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ABSTRACT

The paper discusses the affective influences on students of problem solving instruction using computers, and the relationship of affective influences to student belief systems. It reviews affective issues and problem solving, and analyzes affect with particular reference to the work of G. Mandler. The paper considers types of congnitive processes, instructional environments and student belief systems. The issues are synthesized in a discussion on affect, computers, and problem solving. The implications for research are discussed. (JM)

TECHNOLOGY AND THE ROLE OF AFFECT
IN TEACHING MATHEMATICAL PROBLEM SOLVING

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ABSTRACT

Affective influences on the cognitive processes involved in mathematical problem solving play an important role in instruction. These influences are particularly important in computer-based mathematics instruction. Recent work in cognitive psychology suggests useful approaches to the study of affect in this instructional environment. This paper discusses the characteristics of these affective influences and their relationship to the belief systems of students.

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TECHNOLOGY AND THE ROLE OF AFFECT IN TEACHING MATHEMATICAL PROBLEM SOLVING

> The "emotions" are excellent examples of the fictional causes to which we commonly attribute behavior. (Skinner, 1953)

Research on the role of technology in mathematics education is at an early stage of its development. Initial explorations of ways to incorporate computers in instruction on problem solving appear promising (P. W. Thompson, 1985; Schwartz, 1986), but have yet to reach their full potential. As we see progress in research on technology and the teaching of problem solving, we need to remember that good instruction requires more than good software, and more than attention to the cognitive needs of students. We also need to keep in mind the affective influences on both students and teachers. In this paper, we discuss affective influences on students in the context of problem solving instruction using computers.

Recent research has made substantial progress in characterizing the cognitive processes that are necessary for students to do well in mathematical problem solving. However, the influence of affective factors on these cognitive processes has yet to be studied in detail. In fact, general discussions of how information processing technology shapes thinking sometimes avoid affective issues entirely (Perkins, 1985). This paper will first discuss a general theoretical framework for investigating the affective factors that help or hinder performance in mathematical problem solving using computers, and then suggest how research along these lines could help resolve some troubling research questions.

AFFECTIVE ISSUES AND PROBLEM SOLVING

When students are given a non-routine mathematical problem to solve, their reactions express a lot of emotion. If they work on the problem over an extended period, the emotional responses frequently become quite intense. Most students will begin work on the problem with some enthusiasm, treating it like a puzzle or game. After some time, the reactions become more negative. The students who have a plan to solve the problem may get stuck trying to carry out the plan. Usually they become quite tense, and often try to implement the same plan repeatedly, getting more frustrated with each unsuccessful attempt. If the students obtain a solution to the problem, they express feelings of satisfaction, even joy. If they do not reach a solution, they may angrily insist on help so that they can reduce their frustration.

Students who are solving mathematical problems with the aid of a computer also have intense emotional reactions. Here the situation is complicated by the difficulty of working with a machine. If the student is a novice at working with computers, the emotional reaction may be particularly stressful. On the other hand, if the student enjoys working with computers, the reaction may be very positive.

Student responses to computer-assisted instruction in non-routine problem solving tend to be quite intense, whether they are positive or negative. Despite these rather intense reactions by students, software developers have generally neglected affective issues in teaching problem solving. In fact, there is little instructional material related to mathematical problem solving that deals with affect in any substantive way. The role of affect remains a major underrepresented theme in research on problem solving (Silver, 1985).

AFFECTIVE ISSUES AND COGNITIVE PSYCHOLOGY

Research on problem solving needs a theoretical framework for affective issues that is compatible with the current emphasis on cognitive processes in problem solving. Mandler's work on "mind and emotion" (1975, 1984) provides a sound foundation for such a framework.

Mandler is a well-known cognitive theorist who also works on affective issues. He emphasizes the role of interruptions in an individual's plans or planned behavior as a major source of emotion. In psychological terms, these plans result from the activation of a schema that produces an action sequence, and this sequence has "a tendency to completion" (Mandler, 1984, p. 173). When the sequence is interrupted, the normal pattern of completion cannot occur. The result of the interruption is the physiological arousal of the individual. This arousal might be observed as muscle tension or rapid heartbeat, for example. Along with the arousal, the individual evaluates the meaning of the interruption, and interprets it as surprise, frustration, joy, or some other emotion.

Mandler's analysis of emotion seems particularly appropriate for instructional environments that promote mathematical problem solving. A problem is typically defined as a task where the goal is not immediately attainable, and where there is no obvious algorithm for the student to use. In other words, the problem solver is blocked. These blockages are a central part of mathematical problem solving, and constitute a major source of affect in problem solving. Given the inherent difficulties involved in producing good software, using computers in teaching problem solving is likely to generate even more blockages, more interruptions, more arousal, and more intense affective responses from students.

AN ANALYSIS OF AFFECT

When we observe problem solvers at work, we see the influence of affect. In order to describe what is happening during problem solving, we need to identify the major affective components of the problem solver's experience. Building on Mandler's (1984) work, the following major dimensions of affective factors in problem solving are proposed:

1. **Magnitude and Direction of the Emotion**. Affective influences vary in their magnitude (or intensity) and direction (positive or negative). The emotions observed in problem solving (frustration, joy) are generally more intense than the attitudes that research on the affective domain has usually addressed. Although we tend to emphasize the negative aspects of affect (e.g., the frustration of getting stuck on a problem), more positive emotions (such as the satisfaction of finding a solution, or the joy of making a conjecture) are also powerful influences on problem solving. Computers add to the magnitude of affective reactions to

problem solving. As Turkle (1984) notes, computers have a special "holding power" for students. This holding power is demonstrated by the intense concentration of individuals (including adults as well as students) at terminals; these individuals may be either at work or at play, but they are intensely involved in what they are doing.

2. <u>**Duration of the Emotion**</u>. As Kagan (1978) points out, duration is an important feature of an affective response. Affective reactions to getting stuck on a mathematical problem are typically intense, but generally not long in duration. Students who are frustrated will find a way to relieve that frustration relatively soon. If they are experienced problem solvers, they will try to think of a new heuristic that will help them over the blockage, or perhaps argue that they need some incubation time so that their unconscious mental processes can work on the problem. If they are novice problem solvers, they may just quit at the first sign of frustration and wait for help. The emotional ups and downs that are a part of solving non-routine problems are quite different from the typical studies of attitudes toward computers or some other subject. Most researchers assume that attitudes are stable and lasting; they are treated as though they are traits of the individual that are difficult to change. But for good problem solvers, the emotions change frequently. And the influence of these emotions on problem solving processes is expected to be an important factor in student performance.

3. <u>**Awareness of the Emotion**</u>. Novice problem solvers are frequently unaware of the strategies that they use in solving problems; they lack self-awareness (Kilpatrick, 1984), or the ability to reflect upon and analyze their own cognitive processes. Similarly, these problem solvers are unaware of the affective factors that are influencing them. The tendency of mental processes to become automatic (and hence unconsious) is well known, and seems to apply to the affective domain as well as the cognitive. It seems clear, for example, that some problem solvers automatically decide to quit whenever their first attempt at a solution is unsuccessful. Automatic responses can be controlled, even in cases of serious mental disturbances (see, e.g., Beck & Emery, 1985), so it should also be possible

for students who are relatively normal to control this tendency toward automaticity. First, however, students need to develop their awareness of the affective factors that are influencing their thinking. Once these emotions are brought to consciousness, students have the opportunity to bring them under control.

4. <u>Control of the Emotion</u>. Although there are some students with uncontrollable fears of mathematics or computers, most students do have the capacity to exercise some control of their emotions most of the time. But there are occasions when students will lose control of their emotions. For example, problem solvers sometimes get in a rut, and try the same solution strategy over and over and over again. They reach a point in the solution process where they are blocked, and their emotions take over. Their limited mental capacity (whether you call it short-term memory, M-space, or something else) is totally taken up with evaluating the physiological arousal that comes from the interruption (Mandler, 1984). In this situation, no new approaches can be considered; there is no room in short-term memory to generate new approaches. However, with appropriate instruction, students would be able to recognize occasions when their mental capacity is overloaded, and take appropriate steps to reduce their frustration and regain control of their problem-solving processes. For further discussion of the issue of control and metacognitive processes, see Garofalo & Lester (1985), Silver (1985), and Schoenfeld (1985).

5. <u>Types of Cognitive Processes</u>. Affective reactions may have different influences on different cognitive processes. Metacognitive and managerial processes seem particularly susceptible to the influence of the emotions. When short-term memory is overloaded with affective concerns, problem solvers have difficulty identifying an appropriate heuristic to use, and making managerial decisions about how long to use it. The student is likely to choose some strategy that doesn't require much mental effort--e.g., do some calculations--even if that strategy does not bring them any closer to a solution. Further, the influence of affect is different at the entry or planning phase of problem solving than it is during the review (or looking back) phase. During the entry phase, the most typical

emotion is frustration, where an inexperienced problem solver repeatedly uses the same plan and repeatedly finds that the path to a solution is blocked. In this case, dealing with this negative emotion (frustration or fear of failure) uses up the mental capacity of the students, and they cannot deal with the problem effectively. During the review phase, the problem solver typically feels satisfaction with finding an answer—any answer—and then neglects to check the answer for reasonableness or to look for more elegant solutions. In this case, the emotion is positive, but it also interferes with the students' ability to solve the problem. Thus we see that the emotions (negative in one phase, positive in the other), can keep students from performing at their best in problem solving. Metacognitive and managerial processes seem especially important when the computer is used as an instructional tool. Now problem solvers not only have to keep in mind how to manage their own problem-solving resources, but they also have to manage the resources of the computer.

6. **Types of Instructional Environments.** Within the environment of computer-assisted problem-solving instruction, there are still many varieties of instructional settings. The influence of affect appears to be quite different in small-group, rather than individual, instruction. Affective influences will also vary according to the amount of guidance provided by the teacher or the software, the kinds of assessment that students expect, and many other factors.

7. **Student Belief Systems**. Silver (1985) and Schoenfeld (1985) have argued persuasively that student belief systems are an important factor in mathematical problem solving. There are a number of different kinds of beliefs that play a special role in problem solving. Student beliefs about the content of the mathematics curriculum, for example, suggest that they see very little relationship between mathematics and any kind of higher-order thinking skills. Teacher beliefs about mathematics may suggest similar views (A. G. Thompson, 1985). The relationship of beliefs about mathematics to beliefs about self have important implications for problem solving. These beliefs appear to be related to the area of causal attributions (see, e.g., Reyes, 1984), but the relationships among affect,

beliefs, and causal attributions are yet to be clarified.

AFFECT, COMPUTERS, AND PROBLEM SOLVING

Technological tools, particularly computers, appear to have a special influence on affective factors, expecially in the context of problem solving. It seems that most people are initially quite anxious when they have to deal with a computer. This reaction seems to be as common among the mathematicians in my department as it is among the general population. It also seems to be more common among adults than children. I think of this reaction as "machine anxiety", since it generally centers around a fear of breaking the machine.

Although most people get over this reaction after an hour or two of work on a computer, there are some cases where machine anxiety can last for a much longer time. Since this extreme level of machine anxiety is not very common among students, we will not deal with it here. Also, since the computer is becoming increasingly omnipresent in our society, it seems reasonable to expect that this initial machine anxiety will continue to decline in importance.

Once students get over the initial machine anxiety, there are still a number of special affective difficulties that involve the computer. Turkle (1984), for example, reports that young children go through various stages of beliefs about computers. They wonder initially (age 6) if the computer is alive, but later (age 10) decide that a computer can't be alive since it does not show emotion. Older students may have a desire for mastery over the computer that interferes with their interest in mastery over the mathematics that we want them to learn. Beliefs about the reliability (or unreliability) of computers and the instructional software may also influence how students perform on problem-solving tasks.

When the computer is used as a tool for teaching mathematical problem solving, a variety of other issues related to affect become important. The Supposer software, for example, provides an interesting way to encourage students to sense the joy of making conjectures (Schwartz, 1986). It also has an impact on the sociological structure of the classroom, including the role of the teacher, as well as the role of the student (Kaput, 1986). This new software, as well as other software packages, could have a significant impact on the belief systems of both students and teachers about what mathematics is, and about what kinds of mathematical knowledge are important.

Work with students at San Diego State University provides some evidence on several of the points that have been discussed above. In interviews with prospective elementary teachers who were working with a transformational geometry microworld (P. W. Thompson, 1985), we were surprised to hear that a computer could have a personality. Other comments were more along the lines of what we expected. Students discussed the fear of getting stuck. "It's when there's no progress. It's when you keep running into brick walls." They also expressed concerns about making mistakes--"I didn't want to make a mistake on the screen"--even though erasing the screen was a simple and well-known maneuver. But when they solved a problem they noted the joy and satisfaction that they felt, and pointed out that solving a hard problem independently is much more satisfying that solving an easy problem, or solving a harder problem with help from the teacher. Additional interviews are now underway, and more data will be available at a later date.

In summary, we need to investigate affective influences on problem solving with special care in an environment that uses the computer as an instructional tool. The presence of the machine itself is enough to cause an emotional reaction from many students. After the student becomes accustomed to the machine, however, the nature of the problem itself becomes more important for most students than the instructional environment. The computer also may have a particularly strong impact on the beliefs of both students and teachers.

SUMMARY AND IMPLICATIONS

In planning research on the topic of technology and the teaching of problem solving, we need to incorporate work on affective influences. We need to analyze the characteristics of students' affective reactions, including their magnitude and direction, their duration, the level of awareness and control of the emotions, and their influence on cognitive processes in various instructional settings. We also need to investigate the belief systems that students exhibit, including their beliefs about computers, about problem solving, and about themselves. An analysis of these dimensions of affective influences in the problem solving environment should help us develop a theoretical structure witn a sound basis in cognitive psychology. Such a framework can provide direction to research on the uses of computers in the teaching of mathematicai problem solving.

The development of such a theory has important implications for mathematics education. It should help to improve instruction in mathematical problem solving for all students, especially for those who find problem solving particularly stressful. Moreover, research on affective issues should help explain the persistent and troubling differences between boys and girls in their enrollment in computer classes, as well as on their performance on problem solving tasks. Previous efforts to explain these differences have concentrated on factors like spatial ability, course enrollment patterns, and biological issues (McLeod, 1985). None of these approaches has been very helpful in explaining sex differences in problem solving achievement. More detailed investigations of affective factors may be useful in determining the sources of these differences in performance (Reyes, 1984).

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