Using the Outdoors to Enrich the Teaching of Mathematics.

Written for teachers in elementary and middle schools and for personnel at outdoor education centers, this publication is the second in an ERIC/CRESS series on utilizing outdoor education methodologies to enhance the academic curriculum. The resource guide suggests ways of getting students interested in arithmetic and mathematics and how to make teaching math fun. Fifteen outdoor activities are keyed to the basic mathematics skills which the National Council of Teachers of Mathematics (NCTM) advocates each student acquire before completing high school. The skills include problem solving; applying mathematics in everyday situations; alertness to reasonableness of results; estimation and approximation; appropriate computational skills; geometry; measurement; reading, interpreting, and constructing tables, charts, and graphs; using mathematics to predict; and computer literacy. Each lesson is presented as an "idea" and contains descriptions of the NCTM skill, purpose, materials required, specific activity, and procedures. The lesson titles are: "Outdoor Shapes," "Hull Gull," "Arithmetic Treasure Hunt," "Cricket Thermometers," "Popcorn Scramble," "Seesaw Equations," "Practice with Dimensions of Circular Objects," "Outdoor Sets," "Ant and People Race," "Can You Locate the Center of an Acre?" "How Many Deer and Moose," "How Tall Is That Tree?" "Lumberjack Lesson," "Hunting Buried Treasure," and "Building and Using a Measuring Wheel." (NEC)
Using the Outdoors to Teach Mathematics
USING THE OUTDOORS TO ENRICH
THE TEACHING OF MATHEMATICS

by

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Preface

Written for teachers in elementary and middle schools and for personnel at outdoor education centers, this publication is the second in an ERIC/CRESS series on utilizing outdoor education methodologies to enhance the academic curriculum. The resource guide suggests ways of getting students interested in arithmetic and mathematics and how to make teaching math fun.

Recent events have highlighted the need to intensify and promote mathematics education in American schools. Since use of outdoor methodologies provides opportunities for students to see math in action, these kinds of activities are particularly appropriate to enhance computational skills and problem solving abilities that are required in later school years. These learning experiences provide the sound base for study recommended by the National Commission on Excellence in Education for those crucial first 8 years of schooling.

Dr. Milton Payne brings to this work 20 years of experience in the areas of science education, outdoor education, middle school teaching, and individualized instruction. He currently teaches several courses in outdoor education using a multidisciplinary approach and has conducted workshops and seminars in more than 30 school districts including Houston, Fort Worth, Texarkana, and Tyler. Dr. Payne was also one of the founders of the Texas Outdoor Education Association.

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Introduction

"I hate math!" If arithmetic and mathematics teachers had a penny for every time a student has made this statement, they would all be rich. Some enterprising schools have even had contests to find out whether girls or boys, or a particular race of people, or kids with certain kinds of grandparents hate math most. Parents sometimes have been known to get into furious arguments with their neighbors about whose kids hate math the most.

Everyone's getting tired of hearing about how much people hate math. It's time for a change. We need some buttons and bumper stickers that say, "I Love Math!" We need to get a big film company to produce a TV series with the hero a math teacher. We need a new image. What is the favorite subject for all kids? Recess. Right! Let's change math to recess. Can you imagine the shock parents are going to get when little Johnny comes home from school and tells how much fun he had in math today?

First, we'll have to find out as much as we can about recess. If you visit a school, the first thing you notice is that when recess is announced, the kids almost kill one another trying to see who can get out of the classroom first. It's as if being the second person out of the room at recess is even worse than having to sit a whole period working arithmetic problems. The next thing you will notice about recess is the kids are outside doing something. It's sometimes hard to tell what they are doing, but whatever it is, they are constantly busy at it. The third thing you notice is that they hate to come back in the room after recess. In fact, they play a game called "Who can make it back to class last?" And the winner is usually the kid who was winner at "Who can get outside first?"

That little boy or girl who always wins in "Who can get outside first?" is a very likely candidate for "mathophobia." This is a child
who is disinterested, bored, fearful, anxious, or otherwise negatively involved with math. The impact of such negative attitudes and feelings in a learning environment is widely acknowledged to be great. In fact, Lazarus (1974) identified fear of mathematics as one of the most serious problems facing today's mathematics teacher. Tobias (1978) stated that although math anxiety is no new development, it is of increasing concern to educators. Hodges (1983) and Sherard (1981) both discussed the problems of children's fear of mathematics. Brush (1980) reported that many students are just plain bored with mathematics and find their math classes dull and lacking variety.

This resource guide is about variety. It's about how to get kids interested in arithmetic and mathematics. It's about how to make teaching math fun.
Outdoor Teaching and the Goals of the Mathematics Curriculum

Outdoor teaching is not a new idea. People were taught outdoors before "schools" were developed. Children probably learned to count using pebbles; they measured patterns among flowers and leaves; they sorted and classified sea shells; they compared how much fluid different sizes of coconut hulls would hold; they grouped garden vegetables in sets; they computed the amount of food necessary for each individual in their tribe to be able to survive the winter. It is only in recent times that we have moved learning into classrooms.

And classrooms are great inventions. They allow us to enjoy climate control, provide wall plugs for film projectors, and give us a place to hang chalkboards and pictures of the outdoors. But recess is still the favorite time of the school day. So let's get back to how we go about turning math into recess.

Before we take mathematics outside, we must first come to grips with what we are supposed to be doing to kids in math class anyway.

When we read what experts say about what is supposed to be happening with kids and math, the first thing we find out is that whatever it is, it isn't happening nearly as much as people would like.

· William Lamon (1972) in the book, Learning and the Nature of Mathematics, reviewed the results of mathematics instruction from the 1920s through the 1960s and concluded:

But history has clearly shown that traditional mathematics teaching has produced and accomplished little with regard to people's ability to think and respond successfully in a mathematical environment. In other words, traditional approaches to mathematics have proven to be inappropriate, and modern approaches demonstrate distinct inadequacies. (pp. 7-8)
His research indicates that neither "traditional" nor "modern" approaches to mathematics instruction seem to make much difference in people's ability to use mathematical problem solving in everyday life situations.

Recent results from state mathematics assessment tests, standardized achievement tests, the Scholastic Achievement Tests, and the National Assessment of Educational Progress indicate that achievement for girls and minority groups is depressingly low. Achievement in the area of problem solving is low for all groups of students. An excellent review of this type of data can be found in a report by the Education Commission of the States (1979).

In our reading, we discover that most mathematicians and mathematics educators agree that the main reason we have kids go to arithmetic class is to acquire computational skills and mathematical thinking or problem solving ability. And, as we have read, many don't think the job is getting done very well.

There must be a reason kids aren't learning math. We know that many of them are afraid of it, are anxious about it, and are bored with it, but why is it they have gotten into this shape? More than 20 years ago, a group of mathematics experts comprised of 65 mathematicians and mathematics educators (Study Group in Theoretical and Applied Mathematics Curricula, 1962), stated what they thought was the main reason for low achievement in mathematics:

> Therefore, to introduce new concepts without a sufficient background of concrete facts, to introduce unifying concepts where there is no experience to unify, or to harp on the introduced concepts without concrete applications which would challenge the students is worse than useless; premature formalization may lead to sterility. (p. 190)

These professionals didn't think that enough concrete application of ideas and skills was being used by teachers. To most kids, arithmetic was something you did in school. They didn't see much use of it during
recess or in their homes. The little kids learned that they must know their numbers so that they could find the channel where Superman would be Saturday morning—but beyond that, they couldn't really see much sense in spending all that time with numbers. And algebra? Every high school student knows you have to take algebra to go to college. It's like diagraming sentences—don't ask about why. Just do it because everybody's always done it.

But that was 23 years ago. What are today's experts saying about why kids hate math—why don't they learn it very well? When we read Gagne (1977); Bruner (1966); Crosswhite, Higgins, Osborn, and Shumway (1973); Skemp (1972); Piaget (1964); Davis (1972); and Rosenbloom (1972), it appears that a host of mathematics experts throughout the past 20 or 30 years have been saying that effective and meaningful mathematics learning must be based upon real, concrete, hands-on, motivational learning experiences.

The highly respected Zolton Dienes, mathematician, psychologist, and educator, put it best perhaps when he explained that children learn mathematics by "doing." They must "experience" mathematics, and they do so best with the aid of a wide variety of objects and situations. He said that children should be taught mathematics by familiarizing them with concrete examples of the concepts or skills to be learned. The challenge of improving mathematics instruction, according to Dienes, lies in the hands of future teachers. Dienes (1972) states:

[Teachers who are] responsible for creative mathematical interaction between child and environment must learn some new classroom techniques...when a child is making an inquiry, it is not the teacher who should provide the answer but the situation, which should be set up [so] that the truth is discoverable...it will also be necessary to concentrate on the techniques of getting the children to work in small groups...discussion between peers is a very important ingredient in learning...very little discussion takes place between child and child in matters of mathematical importance. This is probably because situations
have not been contrived in which mathematics becomes important to children. (pp. 56-66)

Dienes further suggests several instructional strategies which are easily implemented outdoors: placing children in small groups to work and share ideas, creating experiences important to children, and creating situations in which the student can discover and use a wide variety of concrete materials.

Mathematics laboratories and learning centers are means of providing children with concrete objects and meaningful learning experiences. The outdoors should be thought of as an extension of the classroom—a giant mathematics laboratory and learning center!

As mentioned earlier, mathematics instruction seeks to enhance computational skills and problem solving ability. The National Council of Teachers of Mathematics (NCTM, 1980) has divided these 2 broad areas into a set of 10 "basic mathematical skills" which each student should acquire before he/she completes high school:

1. Problem solving.
2. Applying mathematics in everyday situations.
3. Alertness to the reasonableness of results.
4. Estimation and approximation.
5. Appropriate computational skills.
7. Measurement.
8. Reading, interpreting, and constructing tables, charts, and graphs.
10. Computer literacy. (p. 6)

The outdoor activities included in this guide have been keyed to this set of basic skills.
Exploratory and Systematic Mathematics Teaching--Outdoors

There are basically two types of teaching strategies in the area of mathematics instruction--"systematic" teaching and "exploratory" teaching.

"Systematic" instruction uses deductive methods. In practice, it is most often used in teaching children the fundamental operations and developing their mathematical skill. It implies the use of systematic reinforcement, drill, and practice. This type of instruction is usually best done within the classroom. It is functionally better served with books, paper, pen, and desks. There are, however, many outdoor mathematics activities which are highly effective reinforcers of facts and rules even though this outcome may be secondary to the primary intent of the outdoor activity.

"Exploratory" teaching has as its main goal the development of mathematical thinking and problem solving ability. It is primarily inductive. It is often called the "guided discovery" approach. In exploratory, or guided discovery teaching, the teacher establishes a situation or problem and the children (with guidance from the teacher, as necessary) develop methods and procedures for solving the problem. It encourages divergent rather than convergent thinking. Children are encouraged to be creative, to use their imaginations. The teacher functions as a resource person and as a provoker of thinking. This method of instruction is effectively served with a wide variety of outdoor activities. Such activities involve real life situations, have great variety, and include manipulation and experimentation with concrete objects. These procedures furnish the teacher with a means of implementing modern research findings in mathematics which conclude that frequent use of physical objects and real-life situations is essential for developing abstract concepts and for meaningful transfer of learning.
Activities for the Outdoor Mathematics Laboratory

Paul Rosenbloom, developer of the Minnesota Mathematics and Science Teaching Project and recipient of many honors in mathematics, has identified mathematical work as consisting mainly of "observation and experiment, guessing at what might be true, feeling what ought to be true, testing hypotheses, looking for analogies, building mental pictures, and trying out ideas without any certainty of success" (1972, p. 88).

It is within the context described by Rosenbloom that the outdoor mathematics laboratory makes its greatest contribution to the goals of mathematics instruction. The activities which follow are intended to function as illustrations of how concepts and skills which are generally taught indoors with books, chalkboard, paper, and pen can be taught on the school grounds.

The activities described are not identified for a particular level. The maturity and mathematical background of children vary so much that what is appropriate in a given classroom in a given school may be totally inappropriate for another classroom in another school.

The activities are listed somewhat in order of complexity; however, it is strongly believed that each activity contains ideas which will be useful in helping teachers of all grade levels to become more proficient in recognizing the wide range of possible lessons adaptable for outdoor instruction.

You will probably need to modify each activity to fit your particular needs. The activities are designed to be springboards for your own thinking, not to be finished or completed lessons. Each lesson is presented as an "idea" which includes some specific math skills; but, more that that, it is presented as a way, a method, of teaching. As such, the lessons should function as a stimulus for your own creativity.
Textbooks, resource guides, and other curriculum materials contain a wealth of ideas and activities about mathematics. It is hoped that the activities presented in this guide will help you learn to read an activity from a textbook or curriculum guide and ask, "Could this activity be effective if taught using outdoor resources?" When you find yourself saying "yes" for some of them, then you are on your way to some exciting and rewarding experiences in teaching. As the old saying goes, "Try it—you'll like it!"

Notes Concerning Activities

1. Each of the following activities is keyed to at least one of the basic skills identified by the National Council for Teachers of Mathematics and listed on page 6 of this guide.

2. When collecting is called for, you should establish guidelines for what should not be collected and what should be done with the collections upon completion of the activity.

3. A whistle is helpful in most activities in the outdoors—use it as the signal for children to complete a task and join you at a specified location.
OUTDOOR SHAPES

A. **NCTM Skill:** Geometry.

B. **Purpose:** To provide practice naming and recognizing five geometric shapes.

C. **Materials:** Each child should have an envelope containing the following shapes cut from colored construction paper:
   - circle--red
   - square--yellow
   - triangle--blue
   - ellipse--green
   - rectangle--orange

D. **Activities and Procedure:** After the shapes have been introduced in the classroom, take the students outside to see how well they have learned them. Have each child bring along his envelope of colored shapes. Before leaving the room, tell the children which shape you will be looking for first, and have them take that shape out of their envelopes.

   As you walk along, ask questions: "Have you found something shaped like a circle?" When someone has, stop and compare it to the shape the children are holding.

   After your discussion of circles, have the children put their circle shapes back into the envelopes and take out the next shape you choose. "Let's try to find something square. What color is your square?"

   When someone in the group has found a real object corresponding to each shape, seat the children. Then encourage each child to look across the school grounds to find a shape he/she can tell the others about. From a distance the children may see different shapes of trees; the school bus may look like a rectangle, the frame of the swing set may look like a triangle, and so on.
This is a good time to let them lie back and watch the cloud shapes. Here are shapes that change; encourage the children to compare these shapes with the ones in their envelopes.

Outdoor geometry can blend so easily with science and language arts. It's easy to see how you could lead this activity into creative story-telling and observing nature.
HULL GULL

A. **NCTM Skills:** Problem solving, appropriate computational skills.

B. **Purpose:** To provide practice in subtracting (literally "taking away") and making a guess.

C. **Material:** Small pebbles.

D. **Activities and Procedure:** Take the children to a graveled area of the school grounds and let each child select 10 small pebbles. (They must be small enough so that all 10 will fit into a closed fist.) When the children have been seated, arrange for each child to have a playing partner. As the players sit facing each other, the first player hides a few pebbles in his/her hand, shakes them (if his/her hand is large enough), and announces that he/she is ready to play by saying:

   First: "Hull Gull."
   Second: "Handful?"
   First: "How many?"

   The second player guesses. If the guess is right, the second player gets the first player's pebbles. If the guess is incorrect, the second player gives up the difference between the guess and correct number. (That is, if the first player is holding five, and the second player guesses seven, the second player must give up two pebbles.)

   Then the second player hides an unknown number of pebbles in his/her hand, and it is the first player's turn to guess the amount. And so the game continues until one player gets all the pebbles.

   This is an especially good arithmetic game because it requires no equipment and, once learned, is enjoyable enough that the children will want to play it on their own time. (Note: This is a very old game, said to have had its origin in ancient Greece.)
ARITHMETIC TREASURE HUNT

A. **NCTM Skill:** Problem solving.

B. **Purpose:** To provide experience in counting, recognizing numerals, and grouping into sets. The activity can also be expanded to include similarities and differences and counting for addition.

C. **Material:** Each child will need a large paper bag with the numerals 1, 2, 3, 4, and 5 printed vertically on it. (This provides a good way to recycle grocery bags!)

D. **Activities and Procedure:** Explain to the children that the class is going on a treasure hunt. Give each child a bag to hold his/her treasures. These are to be nature treasures, and they are to be sets of treasures. There must be a group of two things that are alike, another group of three things that are alike, a group of four things, a group of five things, and a group that has only one thing in it. Each child determines the nature of his/her treasure. Elaine may have three rocks in her set of three, while Bill can choose to have three leaves in his set of three.

Take the children to a suitable area on the playground and give them a few minutes to gather their sets. When everyone is back and seated in a circle, have each child take the treasure from the sack and arrange his/her sets to correspond with the numerals written on the sack. You can now conduct whatever discussion about the sets that you wish.

This activity can be done in a small group or with partners. It can be kept simple, then expanded as the children's mathematical skills increase. After talking about sets you can re-group the treasures. For instance, put all the leaves together, and then make new sets of them. How are these things alike? In what ways are they different? How can we make new sets from the leaves?"
You can have each child choose a partner and combine sets.
"Joe had one stick and Jenny had a set of three sticks. Count how many sticks you both have together."
CRICKET THERMOMETERS

A. **NCTM Skills:** Appropriate computational skills; measurement; reading, interpreting, and constructing tables, charts, and graphs.

B. **Purpose:** To develop skill in using the thermometer, keeping accurate records, calculating percentages, and comparing the Fahrenheit to the Celsius scale.

C. **Materials:**

1. Suitable containers for crickets.
2. Thermometers—Fahrenheit and Celsius if available.
3. Stopwatch—watch with second hand will work.
4. Chart for recording data.

D. **Activities and Procedure:** Children may never think of insects as being measuring instruments; yet folklore says that we can tell the temperature by listening to crickets chirp. How accurate would a cricket thermometer be?

The most enjoyable part of this activity for the children will be, of course, catching the crickets. Where can we find crickets? Let the children answer—they'll know—under leaves, in thick grass, under boards, in the garden. Can we find them by listening to their chirpings? Don't overlook this aspect; it helps control the noise the children make as they are searching! Divide the class into groups of 4 or 5—preferably with a fearless naturalist in each group. Provide each group with a container with a lid. Plastic freezer containers are good for this. The cricket can be transferred to a see-through glass home after the scrambling to catch the cricket has been done. Plan which groups will go to which areas of the school grounds. Have all groups report back to a central location after 5 minutes or so. If crickets cannot be found on the school site, have children collect them after school. Have a speaker from each group share experiences with the others. Did you

While the children are outside, they can prepare homes for their crickets. A jar with soil and a bit of rooted grass or small weeds will do fine. The cricket will also appreciate a few leaves to hide under. A removeable lid with air holes in it will keep the cricket in and will allow the children to feed it small bits of apple.

If you are lucky enough to have a cricket that is already chirping, the class can record its first cricket temperature reading right there on the school grounds. You will, of course, have brought along a thermometer and your watch with the second hand on it "just in case."

To determine the temperature by cricket chirps, count the number of chirps in 1 minute, divide by 4, and add 40. The answer is the approximate temperature in Fahrenheit degrees. The number of times the cricket chirps in 10 seconds will give you the approximate temperature in Celsius units.

Check the cricket for accuracy by putting the thermometer in its home and reading it.

You will want to keep temperature records for several days, so plan to have the crickets as classroom pets for a while. From among the crickets brought in, hopefully you will have at least one good "chirper." If you have more than one, so much the better; each group can monitor its own cricket.

Here is a sample chart on which to record your data:
<table>
<thead>
<tr>
<th>Date and Time</th>
<th>Cricket Temperature</th>
<th>Thermometer Temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Let the children compute their results as a percent:

Times tried: ______________________
Times correct: ___________________
Percentage correct: ______________

You will want to vary the temperature readings by checking at different times during the day and in different locations (both inside and outside). Note: This activity is obviously a multifaceted one. It meshes well with science and with language arts. Don't neglect to leave a copy of *The Cricket in Times Square*, by George Selden, lying around the classroom. Children will enjoy reading it.

You may wish to read aloud the following quotation from "Cricket As Pets," which appears on pages 15 through 17 of *A Junior Naturalist's Workbook*, by John Gardner (1969):

For hundreds of years the Chinese and Japanese have kept crickets as pets in order to enjoy their merry chirp and so that they would have good luck throughout the year.

Fall is the best time to look for crickets. In fact, literature tells us tales of the chirp of the cricket as describing the coming of autumn.

You can probably find several crickets living in your yard or a nearby field, under stones and in burrows where they live on grass and clover. Only the male cricket chirps, so you
may have to collect two or three in order to insure the cricket song. He has his wing covers developed into a sort of violin, which he rubs back and forth to develop his song.

When the cricket is ready to make his song, he lifts one wing about 45 degrees and draws the scraper of the under wing against the file of the upper wing. To insure that his musical equipment will not wear out, the cricket can change from wing to wing whenever he wants to do so.

A FEW REMINDERS:

1. Place your cricket cage in a sunny window.
2. Water the dirt lightly to keep the grass or plant growing.
3. Keep your cricket cage clean and free from decaying food.
4. Remember your cricket is a living thing, so treat it kindly.
THE POPCORN SCRAMBLE

A. **NCTM Skills:** Appropriate computational skills; reading, interpreting, and constructing tables, charts, and graphs.

B. **Purpose:** To provide practice transferring data to a graph and applying the concepts of ratio and proportion.

C. **Materials:**
   1. Colored popcorn kernels.
   2. Poster board.

D. **Activities and Procedure:** Select an area of the school grounds which is sparsely covered with short grass. Take the class to this area and have them form a circle. Each student should be about 15 feet from the center. Stand in the center and explain that you are going to distribute popcorn kernels inside the circle and that at the word "scramble" they are to pick up as many kernels as they can within the time limit you will set.

   To prepare for this activity you will need to have selected popcorn so that you have about 65 kernels of each color you have chosen to use. It is best to count the kernels by hand, but if you are rushed for time, you can borrow a graduated cylinder from a science teacher and get a good estimate. Ten milliliters will hold about 65 kernels.

   You also should have at least six different colors. Natural corn is fine as one of your colors. If you wish, you can color additional corn using dye, paint, or food color. (Color one set bright or blaze orange. Students usually collect more of this color during the activity. Develop the idea of why "blaze orange" is recommended as winter clothing color for hunters.)

   Explain to the children that as soon as you have scattered the corn in the study area you will say "scramble" and then count
rapidly to 10. When they hear "10," they must stop collecting. Ask if they have questions. If not, then scatter the corn, get out of the circle!, and then give the signal. You should allow no more than 5 or 6 seconds for your count.

Ask each student to develop a chart which represents his/her collection of popcorn. The following is an example:

**Popcorn Scramble Data**

Student: Bob Jones  
Number of Kernels: 11  
Distribution by Color:

- White -- 2  
- Green -- 1  
- Orange -- 4  
- Red -- 2  
- Brown -- 1  
- Natural -- 1

Have students share results of their scramble. Ask whether there is a way to communicate the results of the total group. Discuss the use of a graph.

Using a large sheet of poster board, ask students to help develop the graph. What colors were found? Have students compute the number of kernels for each color. What was the total of all colors? The following is an example:
You might wish to discuss topics suggested by the results of the data from the "scramble." Would brown be the color found least on another area of the school grounds? What is meant by "protective coloration?"

Have the students make inferences about the total number of popcorn kernels you scattered in the study area. In this case, if a sample of each color in your original set of 8 colors was found and you had counted 65 of each color, the total number of kernels would be 520. Have the students compute the ratio of total found to total distributed. In this case, 173 were found. Therefore, the ratio of "collected" to "distributed" is 173:520.

You may also want the students to determine the percent of the total found as well as the percent of each color found. "If there were 65 blue kernels and we found 23, what percent of the blue kernels did we find?"
Or you may wish to have the students determine what percent of the total of all colors is represented by the number of orange found: "If there were a total of 520 distributed kernels, and we found 37 orange kernels, what percent of the total is represented by the orange kernels found?"
SEESAW EQUATIONS

A. **NCTM Skills:** Appropriate computational skills, measurement.

B. **Purpose:** To provide practice with multiplication and equations.

C. **Materials:**
   1. Bathroom scales.
   2. Seesaw.
   3. Tape measure.

D. **Activities and Procedure:** This activity should be done at a time during the day when your class can get access to the playground seesaws. Divide the class into groups according to the number of seesaws available; six to eight per group is fine.

   It is necessary that each child who participates in this activity knows his/her weight. If you can collect several bathroom scales, they can be used by the children during the activity.

   Select four children to help you demonstrate the activity. Among the four include one of the lighter students and one of the heavier students. The other two students should be included because they have the same weight.

   Review the use of the symbols $>$, $<$, and $=$. Choose a well-balanced seesaw. Ask the two children with the same weight to climb on the seesaw, one at each end. The seesaw should balance. Ask the children which symbol should be selected to represent the seesaw with the first two students on it. The $=$ symbol should be chosen to represent equal weight and balance of the two children.

   Now have the children measure the distance from the fulcrum point to child A. Do the same for child B. The distance should be approximately equal.

   Introduce the formula: \( \text{distance} \times \text{weight} = \text{distance} \times \text{weight} \). Illustrate the formula using the seesaw and the two children. If
child A and child B each weigh 70 pounds, and if the distance from
the fulcrum point to each child is 6 feet, then \( dxw = dxw \) is
found by substituting:

\[
(6)(70) = (6)(70) \\
420 = 420
\]

The equation is solved.

Next, demonstrate the seesaw equation using the other two
children. Assume that child C weighs 100 pounds and child D weighs
50 pounds. Ask each child to get on the seesaw. Ask the children
which symbol illustrates the relationship of child C to child D.
The symbol \( \geq \) should be used. Ask the students about the
relationship of child D to child C. The symbol \( \leq \) should be used.

Explain that the seesaw equation is not balanced. Ask the
children how, according to the formula \( dxw = dxw \), the seesaw
could be balanced. Discuss the answers suggested. They will
usually suggest (1) add more weight; (2) move the seesaw on the
fulcrum; (3) change the distance between the students.

Ask the students to compute the distance that child C needs to
be moved according to the formula:

\[
dxw = dxw \\
6 \times 100 \neq 6 \times 50
\]

Let \( x \) equal the distance child C must be from the fulcrum in
order to balance the seesaw; then:

\[
(x)(100) = (6)(50) \\
100x = 300 \\
x = \frac{300}{100}
\]

\( x = 3 \)
Therefore, child C must be 3 feet closer to the fulcrum. Move him that distance and the seesaw should balance. If the seesaw does not balance, ask the children to identify the probable cause. If calculations have been made properly, then imbalance is most likely due to lack of precision in balance of the seesaw.

When the students understand the procedure, have them go to the seesaws and work out several seesaw equation problems on their own.
PRACTICE WITH DIMENSIONS OF CIRCULAR OBJECTS

A. **NCTM Skills:** Geometry; measurement; reading, interpreting, and constructing tables, charts, and graphs; appropriate computational skills.

B. **Purpose:** To practice measuring skills, charting data, solving for radius and circumference; to reinforce the meaning of circumference, diameter, radius, and π.

C. **Materials:**
   1. Tape measures.
   2. Metersticks or yardsticks.
   3. Yarn.

D. **Activities and Procedure:** Practice in concept development using measurement skills is especially well suited to out-of-classroom areas. Sometimes children fail to translate paper-and-pencil classroom activities to objects in the outside world. Measurement of circular objects illustrates how skills which have been developed indoors can be reinforced with outside practice.

   After completing lessons on geometry or measurement of circles, take the children outside. First ask the children to identify objects which are circular. They should come up with objects such as tree trunks, smoke stacks, automobile tires, flag poles, trash cans, and other examples. Explain that the activity will be to determine the circumference, diameter, and radius of some of these objects.

   You may want to flag the objects for this activity prior to taking the children to the study area.

   Divide the class into teams and set a time limit for the teams to measure the objects and do the necessary calculations.
When the teams have completed the tasks, return to the classroom and have the students organize the data they have collected into a chart on the chalkboard. The chart may be like the one below:

**Measurement of Objects on the School Grounds**

<table>
<thead>
<tr>
<th>Circumference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Team I</td>
</tr>
<tr>
<td>Team II</td>
</tr>
<tr>
<td>Team III</td>
</tr>
<tr>
<td>Team IV</td>
</tr>
<tr>
<td>Team V</td>
</tr>
<tr>
<td>Tree Trunk</td>
</tr>
<tr>
<td>Trash Can Lid</td>
</tr>
<tr>
<td>Flag Pole</td>
</tr>
</tbody>
</table>

Develop a similar chart for radius and diameter measurements. To determine the diameter of a tree, use three yard- or meter-sticks. Have the team members hold two of the sticks horizontally on opposite sides of the tree. The sticks should be parallel and at the same height. Extend the ends of the sticks in one direction beyond the tree trunk. With the third stick measure the distance between the extended portion of the two parallel sticks. This is an approximation of the diameter.

Another method which can be used is to take yarn, string, or a tape measure, wrap it around the tree, and measure the length required to encircle the tree. The diameter is about one-third of the circumference.

**Note:** This activity can be extended to upper grade skills by using the geometric formulas $C = \pi d$ (or $C = 2 \pi r$) and $r = C/2\pi$. Make one measurement and solve for the other. For example, if the radius is 6 inches, solve for the circumference: $C = 2 \times 3.14 \times 6$. If the circumference is 10 inches, solve for the radius: $r = 10/2 \times 3.14$. 

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Another use you could make of their measurements would be to discover what $\pi$ (pi) represents. Have the children look at the data gathered on circumference and diameter and see if they can find any special mathematical relationship between each pair of measurements. If no one finds any, instruct them to divide each circumference by its diameter and see what they get. In each instance the answer will be $3.14$ or $3 \ 1/7$. This relationship is constant regardless of the size of the circumference and the diameter. It is identified by the symbol $\pi$ and used when working mathematically with circles.
A. **NCTM Skills:** Problem solving, appropriate computational skills.

B. **Purpose:** To develop the concepts of sets, relationships, and attributes, including grouping and regrouping sets.

C. **Materials:**
   1. Paper bags.
   2. Yarn.

D. **Activities and Procedure:** Explain to the children that you are going to take them on a walk around the school grounds. Tell them that each of them will be given a sack and will get to collect some objects while on the outdoor hike. Ask the children to name some things they might find.

   Group the children in teams of two or three. Assign each team a task. For example, Team I is to find three rocks of different sizes, three different-shaped leaves, and three things which have different colors. Team II might be looking for three rough objects and three smooth objects. Team III might be looking for three things that are brown and three things smaller than a marble. Team IV might be looking for three things that are made of paper and three things that are made of wood. The tasks will depend upon the ages of your children and the environment of your school grounds.

   When the children have collected the objects, have them sit down in a circle around you. Members of each team should sit together. Give each team one piece of yarn for each task it has been assigned. Each piece should be a different color and about 2 feet in length.

   Tell the children to lay each length of yarn down on the ground in the shape of a circle. The students should lay the yarn
in front of them so that it will be easy for the other children to see all the circles.

Tell them to group the objects they have collected in the circles of yarn. For example, Team I would place the three rocks in one circle of yarn, the three different-shaped leaves in another circle of yarn, and the three different-colored objects in the third circle.

Call upon certain children to tell what they have in one of their circles. Ask why they have the objects together. Have as many children as time will allow explain what is alike or different about the objects in each set or circle.

Give each team an additional piece of yarn for each set of objects they have. Visit each team and make one circle of yarn near the sets they have in front of them. Ask them to take the extra pieces of yarn you have given them, and lay each piece down so that it joins each one of their sets to the empty set. The circles might look like the following figure:
Ask the children to think of a property or attribute which is true of one object from each of the sets they have made. If they have two different sets, they should try to find an object from set one that matches an object in set two in some way. If they have three sets, they are to try to match attributes from three objects (one from each set). Teacher guidance will depend on student need.

When they have done this, ask them to place these matched objects in the empty set you have formed. Again have the children share about the groupings they have made, explaining the similarity of the objects in the new sets.

Place a large (3-foot diameter) circle of yarn in the center of the circle of children. Ask one child to select one object from the new set his/her team has made. Ask the student to bring the object and place it in the center circle. When this is done, give the student enough yarn to connect the center circle to the set from which he or she selected the object. You may wish to cut these pieces of yarn to length as the activity unfolds. The circles will look like the figure on the following page:
Ask another group to find one object from their sets which is like the object which is now the center set. You may wish to have the groups do this in order from right to left or some other such scheme to avoid confusion.

Through this process the children visualize a large number of similarities and differences. They practice combining parts into a whole and also separating a whole into parts.

Depending upon the ages of the children, you may also want to count the number of objects in the sets, explain an "empty set," and discuss the union and intersection of sets and the attributes of a wide variety of objects.
THE ANT AND PEOPLE RACE

A. **NCTM Skills:** Problem solving; appropriate computational skills; measurement; reading, interpreting, and constructing tables, charts, and graphs.

B. **Purpose:** To provide practice in graphing, computing speed, and comparing by simple ratio.

C. **Materials:**
   1. Paper towels or plastic spoons.
   2. Stopwatch.
   3. Chalk--colored if possible.
   4. Rulers.
   5. Tape measures.
   7. Poster boards (2).
   8. Strings (2 long pieces and 1 short piece for each child).
   9. Paper and pencil for each child.

D. **Activities and Procedure:** In order for each child to be able to participate in and to enjoy this activity, it must be well-organized before you leave the classroom. Make sure that the children understand the three basic parts of this activity. There will be an ant race, a people race, and the recording and computing of data. Divide the class into two groups. One group can run the ant race while the other does the people race. Then they can switch. Have the basic outlines for a bar graph on each poster board—one for the ant race and one for the people race.

   For the ant race, you will need a paved or concrete (or non-grassy) area with ants nearby. Let each child in the first "ant group" use the paper towel or plastic spoon to capture and bring to the cement area a speedy-looking ant. Each child should measure his/her ant and record its length on paper before the race. (Decide beforehand what units of measurement you will be using and provide
suitable measuring instruments.) Give each child a piece of chalk, a string, and a ruler. Have students spread out on the concrete area so that each ant will have plenty of room to "race." Then you (or a student timer) say "Go!" and each child releases his/her ant and traces the path it takes with chalk. After 10 seconds, call "Stop!" Using the string to follow the winding path of the ant, have the students measure and record how far the ant traveled in the allotted time. Let each child record the name of his/her ant and the distance it traveled on the ant race graph.

For the people race, you will need a large, smooth area with a length of at least 80 feet. Have the children line up with their heels on the starting string. (To make measuring easier, you may want to put another long string parallel to the starting string 50 feet away.) When the timer says "Go!" each child is to walk as fast as he/she can in a straight line for 10 seconds. (Use whatever time you used for the ant race.) When the timer says "Stop!" each racer is to stop where he/she is, mark that place, and measure how far it is from the starting line. Have each child record this on paper, then plot his/her name and the distance traveled on the people race graph.

The third part of this activity can be done by each group at the conclusion of each race or when you call the students together to discuss the great "races." Each student should have on paper how long his/her ant was and how far it traveled. Have each child place on the record his/her own height and how far he/she could walk in the allotted time. Now we are to the real "race." Who went farther according to size? You? Or your ant? Have each child compute how many body lengths he/she covered, and how many his/her ant covered. How do these figures compare? Can the students figure a mathematical way to make themselves as small as the ants? To make the ants as large as people? From here you can compute inches per hour, feet per hour, miles per hour (or centimeters, meters, and kilometers per hour). How far might an ant travel in a day?
This activity should be a successful experience for each child. Even if he/she can't figure a mathematical method to compare his/her height to the distance he/she "raced," a student should be able, using all the measuring devices available, to measure physically: "I am this long. It takes how many of these measures to equal the distance I raced?" On the other hand, your mathematical "geniuses" can carry their computations as far as they wish. That is, you can just stop here with a pleasant introduction to the idea of ratio, or you can go on to express it in different ways.

Note: The ant race may be conducted on bare ground and the ant's path traced with a stick if a concrete area is not available.
POSTER BOARD GRAPHS -- EXAMPLES

ANT RACE

Inches Traveled

<table>
<thead>
<tr>
<th>Inches Traveled</th>
<th>Lightning</th>
<th>Silverstreak</th>
<th>Speedy</th>
<th>Big Red</th>
<th>Creepy</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
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</tbody>
</table>

Name of Ant: Lightning, Silverstreak, Speedy, Big Red, Creepy

PEOPLE RACE

Feet Traveled

<table>
<thead>
<tr>
<th>Feet Traveled</th>
<th>Stephen</th>
<th>James</th>
<th>Renee</th>
<th>Susan</th>
<th>Eric</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
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</tbody>
</table>

Student: Stephen, James, Renee, Susan, Eric
CAN YOU LOCATE THE CENTER OF AN ACRE?

A. **NCTM Skills:** Geometry, measurement, appropriate computational skills.

B. **Purpose:** To reinforce several mathematical skills while developing the idea of the dimensions of an acre. Children often read about an acre of land. They work area problems, geometry problems, and other problems having to do with acres. Few children (and perhaps few adults) really have an accurate concept of the size of an acre.

C. **Materials:**
   1. Several compasses.
   2. Measuring tapes (50- or 100-foot tapes are the best).
   3. Yarn.

D. **Activities and Procedure:** This activity requires a large area of the school grounds. Depending upon the size of your grounds, you will need to have the students conduct the study in teams of nine. Each team needs an acre of ground.

   **Note:** The directions given for compass readings will need to be adjusted to fit the layout of your school grounds.

   Explain to the students that the task of each group is to locate the center of an acre. To do this, they will first need to lay out an outline for a square acre.

   Each angle created in this activity will be 90 degrees. Use the "three, four, five right triangle" method (Pythagorean theorem) to demonstrate the concept of a right angle as follows:

   Cut three pieces of yarn:
   1. 12 feet long.
   2. 16 feet long.
   3. 20 feet long.
Form the pieces of yarn into a triangle. The Pythagorean theorem holds true for this triangle, thus defining it as a right triangle. The formula defining a right triangle follows (a, b, and c represent the pieces of yarn from shortest to longest):

\[ a^2 + b^2 = c^2 \]

If \( a = 12 \), \( b = 16 \), and \( c = 20 \), then:

\[ a^2 = 12 \times 12 = 144 \]
\[ b^2 = 16 \times 16 = 256 \]
\[ c^2 = 20 \times 20 = 400 \]

\[ 144 + 256 = 400 \]
\[ 400 = 400 \]

The angle formed by \( a \) and \( b \) measures 90 degrees.

The following four steps should be followed to complete this activity.

**Step I**

Divide each team of nine into three groups of three.

- **Group 1** stays at the starting point.
- **Group 2** measures 69.5 yards north.
- **Group 3** measures 69.5 yards east.

![Diagram of starting point and directions](image-url)
Step II

Group 2 sends one member 69.5 yards east.
Group 3 sends one member 69.5 yards north.

Step III

One member of each group measures 49 yards towards the group farthest away from you (diagonally across the acre). The point where the four students meet (point of intersection of lines a and b) is the center of the acre.

Step IV

All group members look at the location of all other group members. They should be able to see what an acre looks like and where its center is located.

You may find the following background information about measurements of an acre useful:
1. one acre = 4840 square yards

2. one square acre has four equal sides, each = 69.57 yards (4840 = 69.57^2)

\[
\text{69.57 yds.}
\]

3. .57 of a yard = 57/100 of 36 inches = 20.5 inches
   \[57/100 \times 36/1 = 2052/100 = 20.52 = 20.5\]

4. Pythagorean Theorem (all right triangles):
   \[a^2 + b^2 = c^2\]
   The sum of the squares of the shorter sides (legs) equals the square of the longest side (hypotenuse).

\[
\begin{align*}
a^2 + b^2 &= c^2 \\
69.57^2 + 69.57^2 &= c^2 \\
4839.98 + 4839.98 &= c^2 \\
9679.96 &= c^2
\end{align*}
\]

If \(c^2 = 9679.96\), \(\sqrt{c^2} = \sqrt{9679.96}\)

Then \(c = 98.38\)
Therefore, the longest side measures 98.38 yards and the midpoint of the longest side, at 49.19 yards, would be the center of the square acre.
A. **NCTM Skills:** Problem solving; estimation and approximation; appropriate computational skills; reading, interpreting, and constructing tables, charts, and graphs; using mathematics to predict.

B. **Purpose:** To develop the concepts of quadrant study and sampling technique. Children use estimation and computation and are introduced to a variety of elementary statistical concepts.

C. **Materials:**
   1. A half-pint jar.
   2. A pound of popcorn kernels (deer).
   3. 200 dried lima beans (moose).
   4. Index cards.
   5. Ruler, yard- or metersticks, tape measures.

D. **Activities and Procedure:** Prior to class, locate an area of the school grounds that has sparse grass cover. Measure plot "A" and flag each corner. Plot "A" should be a 10' x 15' rectangle. Measure plot "B" at 20' x 10' and flag each corner.

   Sprinkle one half-pint of popcorn kernels (to represent deer) in each plot. Scatter them by hand as evenly distributed as possible. There are about 1,700 kernels in one half-pint.

   Scatter 100 dried lima beans (to represent moose) in each plot.

   Draw a rectangle on the chalkboard. Review the procedure for computing the area of a rectangle—length times width. Have the students do a few problems dealing with area before going to the study plots.

   Explain to the class that they are going to be biologists and are to determine how many deer and moose are on nearby "ranches." Divide the class into two groups. Subdivide each group into teams.
of three or four students. Group I will be assigned to one of the plots, and Group II will work with the other plot.

Tell the students that several teams will be working on each "ranch" and that each team is to work independently. Explain that each team is to calculate the number of deer and moose on the ranch it is assigned. No deer or moose are to be picked up—they must be left where they are found. If a team takes out deer or moose or disturbs the distribution, other teams may obtain false counts.

Explain that each team is to decide how to determine (a) how many deer are in the area and (b) how many moose are in the area. Once they have decided upon a procedure, they should carry it out. Each team's study should result in a single number for the deer estimate and one for the moose estimate. Ask each team to write both of these numbers on the index card you will provide. Once a team completes the study, it should come and join you but should not, as yet, reveal its findings.

When each team has completed the study, use a poster board to record the results. On the poster board, make a column including the team name for each group. Label a column for "deer" and a column for "moose."

Ask Team A of Group I to report on the exact procedure used to estimate the deer and moose populations. After the team explains the procedure, one member writes on the chart the team's estimates of deer and moose. Each team, in turn, reports and records the findings.

The completed chart might look as follows:
### Deer and Moose Count

<table>
<thead>
<tr>
<th>Team</th>
<th>Deer</th>
<th>Moose</th>
<th>Team</th>
<th>Deer</th>
<th>Moose</th>
</tr>
</thead>
<tbody>
<tr>
<td>Team A</td>
<td>642</td>
<td>8</td>
<td>Team A</td>
<td>1968</td>
<td>75</td>
</tr>
<tr>
<td>Team B</td>
<td>1295</td>
<td>114</td>
<td>Team B</td>
<td>3695</td>
<td>18</td>
</tr>
<tr>
<td>Team C</td>
<td>4059</td>
<td>68</td>
<td>Team C</td>
<td>2696</td>
<td>29</td>
</tr>
</tbody>
</table>

When the data chart is completed, reveal the actual numbers of deer and moose on each ranch.

Discuss with the class the usual procedure in doing a census study. First, compute the area of the study plot. In this case, Ranch A is 10' x 15', or 150 square feet in area. Ranch B is 20' x 10', or 200 square feet in area.

Next, decide upon the "area" (one square foot, for example) for each sample and the number of samples to be taken. Mark off the area, and count the objects of the survey (deer and moose, in this case) which are in the sample. Compute the average (mean) of the sample plots. If the average is 22 deer, then the number in a 150-square-foot area is 22 x 150, or 3,300 deer. This is an estimate derived by the random sampling technique.

Explain to the class that the accuracy of the random sampling technique is dependent upon the accuracy with which the area of the study plot is computed, the accuracy of the count of each random sample, the number of samples taken, and how evenly the population is distributed over the area. Six samples should lead to a more accurate estimate than two samples.

Follow this activity with another "deer-moose" study a few days later. This time, use one plot measuring 50' x 50'. (If you wish, you can make two study plots of this size.) Scatter a pint of popcorn kernels in the area (about 3,400 kernels) and 500 lima beans. Divide the class into teams again. They can all use the
same study plot this time, or they can work out of two plots as they did previously. Your students should be much more accurate in their estimates this second time, and the range of estimates should be much narrower than that for the first activity.

If your students have practiced computing the area of a circle, you might wish to lay out a circular "ranch" for this second "deer and moose" count. Students can also compute the ratio of deer to moose or possibly project population density of deer in a 2-year period of time. Assume 2 good years; two-thirds of the deer are females (does), and each year each doe has two fawns. Assuming a zero death rate for the 2 years, what would the population become? Questions and problems such as these are usually of interest to students and encourage them to go beyond superficial understanding of the skills and concepts involved.
HOW TALL IS THAT TREE?

A. **NCTM Skills:** Problem solving; appropriate computational skills; measurement estimation and approximation; reading, interpreting, and constructing tables, charts, and graphs.

B. **Purpose:** To determine the height of objects using two methods of estimation.

C. **Materials:**
   1. Poster board.
   2. Craft sticks.
   3. Tape measure.

D. **Activities and Procedure:** Almost everyone has stood at the base of a tall tree and guessed at its height. This activity takes advantage of this curiosity about height.

   Select a tree on the school grounds; or if no trees are on your campus, use a flag pole, water tower, utility pole, tower, or some other tall structure. Take the children to the tree you have chosen and tell them that you are going to allow them about 10 seconds to do a task that you are going to explain. Explain that they should write a number on paper at the moment you say the word "stop."

   Give the instructions quickly. Say something like, "Pencil and paper ready? OK, the task is to tell me how tall this tree is. You have 10 seconds to write a number." Wait no longer than 10 seconds, then say, "Stop!" The purpose of this procedure is to eliminate, insofar as possible, estimation using some standard. This number should represent a guess. You may want to tell the students to use feet or meters as the unit of measure.

   Have students record their guesses on a chart. Discuss the range of guesses. You may want to rank the guesses from highest to
lowest guess. There is no need to record student names as those having the extreme guesses sometimes feel awkward.

Tell the students that the next task is to do something to estimate the height of the tree: not just guess, but collect some kind of data which would be useful in finding out how tall the tree really is. Group the students into teams of two or three. Tell them that their task is to come up with some system to estimate the height of the tree. Each group is to write down one number which represents the group's answer. Tell them they have about 10 minutes to arrive at an estimate.

After 10 minutes, call the groups together and have them announce their estimates, explain how they arrived at their figure, and record the estimate on the chart. Label this data as Estimate I. The range of this data will be much narrower than the range of the original guesses.

Ask the students how, as a whole group, you could arrive at one number which would best represent the height of the tree. They will probably suggest getting an average (mean) from the small group estimates. Do this and record the number on the chart.

Explain to the class that they will now calculate the height of the tree using two accepted methods of tree height estimation. When finished, all estimations will be compared.

Give each student a craft (or popsicle) stick. Select a student to be a "unit" of measure. Ask the student (let's call her Mary) to stand against the trunk of the tree. Have the other students stand about 100 feet from the tree, facing Mary. Tell them to hold the stick at arm's length in front of them. Instruct them to look at Mary so that they see her "standing" next to the stick. She will appear shorter than the stick. Have them mark Mary's height on the stick. This is a "Mary unit."
Tell the students to determine how tall the tree is in Mary units by holding the stick in front of them, looking at the tree, and moving up the tree in increments of Mary units. Then ask them how they would convert from Mary units to a standard unit of measure. Use the tape measure to determine Mary's height. If feet is the unit to be used, have students compute the height of the tree after converting Mary units to feet. You may want students to place estimates on a chart. Determine the range and the mean of these estimates.

Explain that there is another method of computing tree height which you will demonstrate. You must have a bright, sunny day for this part of the activity. It is also best done at mid-morning or afternoon.

Measure the length of the shadow cast by the tree. Measure the length of the shadow cast by one of the students. Let the shadow of the tree be $D$ (assume that it is 10 feet). Let the shadow of the student be $D'$ (assume that it is 2 feet). Now measure the height of the student (assume that it is 5 feet). Let this be $H'$. Height of the tree ($H$) equals the height of the student ($H'$) times the length of the shadow of the tree ($D$) divided by the length of the shadow of the student ($D'$).

$$H = \frac{H'D}{D'}$$

$$H = \frac{(5) (10)}{(2)} = \frac{50}{2} = 25$$

The estimated height of the tree, using this method, is 25 feet.

The problem may also be set up as a simple ratio:

$$5:2 :: H:10$$

$$2H = 50$$

$$H = 25$$
You may now want to compare the results of all trials. You will need to impress upon the students that the first number that they each wrote down was a guess based upon very little data. The other numbers are estimates based upon systems of computation. The actual height of the tree remains unknown.
THE LUMBERJACK LESSON

A. **NCTM Skills:** Appropriate computational skills, measurement.

B. **Purpose:** To introduce cordage, to review circumference, and to measure height.

C. **Materials:**
   1. Tape measure.
   2. Pencil and paper.

D. **Activities and Procedure:** Students should know how to determine the height of a tree before doing this activity.

   Select a large tree on the school grounds or in the neighborhood as the subject for the lesson. If you are on a campus with no trees, use a utility pole. Results may not be accurate, but the mathematical process will be just as effective.

   Have the students determine the height of the tree. They must also measure the circumference of the trunk at "breast height" (the lumberjack unit of measure representing 4 1/2 feet from ground level).

   Knowing the circumference at breast height and the height of the tree, the students compute cordage as follows:

   Let \( C \) represent the circumference.
   Let \( H \) represent tree height.
   \( 3000 \) is the constant.

   \[
   \text{Cordage} = \frac{C^2H}{3000}
   \]

   If a tree has a breast-height circumference of 5 feet and a height of 32 feet, then the cordage for this tree is computed as follows:
The tree contains about one-fourth of a standard cord of wood. (This formula does not take into account wood which may come from the limbs of the tree.)

A standard cord is a stack of cut wood measuring 4' x 4' x 8' or 128 cubic feet. Now ask the students to determine how many trees of this size make a cord of wood.
HUNTING BURIED TREASURE

A. **NCTM Skills:** Reading, interpreting, and constructing tables, charts, and graphs.

B. **Purpose:** To involve students in concrete activities which extend and reinforce concepts dealing with ordered pairs, coordinate systems, relative position, proximity, and mapping. (It is necessary to have previously introduced coordinate systems, ordered pairs, and coordinate grids.)

C. **Materials:**
1. Garden hoses.
2. Tape measures.
3. Graph paper.
4. 5" x 8" note cards (about 200).
5. Clipboards or other firm support to write on.
6. Yarn.
7. Markers—craft sticks with "flag" attached (2" x 2" square of construction paper) or something of this nature.

D. **Activities and Procedure:** This is an involved activity, and you may wish to extend it for more than one class period. The children are going to create a "river" on the school grounds and bury "treasure" along the river. Students should be in teams of four to six members. Each team will conduct the activity independently. Each team will need a water hose (or substitute).

Ideally, you should select a location on the school grounds which is slightly sloping (so your "river" can run downhill). Each team will need a space large enough to contain its "river" and the accompanying coordinate system—an area about 30' x 40' will be adequate.
Take the students to the area of the school grounds selected. Ask several students to describe a river. In their descriptions, terms such as "wandering," "turning," and "curving" should be used. Take one of the hoses and create a curving "river" on the grounds. Ask one of the children to walk to the river "bank" and stand at a position of his/her choice. (Maybe he/she is going to select a good fishing spot on the river bank.)

Once the student has selected a spot, ask other students to describe that location on the river. The task will not be easy as there are no standard reference points to use in trying to identify the location. Ask the children if they can think of some things which might be done to make the location easier to describe. Suggestions may include using a compass to identify directions, measuring the distance of the point to be located from stationary points such as the ends of the river, trees, buildings, etc.

Explain that one way to locate a geographic position is to use a map. (You may wish to demonstrate how to locate a city by using the map's coordinate system or how to find a street on a city map.) Tell the students that each team is to lay out its river and make a "map" of it. Once this has been completed, the team will use the map and coordinate system to locate certain points on the river. Make up a story about the river—possibly the points will be places where buried treasure is located.

Demonstrate what each team is to do. Lay out the river. Suggest that it have several major bends in it that will be easy to recognize. (Older children may want to identify other geographic features along the river such as a huge hill—a rock placed at some point—or a marsh—some grass sprinkled along a bend—etc.). Position the x and y axes using the yarn. The y axis should extend a few feet beyond the "river," and the x axis should be about the same length as the y axis. (See the "Garden Hose River" map diagram on page 56 for positioning of each axis.) Decide upon a scale for measuring units on each axis. For example, 1 foot equals 1 mile.
Have a student start at the "headwaters" of the river, walk along the river, and stop at a spot of his/her choice. Ask another student to walk along the x axis to the point where the first student, who is standing on the river, is directly across from him/her. Do the same with the y axis.

Ask the children to decide what ordered pair of numbers describes that location. They will realize that they must have points located on each axis in order to describe the location. Explain that each team will place index cards along each axis to identify the points on the axis when it lays out the river. For purposes of demonstration make a card with a point for y (for example, +5) and a card with a point for x (-3). This would locate point "E" on the attached "river" illustration. The ordered pair would, therefore, be (-3, +5).

Explain that each team is to set up its river and a coordinate system as you have demonstrated. It is then to "map" the river on graph paper. Explain that the beginning and the end of the river as well as each bend in the river should be mapped. The more points on the graph paper, the more accurately the "Garden Hose River" map will represent the river.

Once the points (ordered pairs of numbers) are established, they should be connected. (See sample map on the following page.) Each team should then compare its map to the actual river. How well does the map reflect the actual river? Are there variations? Where?

Modify the following instructions to fit the age and background of your class. When each team has completed the map, it should make a treasure key with the location of several "buried treasures" which can be found near and along the "river." The "treasure key" has the ordered pairs and the name of the treasure for each location.
Example: (+9, +6) - brown rock
(-7, -3) - gum wrapper
(-4, +9) - apple core

At each of these points, an object (stick, rock, pine cone, leaf, piece of paper) should be placed. Distractor (nontreasure) objects should also be placed in the map area. Each team must also develop a "Buried Treasure" map which identifies the point of the treasure.

Example: (+9, +6) is treasure X on "Buried Treasure" map
(-7, -3) is treasure R
(-4, +9) is treasure Z

The "Buried Treasure" map includes only the ordered pairs for the location of the treasure, not its name. (See example on following page.)

When the teams have completed the buried treasure maps and treasure keys, each team trades its original "Garden Hose River" map and "Buried Treasure" map with another team. Each team will keep the treasure key it has made. The key will be used to check locations of treasures found along its river. Each team is to see if it can locate the other team's buried treasure. When a "treasure" is located, a marker is to be placed at that point, the treasure is collected, and its name and location recorded.

After all teams have collected their treasures, the class should go to each river where the treasure hunters identify the treasures and the location of each. The team that placed the treasure checks the accuracy of the "find" with its key.

This activity can also incorporate simple or more complex mapping skills and compass use. It can be used in many ways to develop many skills.
BUILDING AND USING A MEASURING WHEEL

An excellent tool for measuring distances is the measuring wheel or trundle wheel. A simple measuring wheel is easy to construct and can be useful in many outdoor activities. You or your students can make the device with the following materials:

1. A flat circle cut out of rigid cardboard—a circle with a diameter of 11 3/8 inches yields a circumference of 36 inches; a diameter of 3 3/4 inches yields a circumference of 12 inches; a 32-centimeter diameter yields a 1-meter circumference. Make a small hole in the center of the wheel. With a marker indicate the radius of the wheel. The marking will serve as a reference point for measuring.

2. A light stick with a hole drilled about an inch from one end (a yard- or meterstick is just fine and many already have a hole in one end).

3. A bolt which will extend through the stick and cardboard, two washers, and a self-locking nut (or you can use two washers and a long brad).

Assemble the wheel as shown, tightening the nut or bending the brad until the wheel rolls freely but does not wobble.
By using a plywood wheel instead of a cardboard wheel, two flat sticks on either side of it with a dowel stick through them at the opposite end for a handle, a couple of spacers for strength, and a coat of paint, a person with a little skill at carpentry can convert this basic plan into a more sturdy tool that can be used for many years of teaching mathematics.

Now you have a measuring wheel in addition to your collection of rulers, yardsticks, metersticks, and tape measures. What can you do with it? Its uses are as varied as the skills of the children you teach:

1. Primary students can use it to count as they roll it along to measure how far it is from their room to the bike rack, the bus, the cafeteria, etc. If necessary, show them how to count each revolution as the radius marker comes all the way around and touches the ground.

2. Use it to measure lengths and widths. Does l x w = perimeter? You can use the measuring wheel to find out.

3. How big is an acre? Really big! If an acre is square, there are 208.7 feet on each side. Your students can measure an acre on the school grounds with the measuring wheel. Just how many acres are in your school grounds? That too can be computed (43,560 square feet = 1 acre).

4. Use the measuring wheel to help you lay out a classroom garden. How big is the overall garden space? How do we want to divide it? Into plots? Into rows?

5. The trundle wheel is a natural for measuring around things. Let the students find out how far it is around the merry-go-round. Is there a big tree on the school grounds? Measure around it.

6. Prepare a treasure hunt. The clues should say things like, "Starting from our classroom door, face the flagpole, walk 4 meters, and look for the next clue." The measuring wheel can be used to help follow the directions.
7. They know the softball field has 60 feet between bases. (Do kids ever lay out their own playing fields any more?) Do you think that when you hit a home run you actually run 240 feet? Why or why not? Can you use the measuring wheel to find out more exactly how far a base runner runs?

8. Students with skill in multiplying decimals may want to try a different type of "measuring wheel." Use chalk to mark a white radius marker on a bicycle tire. A standard 26" bike tire has a circumference of approximately 86.4 inches (7.2 feet). A 20" bike tire has a circumference of 69.1 inches, or 5.75 feet. Note that 26" and 20" refer to rim sizes, not actual diameters; inflation and thickness of the tire will influence actual measurements. Let the students measure the distance between their homes and the school with the bicycle "measuring wheel."

9. Working with measuring wheels will provide an excellent opportunity to check accuracy of different measuring instruments. If the wheel is "off" about 1 inch per revolution, will this seriously affect the total measurement? In what instances?

10. The measuring wheel can be used to measure distances longer than the standard tape measure. For instance, mathematical exercises such as the great ant race; measuring shadows of trees, flagpoles, etc., to help determine height; checking on estimates of distances; and making measurements for maps or scale drawings can all be handled easier with the aid of this versatile tool.

11. The measuring wheel may stimulate curiosity beyond the classroom. Have any of the students ever seen the instrument that the highway department uses that is very similar to the measuring wheel? It looks rather like a unicycle with no seat and has its own automatic counter attached. Hopefully, some students will observe that the odometer of a car operates on exactly the same principle as our measuring wheel.
12. Planning and building the wheel itself is a stimulating challenge to students. After a unit on the relationship of diameter to circumference, they should be able to figure what size their wheel needs to be in order to provide the circumference they'd like to have.
References


About the Author

Milton Payne is a professor of science and outdoor education at Stephen F. Austin State University, Nacogdoches, Texas. Dr. Payne taught science in the Waco and Austin public schools. He attended Texas A&M and North Texas State University and received his EdD degree from North Texas State in 1967.

Dr. Payne's primary interests since joining the faculty at Stephen F. Austin in 1967 have been in the areas of science education, outdoor education, middle-school teaching, and individualized instruction. He has conducted workshops and seminars in more than 30 school districts including Houston, Fort Worth, Texarkana, and Tyler.

He has presented papers before the Texas Academy of Science, the Texas Middle School Association, the Texas Outdoor Education Association, and other professional organizations. He was among the founders of the Texas Outdoor Education Association. Articles he has written about science and outdoor teaching have appeared in several regional and national publications.