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ABSTRACT

Many high school and college students seem to dislike mathematics, do poorly in the subject (as evidenced by declining test scores), and avoid scheduling courses in mathematics. Answers to why test scores are declining and why mathematics achievement is low among adults are speculative and as varied as the educators who offer them. However, one answer may lie in the area of cognitive development. Most of the basic mathematics which students are expected to remember for passing high school and college examinations is taught at a time when they are psychologically incapable of abstract conceptualization. Furthermore, students are taught basic mathematics with methods and strategies which require use of abstract symbols at a stage in their life when they are capable of "conceptualizing" only when the concepts are physically structured for them (representations of a mathematical reality constructed in three-dimensional form). Eight areas for curriculum developers to explore or implement in schools (based on knowledge of the psychological development of youth) are offered. They include: excluding concepts and skills from the mathematics programs of 6- and 7-year-olds which they are psychologically incapable of grasping; using real-life mathematical applications; and using Piagetian-oriented teaching methods. (JN)

MATHEMATICS ACHIEVEMENT SCORES IN HIGH SCHOOL: ARE THEY RELATED TO THE WAY WE TEACH IN THE EARLIER GRADES?

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Why Do High School and College Students Dislike Mathematics and Do Poorly In It?

Why do so many high school and college youngsters seem to dislike math? Far too many students are doing poorly in math and they avoid scheduling any courses in the subject. Student scores on the Scholastic Aptitude Test (SAT) of the College Entrance Examination Board have dropped steadily during the last decade. Although the board has not publicly speculated about the causes for the decline, it has shown concern about the significance; so has the public.

National Assessment of Educational Progress studies indicate that a high percentage of educated young adults cannot use a tax table, balance a checkbook or shop for a can of tuna. In a checkbook exercise only 16% of the young adults succeeded (72% indicated that they had previously balanced a checkbook). Only one-third of the college graduates completed the checkbook exercise correctly (100% of them had a checkbook).[14]

An earlier NAEP test of 34,000 17 year old persons and 4,200 adults showed that fewer than half of the total group could engage in unit shopping. Only 10% of the 17 year olds and 20% of the adults could calculate a taxi fare while only 1% of the 17 year olds and 16% of the adults could balance a checkbook.[11]

The Responses to the Question of Poor Mathematics Achievement

Why is the phenomenon of declining test scores and low math achievement among adults taking place? The speculative responses to this question may be as many and varied as there are educators and their critics. A logical starting point may be the fact that the dominant mathematics curriculum writers of the 1960's and 1970's ignored the contributions of the eminent cognitive/developmental psychologists of the century, i.e., William A. Brownell and Jean Piaget. These contributions should have had a profound impact on the mathematics curriculum designs of the last two decades.[6] Instead we continue to frustrate learners and encourage the development of serious mental blocks and math anxiety by the premature presentation of algebra and other topics eg. 3 + ☐ = 5 is an algebraic sentence presented in the first grade; a concept most children cannot internalize because they lack the psychological tools for doing so.[4 (p. 129 and p. 161)] This symbolism has no real meaning for many first graders. The teacher's efforts are wasted and she becomes unfairly frustrated. The learner needs to conceptualize the idea of the conservation or invariance of number before he is able to successfully work with the meaning of numbers, place value, addition and subtraction. This means that most math activities now presented in the first grade should be postponed until the second grade. Additional readiness activities with three dimensional physical objects in the first grade will help learners with the mathematical abstractions required of them in the second grade, i.e., children need many experiences with the concept of grouping before they are introduced to our base ten number concepts such as place value, decimals and fundamental computations The reason Johnny can't add in the first grade may be because he is not yet psychologically ready to add. When it is decided to group learners for teaching them mathematics, they should be grouped according to the developmental stages so well defined by Piaget.[4 (pp. 19, 28, 36, 129, 132-133)] This grouping should be based on Piagetian testing and should include middle and high school learners. The premature presentation of algebra to middle school learners who are cognitively underdeveloped will accomplish little more than severe cases of math anxiety.

According to Piaget, the logical mathematical structures involved in math education are mostly internal. Thus, the conditioning types of teaching methods which are used so often with learners in mathematics are not appropriate.[4 (p. 36)]

"Knowing how to" is not the same as "knowing," i.e., we may "know how to" do addition, subtraction, multiplication or division by memorizing certain steps or processes without having internalized a conceptualization, abstraction, or cognizance of what is taking place.[4 (p. 49)] *Most arithmetic and mathematics teachers in every grade level are mistaking a correct conditioned response for an internalized concept.* "Knowing how to" is a sensory motor preoperational level function, i.e., the mechanized abstract manipulation of familiar symbols for the purpose producing a correct answer; a correct conditioned response.[4 (p. 49)]

"Knowing" involves internalizing a concept or "conceptualization." This is the ability to construct abstractly what is going on physically, i.e., theorizing.[4 (pp. 49-50)] The necessary operating structures for theorizing are not acquired by the learner until the ages of eleven or twelve; give or take two years. This means that some learners might acquire them at ages nine or ten while still many others might not acquire them until the ages of thirteen or fourteen and considerably later for some young adults. The gap between the mechanical or "sensorimotor" level and "theorizing" or abstract level is six years; i.e., learners are doing the basic operations in arithmetic six years before they are able to actually internalize the concepts, eg. they are able to perform an operation or a series of operations correctly in behavioral terms but they have not yet conceptualized the task. The learners are not conscious or cognizant of "Why" they did what they did or why the correct answer was produced.[4 (pp. 50, 60, 139)] Available evidence indicates that teaching for retention is meaningless until the learner has developed the cognitive process necessary for him to correctly interpret what he sees and does.

The presentation of algebra in the seventh grade is premature for those youngsters who will not develop into their formal operations stage until the ages of thirteen, fourteen or later.

In the seventh grade many of the learners have already entered their formal operations stage of development. They are now capable or responding correctly because they have internalized a concept instead of because they memorized a procedure for achieving the correct answer. The learner's new ability to conceptualize abstractly is now accompanied by a very much improved long-term memory. Traditionally, at this point, the learner is expected to remember "forever" all previous mathematics learned in grades kindergarten through grades six which were acquired during his concrete operational stage of psychological development; a period when conceptualization may have taken place only if the concept was physically constructed for him with a three dimensional manipulative device. It is this physical representation of the concept that helps the youngster reconstruct a mathematical operation in his own mind, i.e., conceptualization. It is important to emphasize that most elementary school learners are not taught mathematics with the method of conceptualization just described. If any diversion from traditional, abstract, repetitive and mechanical conditioning takes place at all, it is often, and at best, done with two dimensional charts, graphs, diagrams or pictures. The concrete operational stage of the learner's psychological development is also a period characterized by a limited memory as compared to the long term memory associated with his next level of psychological development, i.e., the formal operations stage. Later, some additional support is sometimes afforded the learner to help him hold on to his acquired mathematics behavior a little longer through sporadic and periodic reinforcement by repetition or reconditioning of math skills learned previously during his middle school years. A few years later when the learner needs his math skills for succeeding in the SAT tests or for an occupation or career, the typical student has forgotten how to invert a fraction, much less why. The student forgets the step-by-step procedure for doing so and probably never knew why he was doing it in the first place. This short-term learning took place because of the learner's cognitive underdevelopment. Indeed, math anxiety itself is at least a partial product of the learner's cognitive underdevelopment.

According to Barbara Berman (1982)[1] Mathematics education is in a state of crisis during an era when the need for mathematically capable persons is increasing annually. Berman cites the work of Piaget when suggesting that a lack of success in mathematics for the average student is because of the teaching approach, not the subject matter itself, and because of a too rapid passage from the concrete to the abstract. Failure in subject areas other than mathematics may be largely attributed to the fact that teachers insist on teaching learners with words and abstract symbols instead of with concrete materials (multiple embodiments). The use of three dimensional manipulative materials increases retention and conceptualization of the material learned.

Concrete materials must become a permanent part of the mathematics program over long periods of time. Learners should not be rushed from the concrete to the abstract. They should not be rushed through math activities. Communication among the learners and with the teacher should be encouraged while multiple embodiment techniques are being used.[1]

Note: The results of a study in grades three to six by Berman and Friederwitzer[2] (1977) indicated that the most frequently used manipulative materials were rulers, the abacus, counting sticks, squares, yardsticks, compasses, protractors and money. Only two of the eight items listed are truly concrete learning devices. These are the counting sticks and the squares.

Carl D. Glickman (1979)[7] discussed growing evidence which suggested that a child's cognitive development and his abstract, symbolic mental processes may be rooted on earlier experiences described as play. Through play a child learns to identify and classify objects according to certain physical properties. A child must be able to classify real concrete objects and label them through play before he can classify, decode and read the letters of the alphabet. The child who can find many labels and use them for real objects can begin to read. The same is true for numerals and mathematical skills.

Glickman's[7] opinion is that the last generation of school children has played less than any other generation. It seems that children gathering together to play with objects, play games, build puzzles, read a story, become involved in make believe situations and roughhousing has been replaced by watching television. Previous generations had space to run, tumble, climb trees, wade creeks, swim rivers and explore a variety of plants and animals. In a nation of urban dwellers, the alternative to watching television for a modern child may be a busy street or a sidewalk. Recesses in urban schools were eliminated long ago.

Piaget's work is beyond theory. It is supported by research findings in the neuro sciences. These findings concern mainly neuron development, i.e., the myelination of specific br in areas and the spine.[8] They include hemispheric specialization in the brain. Piaget's four stages of cognitive development correspond to the six layers of neurons in the neocortex which develop at different times and perform different functions. The evidence described by Virginia R. Johnson (1982)[8] indicated that the cognitive processes described by Piaget correspond to the development of the myelination of nerve fibers in certain areas of the brain and to the Epstein growth spurts.[8,5] The formal operational reasoning is associated with the completion of the myelination in the frontal lobe. Myelination of the nerve fibers of the corpus callosum, which connects the two hemispheres of the brain, makes it possible for the learner to perform Piagetian tasks and to enjoy optimal learning, i.e., Rosemary Kraft[10] found that when children were presented with Piagetian tasks their right hemispheres were activated first. After they had processed the information their brain activity moved to the left hemisphere as they answered the questions or completed the tasks.

The application of cognitive skills can improve problem solving abilities and improve I.Q. scores. Programs and courses in the problem solving skills have been offered by the physics department at the University of Massachusetts, Amhurst, the Department of Philosophy, St. Louis University, Whittier College and the State University of New York at Buffalo English department. The applications of the cognitive skills approach in these teaching situations not only significantly improved the problem solving skills of the students but also resulted in a reported

Note: Myelin is an insulator for neuro axons. In non biological terms, it corresponds to the insulation on electrical wiring. The fatty coating is produced by the glial-specific cells of the brain. Myelination makes possible the speed and intensity of neuro signals.

gain of 6 to 10.5 I.Q. points.[15] These results were confirmed under rigorous experimental conditions at the State University of New York at Buffalo. Is it possible that if college students in science, philosophy, and English can benefit significantly from courses in the logic and problem solving of the cognitive skills approach in teaching that the teachers and learners of mathematics in the schools, too, can benefit? Indeed, the teachers themselves may be the victims of the same teaching methods in mathematics that are now handicapping their students and former students. If psychologically sound teaching methods are to be offered to the next generation of learners, they must first be offered to those who teach mathematics in the schools. Indeed, the number of U.S. adults who have developed the capacity for abstract formal operations thinking has been estimated at less than 50%.[3,8,15] Formal operations thinking is an ability required for the completion of some college courses, yet regular college courses have had little if any effect on the development of this skill. If the schools and colleges made an effort in every class to develop the cognitive processes while stimulating and nurturing skilled reasoning, problem solving, and precise analytical thought, the result might be that all students would develop minds for succeeding in a technological world. This kind of development must first take place and be nurtured in the minds of teachers and instructors.

The Implications For Curriculum Redevelopment

The implications for curriculum redevelopment may provide a solution to the failures of recent mathematics education without resorting to the rote mechanical conditioning methods of recent decades. Indeed, the new math itself may have been a direct response to those less than successful teaching methods. Repeating the question now for past and present math educators, *How often is a correct conditioned response mistaken for an internalized concept?* Evidence provided by low test scores indicate that a giver math skill might be a temporary behavior imposed by mechanical conditioning or behavior imprinting techniques; an imprint of temporary behavior which fails the learner before he is old enough to leave the presence of those who conditioned it. The task of re-educating parents and others concerned with traditional performance expectations seems insurmountable. Those educators involved in curriculum development and who wish to respect present day public demands on math education and, yet provide learners with some of the benefits of what we now know about the psychological development of youth, might consider exploring and implementing these suggestions in the schools:

1. The schools need to consider a fundamental reconceptualization of their objectives for teaching mathematics which rests on scientific theory and recognizes that logical-mathematical knowledge is lasting knowledge and must be constructed from within by each individual learner.[9] The most important

objective in teaching mathematics should be the strengthening of the process of thinking and reasoning for achieving a solution, not the short-term success of the mechanical ready-made methods of achieving the correct written answer.

2. Textbooks and computer software intended for teaching mathematics should respect the psychological development of young learners and teaching methods which are compatible with Piaget, i.e., concepts, skills and algorithms should not be presented to learners who are not yet mature enough to learn them.[9 and 12]

3. The popular spiral curriculum is based on learning by periodic association with a concept or a skill. The concept or skill to be learned is repeated after a span of weeks or months. Each repetition builds on past experience. The spiral curriculum for the cognitive underdeveloped learner might mean repeated failure every 3, 6, 9, 12, 26 or 52 weeks. The [illegible] programs of six and seven year old learners especially should not contain concepts and skills for which they are psychologically incapable of grasping.[12]

4. The structuring of all elementary school math concepts should be done with instructional methods which include practical applications. These real life practical applications might include:

a. the computation of each learner's home utility bills for fuel, telephone, electricity, water and automobile transportation.

b. the practice-writing of personal checks for paying household expenses.

c. planning a weekend class trip in terms of specific itemized costs.

d. computing or estimating the child's cost to his parents for one day, one week, one month and one year in terms of food, clothing, shelter, schooling, transportation and entertainment.

e. designing a personal budget for one month.

5. Problem solving skills should be taught to learners in the intermediate, middle and high school grades. These include the skills in sorting (right brain processing) the information given in the problem and the logical procedures for its solution (left brain activity).[8] The learner should be given the kind of information which can be comprehended and processed by the right hemisphere of his brain as well as the left simultaneously and in a complimentary way.[8]

6. Diagnostic teaching in mathematics, too, should respect the work of Piaget and the processes of cognitive development and not merely focus on computational errors and the reinforcement of the proper steps toward the correct answer. Diagnostic teaching should provide indicators of the learner's psychological development and readiness for the math concept being taught.

7. The child's home and school environments should be designed to encourage playing.

8. Learners should be encouraged to express and explain their ideas by sketching them as well as writing.

Changes in curriculums have been known to take years, decades or not to occur at all. What can be done in the classroom now to help middle and high school learners attain long term math retention and thereby improve their achievement in mathematics? Responses to this question might include special sixty day teaching efforts in the seventh and ninth grades for the purpose of helping learners structure the logic necessary for them to conceptualize the arithmetical operations presented in the first six grades. Seventh and eighth grade concepts and skills would be included for ninth graders. A similar program could be designed for upperclassmen who are closer to a trade, career, or college than their younger peers.

The key word in these special efforts is "structure" not "review." The difference is in the new emphasis on abstract conceptualization supported by the continued use of three dimensional manipulative teaching devices or models, eg. blocks, beads, abacus, rods and place value devices. The two dimensional tools might include diagrams, sets, designs, arrays, or number lines which relate algorithms or equations to specific visible corresponding representations. Two dimensional videos and three dimensional models should be coupled with any abstract methods of instruction which utilize alternate expanded algorithms, eg. the teacher should explain and expand algorithms and the processes for their solution as prime factors and components, patterns and sequences, arrays and properties. Other abstract learning devices should include an awareness of inverse and symmetrical relationships, i.e., subtraction as the inverse of addition, multiplication as a form of addition, division as the inverse of multiplication, division as repeated subtraction, fractions as decimal fractions and metrics as decimal fractions.

SUMMARY

Most of the basic mathematics fundamentals which the learner is expected to remember for passing high school and college exams are taught to him prematurely at a time when he is psychologically incapable of abstract conceptualization. Furthermore, the learner is taught the basic math fundamentals with methods and teaching strategies which require the use of abstract symbols at a stage in his life when he is capable of "conceptualizing" only when the concepts are physically structured for him, i.e., representations of a mathematical reality constructed in three dimensional form.

Those who insist on defending the rote memorization and conditioning methods for acquiring the basic mathematics fundamentals do so knowing that they are imprinting skill behavior on the short term memories of children whose psychological development is incomplete. They are forcing the child to use inappropriate neural networks. Long range memory develops between the sixth and ninth grade, ages nine to fourteen.

Among a child's limitations to think logically are his egocentrism of thought and the lack of consciousness of his own thoughts. He views problems from his own perspective and not that of others. Children are limited in engaging a conscious thought process for the purpose of solving problems, i.e., retracing their steps to defend a solution. When a child's thought processes are not entirely conscious he is limited in his ability to form definitions or generalizations. A child's incomplete psychological development limits his ability to engage the four basic ways that propositions in logic can be combined, i.e., by conjunction, disjunction, negation and implication. Not until the ages of eleven or twelve for some and thirteen to fourteen or later for others do learners possess a complete process to determine the correct solution from all possibilities.[4 and 13]

Piaget had a hypothesis concerning memory, what is remembered depends on the subject's stage of intellectual development. If he is correct, a child's memory performance at any given time is a function of his developmental stage, i.e., his memory and recall actually improve when he reaches each new developmental level. Test results tend to confirm Piaget's hypothesis.[13]

Research findings in cognitive development strongly suggest a major theme. Learners need teachers of mathematics who help them to conceptualize "why" the answer is correct as well as "how" to compute. Learners are able to internalize mathematical concepts, skills and defend a correct answer through logic. Logic for most elementary school learners must first be structured in physical form.*

Learners need mathematics teachers who recognize that long-term memory improves significantly after the sixth grade and who continue to include two and three dimensional devices when explaining skills and concepts.

Cognitively developed high school students will make the necessary mental connections to score well on SAT mathematics tests. They will be prepared for college, career or occupational training.

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*NOTE: Whenever possible the learner should be encouraged to structure the logic of a mathematic skill or concept in physical form for himself.

NOTE: The effectiveness of teaching the step-by-step routines for solving mathematics problems on any level might be judged in light of the high school mathematics test scores of the last decade. The psychological research in the teaching of mathematics to young learners, including the work of Jean Piaget, supports the teaching methods which include the use of three dimensional manipulatives, including the tripoint progression.

Analysis of the results of mathematics test scores of college freshmen who entered the City University of New York indicated that most students, among those who were able to correctly perform a given computation of a decimal concept, were not able to interpret its meaning.* How much longer will it be for these students before they forget the step-by-step routine?

A research suggestion might be to compare the learners' abilities to recall the skills for problem solving over a 6, 8 or 10 year period, i.e., how many students will be able to recall the step-by-step routine for solving a problem? How many students will be able to recall the meaning of the computation or be able to interpret the concept? How many students will be able to recall both the step-by-step routine and the interpretation of the concept? How many students will not be able to do either?

If skillful methods are essential to effective teaching, then where does the pursuit for excellence begin?

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This film series is not a specific footnote but I have a respect for the ideas and methods presented by Joseph Moray.