td></td>
<td></td>
<td>Negative: Correlation</td>
</tr>
<tr>
<td>That B</td>
<td></td>
<td></td>
<td>That not B</td>
</tr>
<tr>
<td>:: B (factually)</td>
<td></td>
<td></td>
<td>not B (factually)</td>
</tr>
<tr>
<td>OR</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Falsification</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Positive: Negation</td>
<td></td>
<td></td>
<td>Negative: Reciprocation</td>
</tr>
<tr>
<td>That not B</td>
<td></td>
<td></td>
<td>That B</td>
</tr>
<tr>
<td>:: Not B (factually)</td>
<td></td>
<td></td>
<td>B (factually)</td>
</tr>
</tbody>
</table>
Figure 2
Doubling the Square on the Base of the Diagonal