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ABSTRACT

The Auto-Regressive Integrated Moving Average (ARIMA) Models, often referred to as Box-Jenkins models, are regression methods for analyzing sequential dependent observations with large amounts of data. The Box-Jenkins approach, a three-stage procedure consisting of identification, estimation and diagnosis, was used to select the most appropriate stochastic model for describing undergraduate grade point average. The findings, based on a half-century of data from two universities, showed that meaningful mathematical models can be created to describe the time series of changes in the yearly grade point average. The models are tentative because of the small number of available observations and their relative complexity. A mathematical description of the time series of grades is complex enough to suggest that no simple answer may suffice. The data appears to be best modeled by an approach that postulates random shocks persisting for only a finite time, yet each of which can be represented as an exponentially weighted average of all previous observation. (JAZ)

TIME SERIES ARIMA MODELS
OF UNDERGRADUATE GRADE POINT AVERAGE

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Abstract

The Box-Jenkins approach to time series analysis, a regression method for analyzing sequential dependent observations, was used to select the most appropriate stochastic model for describing undergraduate grade point averages. The technique, applied to approximately a half century of data from two universities, suggested that the moving average model provided the optimal fit. Suggestions were made for further exploration of GPA data.

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TIME SERIES ARIMA MODELS
OF UNDERGRADUATE GRADE POINT AVERAGE

Whenever a phenomena is observed over time, it is often useful to search for temporal patterns within the data. Economists have studied stock market prices, sociologists have examined population levels, and psychologists have investigated changes in the incidence of depression. For such purposes, a variety of time series analysis procedures have been developed, derived primarily from the theory of multiple regression. These techniques require data gathered from at least fifty time periods (McCleary and Hay, 1980, p. 20). Since archival data covering this many time periods is not as commonly collected in education as in some other fields, these mathematical approaches are not as widely used in educational research. It is the purpose of this paper to illustrate such an application, using undergraduate grade point averages.

Although educational institutions evaluate their students each term, a single group of pupils is not often evaluated fifty times on the same variable, as would be required for a time series analysis. However, a meaningful time series can be realized by obtaining the average grades given during each of the grading periods across a lengthy time span. For about the last half century, many universities and colleges have adopted a 5-point grading scale, using either the letters A through E or the numbers 1 through 5. Some of the institutions calculated, at each grading period, the average of grades awarded to their students, with the intent of maintaining reasonable consistency in their grading standards both among their departments and across time. Approximately fifteen years ago, reports began appearing that a conspicious increase was occuring each year in the grading patterns at many institutions (Birnbaum, 1977). Although that pattern appears to have abated during the past few years (Suslow, 1977), grades remain at a noticably higher level than prior to the increase.

A variety of factors have been suggested to explain the phenomena of institutional grade average fluctuation (Birnbaum, 1977), but there has been a lack of data that support the proposed explanations. Rogers (1983) examined several independent variables (demographic and economic) for the possibility of explaining temporal variation over an extended time frame, but found each of them lacking in explanatory power.

Any "explanation" of a phenomena implies that the phenomena can be adequately described. Mathematical models, and regression models in particular, are appropriate for such a description, but an examination of the literature suggests that most authors rely solely on visual graphs rather than employing mathematical modeling. It was the purpose of this study to use a stochastic time series approach to generate mathematical models that might appropriately describe the entire sequence of grade point data.

Method

Sample

Grade point average data were collected from two midwestern universities for about a fifty year span. For the first, hereafter called University A, data was collected for each year from 1929 through 1982. This data is plotted as a time series plot in Figure 1. For the second institution, hereater called University B, data was collected each year from 1932 to 1982, except for the years 1943 through 1946, when no data was available. This data is plotted in Figure 2.

Procedure

These data were analyzed with the time series analysis procedures brought together in 1970 by George E. P. Box and Gwilyn M. Jenkins, in their volume entitled Time Series Analysis: Forecasting and Control (revised edition 1976). These Auto-Regressive Integrated Moving Average (ARIMA) models (often referred to as "Box-Jenkins" models) require a large amount of data. However, when data are collected over an extended time period, as in this study, there is the possibility that the social meaning of the data could change over time. Thus, it becomes difficult to assign the same interpretation to the data at the beginning and end of the series. Nonetheless, the study of temporal patterns is an intriguing one, and with the development of appropriate computer software, the Box-Jenkins methods have become available to a much wider audience.

McCleary and Hay (1980) have prepared a treatise designed to encourage the use of the Box-Jenkins analysis for social science data, and to explicate strategies for both analyzing the data on the computer and presenting the computer output. Their strategies undergird the analysis in this study. The data was processed on a Harris computer, using MINITAB (Ryan, et al., 1982). Other approaches and other computer programs could have been used, but this was the one available for this project. The reader will need to interpret the methodological procedure of this study in that light.

The empirical identification procedures recommended by Box and Jenkins require an analysis of the autocorrelation function (ACF) and the partial autocorrelation (PACF) of the time series. The graphed ACF and PACF for both of the University time series are shown in Figures 3 and 4. The ACF is a set of correlations, each one of which represents the correlation between the original sequence and itself when lagged k units. For observations close together, e.g., 1 or 2 lags, we most often find a higher correlation than for observations further apart, as is typified in Figures 1 and 2, where the correlations are slowly dying out as the lags increase. This dying out phenomena is a consequence of the fundamental tenent of the ARIMA model, namely that the effect of any given input to the system declines over time. (Note that this is just the opposite of a time series of a bank savings account where, assuming a constant interest rate, the compounded interest from the first dollar invested is always larger than that from any subsequent dollar invested.) When the data is properly modeled, the residuals (errors resulting from the model) should be randomly distributed, and thus yield an ACF with with values that are all statistically non-significant. The goal of the Box-Jenkins approach is to find such a model.

The Box-Jenkins approach is a three stage procedure to build a model, consisting of Identification, Estimation, and Diagnosis. Each of these will be illustrated in the following analysis. The cycle iterates until an interpretable solution is found.

University A

Identification.

An examination of the ACF of the raw data (Figure 3) shows that the ACF falls to zero slowly, indicating that there is a strong systematic trend in the data. The most common method for removing this trend is to transform the data by replacing each observation with the difference between it and the preceding observation. When this difference transformation is complete, the ACF is again computed. Figure 5 shows the ACF for the differences. The values are much smaller, indicating almost random data. However, there are some spikes, which may be due to sampling error or to some systematic process, so further analysis is required.

The PACF is interpreted similar to the ACF, except that each value is the correlation between observations _k_ units apart _after_ the correlation at intermediate lags has been controlled or "partialled out". The PACF in Figure 3 shows a single spike, which may be the result of what is called a moving average (MA) component. This moving average component can be conceptualized as a random "shock" which is added to each observation to obtain the predicted value for the next observation.

The distinguishing characteristic of a moving average process is the finite duration of the shock. The shock persists for q observations and then is completely suppressed (McCleary and Hay, p. 61). Such a "shock" might be the result of the new grades that are added each term for each particular student. Since the majority of students will leave the institution after four years, the impact of any particular student will vanish when that individual leaves.

From the ACF and PACF we can now tentatively "identify" the model as an ARIMA (0, 1, 1). The zero indicates that there is no auto regressive (AR) term, the middle 1 indicates that differencing is to be used (this is the Integrative (I) term), and the last 1 indicates a moving average (MA) term.

Estimation.

When the estimates of the parameters were computed, it was found that the (0, 1, 1) model produced a t-value of only 1.23 for the MA term. Since this value was not statistically significant at the .05 level (nor anywhere near there), the model was rejected, and the procedure returned to the identification stage.

Identification.

It might be useful at this point to emphasize that since the estimated ACF and PACF are based on very small samples, they are subject to relatively large sampling errors. Consequently, any identification is very tentative.

Because the ACF and PACF for first differences appeared rough, it seemed appropriate to take second differences, i.e., differences between the difference scores. Figure 6 shows the resulting ACF and PACF. They appear more interpretable, suggesting a (0, 2, 1) model. An examination of Figure 1 also suggested that the variance was not constant across time. To attempt to correct this, a logarithmic transformation of the data was performed.

<u>Estimation</u>.

Table 1 shows the results of estimating the (0, 2, 1) model. The moving average parameter of .9767 satisfies the stationarity requirement that its absolute value be less than 1.0, and is also statisti[illegible]ally significant at less than the .05 level.

<u>Diagnosis</u>.

The simplest diagnostic procedure is to compare the results of the given model and alternative models. In this way, it can be shown that a parti[illegible]ular model is optimal in that neither a simpler nor a more complex model will suffice. The simpler model (0, 1, 1) was already shown to be inadequate. The more complex model (0, 2, 2) yielded a statistically insignificant second MA term, so it was rejected. The (1, 2, 1) model was also tested, but the AR term was insignificant. Thus, the ARIMA (0, 2, 1) model was accepted as the "best" fit.

The equation generated by this procedure can be conveniently written in the following form: $(1-B)^2y_t = (1-.9767B)a_t$ where B is the backshift operator, and a_t is the random-shock ele[illegible] (McCleary and Hay, (1980), p. 46, 64). The backshift operator is defined [illegible]$_t = y_{t-1}$ and follows the usual algebraic rules. The operator (1-B) re[illegible]ents first differences and $(1-B)^2$ represents second differences.

The random shock element a_t is the stochastic component in the equation. In the ARIMA model this moving average component can be shown to be mathematically equivalent to the exponentially weighted average of all previous observations (Pankratz, 1983, p. 49, 109; McCleary and Hay, (1980), p. 63).

University B

Identification.

An examination of the estimated ACF and PACF of the raw data (Figure 4) suggests that this data is also non-stationary and needs to be differenced. The single spike on the PACF suggests a (0, 1, 1) model.

Estimation.

The (0, 1, 1) model produced an estimate of the Moving Average parameter with a t-value of .23. Since this was far from statistical significance, modifications needed to be made. Second differences were used, since the data appeared to approximate a quadratic trend. The (0, 2, 1) model produced a parameter with a t-value of 11.12, which was highly significant.

Diagnosis.

The model was first diagnosed by comparing it with a more complex model. Accordingly, a (0, 2, 2) model was tested. It produced significant t-values for both MA parameters, as shown in Table 1. To compare the two models, the mean squares of the residuals was computed. The (0, 2, 1) model yielded MSR = .0011274, while the (0, 2, 2) model yielded MSR = .0009977. Finally, a (1, 2, 2) model (yet more complex) was tested, but it yielded MSR = .0011641. Consequently, the (0, 2, 2) model was favored, since it yielded the smallest MSR.

The ACF and PACF for the Residuals of model (0, 2, 2) are shown in Figure 7. No spikes are shown at lag 1 or any other lags. The residuals appear to meet the diagnostic criteria, so the model is accepted.

The model can be conveniently written as $(1-B)^2 y_t = (1 - 1.1475B + .5302B^2)a_t$.

Seasonality

The data analyzed above were yearly data, but many social science data are collected on a monthly or quarterly basis and show strong seasonal components. To test the hypothesis that a seasonal trend occured in grade averages within each year, data was obtained from University B for each quarter term for a 44 year span. The raw data plot is shown in Figure 8, where the numbers 1, 2, and 3 designate Fall, Winter, and Spring terms, respectively. A strong seasonal trend appears to be a noticable feature, so a time series seasonality analysis was performed. Seasonality is defined as "any cyclical or periodic fluctuation in a time series or repeats itself at the same phase of the cycle or period" (McCleary and Hay, 1980, p. 80).

Identification

Figure 9, showing the estimated ACF and PACF of the first differenced data, indicates a strong seasonal trend. The seasonality factor is known to be three. Further, most social science process yield regular and seasonal factors of the same type. Since the previous analysis of University B suggested ARIMA (0, 2, 2) model, it was decided to first test the ARIMA $(0, 2, 2)(0, 2, 2)_3$ model. However, that model produced an estimate of a Moving Average parameter with a t-value of only .41. Modifications were then made upon the model. After several trials, the ARIMA $(0, 1, 1),(0, 1, 1)_3$ model was found to produce statistically significant terms, as shown in Table 3, with MSR = .0006550.

Diagnosis

The $(0, 1, 1),(0, 1, 1)_3$ model was diagnosed by comparing it with several competitive models. Adding a second MA parameter proved non-significant.

Using an AR term in either the regular or the seasonal component (which might be inferred from the slow decay in the ACF) yielded a significant term, but the MSR term was slightly larger. A simpler model, postulating only a seasonal term (0, 1, 0), $(0, 1, 1)_3$ also produced a larger MSR.

The ACF and PACF for the residuals of the model are shown in Figure 10. Since no spikes are observed, the model is accepted.

A time series with both regular and seasonal components is expressed as a multiplicative model. The accepted model is written as:

$(1-B)(1-B^3)y_t = (1-.37B)(1-.77B^3)a_t.$

Conclusion

This paper has suggested that meaningful mathematical models can be created to describe the time series of changes in the yearly grade point average at a university. The models are very tentative, partly because of the small number of available observations and also because of their relative complexity.

While this paper has not answered the questions about the so-called "grade inflation," it has indicated that a mathematical description of the time series of grades is sufficiently complex to suggest that no simple answer may suffice. The data is unstationary, as shown by the need for differencing. It further appears to be best modeled by an approach that postulates random shocks that persist for only a finite time, yet each of which can be represented as an exponentially weighted average of all previous observations. This perhaps reflects both the influx of new students and the persistent effects of traditional grading practices.

Data for this study was available for only two institutions of higher education, so the generalizability of the results is limited. Studies with data from other institutions would serve to indicate the existence of general patterns across institutions.

References

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Ryan, T. A., Joiner, B. L., and Ryan, B. F. (1982). Minitab reference manual. University Park, Pa: Pennsylvania State University.

Table 1. Parameter estimates for the ARIMA (0, 2, 1) model. University A.

FINAL ESTIMATES OF PARAMETERS

NUMBER	TYPE	ESTIMATE	ST. DEV.	T-RATIO
1	MA 1	0.9767	0.0439	22.26

DIFFERENCING. 2 REGULAR

RESIDUALS. SS = 0.0191286 (BACKFORECASTS EXCLUDED)
DF = 51 MS = 0.0003751
NO. OF OBS. ORIGINAL SERIES 54 AFTER DIFFERENCING 52

Table 2. Parameter estimates for the ARIMA (0, 2, 2) model. University B.

FINAL ESTIMATES OF PARAMETERS

NUMBER	TYPE	ESTIMATE	ST. DEV.	T-RATIO
1	MA 1	1.1475	0.1224	9.38
2	MA 2	-0.5302	0.1220	-4.35

DIFFERENCING. 2 REGULAR
RESIDUALS. SS = 0.0429018 (BACKFORECASTS EXCLUDED)
DF = 43 MS = 0.0009977
NO. OF OBS. ORIGINAL SERIES 47 AFTER DIFFERENCING 45

Table 3. Parameter estimates for the ARIMA (0, 1, 1), $(0, 1, 1)_3$ model University B.

FINAL ESTIMATES OF PARAMETERS

NUMBER	TYPE	ESTIMATE	ST. DEV.	T-RATIO
1	MA 1	0.3704	0.0814	4.55
2	SMA 3	0.7728	0.0564	13.70

DIFFERENCING. 1 REGULAR 1 SEASONAL DIFFERENCES OF ORDER 3

RESIDUALS. SS = 0.0825284 (BACKFORECASTS EXCLUDED)
DF = 126 MS = 0.0006550
NO. OF OBS. ORIGINAL SERIES 132 AFTER DIFFERENCING 128

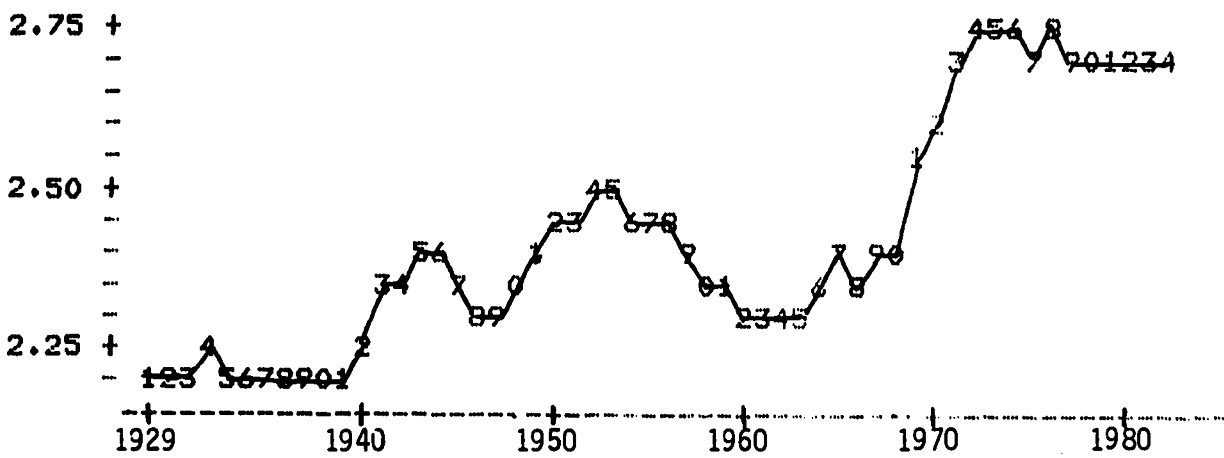

Figure 1. Grade Point Average (GPA) at University A, by year, from 1929 to 1982. (Prior to 1944 the data is for the whole year; afterward it is for fall term.)

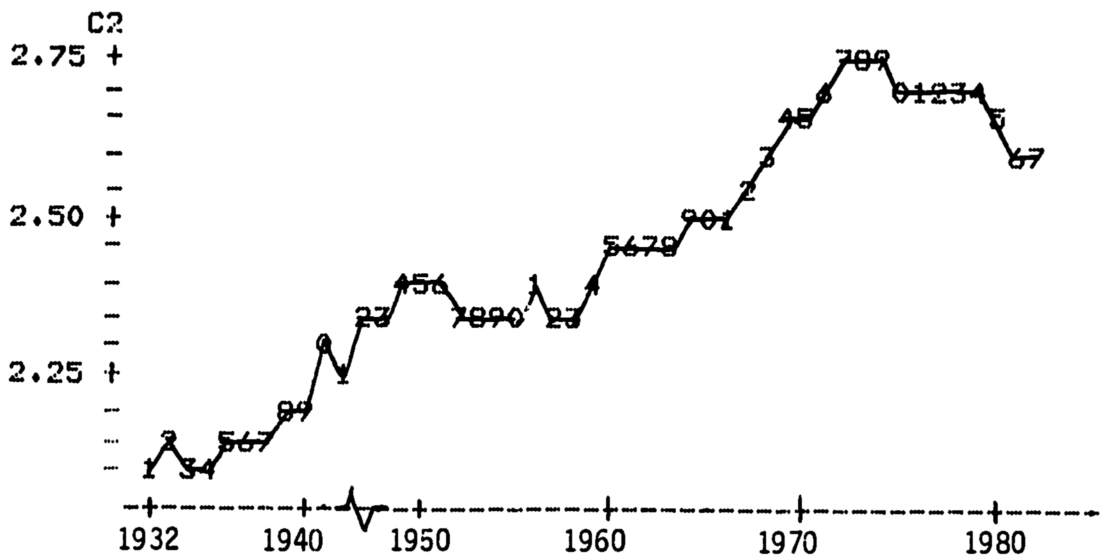

Figure 2. GPA at University B, by year, from 1932 to 1982 (fall term). Far 1943-1946, data are not available.

Autocorrelations

```
             -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
               +----+----+----+----+----+----+----+----+----+----+
  1   0.931                              XXXXXXXXXXXXXXXXXXXXXXXX
  2   0.849                              XXXXXXXXXXXXXXXXXXXXXX
  3   0.753                              XXXXXXXXXXXXXXXXXXXX
  4   0.666                              XXXXXXXXXXXXXXXXXX
  5   0.567                              XXXXXXXXXXXXXXX
  6   0.484                              XXXXXXXXXXXXX
  7   0.392                              XXXXXXXXXXX
  8   0.312                              XXXXXXXXX
  9   0.221                              XXXXXXX
 10   0.119                              XXXX
 11   0.026                              XX
 12  -0.042                             XX
 13  -0.084                            XXX
 14  -0.112                           XXXX
 15  -0.101                           XXXX
 16  -0.083                            XXX
 17  -0.060                            XXX
```

Partial Autocorrelations

```
             -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
               +----+----+----+----+----+----+----+----+----+----+
  1   0.931                              XXXXXXXXXXXXXXXXXXXXXXXX
  2  -0.140                          XXXXX
  3  -0.135                           XXXX
  4   0.028                              XX
  5  -0.145                          XXXXX
  6   0.064                              XXX
  7  -0.137                           XXXX
  8   0.017                              X
  9  -0.134                           XXXX
 10  -0.185                         XXXXXX
 11   0.051                              XX
 12   0.066                              XXX
 13   0.126                              XXXX
 14  -0.008                              X
 15   0.237                              XXXXXXX
 16  -0.010                              X
 17  -0.034                             XX
```

Figure 3. Estimated ACF and PACF for GPA. University A.

Autocorrelations

```
             -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
              +----+----+----+----+----+----+----+----+----+----+
 1   0.939                             XXXXXXXXXXXXXXXXXXXXXXXX
 2   0.899                             XXXXXXXXXXXXXXXXXXXXXXX
 3   0.827                             XXXXXXXXXXXXXXXXXXXXXX
 4   0.749                             XXXXXXXXXXXXXXXXXXXX
 5   0.673                             XXXXXXXXXXXXXXXXXX
 6   0.595                             XXXXXXXXXXXXXXXX
 7   0.511                             XXXXXXXXXXXXXX
 8   0.436                             XXXXXXXXXXXX
 9   0.356                             XXXXXXXXXX
10   0.291                             XXXXXXXX
11   0.220                             XXXXXXX
12   0.174                             XXXXX
13   0.129                             XXXX
14   0.093                             XXX
15   0.060                             XX
16   0.032                             XX
```

Partial Autocorrelations

```
             -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
              +----+----+----+----+----+----+----+----+----+----+
 1   0.939                             XXXXXXXXXXXXXXXXXXXXXXXX
 2   0.142                             XXXXX
 3  -0.263                      XXXXXXXX
 4  -0.180                         XXXXX
 5   0.003                             X
 6   0.000                             X
 7  -0.113                          XXXX
 8  -0.002                             X
 9  -0.042                            XX
10   0.043                             XX
11  -0.069                           XXX
12   0.116                             XXXX
13   0.039                             XX
14  -0.040                            XX
15  -0.060                            XX
16  -0.006                             X
```

<u>Figure 4</u>. Estimated ACF and PACF for GPA. University B.

Autocorrelations

```
              -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
                +----+----+----+----+----+----+----+----+----+----+
  1    0.128                             XXXX
  2    0.141                             XXXXX
  3    0.046                             XX
  4    0.029                             XX
  5   -0.322                     XXXXXXXXX
  6   -0.008                             X
  7   -0.111                          XXXX
  8   -0.065                           XXX
  9    0.086                             XXX
 10   -0.166                         XXXXX
 11   -0.157                         XXXXX
 12    0.008                             X
 13   -0.104                          XXXX
 14   -0.265                      XXXXXXXX
 15    0.025                             XX
 16   -0.085                           XXX
 17   -0.035                            XX
```

Partial Autocorrelations

```
              -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
                +----+----+----+----+----+----+----+----+----+----+
  1    0.128                             XXXX
  2    0.126                             XXXX
  3    0.015                             X
  4    0.004                             X
  5   -0.345                    XXXXXXXXXX
  6    0.071                             XXX
  7   -0.038                            XX
  8   -0.025                            XX
  9    0.158                             XXXXX
 10   -0.354                    XXXXXXXXXX
 11   -0.100                          XXXX
 12    0.068                             XXX
 13   -0.145                         XXXXX
 14   -0.133                          XXXX
 15   -0.081                           XXX
 16   -0.162                         XXXXX
 17    0.043                             XX
```

Figure 5. Estimated ACF and PACF for first differences. University A.

Autocorrelations

```
           -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
             +----+----+----+----+----+----+----+----+----+----+
 1  -0.505               XXXXXXXXXXXXXX
 2   0.073                           XXX
 3  -0.063                          XXX
 4   0.196                           XXXXXX
 5  -0.387                  XXXXXXXXXXX
 6   0.251                           XXXXXXX
 7  -0.097                          XXX
 8  -0.057                           XX
 9   0.237                           XXXXXXX
10  -0.139                         XXXX
11  -0.085                          XXX
12   0.143                           XXXXX
13   0.037                           XX
14  -0.265                     XXXXXXXX
15   0.217                           XXXXXX
16  -0.097                          XXX
17   0.036                           XX
```

Partial Autocorrelations

```
           -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
             +----+----+----+----+----+----+----+----+----+----+
 1  -0.505               XXXXXXXXXXXXXX
 2  -0.244                    XXXXXXX
 3  -0.201                     XXXXXX
 4   0.118                           XXXX
 5  -0.318                  XXXXXXXXX
 6  -0.139                         XXXX
 7  -0.131                         XXXX
 8  -0.287                     XXXXXXXX
 9   0.236                           XXXXXXX
10  -0.073                          XXX
11  -0.179                        XXXXX
12   0.039                           XX
13  -0.018                           X
14  -0.072                          XXX
15  -0.028                           XX
16  -0.206                     XXXXXX
17   0.021                           XX
```

Figure 6. Estimated ACF and PACF for second differences. University A.

Autocorrelations

```
            -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
              +----+----+----+----+----+----+----+----+----+----+
 1    0.157                            XXXXX
 2   -0.056                           XX
 3   -0.098                          XXX
 4   -0.023                           XX
 5   -0.122                         XXXX
 6   -0.094                          XXX
 7   -0.131                         XXXX
 8   -0.065                          XXX
 9   -0.055                           XX
10   -0.094                          XXX
11   -0.012                            X
12    0.125                            XXXX
13    0.152                            XXXXX
14    0.064                            XXX
15   -0.086                          XXX
16   -0.013                            X
```

Partial Autocorrelations

```
            -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
              +----+----+----+----+----+----+----+----+----+----+
 1    0.157                            XXXXX
 2   -0.082                          XXX
 3   -0.078                          XXX
 4    0.001                            X
 5   -0.135                         XXXX
 6   -0.065                          XXX
 7   -0.131                         XXXX
 8   -0.065                          XXX
 9   -0.079                          XXX
10   -0.142                        XXXXX
11   -0.034                           XX
12    0.059                            XX
13    0.065                            XXX
14    0.003                            X
15   -0.123                         XXXX
16    0.002                            X
```

Figure 7. Estimated ACF and PACF for residuals from Arima (0, 2, 2) model. University B.

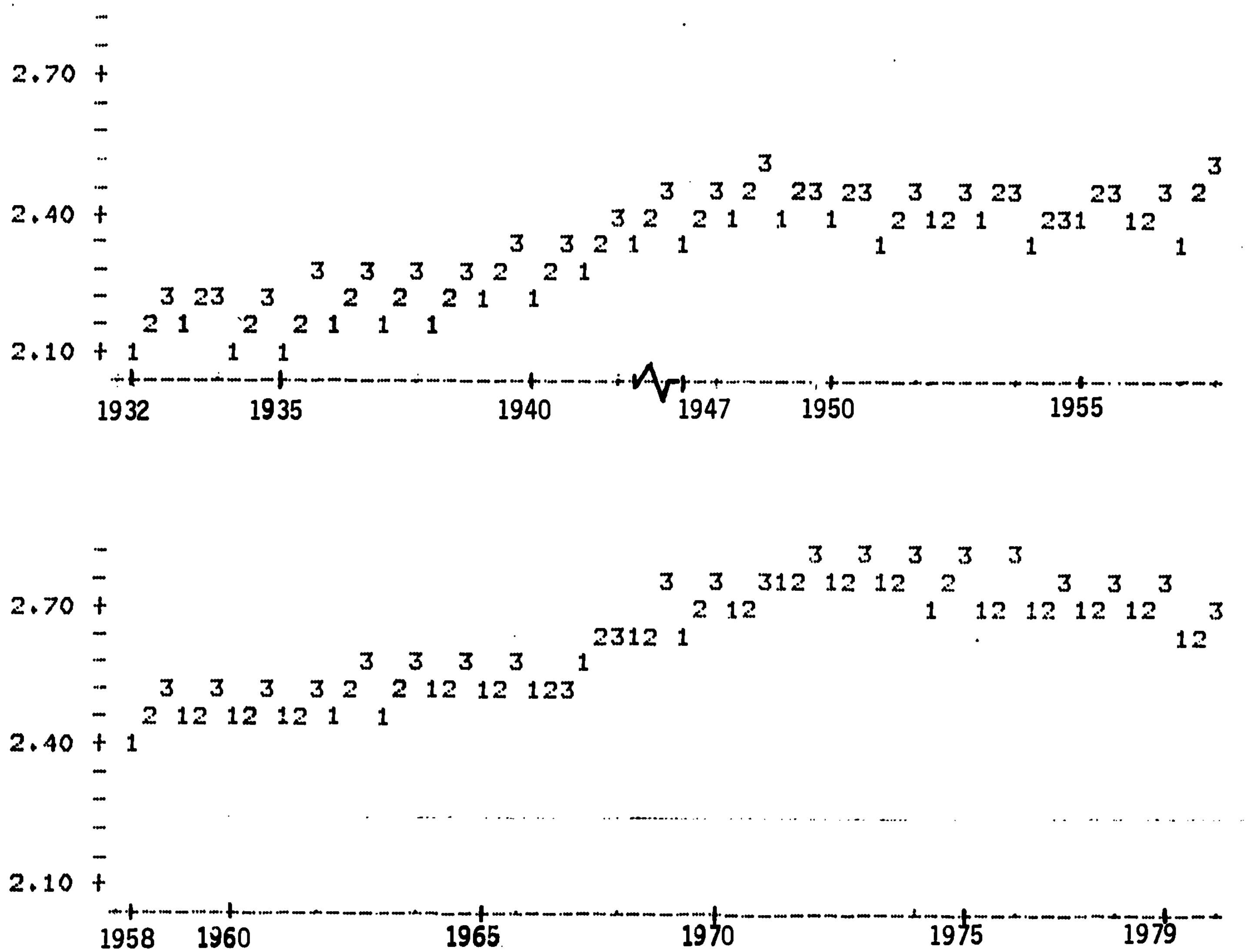

Figure 8. GPA at University B, by quarter, from fall term 1932 to spring term 1982. For 1943-1946, data are not available.

Autocorrelations

```
             -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
               +----+----+----+----+----+----+----+----+----+----+
  1  -0.461                 XXXXXXXXXXXXX
  2  -0.426                  XXXXXXXXXXXX
  3   0.857                          XXXXXXXXXXXXXXXXXXXXXX
  4  -0.419                   XXXXXXXXXXX
  5  -0.427                  XXXXXXXXXXXX
  6   0.808                          XXXXXXXXXXXXXXXXXXXXX
  7  -0.375                    XXXXXXXXXX
  8  -0.396                   XXXXXXXXXXX
  9   0.762                          XXXXXXXXXXXXXXXXXXXX
 10  -0.351                    XXXXXXXXXX
 11  -0.383                   XXXXXXXXXXX
 12   0.729                          XXXXXXXXXXXXXXXXXXX
 13  -0.351                    XXXXXXXXXX
 14  -0.374                    XXXXXXXXXX
 15   0.707                          XXXXXXXXXXXXXXXXXXX
 16  -0.344                    XXXXXXXXXX
 17  -0.343                    XXXXXXXXXX
 18   0.644                          XXXXXXXXXXXXXXXXX
 19  -0.291                      XXXXXXXX
 20  -0.360                    XXXXXXXXXX
```

Partial Autocorrelations

```
             -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
               +----+----+----+----+----+----+----+----+----+----+
  1  -0.461                 XXXXXXXXXXXXX
  2  -0.811         XXXXXXXXXXXXXXXXXXXXX
  3   0.471                          XXXXXXXXXXXXX
  4   0.044                          XX
  5  -0.256                       XXXXXXX
  6   0.061                          XXX
  7   0.037                          XX
  8   0.043                          XX
  9   0.102                          XXXX
 10   0.020                          X
 11  -0.007                          X
 12   0.108                          XXXX
 13  -0.037                         XX
 14  -0.090                        XXX
 15   0.023                          XX
 16  -0.039                         XX
 17   0.029                          XX
 18  -0.175                      XXXXX
 19   0.042                          XX
 20  -0.115                       XXXX
```

Figure 9. Estimated ACF and PACF for first differences on quarter data. University B.

Autocorrelations

```
          -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
            +----+----+----+----+----+----+----+----+----+----+
 1  -0.021                          XX
 2   0.008                           X
 3   0.179                           XXXXX
     0.029                           XX
 5  -0.112                        XXXX
 6  -0.015                           X
 7   0.083                           XXX
 8   0.018                           X
 9  -0.037                          XX
10   0.091                           XXX
11   0.077                           XXX
12  -0.020                           X
13  -0.082                         XXX
14   0.001                           X
15   0.032                           XX
16  -0.126                        XXXX
17   0.054                           XX
18  -0.144                       XXXXX
19   0.052                           XX
20  -0.125                        XXXX
```

Partial Autocorrelations

```
          -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
            +----+----+----+----+----+----+----+----+----+----+
 1  -0.021                          XX
 2   0.008                           X
 3   0.179                           XXXXX
 4   0.038                           XX
 5  -0.118                        XXXX
 6  -0.056                          XX
 7   0.078                           XXX
 8   0.069                           XXX
 9  -0.022                          XX
10   0.046                           XX
11   0.059                           XX
12   0.007                           X
13  -0.105                        XXXX
14  -0.047                          XX
15   0.053                           XX
16  -0.067                         XXX
17   0.053                           XX
18  -0.202                      XXXXXX
19   0.077                           XXX
20  -0.121                        XXXX
```

Figure 10. Estimated ACF and PACF for residuals from the ARIMA model $(0, 1, 1), (0, 1, 1)_3$. University B.