DOCUMENT RESUME

ED 266 007 SE 046 384

AUTHOR Klauer, Elizabeth; Rule, Ann
TITLE Teaching Strategies in Algebra: The Effectiveness of Relating and Sequencing Algebraic Concepts.
PUB DATE [85]
NOTE 14p.
PUB TYPE Guides - Classroom Use - Guides (For Teachers) (052)

EDRS PRICE MF01/PC01 Plus Postage.
DESCRIPTORS *Algebra; *Error Patterns; *Mathematical Concepts; *Mathematics Instruction; Secondary Education; *Secondary School Mathematics; Sequential Learning; *Teaching Methods

ABSTRACT

Structuring the algebra course to provide a link between a student's existing knowledge and the new topic being presented is discussed. Developing relationships among topics is suggested through examples describing a mathematical problem and effective teaching approaches. Stress is placed on teachers reflecting on mistakes pupils are likely to make with particular content and recalling relevant previously learned material. (MNS)

Teaching Strategies in Algebra: The Effectiveness of Relating and Sequencing Algebraic Concepts

Elizabeth Klauer and Ann Rule

St. Louis University

Background

The concern over the mathematical literacy of students has spread through all levels of school including the high school. Research has been continuous since World War II concerning levels of high school students' mathematical competency (Beckmann, 1978). For many students entering ninth grade, the school curriculum suddenly calls for them to take an algebra course. It is in algebra that a student is confronted with many abstract concepts, for example, the use of a variable. This transition from the concrete to the abstract is difficult for some students (Alexander, 1978). As a result, the algebra teacher is faced with the dilemma of how to best help those students drilled in basic mathematical skills make the needed transition.

One way is for the teacher to begin showing relationships among various concepts in order for a student to learn how to organize his/her previously learned skills. It has been shown that even if a student is verbally told how previous learning may be helpful in a new situation, transfer is significantly increased (Dorsey and Hopkins, 1930). In 1957, Kittell studied the effects of varying amounts of direction on sixth graders' discovery of known principles on transfer to other situations and then on these students' retention of learned principles. His conclusion was that "furnishing learners with information in the form of underlying principles promotes transfer and retention of learning principles and may provide the background enabling future discovery of new principles."

In a subject such as algebra, it is not only important for a student to learn the various concepts as they are covered, but also to retain that knowledge. Many of the topics are dependent upon ideas previously covered. Such a course provides a good example for Gagne's Principle Learning theory to be applied. He says, "Learning principles not only produces a capability commonly referred to as 'understanding'; at the same time it establishes a capability that is retained well for relatively long periods of time."

Gagne (1965) defines a principle as a chain of two or more concepts and says that if a principle is to be learned, then the person must already understand the concepts being chained. More specifically, if learning at any level is to occur at its utmost, then one must pay careful attention to the prerequisites for this learning. This is what he later calls the organization of knowledge as a hierarchy of principles. A student is "ready" to learn something new once he/she has mastered the prerequisites. Someone neeos to specify and order these prerequisites for a topic to be learned and, as Gagne says, this is the role of the instructor.

In algebra, it is possible for topics to be covered as a hierarchy of principles. In order for this to be done, the teacher may find it beneficial to first take a close look at how students solve or fail to solve algebraic problems (Carry, 1980). If a teacher looks at the mistakes made by a student or the problems which are encountered, he/she would be able to see the prerequisites needed for certain concepts to be learned. This could lead to more successful ways to sequence instruction in order to point out more relationships between the concepts.

Structuring a course to provide a link between a student's existing knowledge and the new topics being presented is not always easy. Generally the more experience a person has had in teaching, the more likely he/she will be able to describe the majority and sometimes pattern of the mistakes most frequently made. Whether or not that knowledge is used to help students correctly reason through an example and avoid these mistakes is the question. Avoiding errors will certainly involve more than just rote learning on the part of the student. Relationships among topics must also be shown to the student, building on his/her previous knowledge.

Such organization and teaching patterns suggested to be used by the algebra instructor can be seen in the following examples. Because simplifying various types of expressions is a major part of any first year algebra course, the first example wil deal with simplification of numerical expressions. Many students will have a difficult time with such problems unless they are aware of how to relate previously learned arithmetic properties and axioms to larger numerical expressions.

Examples

Look at the problem in Table 1.

Insert Table 1 about here

The correct answer, of course, is 4. However, at first, it is very likely that a variety of answers, such as the two listed, would be received. To more clearly see the pattern of these students' errors and hence know what axioms and rules concerning exponents precisely need to be reviewed, try to answer the following questions. If the same students (A and B) computed $2\cdot4-3^2+4$, what would student A answer? student B?

If student A is consistently making the same kind of computational error, he/she would have gotten 21. Student B would arrive at an answer of 76 by computing $2\cdot4(-3)^2+4$. Although these answers seem, at first, to be quite different from the correct answer of 3, they really indicate a misunderstanding of certain axioms, exponents, and the order of operations. These same students, in all probability, would correctly compute $12\div3\cdot5+4^2$ and arrive at an answer of 36. Why? As will be shown later, one of the most confusing parts of $12\div3\cdot5-4^2$ to a student is how to simplify the last section containing -4^2. Most, if not all, of them will know that $+4^2=16$ by this time and therefore, have less trouble with $12\div3\cdot5+4^2$.

Teaching Approach

What can a teacher do to help students avoid making such mistakes rather than trying to correct them after they are made? It is necessary for the teacher to be aware of the various concepts needed to simplify such examples and to demonstrate how such previously learned facts aid in the simplification of expressions. The following examples will illustrate what is needed to accurately simplify those similar to $12\div3\cdot5-4^2$.

First of all, students must understand the relationship between addition and subtraction as is shown in the examples $6+(-3)=6-3$ and $12-9=12+(-9)$. They also need to remember that

$(-1)(6)=-6$ or $-9=-1(9)$. From this last fact, $-9=-1(9)$, and by recalling what the exponent indicates in 4^2, i.e. $4\cdot 4$, you can show that $-4^2=(-1)(4^2)=(-1)(4)(4)=-16$. Ask, then, what -5^2 equals and what -10^2 equals. This should then lead to the correct answer of $(-4)^2$. Discuss that $(-4)^2=(-4)(-4)=(-1)(-1)\cdot(4)(4)=16$. Ask if this was the same as -4^2.

The next topic which needs to be recallled is the commutative axiom for addition $(a+b=b+a)$. By the commutative axiom for addition, this means that $2+3=\square$? What about $4+(-2)$? (Remind students that subtraction is not commutative.) Then, if $-4^2=-16$, $-4^2+2=-14$. What is $-3^2+2=\square$? (-7) By the commutative axiom, the expression could be rewritten as $2+(-3^2)$ or $2-3^2$. Students should notice that in $2+(-3^2)$, the entire value of -3^2 is in the parentheses. (Note: most textbooks will use the notation $2+(-3)$ rather than $2+-3$.) Remember the difference between (-3^2) and $(-3)^2$. You may want to do a few more examples such as $4-6^2=\square$, $-7^2-5=\square$, and $-4-3^2=\square$.

Finally, go through the steps in the order of operations.

1. Simplify expressions within grouping symbols,
2. simplify exponents,
3. simplify products and quotients in order from left to right, and
4. simplify sums and differences in order from left to right.

After reviewing all of the above, then introduce the example $12\div 3\cdot 5-4^2$. Do this by carefully going through the steps in the order of operations. Ask students what to do first, second, etc. Eventually lead to an example such as

BEST COPY AVAILABLE

$5(4^2-2^2)\div6+4\div(-4)=\square$. (9)

Depending upon the class, the teacher may wish to discuss, at this point, an example such as $3(4-1)=9$. Solve $6-3(4-1)=\square$. If students understand the order of operations, they should get an answer of -3. What if they get an answer of 9? What did they do wrong? Here it is probably necessary to also review the distributive axiom. First ask what $3(4-1)$ equals and then $-3(4-1)$. It is sometimes beneficial to show that by using the order of operations, you will get $-3(3)=-9$. Students should also see that the same answer is arrived at by using the distributive axiom: $-3(4)-3(-1)=-12+3=-9$. (One of the mistakes which occurs is the failure to multiply $-3(-1)$.) Finally, in the example, $6-3(4-1)=\square$, indicate that according to the order of operations, after simplifying inside the parentheses, multiplication is done. Discuss that the example really states $6-3(3)$, not $(6-3)(3)$.

Later in the year, algebra students will be simplifying problems similar to the one shown in Table 2.

Insert Table 2 about here

Before students are able to add or subtract any two fractions, they must be able to do so when the fractions have the same denominator. If the example is to be simplified, i.e., expressed in simplest form, then the correct answer is 6.

Although answer A shows the correct addition of the original two fractions, it indicates a lack of understanding about the simplification of a quotient. This student has failed to recognize that $6x+6y=6(x+y)$. For answer B, the student knew how to add the two fractions to get $\frac{6x+6y}{x+y}$, however, he/she then incorrectly attempted to simplify that quotient. After arriving at $\frac{6x+6y}{x+y}$, the student cancelled in the following manner to get an answer of 12: $\frac{6\cancel{x}+6\cancel{y}}{\cancel{x}+\cancel{y}}$. For answer C, the student separated $\frac{6x+6y}{x+y}$ into $\frac{6x}{x}+\frac{6y}{y}$ and then subracted similar terms, $6x-x+6y-y$.

What should an algebra teacher review before presenting the example in Table 2? Although the two fractions being added already have a common denominator, many previously learned facts and rules need to be recalled for these to be correctly added and the answer expressed in simplest form.

First, remind students of how to add and then simplify the answer to an example such as $\frac{1}{4}+\frac{1}{4}=\frac{2}{4}=\frac{2}{2\cdot 2}=\frac{\cancel{2}}{\cancel{2}\cdot 2}=\frac{1}{2}$. Point out that when the fractions with a common denominator are initially added, it is only the numerator which changes. The addition rule for fractions states that $\frac{a}{c}+\frac{b}{c}=\frac{a+b}{c}$ $(c\neq 0)$. Ask what the answer is to $\frac{1}{7}+\frac{4}{7}=\square$. ($\frac{5}{7}$) Can this answer be simplified? Why or why not? Most, if not all, of the students should realize that a quotient such as $\frac{5}{7}$ cannot be simplified because the numerator and denominator have no common factors other than 1 or −1 which may then be cancelled.

The concept of cancellation, or what can be cancelled in a particular quotient, frequently poses a problem for algebra students. Perhaps part of this is due to a lack of understanding about what a factor means and that in a quotient, only factors which the numerator and denominator have in common may be cancelled. Students must understand that in 2x (2 times x), the 2 and the x are factors. In 2x+7, however, neither the 2 nor the x is a factor of the expression.

Once this is reviewed, have students simplify $\frac{108}{180}$ and then $\frac{x}{x}$. When simplifying $\frac{x}{x}$, $\frac{x^2}{x^2}$, $\frac{3xy}{3xy}$, or something similar, students sometimes fail to realize that what remains is $\frac{1}{1} = 1$. Although seemingly trivial, it should also be shown that 1 or -1 is a factor of every number and polynomial, for example $x+y=1(x+y)$. In this way, it can further be explained that 1 and $x+y$ are the factors of $x+y$, not x and y taken separately. As a result, the class should be asked what $\frac{x+y}{x+y}$ equals when simplified. When explaining that this reduces to 1, factor the numerator and denominator first, $\frac{1(x+y)}{1(x+y)}$. Then cancel factors and arrive at the answer of 1.

After once again reminding students that you only cancel factors common to a numerator and denominator, present the problem of $\frac{3x+6}{x+2}$. Ask them to factor this and then cancel if possible. The correct answer should be 3. See how long it then takes them to simplify the sum $\frac{3x}{x+2} + \frac{6}{x+2}$. Finally, have them work on the example in Table 2.

Summary

No matter what topic is being covered in an algebra class, a teacher may wish to follow these pointers:

1. think about what mistakes a student may make with a particular example and why he/she may make them
2. introduce the topic by recalling previously learned material which will be relevant to that example.

In this manner, you are then bringing a student's existing knowledge together in an attempt to present the next topic in some kind of a logical sequence. The research, as stated earlier, on such instructional sequencing indicates that this type of strategy will be beneficial to the student both for particular topics and in relating many of the concepts studied. You cannot anticipate all of the errors that a student may possibly make on an example, but certainly by recognizing some of them you can learn to present material in such a way that many major problems can be avoided.

Bibliography

Alexander, David. (1978). A Clinical Diagnostic Investigation of Fundamental Reasons for Conceptual Difficulties with Algebra. (State University of New York at Buffalo, 1977.) Dissertation Abstracts International, 39A (August 1978), 764.

Beckmann, Milton W. (1978, February). Basic Competencies - Twenty-Five Years Ago, Ten Years Ago, and Now. Mathematics Teacher 71.

Carry, L. Ray (1980). Psychology of Equation Solving: An Information Processing Study. University of Texas, Austin. (ERIC Document Reproduction Service No. ED 186 243.)

Dorsey, M.N. and Hopkins, L.T. (1930). The influence of attitude upon transfer. Journal of Educational Psychology, 57, 410-417.

Gagne, Robert M. (1965). The Conditions of Learning. New York: Holt, Rinehart and Winston.

Kittell, J.E. (1957). An experimental study of the effect of external direction during learning on transfer of retention of principles. Journal of Educational Psychology, 48, 391-405.

Table 1

Possible Student Answers

to the Simplification of a Numerical Expression

Example: $12 \div 3 \cdot 5 - 4 =$

A.) 36

B.) 84

Table 2

Possible Student Answers

to the Simplification of an Algebraic Expression

Example: $\frac{6x}{x+y} + \frac{6y}{x+y}$

A.) $\frac{6x+6y}{x+y}$

B.) 12

C.) 5x+5y