A set of teaching and workshop materials has been developed to illustrate the different uses of mathematics in high school mathematics and science classes. It consists of this packet of workshop activities with detailed instructions on how to use them in a combined mathematics-science workshop and a related set of problems (with solutions) that focus on the mathematical topics most commonly used in science classes. The packet includes: (1) background information on the workshops; (2) description of materials provided; (3) workshop responses; (4) a description of four types of workshops; (5) directions for and strategies used in leading the different types of workshops; (6) things to look for during workshop discussions; and (7) viewgraph (overhead transparency) masters. (JN)
Mathematics and Science - Language, Communication and Problem Solving

A MATH-SCIENCE WORKSHOP

by

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TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)."
We thank the many teachers throughout northern New England who participated in our summer institutes and academic year workshops. Their enthusiasm gave us the incentive to assemble this guide. We particularly thank

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VIEWGRAPH A: Introduction
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VIEWGRAPH B  Treatment of math in Algebra 2 texts
VIEWGRAPH C  Problem solving procedure
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Background

In recent years, we at the University of Maine as well as our colleagues across the nation have observed that students in beginning physics and chemistry courses have great difficulty solving word problems and applying their knowledge of mathematics. To address this problem we ran four National Science Foundation Summer Institutes which brought together mathematics and science teachers primarily from Northern New England.

Although the focus of these Institutes was on teaching problem-solving skills, it quickly became apparent that there are other factors affecting the problem that we and others had not been aware of. The first of these is that there are two distinct mathematical 'languages' being used in the school. Our experience showed us that the two different and distinct 'languages' of mathematics used by mathematics and science teachers is a serious stumbling block that students encounter in learning how mathematics is used in the physical sciences. While the mathematics used in the mathematics classroom is the same as that used in the science classroom the way it is said and written, differs significantly.

Since the teacher, and sophisticated student, 'translate' easily and speak both languages fluently, they are not even aware that the mathematical 'languages' used differ from science to math classes. The use of different mathematical languages is, to the unsophisticated student, a very serious problem. This different use often leads to the feeling among students that mathematics and
physical science are entirely unrelated subjects. Most science
and mathematics teachers are not aware that they are using dif-
ferent mathematical languages and that their textbooks are also
addressing the same mathematical problems in different languages.
There is a lack of communication between science and mathematics
teachers as regards what they are doing mathematically in the
classroom and how they are doing it.

The second problem is that of unfamiliarity and has two
aspects. In the last 15 years the language, notation, and termin-
ology used in mathematics as well as the focus of the mathematics
curricula have changed considerably. Teachers of science are, in
general, unfamiliar with the 'modern' aspects of the mathematics
curricula and fail to take advantage of it in their classes. On
the other hand, teachers of mathematics who graduated from
college since the mid 1960's, in many cases, have not taken many
physical science courses. Thus, they are unable to illustrate
mathematics with applications or motivate it with realistic
problems. The result is a 'pure' mathematics which, to the
student, is completely divorced from science. Add this
background (or lack of background) to the fact that over the past
20 years many small schools in northern New England were combined
into larger regional schools with 'separate' mathematics and
science departments and one can understand why the lack of
communication and the different languages and notation has
arisen.
Our experience in the past several years in running summer institutes combined with academic year workshops and conferences has given us considerable insight into the problems and possible solutions involved in furthering our goals to enhance communication and problem solving in mathematics and science. Drawing on these past experiences, we have assembled this packet of information to be distributed to all high schools in northern New England. These materials are designed to:

1. encourage more meaningful communication between mathematics teachers and science teachers in high schools.
2. encourage more effective use of mathematics in science courses in high schools.
3. encourage greater emphasis on problem solving in mathematics and science classes in high schools.

It is with the hope of addressing these problems that these workshops were developed.
Materials

A set of teaching and workshop materials has been developed to illustrate the different uses of mathematics in high school mathematics and science classes. It consists of:

(1) This packet of workshop activities with detailed instructions on how to use them in a combined mathematics-science workshop. Each activity is meant to stimulate discussion among the mathematics and science teachers -- that discussion being the important outcome of the workshop. The actual activities suggested are fun to carry out and need no preparation on the part of the teachers. The results are surprisingly revealing and will show dramatically what the different languages of mathematics are and emphasize once again the need for serious problem-solving activities in both mathematics and science classes.

Also included in this workshop packet are:

(a) a short list, with examples, of some things to look for in the discussion accompanying a workshop. This list is of value to the workshop leader who should read it before presenting the workshop.

(b) supplementary viewgraph masters that can be duplicated and used during the workshop at the leader's discretion. They are suitable for initiating a discussion at a general staff meeting. These viewgraph masters include:

(i) a list of what is and is not included in a
(ii) a problem-solving strategy for students to use
(iii) a list of some difficulties students have in science.

(2) A separate packet of problems, with solutions, that focus on the mathematical topics most commonly used in science classes. These may be used in mathematics and science classes as student assignments or as illustrative or example problems. Their use will foster an appreciation of mathematics as an essential tool in the sciences and will encourage the transferability of knowledge and skills gained in mathematics classes to science classes. These are meant as sample problems only. Mathematics teachers and science teachers are urged to meet regularly with each other to develop other problems that unify the mathematics and science instruction in their schools.

Before beginning the workshops the leader or facilitator should make overhead transparencies of all pages appearing at the end of this packet labelled "VIEWGRAPH". There is a total of six such viewgraph masters. They are labelled VIEWGRAPH 1, 2, A, B, C and D. Instead of transparencies, simple photocopies can be made of these viewgraph masters and distributed as needed to the workshop participants. The choice of transparencies or photocopies is left to the workshop leader.
Workshop Responses

Through actual workshop experience we have found that examples of different mathematical languages quickly surface, such as:

1. use of exact numbers in mathematics versus approximate numbers in science, e.g. expressing an answer as $10\sqrt{3}$ ft rather than 17.3 ft.

2. use of all four quadrants in graph plotting in mathematics rather than just one quadrant as is common in science.

3. neglect of units in mathematics classes.

4. use of graphs in mathematics to illustrate a functional relation versus use of graphs to present data in science.

5. use of $x$ and $y$ as variables in mathematics versus the use of many different letters as variables in science.

These are just a few of the different uses of mathematics that we have found. Many other differences will emerge during the workshops. Each difference can lead to informative discussions between mathematics and science teachers regarding items such as the interfacing of the mathematics and the science curricula, the way mathematics is used in the two areas, the mathematical difficulties experienced by students in science classes, etc. THESE WORKSHOPS WILL BE COUNTED SUCCESSFUL IF THEY LEAD TO AN ONGOING DIALOGUE BETWEEN THE MATHEMATICS AND SCIENCE TEACHERS IN A SCHOOL.
Types of Workshops

We have identified four types of workshops that can lead to fruitful and informative discussion among science and mathematics teachers. Each type of workshop is designed to be non-threatening. They seek to have teachers present ideas and examples exactly as they do everyday in their classrooms. AT ALL TIMES THE TEACHERS SHOULD KEEP IN MIND WHAT THEY DO IN A CLASSROOM SITUATION -- HOW THEY EXPLAIN A CONCEPT -- WHAT EXAMPLES THEY USE -- WHAT BACKGROUND KNOWLEDGE THEY ASSUME.

Some workshops require teachers to prepare materials singly, other workshops seek cooperation among several teachers in a group. All workshops seek, at some time, to have all teachers participate in a general discussion drawing on their classroom experience. THE MAIN TASK OF THE WORKSHOP LEADER IS TO INITIATE THIS DISCUSSION IF IT DOESN’T HAPPEN SPONTANEOUSLY.

The workshop leader or facilitator does not need much, if any, scientific background. The purpose and emphasis of these workshops is participant discussion. The leader simply organizes the workshop and at most asks the participants for comments or differences in individual approaches. No technical knowledge is required. To help the leader there is a later section describing some of the things to look for. This section follows the workshop directions.

Of the four types of workshops that we will explain, the lead-off workshop can be of TYPE 1, 2 or 3. TYPE 4 should be delayed until one or more of the other types have been used.
TYPES 1, 2, and 3 are designed to bring out the differences between the mathematics used in science and mathematics classes. TYPE 4 is meant more as an in-depth comparison of these differences. We give several topics (problems) for each type of workshop so that a given type of workshop can be repeated several times on separate occasions, each time treating a different topic. No more than one or two topics (problems) should be covered at any one workshop.

TYPE 1. A problem is presented that participants should solve by working in groups. Each group should consist entirely of mathematics teachers or entirely of science teachers. No group should be larger than four teachers. Each group's solution is put on a transparency and presented in turn to the whole audience for discussion. This type of workshop is appropriate for eight or more participants. Examples of problems are displayed on VIEWGRAPH 1.

TYPE 2. A very general task is assigned. Each teacher uses a transparency to illustrate his/her approach to the task. A large variety of responses can be anticipated. All transparencies are then presented to the audience for discussion. This workshop works best with 10 or fewer participants since it is time consuming to run through each participant's response. Examples of tasks that can be assigned are displayed on VIEWGRAPH 2.
TYPE 3. The audience is divided into groups of no larger than four teachers, each group consisting entirely of mathematics teachers or of science teachers. Each math group prepares a problem for a science group to solve and also prepares its own solution. Each science group similarly develops a problem for a math group to solve. These problems should be typical of those presented in the classroom. One of the math problems is selected and presented to all science groups to solve and one of the science problems is selected and assigned to all math groups. Each group works out its own solution on a transparency for presentation to the whole audience. We than have two problems with the two originator’s solution and with the several groups’ solutions. This workshop is appropriate for eight or more participants.

TYPE 4. This type of workshop involves all participants in a discussion situation focusing on topics such as student difficulties in problem solving, homework policies in various classes, techniques taught in Algebra I, etc.

Before presenting a workshop the leader or facilitator should read through the instructions for the type of workshop and select a problem, topic or task from the appropriate viewgraph master. The first action of the workshop is to place VIEWGRAPH A on the overhead projector so that the participants have an idea of the purpose of the workshop.
DIRECTIONS FOR LEADING A WORKSHOP-TYPE I
(participants work in small groups)

An overhead projector, blank transparencies and pens are needed.
From VIEWGRAPH 1 select a problem to start with that you feel is appropriate and have available overhead transparencies of the VIEWGRAPH A and VIEWGRAPH 1 masters.

1. Show VIEWGRAPH A on the overhead projector.
   Allow the participants time to read the INTRODUCTION.

2. Separate the participants into science teacher groups and math teacher groups (not more than four in a group).

3. Put VIEWGRAPH 1 on the overhead projector exposing only the problem you have selected and show it to the workshop participants. Read the selected problem aloud to the participants. Then give the following instructions:

   "In your group work together to solve this problem. Come up with the best explanation to the problem that you can. This does not have to be the shortest or most elegant solution but rather the best solution for the classroom. WE WANT THE 'BEST' EXPLANATION, AS YOU PRESENT IT TO YOUR STUDENTS IN YOUR CLASSROOM.

   Your explanation may require definitions, terminology, and/or techniques not commonly used by others in the room. PLEASE USE THE TERMINOLOGY, NOTATION, ETC., JUST AS YOU DO IN THE CLASSROOM.

   Write your group's explanation on a transparency. Someone from each group should be appointed to present the explanation before all of us later."
4. Allow time for the groups to complete their solutions on the transparencies.

5. For each group: Have one individual use the transparency and explain the solution **USING THE TERMINOLOGY AND NOTATION AS THEY ARE USED IN THE CLASSROOM.** Now solicit questions or comments. At this point there may be comments on the notation, or the terminology, used.

6. Now continue through the groups' presentations. As each group presents, all participants should be asked to comment on similarities and differences between the explanations or solutions. These comments stimulate an open discussion.

**NOTE:** THIS DISCUSSION IS THE MOST IMPORTANT PART OF THE WORKSHOP.

Typically, the participants will observe and comment on differences in notation, terminology, level of mathematical expectation, etc. **YOUR MAIN TASK AS WORKSHOP LEADER IS TO INITIATE THIS DISCUSSION IF IT DOESN'T HAPPEN SPONTANEOUSLY.** At the end of this section on workshop directions, is a section describing some of the things to look for when leading the discussion.

7. The workshop should end with some positive development such as an agreement to have another workshop or to have teachers share curricular ideas with each other. Keep in mind the fact that students are the ultimate beneficiaries of the workshop.
DIRECTIONS FOR LEADING A WORKSHOP - TYPE 2
(each participant prepares a transparency)

An overhead projector, blank transparencies and pens are needed. From VIEWGRAPH 2 select a problem to start with that you feel is appropriate and have available overhead transparencies of the VIEWGRAPH A and VIEWGRAPH 2 masters.

1. Put VIEWGRAPH A on the overhead projector.
   Allow the participants time to read the INTRODUCTION.

2. Put VIEWGRAPH 2 on the overhead projector, exposing only the task you have selected and show it to the workshop participants. Read the task aloud.

3. Give the following instructions:
   "Use the transparency to perform the task shown. Your work should use the notation, terminology, etc. AS YOU USE IT IN YOUR CLASSROOM. Your choice of illustration should come from one of the courses you are currently teaching. Please put on the transparency everything EXACTLY as you do in the classroom for your students.

4. Allow time for the participants to complete their transparencies.

5. Collect the transparencies.

6. One at a time, place the completed transparencies on the overhead projector. As each illustration is viewed, you should solicit questions or comments -- particularly on the notation, terminology or form used. There should be considerable differences between illustrations that will stimulate discussion. NOTE: THIS DISCUSSION IS THE MOST IMPORTANT PART OF THE WORKSHOP.
Typically the participants will observe and comment on differences in notation, terminology, form, level of mathematical expectation, etc. **YOUR MAIN TASK AS WORKSHOP LEADER IS TO INITIATE THIS DISCUSSION IF IT DOESN’T HAPPEN SPONTANEOUSLY.** At the end of this section on workshop directions is a section describing some of the things to look for when leading the discussion.

7. The workshop should end with some positive development such as an agreement to have another workshop or to have teachers share curricular ideas with each other. Keep in mind the fact that students are the ultimate beneficiaries of the workshop.
DIRECTIONS FOR LEADING A WORKSHOP - TYPE 3
(groups prepare problems for other groups)

An overhead projector, blank transparencies and pens are needed. Have available an overhead transparency of the VIEWGRAPH A master.

1. Put VIEWGRAPH A on the overhead projector. Allow the participants time to read the INTRODUCTION.

2. Separate the participants into groups of not more than four teachers. Each group should consist entirely of either mathematics teachers or science teachers.

3. Instructions to be given:
   
   "Each math group is to prepare a problem for a science group to solve. This problem should involve mathematics that you feel has some use in a science course and is typical of those you use in your classroom.
   
   Each science group is to prepare a problem for a mathematics group to solve. This problem should be typical of those you use in your classroom. Do not try to go out of your way to 'stump' the other group but use the mathematics and science that might be expected of your students.
   
   Write the problem down exactly as you do in class on a transparency USING NOTATION AND TERMINOLOGY COMMON TO YOUR CLASSROOM INSTRUCTION.
   
   When you have prepared your problem, solve it as a group exactly as you do in the classroom and put your
solution on another transparency. **DO NOT DO ANYTHING DIFFERENT FROM WHAT YOU DO IN CLASS.** This solution should use the notation, terminology, etc., exactly as you use it in your classroom."

4. Choose one of the problems developed by the mathematics groups and one of the problems developed by the science groups. (The other problems will not be used.) Give each science group the math problem and each math group the science problem.

5. Read these instructions.

   "In your group work together to solve the problem. Come up with the best explanation to the problem that you can. This does not have to be the shortest or most elegant solution but rather the best solution for the classroom. **WE WANT THE 'BEST' EXPLANATION AS YOU PRESENT IT TO YOUR STUDENTS IN YOUR CLASSROOM.** Your explanation may require definitions, terminology, and/or techniques not commonly used by others in the room. **PLEASE USE THE TERMINOLOGY, NOTATION, ETC., JUST AS YOU DO IN THE CLASSROOM.**

   Write your group's explanation on a transparency. Someone from each group should be appointed to present the explanation before all of us later."

6. Allow time for the groups to put their solutions on overhead transparencies.

7. Now have an individual from one of the groups use the transparency to present that group's solution to the problem given it.
Solicit questions or comments. At this point there may be comments on the notation or the terminology used.

8. Repeat 7 for other groups that were given the same problem.

9. Next have an individual from the group that developed the problem present that group's explanation to the problem. All participants should be asked to comment on similarities and differences between the solutions or explanations. These comments will stimulate an open discussion. **NOTE: THIS IS THE MOST IMPORTANT PART OF THE WORKSHOP.**

Typically the participants will observe and comment on differences in notation, terminology, level of mathematical expectation, etc. **YOUR MAIN TASK AS WORKSHOP LEADER IS TO INITIATE THIS DISCUSSION IF IT DOESN'T HAPPEN SPONTANEOUSLY.**

At the end of this section on workshop directions is a section describing some of the things to look for when leading the discussion.

10. Repeat 7, 8 and 9 using the other problem all groups worked on.

11. The workshop should end with some positive development such as an agreement to have another workshop or to have teachers share curricular ideas with each other. Keep in mind the fact that students are the ultimate beneficiaries of the workshop.
DIRECTIONS FOR LEADING A WORKSHOP - TYPE 4
(Open discussion focusing on various topics)

From the list at the bottom of this page, select a topic to start with that you feel is appropriate. Have available an overhead transparency of the VIEWGRAPH A master.

1. Show VIEWGRAPH A on the overhead projector. Allow the participants time to read the INTRODUCTION.
2. Read the chosen task to the participants.
3. Use a transparency and the overhead projector to keep a summary of the discussion.

Possible topics:
1. Ask the participants to call out the math topics covered in Algebra II and the math needed in Physics and Chemistry. Keep two running lists on a transparency.
2. Ask the participants to volunteer reasons why students have difficulties in problem solving. Keep a running list of these difficulties. Then ask for remedies to each of these difficulties.
3. Have the math teachers explain what is done in Algebra I, for example, and have the science teachers state how they use or could use these topics.
4. Have each participant state his/her homework policy. How much is assigned? Is it graded? Does it count in the final grade?
5. Have participants state how computers are used, if at all, in their classes.
6. Ask the participants how they grade tests and what they look for in grading.
7. Have each participant present a favorite teaching trick.
SOME THINGS TO LOOK FOR IN THE DISCUSSION
WHEN COMPARING DIFFERENT SOLUTIONS

A. Different notation for functions.

Some science teachers refer to an equation like \( V = IR \) as a function from which \( V \) can be found. In mathematics classes \( V = Ir \) is an equation. Specification of the domain and range are necessary before a function can be defined.

For example: \( V(R) = IR \) describes a function with domain \( R \).
\[ V(I) = IR \]
describes a function with domain \( I \).

Usually 'small' letters (not caps) indicate elements in the domain. So that in math class these might be written \( V(r) = Ir \) or \( V(i) = Ri \).

B. Unconventional form when approaching a topic.

In science \( v = v_0 + at \) is the usual way to express the relationship of velocity and time given an initial velocity, \( v_0 \), and an acceleration, \( a \). This relationship is linear. Mathematics teachers almost universally write linear relations as \( y = mx + b \). Note the form difference:

\[
\begin{align*}
v &= v_0 + at \\
y &= mx + b
\end{align*}
\]

C. Different perspective.

The science teacher often writes \( a = c \sin \theta \) as the relationship shown in the diagram. The mathematics teacher might write \( \sin \theta = a/c \).

D. Coefficients.

In mathematics numerical coefficients are generally used while in science letters are used.
E. Variables.

Science teachers view variables as physical quantities while mathematics teachers treat them as numbers.

F. Notation on variables.

In math classes, the letters used to signify variables are almost always x and y. In science classes, many other letters are used (even caps as in \( \pi = ma \)).

Example: \( F = ma \). To the math student, a linear equation in this form implies slope is \( m \), where \( F \) indicates range and \( a \) the domain. To the physics teacher, \( a \) may be the constant and \( m \) the variable.

G. Coordination of the plane.

In approaching a motion problem the first thing in math is to draw a coordinate system. Science teachers often don’t set up a coordinate system.

Example:

\[ \begin{align*}
\text{Science} & \quad \text{Math} \\
\begin{array}{c}
\text{W} \\
\text{\_\_\_}\end{array} & \quad \begin{array}{c}
\text{\_\_\_}\ \text{X} \\
\text{W} \\
\text{y} \\
\end{array}
\end{align*} \]

H. Difference in format.

Math: In presenting systems of equations, the equations are always grouped together and handled as a group.

Science: The equations may be written in different places, and the instructor works with only one before substituting in the other.

I. Math instructors usually deal in the 'general' case while science teachers deal with specific.

Example: Logarithms are defined in base \( b \) in math classes while in science classes only base 10 and base \( e \) are used.
A. Convert 68°F to degrees Celsius, given that 0°C is 32°F and 100°C is 212°F.

B. An automobile has a constant acceleration of 5 km/hr/s. If it begins from rest how fast will it be going after 10 s?

C. How much does it cost to operate an electric heater rated at 2 kilowatts for 5 hours if electrical energy costs 8¢ per kilowatt hour?

D. A 2 kilogram stone has a volume of 400 cm³. What is the volume of a stone of mass 7 kilograms?

E. Convert 12 cm to meters.

F. The following data on silver was found:

<table>
<thead>
<tr>
<th>Mass (g)</th>
<th>Volume (mL)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.095</td>
</tr>
<tr>
<td>1.8</td>
<td>0.17</td>
</tr>
<tr>
<td>2.4</td>
<td>0.23</td>
</tr>
</tbody>
</table>

What is the volume of 1.5 g of silver?
A. Draw a graph that you use in your classroom.

B. Illustrate a linear relation that you use in your classroom.

C. Give an example of how you use ratios and proportions in your classroom.

D. Show how you use units in your classroom.

E. Give an example of how you use percents in your classroom.

F. Give an example of a relationship you use in your classroom.
INTRODUCTION

The purpose of this workshop is to encourage an interaction between mathematics and science teachers with the goal of improving the problem-solving abilities of our students. **THIS WORKSHOP IS DESIGNED TO BRING OUT THE DIFFERENCES IN THE WAY MATHEMATICS IS TAUGHT IN MATH CLASSES VERSUS SCIENCE CLASSES.** Once we teachers realize that there are such differences the first step will have been taken to foster in our high school a more coherent picture of mathematics to our students. This should increase the transferability of math knowledge of our students from math classes to science classes. Communication between math and science teachers will also bring home the point that mathematics can't successfully be taught as an isolated, intellectual subject; it will be much better appreciated by our students if they see science applications of math presented in their math classes. Therefore, approach this workshop with the view that the more you know what your colleagues are doing the better off our students will be.
Treatment of Math Topics in Math Textbooks

1. In solving a single algebraic equation and simultaneous algebraic equations math texts

**DO**
- Use $x$ and $y$ as standard variables
- Use numerical coefficients
- Treat variables as numbers

**DON'T**
- Use letters other than $x$ and $y$ as variables
- Write coefficients as letter parameters
- Treat variables as physical quantities
- Discuss the dependence of the answer on the input parameters

2. In presenting and solving word problems math texts

**DO**
- Set up word equations first
- Translate word equations to algebraic equations with numerical coefficients
- Carefully define the unknown quantity

**DON'T**
- Use symbols to represent all physical quantities
- Construct equations using symbols as coefficients

3. In presenting logarithms and exponentials math texts

**DO**
- Discuss $a^x$
- Solve $6^{2x+5} = 2$
- Treat $\log_{10}$ primarily

**DON'T**
- Mention $e^t$
- Single out $\ln$

4. In discussing trigonometric functions math texts

**DO**
- Use sine, cosine, etc. in terms of an angle with respect to the $x$-axis (standard)
- Discuss relations in a right triangle
- Discuss the period of a trig function as an angle

**DON'T**
- Use symbols other than $a$ for the angle and $x$-$y$ for the coordinate axis
- Use a right triangle in an unconventional orientation
- Use $\omega t$ for the angle and discuss the period in terms of $t$. 
Problem Solving Procedure

1. Read the problem carefully.
2. Restate the problem in your own words.
3. State in words what the givens and unknowns are.
4. Make a diagram with the givens and unknown clearly identified.
5. State in words how you will solve the problem by outlining your plan of attack and by listing the principles you will use. Do not list equations.
6. Identify all explicit and implied givens by their mathematical symbols and assign them their numerical values.
7. Identify the unknown by its mathematical symbol.
8. Identify the mathematical relationship that exists among the givens and the unknown.
9. Solve this relationship for the unknown in terms of the givens.
10. Substitute the numerical values, including units, of the givens and work out the arithmetic. Check for dimensional consistency by seeing if the units that remain after cancellation are proper for the unknown.

If you get stuck:

11. Check to see if all the information, given and implied, in the problem has been used in identifying the givens and assigning them values.
12. Define all givens and the unknown in words.
13. List all equations you can that involve the givens and the unknown, even if some involve additional unknown quantities.
14. See if one of those equations involves just the unknown and the givens. If so, go to step 9.
Difficulties Students Have in Science

1. Simple arithmetic: \( \frac{5}{2/3} = ?, \frac{3/4}{2} = ?, \frac{5}{0.2} = ?, 5^2 \times 5^{-3} = ? \)

2. Use of units, (a) cancellation: \(200 \text{ m} \times \frac{100 \text{ cm}}{1 \text{ m}} = 2 \times 10^4 \text{ cm}\)
   
   (b) appreciation of cm\(^2\): \(9 \text{ cm}^2 = (3 \text{ cm})(3 \text{ cm})\)

3. Use of trigonometry of a right triangle in an unconventional orientation:

   \[
   F_x = F \cos \theta \\
   F_y = F \sin \theta
   \]

4. Rate problems: given a line charge of \(s = 10^{-9} \text{ Coulomb/meter}\) how much charge is in a distance of \(d = 2\text{m}\)?
   
   Charge = \(sd = 2 \times 10^{-9} \text{ Coulomb}\).

5. Solving equations in terms of letters, with numbers substituted only at the end:
   
   \(x = vt\) and \(y = \frac{1}{2}at^2 + y = \frac{1}{2}a(x/v)^2\)

6. Graphs, (a) sketching \(Z = [R^2 + (\omega L - \frac{1}{\omega C})^2]^{1/2}\)
   
   versus \(\omega\).

(b) understanding slope

7. Visualizing two dimensional projections in three dimensions