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AUTHOR	Greeno, James G.
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ABSTRACT

This project addressed the question: What knowledge and processes enable individuals to construct and modify representations of novel, nonroutine problems? The theoretical goal was to extend the information-processing theory of problem solving to include processes that have been characterized as restructuring of problems and productive thinking by Gestalt psychologists such as Duncker (1935/1945) and Wertheimer (1954/1959). Three lines of research were carried out. One investigated different kinds of knowledge that result in problems that are routine, semiroutine, or nonroutine, according to distinctions in a simulation model that was developed in the project. Experiments distinguished between functional knowledge, involving relations between components of a device, and component knowledge, involving information about behavior of components independent of their interconnections. A second line of work studied processes of generating representations of physics problems, and identified important interactions between everyday experience-based knowledge about motions of objects and knowledge of theoretical principles. The third line of research studied processes of understanding and solution of back-of-the-envelope problems, and identified use of general methods for reasoning about quantities that are combined with domain-specific information. (Author/RH)

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James G. Greeno

University of California, Berkeley

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James G. Greeno

University of California, Berkeley

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This project investigated knowledge that supports the generation of representations in problem situations. In an earlier period of research, Gestalt psychologists who studied problem solving emphasized processes of reformulating and restructuring problem representations. In tasks studied by Duncker (1935/1945), Wertheimer (1945/1959), and others, the main requirement of solving problems is achieving an adequate understanding of the problem, and when that is achieved, solution of the problem requires very little search or other further effort.

The research in this project extended current theories of problem solving to account for some generative aspects of the representation of problems. The theory of problem solving, developed by Newell and Simon (1972) and others, and reviewed by Greeno and Simon (in press), largely concerns knowledge for application of known operators and search for solutions in a problem space. Problem solvers also must construct the problem spaces in which they work, and processes of understanding routine problems have been studied and simulated by Hall, Kibler, Wenger, and Truxaw (1986), by Hayes and Simon (1974), by Kintsch and Greeno (1985), and by Novak (1975), among others. The present research concerned representation of problems when the initial representation is not adequate to support solution by application and search involving known operators.

The project was begun in March, 1984, at the University of Pittsburgh, in collaboration with Lauren Resnick. Two lines of research were included in the project: study of restructuring in insight problems of the kind investigated earlier by Gestalt psychologists, and restructuring of representations in intuitive physics. We began working on tasks involving the motion of a pendulum, a device that has been used both in insight problems (Maier, 1931) and in intuitive physics (Caramazza, McCloskey, & Green, 1981). Michael Ranney conducted preliminary protocol studies on the naive physics task of predicting what will happen if the bob of a pendulum is disengaged at different positions as it is swinging; Ranney has continued on this problem and has completed his dissertation at Pittsburgh based on the work that he continued subsequently.

In the fall of 1984 Greeno moved to Berkeley and continued working on the project with new staff and graduate students who joined the project. Research was conducted on three specific questions: (1) knowledge required for representing and solving different kinds of problems, with problems involving reformulation as one of the categories; (2) knowledge and processes of experienced physicists used in developing mental models to represent novel problems; and (3) knowledge and processes used in representing and reasoning in informal "back-of-the-envelope" problems that involve estimation of quantities.

1. Knowledge for different kinds of problems

Greeno and Daniel Berger have developed a characterization of knowledge needed for solution of different kinds of problems. The task they addressed initially was development of a model that would simulate knowledge involved in solving traditional "insight" problems, such as the candle problem (Duncker, 1935/1945) or the two-string problem (Maier, 1931). The model they developed is a generalization of a model of the candle problem that Weisberg and Suls (1973) developed earlier, and the extensive empirical tests that Weisberg and Suls conducted lend considerable plausibility to the general features of the model for the class of problems involving insightful reformulation.

While Weisberg and Suls developed a specific model for the candle problem, Greeno and Berger developed a general characterization of levels of knowledge used in different stages of solving problems that involve insightful restructuring. The levels of knowledge are examples of strong, medium, and weak methods, in Newell's (1980) sense. The most specific knowledge is knowledge of procedures. Procedures have conditions of applicability and actions that change the situation, leading to a solution. A second level of knowledge is functional knowledge, which includes knowledge of the consequences and requisite conditions for performing actions or using objects. Functional knowledge is the kind of knowledge used in systems for planning, including Sacerdoti's (1977) and other subsequent planners.

We say that a problem is routine or semiroutine for a problem solver if the problem solver's knowledge of procedures and functional knowledge are sufficient to solve the problem. These categories of problems include problems that are solved within a single problem space, as this was characterized by Newell and Simon (1972). By this criterion, routine problems include exercises in arithmetic, where instructions specify the operation to be performed. Exercises such as geometry proofs and other similar problems in school mathematics and science are semiroutine requiring functional knowledge that is organized according to planning schemata (e.g., Greeno, Magone, & Chaiklin, 1979). Puzzles that are solved by means-ends analysis or other search heuristics are also semiroutine, involving selection of known operators to achieve definite goals.

Greeno and Berger characterized as nonroutine problems tasks in which the problem solver's knowledge of procedures and functional knowledge are insufficient to solve the problem. In many of the insight problems studied by Gestalt psychologists the required new material involves inferring a possible function for an object that is not stored as functional knowledge in the person's memory. The potential use of the object therefore has to be discovered through a deeper inference than is the case when functional knowledge is adequate. For example, in the candle problem, functions of support and fastening are probably associated with many of the objects in the situation, such as string and tacks, but not with the box. The potential function of the box as a support has to be inferred from its properties -- its flatness, stability, and so on.

In the terms of Greeno and Berger's analysis, new functional knowledge has to be generated by the problem solver in order to solve the problem. This terminology is consistent with Duncker's (1935/1945) discussion, which emphasized modifying the problem space by finding new functional relations. Another way to state the idea is that insight problems require the creation of new problem-solving operators that augment the problem space, which shows how this notion extends Newell and Simon's (1972) theory of problem solving.

Greeno and Berger have implemented programs that simulate solution of nonroutine problems, providing evidence for the sufficiency of their hypotheses. A more interesting question is whether the distinctions in their theory of knowledge requirements for different kinds of problems correspond to significant distinctions between the knowledge of different human problem solvers that influence their success in problem solving. This question has been pursued in two experiments. The first is completed, and the second is currently being conducted.

The experiments on knowledge for problem solving are related to studies by Kieras and Bovair (1984) who have investigated the influence of knowledge that they call a "device model" on capabilities of subjects to learn to operate a fictitious machine. Greeno and Berger invented a device that has components like those of a standard stereo system, but is disguised as a vehicle with alternative sources of energy. The use of a fictitious device enables us to give subjects specific kinds of background knowledge and examine the effect of that knowledge on their ability to solve problems or to learn procedures for operating the device. Components of the device are displayed on a computer screen, with displayed switches that can be set using the screen interface. Subjects solve problems by producing switch settings that cause components to be in different states and that produce internal connections among the components.

Our first experiment replicated and refined a result of Kieras and Bovair (1984), which showed that knowledge of a device model can facilitate learning and inference of procedures for operating the device. We refined Kieras and Bovair's concept of a device model, using a distinction introduced by deKleer and Brown (1981) between two kinds of knowledge about a device. One kind of knowledge involves information about the components of a device, including the states that each component can be in and the operations that control those states. The other kind of knowledge involves information about the interconnections and interactions among components. DeKleer and Brown called these structural and functional knowledge, respectively. We retain the term "functional" for information about the relations among components, but use the term "component" to refer to information about the behavior of individual components, that can be stated independently of their interconnections in a device.

Greeno and Berger's analysis agrees with conclusions of Kieras and Bovair (1984), that functional information should play a more important role than information about individual components in allowing subjects to understand the operation of a device. Knowledge of functions provides a framework for planning the solutions of problems, requiring inferences about states of individual components. Knowledge about the states of components can also be helpful in understanding the operation of a device, but that knowledge does not provide the cause-effect connections that correspond to problem-solving operators. Those connections have to be inferred to expand the problem space needed to plan solutions of problems.

This conjecture was tested by giving different groups of subjects (a) information about behavior of components, (b) functional information, (c) neither component nor functional information, and (d) both component and functional information, respectively, as background for learning procedures for operating the fictitious device. Subjects with background knowledge were then given problems to solve, in which they were asked to set switches so that the device would operate using its different energy sources. Some of the switches determined states of individual components, and these switches were discussed in the component instruction. Other switches determined connections between components, and these switches were discussed in the functional instruction. All subjects received training showing the combinations of switch settings for operating the device in its various states, a transfer problem was given, and two of the trained problems were presented again for recall.

Knowledge for solving the problems could be in the form of schemata that associate requirements for components and for setting states of components with goals of operation of the device. For example, for the device to operate, power must be transmitted to the motor, requiring a connection to the motor from a component called the impulse purifier, and a connection to the impulse purifier from the energy source that is specified in the problem. These requirements are achieved by setting switches that determine connections between the various components. There are other requirements involving the states of the motor, the impulse purifier, and the energy source that are achieved by setting different switches.

Information given in the functional instruction could be used to form schemata for forming subgoals involving flow of power and connections between components. Subjects given functional instruction but not component instruction would need to infer requirements involving states of components and infer or learn the switch settings that were needed to determine those states.

Information given in the component instruction could be used to form schemata for achieving goals involving states of the components. Subjects with component instruction but not functional instruction would need to infer the requirements involving connections between components and infer or learn the switch settings that determined the connections.

Kieras and Bovair (1984) concluded that the concept of power flow and knowledge about connections between components are the main requirements for understanding operating procedures of a device like the one used in these studies. Greeno and Berger agree and the information in the functional instruction provides a version of that relevant knowledge. The schemata that subjects could form on the basis of the functional instruction relates directly to the general goals that are specified in problems, and requires inference of lower-level requirements. Using information in the component instruction, subjects are required to infer the functional interconnections among components of the device, which seems harder than inferences about the individual states.

The results confirmed Greeno and Berger's expectation. Knowledge about interactions among components was sufficient for subjects to infer significant portions of the procedures for operating the device, so that subjects with functional information were able to solve problems on the basis of their background knowledge without specific training and were able to transfer to a new problem after they received training on another set of problems. In contrast, knowledge about the individual components was virtually ineffective, causing small and mainly insignificant differences either in combination with functional information (comparing the group with both functional and component information with functional information alone) or in isolation (comparing component information with no background).

The experiment that is now being conducted extends the investigation of effects of having a device model from tasks of learning operating procedures to tasks of diagnostic reasoning. Knowledge for diagnostic reasoning has been characterized in intelligent tutoring systems (Brown, Burton & deKleer, 1982). These characterizations include knowledge of the states that components can be in, including fault states.

We found in our first experiment that subjects who were given functional knowledge were able to infer structural information in tasks involving operation of a device. The question arises, then, whether component knowledge is an important factor in diagnostic tasks, or whether appropriate functional knowledge is a sufficient basis for inferring the more complex component information required for those tasks as well. Our current experiment investigates that question.

To investigate diagnostic reasoning, we have designed a more complicated version of the fictitious device that we used in our first study. The diagnostic problems that we designed using the initial version seemed easy to solve based on functional knowledge, but we want to apply a stronger test of the hypothesis that functional knowledge is sufficient. The initial version of the device had only one level of components, because knowledge of the internal structure of components is irrelevant for operating the device when it works properly. However, knowledge of the internal structures of components is relevant for diagnosis, if the task is to identify which part of a component needs to be replaced.

In redesigning the device for our next experiment, we were assisted by Douglas Towne and Allen Munroe, of the Behavioral Technology Laboratory. Towne and Munroe are developing a system, the Intelligent Maintenance Training System (IMTS), that enables a device to be designed using screen icons and specifications of component behaviors, including behaviors in fault states. Berger visited at BTL and consulted with their programmers in developing the current version of our display, and we are using programs supplied by BTL in our current experiment.

2. Generating mental models of physics problem situations

A second line of work begun when Greeno moved to Berkeley is a study of processes used by experienced physicists in generating representations of problem situations. Jeremy Roschelle and Greeno have conducted a study and analysis of performance in tasks designed to obtain information about generative processes. In one study, diagrams were shown to experienced physicists who were asked the open-ended question, "What's happening?" In another study, problems were presented in different forms, including a form with concrete objects, such as blocks and pulleys, and another form with abstract objects, such as masses and forces. The empirical work was conducted by Roschelle, and Roschelle and Greeno collaborated on a theoretical analysis of the findings.

Roschelle's findings present a quite different picture of expert reasoning than has been indicated by earlier studies such as Chi, Feltovich and Glaser's (1981), and Larkin's (1983). In some previous studies, performance on routine problems has led to a conclusion that novices represent problems mainly in terms of concrete objects and apply formulas whose variables correspond to abstract terms. In contrast, experts apparently apply schematic structures of abstract variables organized according to theoretical principles such as conservation of energy.

The expericenced physicists in Roschelle's study generated representations using a more subtle combination of processes. The protocols for the question "What's happening?" described systems of objects and referred to images of moving objects as well as to theoretical concepts such as forces due to friction. In his experiment using different diagrams, Roschelle found that the representations of experienced subjects were strongly influenced by the concrete objects in the diagrams. This is contrary to expectations based on the idea that experts match abstract schemata to the components of a problem, because the abstract structures of problems were the same in cases that were represented differently because of the concrete objects.

An interpretation of the results of Roschelle's analysis has been developed by Roschelle and Greeno. The findings are the basis of a model of problem representation in which a mental model of the problem situation is generated by the problem solver. The process of forming the mental model includes parsing the components of the diagram into systems that function as units and creating an envisionment by applying qualitative causal knowledge to generate images of objects in motion. This part of the process is similar to the one described by deKleer (1979) in his model NEWTON. The process also uses knowledge of general principles that constrain the situation by known invariances or qualitative dependencies (e.g., "friction opposes relative motion," or "if velocity is constant the forces are balanced").

In Roschelle and Greeno's interpretation, the process of forming a mental model uses informal knowledge to parse the situation into functional systems and to create envisionments of objects in motion. The knowledge base for this process is assumed to be a set of "pieces" of knowledge, including small schemata that recognize configurations of objects (e.g., two blocks connected by a string that passes over a pully) and generate simple motions of systems based on causes such as gravity. These informal knowledge pieces apply at the level of objects, and are similar to "phenomenological primitives" that diSessa (1983) has discussed. The model includes theoretical concepts that are added to the representation, such as forces and accelerations, and knowledge about these concepts is used to overcome ambiguities and impasses (e.g., if forces are balanced, velocity is constant). An important constraint is that the theoretical components of the representation and the object-level components are kept consistent (e.g., if an object moves in one direction, there cannot be an unopposed force in the opposite direction).

The interaction of informal, piecemeal knowledge with knowledge of theoretical concepts has not been a salient feature of previous analyses. It clearly simplifies the situation to say that novices depend on surface features and experts use theoretical concepts. Roschelle and Greeno's hypothesis begins to show how knowledge at various levels of abstraction can interact in the development of an integrated representation. It also has the advantage of describing a system that could be acquired cumulatively, with knowledge of theoretical concepts added to knowledge that is related directly to objects that are experienced, rather than constituting a relatively disconnected structure of knowledge.

3. Reasoning based on general knowledge and methods

A third study conducted in this project investigated reasoning in tasks known as "back-of-the-envelope" problems. Joyce Moore asked question such as the following:

How many leaves fall in North America in a year?

Fueled only by a 2-ounce chocolate bar, how high could you climb, assuming that you convert energy with 40% efficiency?

At what distance would it be faster to send data by a bicycle rider carrying a reel of magnetic tape than to transmit it across a 300-baud line?

Moore gave problems like these to graduate students in three fields: computer science, physics, and psychology.

The data provide useful information about two processes: informal estimation and the use of general methods of quantitative inference. The methods of quantitative inference are like those used by Larkin, Reif, Carbonnell, and Gugliotta (1985) in their model of expert physics problem-solving called FERMI. These methods provide ways of inferring quantities from other quantities using relations such as additive composition, decomposition into subsets specified by proportions of the whole, and multiplication of rates by quantities. Problem solving by Moore's subjects involve relating the unknown quantity to others, either smaller parts or a larger quantity that contains the unknown, or some other related quantity that can be compared to the unknown. A sequence of these relations was formed, using the general quantitative methods, until a quantity was reached that the problem solver either knew or could judge, at least roughly. These judgments were often very approximate -- for example, estimating the number of leaves on a typical tree by estimating the size of the pile of leaves that would fall from a typical tree (presumeably from raking experience) and judging the number of leaves in that pile from the number of layers of leaves that would compose the pile and the number of leaves in a layer from the area of the pile and the size of a typical leaf.

One important conclusion from Moore's study is that the kind of reasoning methods in the FERMI model are not limited to use by problem solvers who are expert in a domain. Most of the problems were solved with similar methods by subjects whether they did or did not have advanced knowledge in the domain of the problem. Knowledge in the domain provided problem solvers with specific knowledge required for estimating quantities and provided some knowledge of specific relations among quantities. In a few cases, problem solvers used formulas that they knew in the domain, but this was rare. Primarily, problem solvers by experts as well as nonexperts in the domain used the same informal general methods for setting subgoals and making inferences about quantities.

4. Reports

Technical reports have been written that report each of the three projects that are summarized in this report. The reports are:

Greeno, J. G., & Berger, D. (1987). *A model of functional knowledge and insight.* Berkeley, CA: University of California, Berkeley, School of Education.

Moore, J. L. (1987). *Back-of-the-envelope problems.* Berkeley, CA: University of California, Berkeley, School of Education.

Roschelle, J. & Greeno, J. G. (1987). *Mental models in physics reasoning.* Berkeley, CA: University of California, Berkeley, School of Education.

A paper based on Greeno and Berger's report was presented at the meeting of the Psychonomic Society in New Orleans, LA, in November, 1986, and a paper based on Roschelle and Greeno's report was presented at the meeting of the Cognitive Science Society in Seattle, WA, in July, 1987.

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