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ABSTRACT

Some recent research findings with implications for improving mathematics instruction are reviewed in this report. The focus is on two topics: computation, in particular subtraction and estimation, and problem solving. Findings about each are listed, with references for each cited study included. Effective algorithms and strategies used by students are given particular importance in the computation section, while both problem characteristics and problem-solving strategies are included in the problem-solving section. (MNS)

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RECENT RESEARCH ON MATHEMATICS INSTRUCTION

What have we learned from research about how to teach mathematics more effectively? By the early 1980s, a number of publications were available which were designed for teachers, discussing research findings which have implications for classroom practice. These include Driscoll (1980, 1981), Dessart and Suydam (1983), Suydam and Dessart (1976), and Suydam and Weaver (1981). These publications have been supplemented by briefer reviews, such as the "Research Reports" in the *Arithmetic Teacher*, designed for quick reading by busy teachers. This Digest continues the pattern, reviewing recent research of particular interest to teachers.

The focus of many of these studies has been on two topics: computation, and in particular subtraction and estimation, and problem solving. These two concerns have garnered continued research attention throughout this century — the first presumably because it has been the prevailing focus of the elementary school mathematics program, and the second because of its difficulty for both teachers and students at all levels.

Computation

Some of the research on computation arises from investigations into how children learn early number ideas — from counting and numeration, the studies move to explore how these are used with the operations of addition and subtraction. The difficulty children have with subtraction makes it a focal point for researchers. From the recent research we have evidence to add to that summarized earlier (e g , see Suydam and Dessart, 1976):

- The *decomposition algorithm* (see Figure 1) currently used in most mathematics programs to teach multidigit subtraction with regrouping or renaming ("borrowing") *continues to be effective* (Paulson, 1984).
- However, *the equal additions algorithm is also effective* (Brown, 1984).

How do *you* do this example? 91 − 24 = 67

You're using *decomposition* if you do it this way
11 − 4 = 7 (ones); 8 − 2 = 6 (tens)
If you do it this way, you're using *equal additions*
11 − 4 = 7 (ones); 9 − 3 = 6 (tens)

Figure 1.

Second graders were more likely to be correct and were faster when using it. Thus, it is plausible to use either algorithm.

- As you would expect, the main source of difficulty when doing subtraction involves "borrowing" (Cebulski, 1984). Children either attempted to borrow incorrectly, or made inversion errors — that is, they ignored the location of the digits and subtracted the smaller number from the larger. When the skills required for borrowing were taught to third and fourth graders in a *step-by-step fashion, with feedback*, their achievement improved.
- Beal (1984) examined the relationship between students' understanding of principles underlying the decimal numeration system and computational errors in subtraction. The types of errors third graders made with subtraction with multiple renaming were linked to their performance on such tasks as regrouping or trading with manipulative materials, writing multidigit numbers, creating smallest or largest numbers with digit cards, and naming the value of a digit. Thus, *careful attention to developing numeration ideas can have a payoff in work with subtraction.*
- Children invent strategies to solve subtraction problems. Carpenter and Moser (1984) reported that children used a variety of modeling and counting strategies even before formal instruction, and these invented strategies continued to be used for several years. *Helping children to merge invented strategies* with the often more abstract ones taught in school requires careful development by the teacher. Baroody (1984) detailed some informal subtraction strategies, and presented some ideas for the teacher to use that incorporate these varied strategies.
- The use of counting strategies to solve subtraction problems was also noted by Steinberg (1984). She taught second graders to use derived fact strategies, in which *known number facts are used to find the solution to unknown facts.* After eight weeks of instruction, children more than doubled their use of derived facts, which involve *more mature ways of thinking* than reliance solely on counting.

Estimation is another important topic once again receiving increased attention. It is especially important for two reasons: (1) it is used far more often than paper-and-pencil skills in everyday life and (2) it is particularly important as both adults and children do more work with calculators and computers. Ways to check the reasonableness of results are vital.

- Estimation skills depend strongly on a student's "number sense" (Threadgill-Sowder, 1984). They need to *develop a "friendliness" for numbers,* including awareness that most computations can be done in many ways, rather than relying on one algorithm.
- Even at the tenth-grade level, counting skill is related to accuracy in estimating, and better counters *feel more confident* about the estimates they make (Newman, 1984).

Reys (1984) reviewed the research on both estimation and mental computation, and presented specific teaching suggestions. This is a worthwhile article for teachers to check.

Problem Solving

Many summaries of research on problem solving have been prepared [besides those in Driscoll and in Suydam and Weaver, see Silver and Thompson (1984) and Suydam (1980, 1982).] Such summaries have noted the shift in emphasis from the problem to the solver; from what types of problems to use, to how to teach students to solve a wide variety of problems. Nevertheless, some research still concerns problem types:

- Problems with logic content or deductive structure were most preferred by secondary students, while least preferred were problems with geometry content and inductive structure (Miller, 1984).
- Novice algebra students had much more difficulty with surface features of problems than did experienced students (Wilde, 1984). Novices rarely attended to the structure of the problem, a characteristic of good solvers.
- Eighth-grade students were better able to solve real-life situational problems than written problems. Errors could often be traced to weaknesses with content and misunderstanding of concepts (Shinatrakool, 1984).
- Problems which include needed information but few words were not as easy to interpret as problems with more words (Moyer, 1984).

Research has identified problem-solving strategies used by students and has indicated that teaching students to use such strategies promotes problem-solving achievement. Current research continues to support this finding:

- Second-grade pupils given lessons on diagramatic modeling and deriving number sentences made significant increases in achievement (Luckinich, 1984).
- Ninth-grade students taught to build tables had higher problem-solving scores than students taught only translation (Keller, 1984). Moreover, table-building appeared to be especially helpful to students at the concrete level of cognitive development — the lower achievers.
- Vissa (198[illegible] [illegible]ght four heuristic strategies to seventh and eighth graders: making a table, making a diagram, analysis by simplification, and guessing and testing. Not only did they learn to use the strategies, but also in solving a novel problem they *exhibited greater flexibility* in trying other strategies when one seemed unproductive.
- A promising *process-oriented instruction program* on problem solving has been developed by Charles and Lester (1984). They found that experimental classes in grades 5 and 7 scored significantly higher on ability to understand problems, plan solution strategies, and get correct results.

How does a teacher's behavior affect problem-solving achievement? Behle (1985) found some differences between successful and less successful teachers of problem solving in grade 7. Successful teachers *asked questions frequently,* while less successful teachers simply demonstrated procedures. Successful teachers *used a variety of problem sources,* while the others simply used textbook problems. Successful teachers *frequently gave additional credit to students for correcting mistakes*; the others did not. Successful teachers more often encouraged students *to think for themselves,* while less successful teachers did much more *for* their students, including setting up problems.

In a different type of stud[illegible], Orehovec (1984) traced the development of mathematical problem solving over a 90-year period. He found that problem-solving models developed decades ago continue to be used. He also concluded that problem solving can be successfully taught. For this to happen, there must be *a willingness on the part of classroom teachers and others to implement fully a problem-solving approach to teaching mathematics.*

Additional information on these studies may be found in the references which follow, and others may be located by checking the annual listing of research in the July issue of the *Journal for Research in Mathematics Education.*

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