This bulletin focuses on three developments related to evaluation. First, the plans and initial work of the Mathematical Sciences Education Board, formed in 1985, are described. The Board's goals, its view of the major challenges in mathematics education facing the nation's schools, and its plans are discussed. The development of a design for studying the impact of testing on mathematics education is then described. The second section of the bulletin provides an overview of a revised publication on evaluation in the mathematics classroom, available from ERIC/SMEAC ("Evaluation in the Mathematics Classroom," published first in 1974). What is included in the publication is noted, with several illustrative examples. A second publication with similar goals ("Mathematics Assessment for the Classroom Teacher," developed by the Virginia Council of Teachers of Mathematics) is also described. In the third section of the bulletin, the need to evaluate mathematical problem solving more effectively is considered. Procedures that can be used in addition to paper-and-pencil measures are listed. Finally, several tests that attempt to assess mathematical problem solving processes are cited, with examples. Sixteen references are listed. (MNS)
Evaluation In Mathematics Instruction

In 1985, an overview of data from state assessments in mathematics was developed (Suydam, 1984). Such assessments have focused attention on evaluation across classrooms, as have the three sets of reports from the National Assessment of Educational Progress (NAEP). Data from the fourth national mathematics assessment will appear in the near future, and these data will surely be reviewed as carefully as the previous assessments have been. Information both on status – how well are students achieving currently on the various mathematical ideas they need to know – and on change – what, if any, progress has been made since previous assessments – is vital information that has been and will be used to improve instruction.

Moreover, data from the Second International Study of Mathematics (SIMS) attained headlines, as did data from the First International Study, when the ranking of the United States placed it well below almost every other country on most of the achievement scales. Concern for what is going on in classrooms has led to many studies in the past decade, and evaluation is one facet that has been investigated.

Aside from state, national, and international assessments, evaluation goes on, day after day, in each of our classrooms. Such evaluation is addressed by the revision of Evaluation in the Mathematics Classroom: From What and Why to How and Where (Suydam, 1989), it attempts to provide useful guidelines and techniques for classroom teachers.

This bulletin will provide an overview of the types of information available from that publication. Then recent work on one aspect of evaluation will be considered – how problem solving can be evaluated. But first, consideration will be given to the plans and initial work of a new agency in mathematics education, with particular note of its planned work pertaining to evaluation and testing, in particular.

Mathematical Sciences Education Board

The Mathematical Sciences Education Board (MSEB) was established in October 1985, at the urging of the mathematics community, as a major unit of the National Research Council (which in turn is the principal operating agency of the National Academy of Sciences and the National Academy of Engineering). The Board is designed to grow into the most comprehensive and far-reaching national leadership effort in mathematics education ever to be undertaken in the United States. Much of the material which follows is drawn from the informational materials distributed by the MSEB.

Through a variety of efforts, the Mathematical Science Education Board is seeking to provide:
- leadership of continuing efforts to improve mathematical sciences education nationally;
- coordination among on-going educational projects;
- service to localities and states, through assistance in determining core competencies for all students and higher standards, and enhancing teacher preparedness;
- recommendations on how to strengthen weak parts of the infrastructure of mathematics education;
- information to enhance public understanding of the rapidly changing character of the mathematical sciences;
- advice to federal, state, and local agencies on long-range goals and needs in mathematical science education. (MSEB, 1989a, p. 2)

The intent is to provide “for sustained attention over many years to the major issues affecting the quality of instruction and learning in the mathematical sciences” (MSEB, 1989a, p. 2). National in scope, the MSEB is action-oriented, concerned not only with making recommendations but also with implementing ideas.

The MSEB views the nation’s schools as facing major challenges in terms of:
- Curricula and instruction in our schools are years behind the times. They have not been modified and updated to reflect the increased demand for employees and citizens who possess well-developed problem-solving and higher order thinking skills in mathematics. Nor do they reflect the greatly expanded uses of the mathematical sciences. The skills that are needed go well beyond the sort of facility with calculation and manipulation most people identify as ‘mathematics.’
- Calculators and computers have had virtually no impact on mathematics classrooms, in spite of their potential to greatly enrich, enlighten, and enlarge school mathematics. Furthermore, there is little prospect that they will, unless there is continued strong national leadership to do two things:
  - Focus attention on the remarkable opportunities computers offer: to displace some tedious drill; to teach more sophisticated concepts; to provide individualized instruction; to use mathematics to solve a plethora of problems heretofore unsolvable by students; and to put more of a sense of wonder and discovery into the learning of mathematics.
  - Chart a course for the development of new curricula, instructional techniques, and tools for evaluation which will be sufficiently widely accepted so that textbook publishers and software companies, etc., will be willing to invest the necessary time and resources.
- Methods of evaluation – especially standardized, paper-and-pencil, multiple-choice tests of basic skills’ – are being used across the country without sufficient reflection and are themselves obstacles to the teaching of new methods and higher order thinking skills, as well as to the use of calculators and computers. The nation is in the grip of a ‘testing
mystique, which has led to the widespread use of such tests in spite of repeated warnings that several premises upon which their use is based are open to serious question.

- Finally, and most important, there is little awareness in the schools -- and virtually none in the general public -- of what lies just ahead: mathematics curricula, instruction, and methods of evaluation. The movement must undergo dramatic change if we are to properly educate tomorrow’s employees and tomorrow’s citizen. (MSEB, 1988a, pp. 2-3)

The Board is planning to move "through a succession of planned stages of change until learning levels in school mathematics have been brought much more into line with the reality of national needs and national potential" (MSEB, 1988a, p. 6). Working cooperatively with the National Council of Teachers of Mathematics (NCTM), the Board plans to develop and publish, by the end of 1988, recommended standards or criteria for excellence in school mathematics, K-12. These standards will cover curriculum, instruction, and evaluation. One part of this massive effort will involve the development of "guidelines for redesigning tests and other assessment mechanisms so that they are properly aligned with the curriculum and provide meaningful evaluation of student achievement." (MSEB, 1988a, p. 7).

The MSEB is currently developing the design for studying the impact of testing on mathematics education. The purpose of the proposed set of studies is to:

- Document information about current tests and testing practices, assess the impact of these practices on various groups, and direct future developments so that testing practices will be less disruptive, more helpful, and more effective (MSEB, 1988b, p. 1).

The report notes that the specific activities planned have emerged from the following perspectives:

- Valid information about student achievement in mathematics is needed by a variety of people (students, teachers, parents, administrators, policy makers) for a variety of purposes (monitoring progress, selection for placement in courses, program evaluation, accountability).

- Both the curriculum and teaching practice in mathematics need to be directed toward the learning of strategies to solve problems, the application of mathematics to practical situations, and the development of thinking skills. Consequently, tests should reflect students’ achievement in these directions.

- Serious questions have arisen about the validity of existing tests for the uses to which they are being put. Standardized tests and state-mandated tests may yield information that is invalid for certain purposes, and they provide little or no information on several important dimensions of achievement.

- Furthermore, the continued use of existing tests appears likely to impede the much-needed reform in curriculum and instruction that the mathematics education community is developing. (MSEB, 1988b, pp. 1-2)

Additional information about the ongoing work of the Mathematical Sciences Education Board, and detailed plans for its various projects may be obtained from:

Mathematical Sciences Education Board
National Research Council
2101 Constitution Avenue, N.W.
Washington, D.C. 20418
(202) 334-3294

Some readers of this bulletin may be wondering why so much attention was directed toward the MSEB. Clearly, it promises to exert a force, operating from the national level, that we have not had in mathematics education. And if the planned efforts succeed, then evaluation processes, and testing in particular, will change. All of us need to keep aware of the work of the MSEB and provide both advice and aid from our varying perspectives in schools at one level or another. The process of education can benefit from collective collaboration and strong, responsive leadership.

Overview of Evaluation in the Mathematics Classroom

Evaluation in the Mathematics Classroom was first published in 1974; after 12 years, it was clearly time to revise it.

In the introductory section, evaluation is illustrated by some classroom vignettes, and the purposes of the booklet are presented:

1. To help readers develop better paper-and-pencil measures for evaluation, and

2. To review with readers other approaches to evaluation.

The idea that evaluation is much more than paper-and-pencil tests is emphasized.

The second section continues with this idea by considering the scope of evaluation. Subsections deal with the scope of mathematics objectives to be evaluated, the scope of evaluation purposes, and the scope of evaluation procedures. Observations, interviews, inventories and checklists, attitude scales, criterion-referenced tests, norm-referenced tests, standardized tests, and diagnostic tests are among the types of evaluation procedures discussed.

The third section of the publication concerns developing tests. How to plan a test is discussed first, with suggestions on listing the objectives to be assessed, the types of items to be constructed, and deciding on the number of items to be written for each objective. Twenty-one general suggestions for writing test items are then presented. Such points as the following are made:

- Use clear, simple statements. Use language that students understand.
- Choose concise vocabulary, and sentence construction that is appropriate to the level of your students.
- Break a complex sentence into two or more separate sentences.
- Design each item so that it provides evidence that an objective has been achieved. Avoid testing for unimportant details, unrelated bits of information, or irrelevant material.
- Begin a test with easy Items. Placing difficult items at the beginning of a test is likely to discourage average and below-average achievers. You can then arrange items so that the test gets increasingly more difficult, or you can mix easy and difficult items.
- Analyze student responses to each item, for diagnostic use.

If such suggestions seem familiar, it may be because they have become familiar in a variety of publications on developing good tests, such as Gronlund (1984).
Following these general points are specific suggestions for developing each type of test that might be used in mathematics classrooms:

- short-answer questions or completion items
- multiple-choice items
- true-false items
- matching items
- essay items

Finally, some related points about testing are considered: item pools, item analysis as it can be done with teacher-made tests, and test bias. Some illustrative examples are included throughout.

The brief concluding comment notes that the goal of evaluation is that of improving instruction. Measuring or assessing testing only indicates: the teacher then has to do something as a result of what has been indicated. Evaluation is one stage in the ongoing process of teaching.

Lists of references, with annotations, are provided in the final section of the publication. While a few of these references provide general background information, most were selected because they were written to help classroom teachers do a better job of evaluating and making use of that evaluation.

Another publication with some similar intentions and information was developed by the Virginia Council of Teachers of Mathematics (VCTM, 1983). Titled Mathematics Assessment for the Classroom Teacher, it was written by a number of persons, each contributing one of the following chapters:

- Legal Ramifications of Evaluation (William Heffty - a city attorney in Richmond, Virginia)
- Construction and Interpretation of Teacher-Made Tests (James Impara)
- Classroom Assessment of Conceptual Understanding (John VanDeWalle and Harold Mick)
- Diagnosis in the Mathematics Classroom (John McGregor)
- Affective Evaluation (Stuart Flanagan)
- Assessing Problem Solving Skills (Ena Gross)
- Grading and Reporting (Josephine Baker)
- Mathematics Achievement: Assessment Strategies and Applications (Anne Polisell Sweet)

Among the noteworthy features of this publication is that it is filled with specific examples of classroom practices and with specific items that illustrate various points. In the face of the note that it was developed in order to help teachers implement one of the recommendations made by the National Council of Teachers of Mathematics in An Agenda for Action:

**Recommendations for School Mathematics of the 1980s** so that the success of mathematics programs and student learning be evaluated by a wider range of measures than conventional testing. (NCTM, 1980)

The Council underlines this recommendation with the following comments: Evaluation is not limited to testing. It includes gathering data and interpreting the data. Testing is one source of data. There are many others. Today, many people use test scores as the sole index of the quality of mathematics programs or of the success of student achievement. Test scores alone should not be considered synonymous with achievement or program quality. (p. 13)

It is imperative that the goals of the mathematics program dictate the nature of the evaluations needed to assess program effectiveness, student learning, teacher performance, or the quality of materials. Too often the reverse is true: the tests dictate the program, or assumptions of the evaluation plan are inconsistent with the program's goals. (p. 14)

When one considers that the first of the Council's recommendations is that "problem solving be the focus of school mathematics in the 1980s," there is realization that tests often fall to assess this focus effectively.

**Evaluating Problem Solving**

In An Agenda for Action, cited above, it is noted that an emphasis on problem solving demands more flexibility and creativity in assessment than is possible within the restrictions of most current test formats. (NCTM, 1980, p. 15)

Moreover, it is recommended that longitudinal evaluation of individual problem-solving ability should be developed. The acquisition of problem-solving skills is a long-term process and should not be evaluated solely with short-term measures. (p. 15)

Such comments promoted awareness of the limitations of many problem-solving programs, as well as of the evaluation of problem solving. The Mathematical Sciences Education Board has reaffirmed that focusing the curriculum on problem solving means that we must do something different so that problem solving is appropriately assessed.

Your goals need to be considered as you plan how to evaluate. If you want children to solve both routine and nonroutine problems, then you must include both types of problems in your evaluation. If you are emphasizing the teaching of problem-solving strategies, then you must plan to assess students' ability to use strategies.

In addition to paper-and-pencil measures, such procedures as the following should be considered:

- Face students with a problem-solving situation and observe how they meet it. Such points as these might be considered:
  - Is there evidence of careful reading of the problem?
  - Do individual children seem to have some means of beginning to attack a problem?
  - Do they apply a strategy, or do they try to use the last procedure they were taught?
  - Do they have another strategy to try if the first one fails?
  - How consistent and persistent are they in applying a strategy?
  - Are careless errors being made, and if so, when and why?
  - How long are they willing to keep trying to solve a problem?
  - How well are they concentrating on the task?
  - How quickly do they ask for help?
  - What strategies does each child use most frequently?
  - Do they use manipulative materials?
  - What do their behaviors and such factors as the expressions on their faces indicate about their interest and involvement? (Reys et al., 1984, p. 33)

- Interview students. Interviews let you delve into how a student goes about solving a problem, and why he or she does it that way.
  - Basically, you need to (1) present the student with a problem; (2) let the student find a solution, describing what he or she is doing; and (3) challenge the student, eliciting specific details on what he or she is doing and why. (Reys et al., 1984, p. 33)

It is helpful to have an exact record of what is said, so notes should be taken or a tape-recorder used. This makes it easier to analyze the student's thinking later, perhaps more carefully and from a different perspective than when involved in the interview.

- Have students describe to a group how they solved a problem. Again, taking notes will be helpful.

- Have one student teach another how to solve a problem. Observing this process not only can provide information on the students' thinking, but also on the teaching strategies they've absorbed.

- Make an inventory or checklist. An inventory can be used to check on whether a student can solve a problem with a specified strategy, or solve a problem using two or three
strategies. The goal is merely the answer; the emphasis is on finding out whether or not students can use the strategies.

Paper-and-pencil tests can also be used to assess the ability to solve problems. In recent years, several tests have been developed that attempt to assess various aspects of Polya's four-step model of problem solving:

- First, understand the problem.
- Second, devise a plan for solving it.
- Third, carry out your plan.
- Fourth, look back to examine the solution obtained. (Polya, 1973)

One test of this type which is widely used was developed by Wearne and Romberg (Wearne, 1977). It provides information about the child's mastery of the prerequisites of the problem-solving questions posed. To provide this additional information, each problem-solving question is preceded by two other questions. One question assesses the child's understanding of the information contained in the item stem of the problem-solving question; a second question assesses the child's knowledge of an underlying concept of the problem-solving question.

Schoen and Oehmke (1980) reported on the development of the Iowa Problem Solving Test. The goal was to "produce an easily administered test that provides information about the problem-solving subskills that is highly correlated with data from individual interview settings" (p. 218). Three scores are provided for each student, dealing with understanding problems, applying the solution strategies chosen, and looking back at the solution. Sample items are included for each subtest; for instance, this is the sample item for "looking back", which requires the student to identify problems that can be solved in the same way as a given problem, to determine the effect of varying the conditions in a given problem, or to evaluate a given solution strategy.

Shelley has 75 marbles, which is 11 more than twice as many as Karen has. To find how many marbles Karen has, Shelley added 75 + 11 and got 86. She then said Karen has 43 marbles. Is Shelley right?

1) Yes.
2) No. She should have multiplied 86 x 2 and got 172.
3) No. She should have subtracted 75 - 11 = 64. Then 52 is the right answer.
4) No. She should have multiplied 11 x 2 = 22. Then 75 - 22 = 53 is the right answer.

A somewhat similar test has recently been developed by Hofmann (1966). Instead of developing three subtests which cluster items of a particular type, she clustered items assessing four stages of the problem-solving process — read, select a strategy, solve, and review and extend — with each problem. An item from her pool of 200 items being verified is:

A pirate found a treasure chest containing silver coins. He buried half of them and gave half of the remaining coins to his mother. If he was left with 4500 coins, which he put in a black sack, how many were in the treasure chest that he found?

1) To solve this problem, you need to know the:
   a) treasure chest had no gold coins.
   b) black sack had half of the original coins.
   c) pirate buried more coins than he gave his mother.
   d) number of coins the pirate found was less than 5000.

2) To solve this problem, you would most likely:
   a) start with the number of coins in the pirate's black sack and then work backwards.
   b) draw a picture of a treasure chest.
   c) pretend the pirate had fewer coins.
   d) guess a number and then check to see if it is correct.

3) The number of coins in the treasure chest when the pirate found it was:
   a) less than 5000.
   b) more than 5000 but less than 9000.
   c) exactly 9000.
   d) more than 9000.

4) After solving the problem, you know that the pirate gave his mother:
   a) 1/2 of the coins in the chest.
   b) 1/3 of the coins in the chest.
   c) 1/4 of the coins in the chest.
   d) 1/5 of the coins in the chest.

Perhaps it is apparent that the type of problems used by Schoen and Oehmke and by Hofmann differ. The problem with the marbles is somewhat similar to items found in many textbooks, while the problem of the pirate and the coins is nonroutine. The type of problem must be considered when selecting a test, for the test must assess what has been taught. Results of Hofmann's test would probably not be comparable for students who had had experience with nonroutine problems and those who lacked such experience.

The need to match test and curriculum has been emphasized by several analyses. Knifong (1980), for instance, found considerable variation in computational procedures and in difficulty level among the word problem sections of eight standard achievement tests for grade 6. Some problem-solving objectives on the North Carolina Minimum Competency Test had little coverage in the two textbook series which were analyzed (and widely used in North Carolina schools), according to Rogers (1981). And in a detailed analysis of the content of four textbooks and five tests for grade 4, Freeman et al. (1983) reported that:

- 6 topics were in all textbooks and tests;
- 3 topics were in books but not tests;
- 3 topics were on tests, with limited attention in books;
- 10 topics were in all books, some tests; and
- 385 topics were 'covered' by a single textbook or a single test!

Thus, it is clearly both important and necessary to assure that what is tested is being taught.

One further comment about the development of multiple-choice tests should be made. In addition to developing tests which assess the problem-solving steps or the use of strategies, it is also possible to develop tests to assess particular understanding about the mathematical concepts involved in
the problem (acknowledging that some would term this “understanding”). One such item is:

100 is multiplied by a number smaller than 1.

The answer has to be:

a) greater than 100
b) less than 100
c) a fraction
d) 0

Items can also be developed to assess such problem-solving skills as:

- Identifying needed information from extraneous information
- Estimating answers
- Recognizing a reasonable answer
- Making valid conclusions from the information given

... and so on through the list of skills students should know.

Once again, the emphasis is on matching what is evaluated with what needs to be evaluated.

Concluding Comment

In several different ways, this bulletin has attempted to promote awareness that there are many methods that should be used in mathematics classrooms to evaluate the effectiveness of instruction. If we are to change perceptions about mathematics, this may be imperative. The Mathematical Sciences Education Board believes that it is. The approach the Board is taking of considering curriculum, instruction, and evaluation may be the key to changing all three. Both assessment data and observations in classrooms indicate the necessity of this.

REFERENCES


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