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**ABSTRACT**

This manual describes the process used in determining the objectives of a project to develop effective remediation strategies for use by teachers of mathematics. It presents two tools developed for use in assessing remedial mathematics teaching: (1) the "Effective Remedial Mathematics Checklist," for use in supervising teachers of remedial math, and (2) the "Math Remediation Methods Questionnaire (MRMQ)," designed to assess what teachers, as a group, know about effective remediation in mathematics. Information is presented on the theoretical background, test development, test administration, scoring, and analysis. Appended are (1) the checklist; (2) the questionnaire, Form A (K-6); (3) the questionnaire, Form B (7-12); and (4) an extensive bibliography. (TE)

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PAPER NO. 417

EFFECTIVE REMEDIATION STRATEGIES  
IN MATHEMATICS

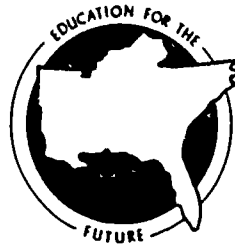
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FALL 1985

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OCCASIONAL PAPERS IN EDUCATIONAL POLICY ANALYSIS

Characteristics of an Effective Remedial Mathematics Teacher  
Effective Remedial Math Teacher Checklist  
Math Remediation Methods Questionnaire

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FALL 1985

Prepared for the

Southeastern Regional Council for Educational Improvement  
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## INTRODUCTION

The unsatisfactory performance of pupils in the area of mathematics and the increasing demand in our technological society for competence in mathematics are topics familiar to all educators. Teachers are faced not only with the task of basic instruction, but currently are charged with the responsibility of providing remediation (in order to meet minimum competency standards) for those students who do not perform adequately. The process of reteaching, or remediation, has yielded contradictory results in research reports. There appears to be a deficit in the area of practicing teachers using effective remediation techniques. The extent to which effective remediation practices in mathematics are utilized in the classroom or taught in teacher-training institutions has not yet been documented.

Three tools have been developed through this project for the purpose of assessing effective remediation practices: the "Characteristics of an Effective Math Remediation Teacher," the "Effective Remedial Mathematics Teacher Checklist," and the "Math Remediation Methods Questionnaire" (MRMQ). The purpose of the "Characteristics of an Effective Math Remediation Teacher" and the "Checklist" is to provide guidelines for supervisors of teachers by describing characteristics of effective teaching in the area of math remediation. The purpose of the "Math Remediation Methods Questionnaire" (MRMQ) is twofold: 1) to provide both an objective measurement of teacher attitude toward teaching mathematics and 2) to be used by school systems to plan inservice activities.

## RATIONALE FOR THE INSTRUMENTS

Because no inventory of characteristics of an effective remedial teacher was available, the development of a list of such characteristics

was considered to be an important contribution to the existing body of literature on math remediation. This list of characteristics of an effective remedial mathematics teacher was developed from a comprehensive review of the literature (Abbott, McEntire, & Wilson, 1985). The characteristics incorporated in the list are grouped into three content areas: knowledge of the structure of mathematics, knowledge of diagnosis, and knowledge of remedial instructional strategies. From the list of characteristics of an effective remedial mathematics teacher, two instruments were developed: an "Effective Remedial Math Teacher Checklist" and "the Math Remediation Methods Questionnaire." The checklist is for use by supervisors/principals in supervising teachers of remedial math. The Math Remediation Methods Questionnaire (MRMQ) is designed to assess what teachers, as a group, know about effective remediation in mathematics. The instrument purports to assess strengths and weaknesses related to mathematics remediation of total groups of teachers in school systems. Two forms were developed: 1) a K-6 form for those teachers in kindergarten through grade 6, and 2) a 7-12 form for those teachers in grades 7-12.

#### THEORETICAL FRAMEWORK

The scarcity of research can be attributed partially to lack of an operational definition for remedial mathematics and lack of descriptions of effective practices. The commonly accepted idea of remediation as

a careful effort to reteach successfully what was not well taught or not well learned during the initial teaching (Glennon & Wilson, 1972).

gives little information as to how the reteaching should differ from the original instruction to insure that better learning occurs the second-time-



around. Efforts to clarify effective remedial teaching in mathematics have been made by a group of mathematics educators who suggest that diagnostic/prescriptive teaching is the most efficient method of remedial instruction (Heddens & Aquila, 1976). The instruments described by this manual attempt to assess effectiveness of remedial teaching in mathematics based on the diagnostic/prescriptive model of teaching.

## INSTRUMENTATION

Items on the two instruments have been developed based on three conceptual areas: knowledge of the structure of mathematics, knowledge of diagnosis, and knowledge of effective remedial instructional strategies. A description of these three areas is as follows:

### Knowledge of the Structure of Mathematics

The hierarchical structure of mathematics is the subject of both past (Whitehead & Russell, 1925 & 1927) and present (Underhill, 1978; White, 1973) research. The inherent logical structure of mathematics that provides a hierarchy of prerequisite learning for any math concept or skill is established by much research. It is, in fact, knowledge of this inherent structure that aids in the diagnosis of problematic learning. Taxonomies and checklists which identify structure and sequencing of mathematical concepts and skills simplify the diagnosis of prerequisite learning deficiencies. It is this broad view that is essential to the remedial teacher who must both diagnose and prescribe for deficient learning. By identifying the breakdown within the structural hierarchy of mathematics, the teacher then can define the remedial needs of the student.

### Knowledge of Diagnosis

Diagnosis of the student's difficulties in learning mathematics is a

necessary step in the remediation process. In order to determine an appropriate remedial strategy, the teacher must be able to assess correctly those concepts which are misunderstood.

Currently, error analysis is the primary method for diagnosis as cited in the related literature (Engelhardt, 1982; Ashlock, 1985; Enright, 1983; Browne, 1906; Roberts, 1968; Reisman, 1972; Ellis, 1972; Cox, 1975a & 1975b; and Radaty, 1979). Analysis of student error is accomplished primarily through examination of the student's written work, both homework and tests, and through oral interviews with the student. Knowledge of appropriate use of various methods for conducting error analysis is considered essential for an effective remedial math teacher.

#### Knowledge of Remedial Instructional Strategies

The most generally accepted definition for remediation is "reteaching". The diagnostic/prescriptive model specifies that the concepts and procedures to be retaught must be identified first; then appropriate instructional strategies can be prescribed. The critical variable is the teacher's ability to correctly diagnose the student's mistake and to select the strategy most appropriate for remediating the specific content which is misunderstood. In order to select the appropriate remedial strategy for the student, a teacher must have a wide knowledge of several methods of teaching-related concepts. The most common mistakes made in selecting remedial strategies include choosing a strategy which: 1) is at a lower level than is appropriate for the student, 2) is at a higher level than can be understood by the student, or 3) involves merely more practice of the skill without having retaught the basic concepts or procedures. Methods for assessing teachers' knowledge in selecting remedial strategies will be discussed in this manual.

## Attitude toward Mathematics Teaching

There has been a recent increase in interest in students' attitudes toward mathematics. This interest is due partially to the importance of mathematics in the curriculum, partially to an assumption that attitudes toward mathematics are generally poor, and especially to the assumption that there is a strong causal relationship between attitude and achievement in mathematics (Kulm, 1980). The relationship between attitude toward mathematics and mathematics achievement is not strong (Wolf & Blixt, 1981), but the relationship, although low, is positive, especially during the late elementary and middle school years (Callahan, 1971).

Since attitude of the student does appear to be related, to some extent, to mathematics achievement, it is reasonable to assume that the attitude of the teacher toward teaching mathematics is an important factor in the effectiveness of his/her teaching. A teacher's attitude toward mathematics is likely to be dependent on several factors including: (1) the teacher's own level of understanding of the mathematics content area; 2) the teacher's own enjoyment of mathematics, 3) the teacher's attitude toward mathematics curriculum materials, and 4) the teacher's understanding of the relationship between student attitude and achievement. The "Attitude Toward Mathematics Teaching" section of MRMQ is designed to measure such teacher attitudinal factors as a component part of the instrument.

### PURPOSES OF THE INSTRUMENTS

#### Purpose of the Effective Remedial Math Teacher Checklist

This checklist is for use as a guideline for assessing actual (observable) teacher behavior in the realm of effective math remediation. It is designed to be used by principals and supervisors as a formative evaluation

tool. The accompanying references may be used by teachers to build their skills in various areas of mathematics remediation.

#### Purposes of the Math Remediation Methods Questionnaire

Information from the Math Remediation Methods Questionnaire can be used for three purposes:

- 1) Needs assessment--to assess strengths and weaknesses of the teacher in the area of math remediation. Results of this test may indicate specific target areas of need for inservice programs that focus on mathematics remediation.
- 2) Self-assessment--to allow individual teachers of mathematics to assess strengths and weaknesses in math remediation. Coupled with the Remedial Mathematics Checklist, the MRMQ can pinpoint areas for instructional improvement.
- 3) Research--to assess the effectiveness of inservice and/or preservice activities directed toward teaching skills in mathematics remediation.

### INSTRUMENT DEVELOPMENT

#### Establishing the Objectives

At the time this project was conducted, there was no description of characteristics of an effective remedial math teacher. Further, there was no review of the available literature on effective remediation practices. Thus, in order to identify specific objectives for the project, a list of the characteristics of a remedial mathematics teacher was compiled after a comprehensive review of the literature (Abbott, McEntire & Wilson, 1985). These objectives were drawn from the available written resources on remedial mathematics, then reviewed and endorsed by a group of math education

researchers and practitioners. The literature review indicated that information about remediation can be grouped into three major areas--structure of mathematics, diagnosis, and effective remedial instructional strategies.

The identified objectives are as follows:

- I. Teachers' knowledge and use of the structure of mathematics as a guideline for possible remedial needs

Can describe and use the structural aspects of mathematics (hierarchies, taxonomies, scope and sequence charts, concept cluster checklist) to locate a sequence of curricular objectives or competency test objectives.

- II. Teachers' knowledge and use of diagnosis as the first steps in remediation.

- A. Uses a variety of survey and analytic tests as well as clinical procedures (error analysis, structured and unstructured student interviews) to diagnose learning problems in mathematics and to evaluate the results of instruction

- B. Uses daily written work diagnostically to discover error patterns and to interview individual students who perform poorly

- C. Uses a variety of teaching experiments, observations and interviews to diagnose functional levels of learning (i.e., concrete, semiconcrete, semiabstract, and abstract)

- D. Uses task analysis to delineate mathematical skills into procedural steps and prerequisite conceptual understanding in order to correctly diagnose the student error in thinking

- III. Teachers' knowledge and use of effective diagnostic/remedial instructional strategies and classroom management techniques.

- A. Can describe and apply theories of how mathematics is learned to methods of instruction

- B. Can describe and use individual learner characteristics to plan instruction

- C. Uses a variety of concrete, semiconcrete, semiabstract, and abstract materials and activities to remediate problems in basic fact mastery, computation, or problem solving

- D. Uses a variety of speaking, reading, writing, and translating activities to remediate problems in using mathematical language
- E. Uses a variety of techniques to manage the mathematics classroom

### Item Selection

Items for the "Effective Remedial Math Teacher Checklist" and the MRMQ are based directly on the "Characteristics of an Effective Remedial Math Teacher." Each item can be identified as measuring one of the objectives. Content of the items is based on the information obtained from the review of related literature.

### Validity of the Math Remediation Methods Questionnaire

Validity of a measuring instrument is defined as the extent to which an instrument measures what it is intended to measure (Nunnally, 1978). Two types of validity that can be examined relate directly to the (1) criterion-related validity (correlation of a test of unknown validity for a particular ability with a test that is known or believed to be valid) and (2) content validity (evidenced by the relationship of the items to the concept being measured).

### Criterion-Related Validity

It was not possible to calculate a correlation coefficient as an index of the criterion-related validity because no similar instrument exists. Information from the body of literature regarding effective teaching and mathematics teaching was considered in developing the objectives used as a basis for the instrument. With the exception of the MRMQ, no known instrument is available for assessing teacher knowledge of effective remediation in mathematics.

### Content Validity

Content validity involves the systematic examination of the content of the test to determine whether it covers a representative sample of the behavioral domain to be measured (Anastasi, 1968). Evidence of content validity was established in two ways: 1) each item measures one or more of the objectives from the "Characteristics of a Remedial Math Teacher"; since this list of characteristics was developed as a result of a comprehensive review of the literature, the items do reflect the content area which was judged as appropriate; in addition, 2) two panels of experts--an advisory panel (composed of State Department of Education personnel and practicing teachers) and a panel of three nationally known math educators--reviewed the items and made suggestions for revision. These suggestions for revision have been incorporated into the final version of the MRMQ. The revised MRMQ was sent to a geographically diverse (Kansas, New Mexico, Florida, South Carolina, North Carolina, Virginia) group of math educators representing either university faculty or state department of education math consultants. These consultants rated each item on both forms as either content valid or not content valid. On the K-6 form, the average percent of items rated as content valid was 95. On the 7-12 form, the average percent of items rated as content valid was 91. An item by item analysis of the K-6 form indicates that over 80% of the raters rated 93% of the items as content valid; no items were rated by 80% of the raters as not content valid. An item by item analysis of the 7-12 form indicates that over 80% of the raters rated 70% of the items as content valid; over 80% rated 1 item as not content valid.

### Reliability of the MRMQ

Reliability refers to the consistency with which a test measures an ability. If a test is highly valid, it will be reliable for those to whom

it is appropriate. Thus, the MRMQ is considered to be reliable for the population of practicing teachers. Studies assessing the reliability of the MRMQ are being conducted during the printing of this manual, and the reliability data will be available by late fall of 1985 in the form of a technical manual from the Southeastern Regional Council for Educational Improvement.

#### DIRECTIONS FOR ADMINISTRATION, SCORING, AND INTERPRETATION OF THE INSTRUMENTS

##### Administration of Effective Remedial Mathematics Checklist

This instrument is to be used by an observer (supervisor) to assess a teacher's use of various types of information essential to effective teaching of math. The observer or supervisor should observe the teacher a minimum of three (3) times in order to sample adequately the identified domain of teaching behaviors. The checklist can be used in various ways: three consecutive days, one class/period per week for three weeks, or one class/period per grading period for three grading periods. It is recommended that the teacher be observed for a minimum of 30 minutes during each observation. The observer should have at least one conference with the teacher to discuss the findings of the observation. Conferences with the teacher also yield pertinent information related to the knowledge base assessed by the checklist.

Specific directions for the observer are:

RATE THE TEACHER'S KNOWLEDGE/BEHAVIOR ON THE RATING SCALE  
PROVIDED FOR EACH OBJECTIVE.

##### Administration of Math Remediation Methods Questionnaire

The Math Remediation Methods Questionnaire (MRMQ) is designed to be taken by individuals.



The directions for the test are:

SELECT THE BEST RESPONSE FOR EACH ITEM BY MARKING  
THE LETTER FOR THE RESPONSE.

The MRMQ is a power test; thus, no time limits have been established.  
The average completion time is 20 minutes for each subject.

#### Scoring of the MRMQ

The MRMQ can be scored by hand, by use of a mainframe computer, or by use of a microcomputer. If computer scoring is desired, opti-scan answer sheets or cards compatible with the computer should be used. If the instrument is administered to large numbers of subjects, computer scoring is advised.

#### Scoring of Attitudes Toward Mathematics Teaching

Responses for the first eleven (11) items of the Attitudes Toward Mathematics Teaching scale are to be added (including scoring of items indicated). The value assigned to each response is as follows:

D = 1      SD = 2      N = 3      SA = 4      A = 5

Reverse scoring should be employed for items 3, 4, 5, 7, 8, 9, 10, i.e.,  
D = 5, SD = 4, etc.

An individual obtaining a score of less than 33 (the median) would be considered as having a positive attitude toward teaching mathematics. An individual obtaining a score of greater than 33 would be considered as not having a positive attitude toward teaching mathematics. Scores close to the median would indicate neither a positive nor a negative attitude, but rather a neutral attitude toward teaching math.

Item 12 was developed for the purpose of providing information to the school system for assessing attitudes toward curricular materials.

### Analyses of Test Results

Analysis of the first three scales of the MRMQ responses yields both scale scores and total number of correct items. This ratio of correct/total items can be expressed either as a percent score or as a raw score.

The "Use of Instructional Strategies" scale can be analyzed further to assess the types of responses generally chosen by the teacher. Responses for this section are coded so that each item has a stem which reflects the election of a strategy for: 1) teaching under or below the student's ability level (U); 2) teaching above the student's ability level (A), 3) using repetition (drill and practice) (R); and, 4) the correct choice (C). The coding following page 13 describes the response options for both the K-6 and 7-12 Forms.

School systems can use data on MRMQ responses to identify the types of inservice activities most needed by their teachers. For example, if a group of teachers consistently selects an "R" response (repetition), an effective inservice program might include the introduction of multiple alternative instructional strategies used in remedial mathematics activities. A group of teachers consistently selecting an "A" response may need inservice activities involving instruction identifying prerequisite learning concepts which are the basis for the mathematical concepts they teach.

### SUMMARY

This manual has described the process used in determining the objectives of the project and two tools developed for use in assessing remedial mathematics

teaching. These tools are: (1) the "Effective Remedial Mathematics Checklist," and (2) the "Math Remediation Methods Questionnaire." Information related to the theoretical background, test development, test administration, scoring, and analysis has been presented.

These instruments have been developed to identify observable characteristics that describe an effective remedial mathematics teacher. It is hoped that the use of these instruments will improve the quality and appropriateness of inservice programs in remedial mathematics and thus the quality of remedial mathematics teaching.

MRMQ Scoring Key

K-6 Form

1. Knowledge of the Structure of Mathematics

- |      |      |      |
|------|------|------|
| 1. c | 4. b | 7. c |
| 2. c | 5. b | 8. b |
| 3. b | 6. a | 9. d |

II. Diagnostic Procedures

- |      |      |      |       |
|------|------|------|-------|
| 1. a | 4. d | 7. b | 10. c |
| 2. d | 5. a | 8. c | 11. c |
| 3. a | 6. b | 9. c |       |

III. Knowledge of Remedial Instructional Strategies

- |      |      |      |       |
|------|------|------|-------|
| 1. c | 4. d | 7. d | 10. c |
| 2. c | 5. a | 8. d | 11. c |
| 3. c | 6. d | 9. c | 12. a |

7-12 Form

1. Knowledge of the Structure of Mathematics

- |      |      |      |
|------|------|------|
| 1. a | 4. a | 7. a |
| 2. d | 5. b | 8. a |
| 3. a | 6. a | 9. d |

II. Diagnostic Procedures

- |      |      |      |       |
|------|------|------|-------|
| 1. c | 4. d | 7. c | 10. b |
| 2. d | 5. a | 8. c | 11. b |
| 3. a | 6. b | 9. a | 12. c |

III. Knowledge of Remedial Instructional Strategies

- |      |      |      |       |
|------|------|------|-------|
| 1. c | 4. c | 7. d | 10. a |
| 2. c | 5. d | 8. c | 11. b |
| 3. a | 6. c | 9. d | 12. c |

## MRMQ Types of Teacher Responses

### K-6 Form - Use of Remedial Instructional Strategies

- |                                 |                                 |                                 |
|---------------------------------|---------------------------------|---------------------------------|
| 1. a. U<br>b. R<br>c. C<br>d. A | 5. a. C<br>b. U<br>c. A<br>d. R | 9. a. A<br>b. U<br>c. C<br>d. R |
| 2. a. U<br>b. R<br>c. C<br>d. A | 6. a. U<br>b. R<br>c. A<br>d. C | 10-12 not apply                 |
| 3. a. A<br>b. U<br>c. C<br>d. A | 7. a. A<br>b. R<br>c. U<br>d. C |                                 |
| 4. a. U<br>b. R<br>c. A<br>d. C | 8. a. U<br>b. A<br>c. C<br>d. R |                                 |

### 7-12 Form - Use of Remedial Instructional Strategies

- |                                 |   |                                  |
|---------------------------------|---|----------------------------------|
| 1. a. U<br>b. R<br>c. C<br>d. A | 6. a. A<br>b. U<br>c. C<br>d. R         | 11. a. R<br>b. C<br>c. A<br>d. U |
| 2. a. U<br>b. R<br>c. C<br>d. A | 7. a. A<br>b. R<br>c. U<br>d. C         | 12. not apply                    |
| 3. a. U<br>b. A<br>c. C         | 8. a. R<br>b. A<br>c. C<br>d. U         |                                  |
| 4. a. A<br>b. U<br>c. C<br>d. R | 9. a. U<br>b. R<br>c. U<br>d. C<br>e. A |                                  |
| 5. a. U<br>b. R<br>c. A<br>d. C | 10. a. C<br>b. R<br>c. U<br>d. A        |                                  |

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## Appendix A

### AN EFFECTIVE REMEDIAL MATHEMATICS TEACHER CHECKLIST

I. Teachers' knowledge and use of the structure of mathematics as a guideline for possible remedial needs.

Can describe and use the structural aspects of mathematics (hierarchies, taxonomies, scope and sequence charts, concept cluster checklists) to locate a sequence of curricular objectives or competency test objectives.

1	2	3
Cannot use to locate objectives	Uses at least one hierarchy to locate objectives	Uses a variety of hierarchies, scope and sequence charts, checklists to locate objectives

II. Teachers' knowledge and use of diagnosis as the first step in remediation

A. Uses a variety of survey and analytic tests as well as clinical procedures (error analysis, structured and unstructured student interviews) to diagnose learning problems in mathematics and to evaluate the results of instruction.

1	2	3
Uses some test	Uses both survey and analytic tests	Uses survey and analytic tests as well as clinical procedure

B. Uses daily written work diagnostically to discover error patterns and to interview individual students who perform poorly.

1	2	3
Never	Occasionally	Frequently



- C. Uses a variety of teaching experiments, observations and interviews to diagnose functional levels of learning (i.e. concrete, semi-concrete, semi-abstract and abstract).

1	2	3
Does not diagnose	Occasionally diagnoses functional level	Frequently diagnoses functional level

- D. Uses task analysis to delineate mathematical skills into procedural steps and prerequisite conceptual understandings in order to diagnose the student error in thinking.

1	2	3
Does not task analyze	Occasionally analyzes tasks into procedural steps	Frequently analyzes task into both procedural and conceptual steps

III. Teachers' knowledge and use of effective diagnostic/remedial instructional strategies and classroom management techniques.

- A. Can describe and apply theories of how mathematics is learned to methods of instruction.

1	2	3
Cannot describe any theory	Describe and applies at least one theory	Describes and applies several theories

- B. Can describe and use individual learner characteristics to plan instruction (i.e. intelligence, attitude, need for structure, anxiety, etc.).

1	2	3
Individual characteristics not used in planning	Used in planning for some students	Used in planning for all students

- C. Uses a variety of concrete, semi-concrete, semi-abstract and abstract materials and activities to remediate problems in basic fact mastery, computation or problem solving.

1	2	3
Uses only abstract, i.e. symbolic,	Uses abstract, semi-abstract, and semi-concrete materials	Uses abstract, semi-abstract, semi-concrete and concrete materials

D. Uses a variety of speaking, reading, writing and translating activities of remediate problems in using mathematical language.

1	2	3
Rarely teaches math as language	Occasionally teaches math as language	Frequently teaches math as language

E. Uses a variety of techniques to manage the mathematics classroom (i.e. time management, interpersonal communication skills, group decision making, peer tutoring, grouping strategies, etc.).

1	2	3
Rarely varies management techniques	Occasionally varies management techniques	Frequently varies management techniques

Appendix B

Math Remediation Methods Questionnaire

Form A, K-6

Gypsy Anne Abbott, Ph.D.

Elizabeth McEntire, Ph.D.

Purpose: This questionnaire is designed to assess what teachers as a group know about effective remediation in mathematics. Individual responses are anonymous. Strengths and weaknesses of a total group of teachers in a school system will be generated from the collective responses so that inservice training programs may be designed to improve the quality of mathematics teaching.

Instructions: Select the best response for each item by marking the letter for the response. (Thank you for your help).

K-6 Form

I. Knowledge of the Structure of Mathematics

1. An initial basic math concept that a child probably learns is:
  - a) classifying
  - b) ordering
  - c) matching
  - d) counting
  
2. Understanding inverse operations enables the student to both categorize and:
  - a) add objects
  - b) count objects
  - c) subtract objects
  - d) order objects
  
3. Concepts of more than, less than, larger, smaller are developed first through:
  - a) matching
  - b) ordering
  - c) adding
  - d) subtracting
  
4. In the learning of mathematics, the student learns to translate:
  - a) from the concrete experiences to representation (  $\Delta \Delta$  ) to the spoken and written word (two)
  - b) from the concrete experiences to representation (  $\Delta \Delta$  ) to the spoken and written word (two) to the written math symbol (2)
  - c) from the concrete experience to representation (  $\Delta \Delta$  ) to the written math symbol (2)
  - d) none of the above
  
5. Being able to count requires that the student understand:
  - a) place value
  - b) one-to-one correspondence
  - c) matching
  - d) seriation

6. Learning to add and subtract does not require that the student understand:
- a) fractional relationship
  - b) place value
  - c) ordering
  - d) set
7. Mathematical operations (+, -, ×, ÷) are more easily understood when:
- a) they are taught simultaneously
  - b) they are taught as separate processes
  - c) they are taught as interrelated
  - d) they are taught sequentially
8. Which of the following numeration concepts developmentally precedes the others:
- a) one-for-many correspondence
  - b) conservation of number
  - c) discrimination of sets
  - d) counting
9. "Regrouping," as in the problem  $193 + 78$ , instruction is conveyed by:
- a) the teacher's description of the procedural steps in symbol manipulation
  - b) the teacher's demonstration of grouping and ungrouping objects
  - c) the teacher's verbal explanation of what the symbol manipulation in regrouping means
  - d) the teacher's demonstration of the relationship between grouping and ungrouping objects, verbal explanation, and symbol manipulation

## II. Diagnostic Procedures

1. An effective way to assess a student's mathematical thinking is to:
- a) watch and ask the student to verbally explain his/her thinking
  - b) have the teacher analyze a written product
  - c) ask the student to correct wrong responses
  - d) have the teacher think aloud as she/he works the problem, then ask the student to correct the problem
2. Error analysis in computation can be done most effectively when:
- a) the teacher knows and understand possible types of errors related to the problem
  - b) the teacher recognizes the alternative algorithms may be available
  - c) the teacher asks a student to explain his/her thinking in solving the problem
  - d) all of the above

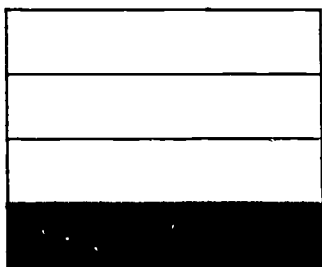
3. Diagnosis of errors in the solving of word problems does not require that the teacher know:
- a student's reading level
  - a student's ability to eliminate extraneous information
  - a student's level of basic fact memorization
  - a student's ability to translate words into math symbols
4. Task analyzing a mathematical task is necessary to:
- determine the types of computation error possible
  - determine the sequence of procedures the student might use to accomplish the task
  - determine pre-requisite underlying concepts
  - all of the above
5. Determining a student understanding of mathematical rules or principles can best be accomplished by:
- asking the student to demonstrate the rule or principle in several ways, i.e. concretely, semi-concretely, semi-abstractly, abstractly
  - asking the student to give an example of a rule or principle
  - asking the student to state the rule or principle
  - asking the student to explain the rule or principle
6. Give the following set of problems completed by Jane Smith, choose the response which likely describes her error:

$$\begin{array}{r} 125 \\ -19 \\ \hline 114 \end{array}$$

$$\begin{array}{r} 71 \\ -25 \\ \hline 54 \end{array}$$

$$\begin{array}{r} 92 \\ -16 \\ \hline 84 \end{array}$$

- the error is procedural, indicating that Jane does not know the steps in regrouping
  - the error appears to be procedural but may indicate that Jane does not have conceptual understanding of place value and subtraction
  - the error is that Jane does not remember and understand "the bottom number is subtracted from the top number"
  - the error is conceptual indicating that Jane does not understand place value in multidigit numbers
7. The student is asked to identify what part of the figure is shaded.



Student responses:

- a.  $1/4$     **b.  $1/3$**     c.  $1/2$     d.  $1/5$

The most probable student error(s) is (are):

- a) misunderstanding of whole-part relationships
  - b) inability to associate the appropriate fraction with part of a figure
  - c) not understanding the word "shaded"
  - d) all of the above are correct
8. John had done poorly on a multiplication fact test. An initial step in successful remediation is:
- a) have him practice with flash cards
  - b) have him complete similar problems in supplemental worksheets
  - c) have him explain his thinking in finding the answers
  - d) have him work several similar problems with a peer
9. If a student can state the procedural steps in doing 2 - digit division, but continues to make errors, diagnosis of the learning difficulty may require:
- a) having the student state the procedural steps as he/she works through a problem
  - b) using concrete or semi-concrete materials to determine the student's functional level of learning with this type problem
  - c) reviewing 1 - digit division with the student to see if errors occur in a simpler problem
  - d) none of the above
10. In the remedial mathematics classroom, it is most important that the student's responses:
- a) be marked as correct or incorrect as quickly as possible so that the grade may be given to the student
  - b) be analyzed by the teacher to determine the error patterns on a teacher-made test
  - c) be explored by the teacher in order to understand the student's level of mathematical thinking
  - d) be followed up by practice on similar problems
11. A strategy frequently used by an effective remedial teacher which is not often used by the regular mathematics teacher is:
- a) presenting multiple examples of the problem type
  - b) talking about examples presented on the board
  - c) task analyzing each problem type
  - d) asking students to demonstrate problems on the chalkboard

### III. Knowledge of Remedial Instructional Strategies

1. Pictures, graphs, and charts should be used in mathematics teaching:
  - a) prior to experience with objects
  - b) each time a concept has not been understood
  - c) whenever they facilitate student understanding
  - d) when introducing mathematical sentences
2. Instruction to remediate deficiencies in mathematics identified by minimum competency tests can best be accomplished by:
  - a) the use of microcomputer software for drill and practice which is at least three grade levels below the child's grade level
  - b) the drill and practice of basic skills related to items from a minimum competency exam
  - c) the review and practice of items requiring the same type of logical thinking as required by a minimum competency exam
  - d) continuing regular curriculum spiraling with occasional emphasis of deficit skills
3. Both motivation and understanding can best be increased by:
  - a) use of more teacher talk time to explain math problems
  - b) use of more student practice time to habituate correct responses
  - c) use of computer software to reteach mathematical concepts and operations
4. Using direct instruction as a technique in remedial mathematics instruction provides for:
  - a) a student's use of concrete objects
  - b) a teacher's verbal explanation of the concept accompanied by written examples on the board
  - c) written examples of needed procedural steps
  - d) instruction to begin with the teacher's discussion of the concept accompanied by the student's verbal and written response
5. A first grade child appears to learn best from touching the given material. However, this child is unable to tell how many objects are in a group. An appropriate remedial strategy might be to:
  - a) have the child handle each object while counting aloud with the teacher
  - b) have the child write the numeral beside each object
  - c) send the child to an independent learning center
  - d) have the child write the answers to several pictorial counting problems on worksheets
6. Jane, a third grader who has maintained a C average in all school subjects including math (the class has been studying multidigit multiplication for two weeks), gave the following answers on the six-weeks' math test on multiplication work:

$$\begin{array}{r} 30 \\ \times 8 \\ \hline 240 \end{array}$$

$$\begin{array}{r} 36 \\ \times 7 \\ \hline 2142 \end{array}$$

$$\begin{array}{r} 72 \\ \times 5 \\ \hline 360 \end{array}$$



Diagnosis of Jane's problem indicates that she has not mastered the concept that one digit must be used to represent one place in a multidigit number. An appropriate remedial strategy would be to:

- a) provide lined paper which only has space for one digit between each set of lines
- b) use drill-and-practice exercises on multiplication facts
- c) demonstrate the concept of hundred and thousand
- d) demonstrate the concept concretely, semi-concretely or semi-abstractly while helping the student write each step on paper

7. Mike exhibited the following errors in his math homework:

$$\begin{array}{r} 845 \\ -124 \\ \hline 761 \end{array} \qquad \begin{array}{r} 231 \\ -190 \\ \hline 141 \end{array} \qquad \begin{array}{r} 372 \\ -191 \\ \hline 281 \end{array}$$

Diagnosis of his responses indicates that he appears to be using a rote procedure for regrouping the numbers. He does not appear to understand the basic process of regrouping. An appropriate remedial strategy is:

- a) demonstrate a set of verbal procedures for crossing-out and renaming the number when borrowing occurs
  - b) have the student practice subtraction problems
  - c) have the student use place value sets independently at learning centers
  - d) have the student practice with place value concrete materials as he writes the symbols for the numbers
8. A teacher overheard May, who obtained a six-weeks grade of "C," state that she "hates math." An appropriate action for the teacher would be to:
- a) assume May's feelings about math are related to her grade
  - b) discuss May's negative feelings about math with her
  - c) discuss attitudes toward math with the whole class
  - d) have May complete a math attitudinal survey during the next math period
9. John, a fourth grade student, was diagnosed on a posttest as needing remedial instruction in double digit division. During two weeks of remediation, he knows the procedural steps, but has difficulty explaining the process. An appropriate evaluation technique for assessing John's skill mastery would be:
- a) having him take a test comprised of single digit division problems on a microcomputer
  - b) having him take the test using a paper-and-pencil format
  - c) having him calculate and explain his answers aloud to his teacher
  - d) having him repeat the posttest which originally indicated his need for remedial work

10. A factor often disregarded in assessing a student's performance in mathematics is:
- a) a student's reading level
  - b) a student's knowledge of basic facts
  - c) a student's attitude toward math
  - d) a student's past grades in mathematics
11. The teachers management of the mathematics classroom is influenced most by:
- a) the teacher's personality and personal rapport with students
  - b) the teacher's use of strict discipline and structure
  - c) the teacher's obvious enjoyment of mathematics and ability to make the content meaningful
  - d) the teacher's use of strict routine in presenting a lesson
12. A distinct difference between developmental theories of mathematics learning and behavioral theory is:
- a) how a student's response is evaluated
  - b) how the teacher selects the content to be taught
  - c) how a student works out a problem
  - d) how the teacher selects mathematical tasks

Appendix C

Math Remediation Methods Questionnaire  
Form B, Grades 7-12

Gypsy Anne Abbott, Ph.D.

Elizabeth McEntire, Ph.D.

Purpose: This questionnaire is designed to assess what teachers, as a group, know about effective remediation in mathematics. Individual responses are anonymous. Strengths and weaknesses of a total group of teachers in a school system will be generated from the collective responses so that inservice training programs may be designed to improve the quality of mathematics teaching.

Instruction: Select the best response for each item by marking the letter for the response. Thank you for your help.

## I. Knowledge of the Structure of Mathematics

1. A basic skill in writing number sentences from word problems is:
  - a) being able to translate words into mathematical symbols
  - b) being able to identify extraneous information
  - c) being able to determine the operations needed for the problem stated
  - d) being able to reduce the problem to its simplest form
2. A student's understanding of decimals is most contingent upon his/her having mastered the concept of:
  - a) one-to-one correspondence
  - b) ordering
  - c) whole/part relationships
  - d) place value
3. Being able to solve area and perimeter problems in geometry requires that the student masters:
  - a) concepts of measurement of distance and space
  - b) rote memorization of formulas
  - c) understanding of relationships between bounded and unbounded figures
  - d) concepts of positions of bodies in space
4. A concept related to reducing fractions is:
  - a) identifying property of multiplication
  - b) identifying property of addition
  - c) the associative property of multiplication
  - d) the commutative property of multiplication
5. Problem-solving skills are highly dependent on the student's:
  - a) ability to perform mathematical computation
  - b) ability to translate the written/oral words into mathematical number sequences
  - c) ability to read at his/her grade level
  - d) none of the above
6. Comprehension of percent problems requires an understanding of:
  - a) equivalence of fractions and decimals
  - b) relationship of base 10 to other bases
  - c) one-to-many correspondence
  - d) none of the above
7. In order to estimate probable answers, the student must have a basic understanding of how to:
  - a) round numbers up or down
  - b) determine place value
  - c) do mental computations
  - d) none of the above

8. The basic skills(s) in geometry is/are:
- a) recognition of patterns
  - b) recognition of open and closed figures
  - c) recognition of figure/ground relationships
  - d) all of the above
9. Mathematical understanding results from:
- a) memorization of basic facts
  - b) knowledge of rules and principles and the ability to calculate
  - c) knowledge of mathematical symbols and their representation
  - d) experiences with the application of rules and principles

## II. Diagnostic Procedures

1. The quickest way to assess the student's error in mathematical thinking is to:
- a) ask the student to correct wrong responses
  - b) have the teacher analyze a written product
  - c) watch the student work and ask the student to verbalize his/her thinking
  - d) model the correct method of thinking, then ask the student to correct the error
2. Error analysis in computation can be done most effectively when:
- a) the teacher knows and understands possible types of errors related to the problem
  - b) the teacher recognizes that alternative algorithms may be available
  - c) the teacher watches and asks a student to explain his/her thinking
  - d) all of the above
3. Correct diagnosis of errors in the solving of word problems does not require that the teacher know:
- a) a student's reading level
  - b) a student's ability to eliminate extraneous materials
  - c) a student's level of basic fact memorization
  - d) a student's ability to translate words into math symbols
4. Task analyzing a mathematical task is necessary to:
- a) determine the types of computation error possible
  - b) determine the sequence of procedures the student might use to accomplish the task
  - c) determine pre-requisite underlying concepts
  - d) all of the above

5. Determining a student's understanding of mathematical rules or principles can best be accomplished by:

- a) asking the student to demonstrate the rule or principle in several ways, i.e. concretely, semi-concretely, semi-abstractly, abstractly
- b) asking the student to give an example of a rule or principle
- c) asking the student to state the rule or principle
- d) asking the student to explain the rule or principle

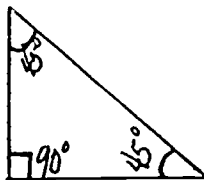
6. Given the following set of problems completed by Tom, choose the response which best describes his error.

$$\begin{aligned}2x + 4 &= 8 \\2x &= 12 \\x &= 6\end{aligned}$$

$$\begin{aligned}4x + 3 &= 9 \\4x &= 12 \\x &= 3\end{aligned}$$

- a) he has skipped a step in solving the problem
- b) he does not understand the concept of equivalence in equations
- c) he does not understand the concept of division
- d) none of the above

7. Given the following problem, describe the following triangle:



The student response was:

- a) scalene triangle
- b) congruent triangle
- c) right triangle
- d) none of the above

The most likely error in the student's thinking is:

- a) the student does not understand the definition of scalene triangle
- b) the student does not understand the concept of congruency
- c) the student does not understand the concept of right triangles
- d) all of the above

8. Given the problem:

Nicole is a tenth grader who earns money by throwing newspapers. She earned \$16.00 during her first week of delivering newspapers. The next week she earned \$21.00. What is the percent of increase or decrease in her earnings?

The student response was:

- a) 76%
- b) 132%
- c) 32%
- d) none of the above

The most likely error(s) in the student's thinking is/are:

- a) the student does not understand the concept of increase or decrease using percent
- b) the student cannot translate words to number sentences
- c) the student does not understand how to compute percent of change problems
- d) the student does not understand the whole/part relationship involved

9. Given the following problem:

A Department store carries dresses and suits in a ratio of 3 dresses to 2 suits. If the store has 140 suits, how many dresses does it have?

The student response was:

- a) 92
- b) 120
- c) 93
- d) 210

The most likely student error was:

- a) the student does not understand how to compare objects in a proportion
- b) the student does not know how to carry out the calculations involved in a proportion problem
- c) the student does not understand the concept of equivalence
- d) the student does not understand how to solve algebraic equations

10. John bought two items of clothing at a local clothing store:  
a) a tie costing \$9.99 and b) a shirt costing \$17.95. Sales tax in his state is 5%. John calculated that his total bill would be \$41.91.

John's error in calculation likely results from:

- a) not performing the addition operation correctly to convert to decimals correctly
- b) not being able to convert correctly from percent to decimal
- c) not understanding place value
- d) not understanding the procedure for rounding numbers

11. Missy purchased a winter coat originally costing \$79.95 at a "20% off" sale. The price that Missy calculated that she would have to pay for the coat was \_\_\_\_\_.

- a) \$15.99
- b) \$95.94
- c) \$53.96
- d) none of the above

The most likely student error is:

- a) not understanding of mathematical operations with decimals
  - b) not understanding of procedures for computing "sales price"
  - c) a multiplication error
  - d) a subtraction error
12. A strategy frequently used by an effective remedial teacher which is not often used by the regular mathematics teacher is:
- a) presenting multiple examples of the problem type
  - b) talking about examples presented on the board
  - c) task analyzing each problem type
  - d) asking students to demonstrate problems on the chalkboard

### III. Knowledge of Remedial Instructional Strategies

1. Pictures, graphs, charts, and simulations should be used in mathematics teaching:
- a) prior to experiences with objects
  - b) each time a concept has not been understood
  - c) whenever they initially facilitate student understanding
  - d) prior to introducing mathematical sentences
2. Instruction to remediate deficiencies in mathematics identified in minimum competency tests can best be accomplished by:
- a) the use of microcomputer software for drill and practice which is at least three grade levels below the child's grade level
  - b) the review and practices of basic skills related to items from a minimum competency exam
  - c) the review and practice of items requiring the same type of logical thinking as items in the test
  - d) continuing regular curriculum spiraling with occasional emphasis on deficit skills
3. A distinct difference between developmental theories of mathematics learning and behavioral theory is:
- a) how a student's response is evaluated
  - b) how a teacher selects the content to be taught
  - c) how a student works out a problem
  - d) how the teacher selects mathematical tasks



4. Both motivation and understanding can best be increased in the remedial mathematics classroom by:
  - a) use of more teacher talk time to explain math problems
  - b) use of more student practice time to habituate correct responses
  - c) use of varied instructional strategies related to the concept being taught and student justification of the responses
  - d) use of computer software to reteach mathematical concepts and operations
  
5. If a student has been diagnosed as not understanding how to write an equation using variables given in a word problem, an effective remedial strategy would be to:
  - a) have the teacher demonstrate several problems on the board
  - b) have the student practice additional problems, correcting wrong responses marked by the teacher
  - c) give the student a test of equations and have him/her find the correct equation for a given word problem
  - d) have the student translate phrases into mathematical expressions
  
6. If a high school student has been diagnosed as not being able to apply geometric formulas for area, perimeter, etc., of polygons, an effective remediation strategy might be to:
  - a) give the student a set of figures and ask him to match figures with appropriate formulas
  - b) have the student practice computation of areas and perimeters for irregular polygons
  - c) have the student draw a set of figures proposing ways to find perimeter or area and justify his/her proposal
  - d) have the student practice perimeter and area problems and correct errors marked by the teacher
  
7. If a 12th grade student does not understand probability theory, an effective remediation strategy would be to:
  - a) have the student construct a wheel of fortune and write probability sentences, defending his/her response
  - b) have the student complete practice worksheets on probability and correct errors marked by the teacher
  - c) ask the student to guess the chance of obtaining the number 6 on one throw of a die
  - d) ask the student to find examples of probability in everyday life and explain them to the class
  
8. During the fourth six-week period, an 11th grade student who is enrolled in Algebra I does not know how to solve algebraic sentences involving two variables. An effective remediation strategy would be to:
  - a) have the student work out similar algebraic sentences by two variables
  - b) have the student solve the two-variable equation when given the value of one of the variables
  - c) have the student practice simpler algebraic equations and gradually increase the complexity
  - d) have the student copy from the textbook the procedures and principles which must be remembered in solving the algebraic number sentence

9. A 10th grade student enrolled in a general math class successfully completed only one of the six items dealing with the mathematical manipulation of decimals on the state minimum competency test. Diagnosis indicates that the student is unable to determine the appropriate place for the decimal point. Appropriate remedial strategy(ies) might include:
- a) using money to demonstrate the meaning of decimals
  - b) using drill and practice software programs with a microcomputer
  - c) using both calculation and estimation to simplify instruction
  - d) using place value board, objects, and number cards to present the concepts of exchange and notation of place value
10. If an 11th grade student whose reading skills are at a 4th grade level does not understand how to apply geometric concepts such as parallel and perpendicular lines to reading the streets on a map, the best remedial strategy(ies) might include:
- a) having the student use fingers to follow streets to determine if the streets are intersecting or parallel
  - b) having the student review concepts of parallel and perpendicular lines using geometric figures
  - c) having the teacher reteaching map skills related to directionality
  - d) having the teacher teaching the plotting of coordinate points on an axis
11. A 10th grade student whose reading skills are below the 40th percentile and whose math skills are at the 70th percentile on the Stanford Achievement Test cannot solve written word problems related to the use of money. Remediation strategies have been employed for three weeks. An appropriate strategy for evaluation might be:
- a) have the student take a test of similar problems on the microcomputer
  - b) have the student use real money in order to solve the test problems
  - c) have the student listen to the problems via tape recorder and then solve the problems on the test
  - d) have the student complete pages in a workbook specifically designed for teaching money
12. The teachers management of the mathematics classroom is influenced most by:
- a) the teacher's personality and personal rapport with students
  - b) the teacher's use of strict discipline and structure
  - c) the teacher's obvious enjoyment of mathematics and ability to make the content meaningful
  - d) the teacher's use of strict routine in presenting a lesson

Appendix D  
An Effective Remedial  
Mathematics Teacher Checklist

References

I. Teacher's knowledge and use of the structure of mathematics as a guideline for possible remedial needs.

Can describe and use the structural aspects of mathematics (hierarchies, taxonomies, scope and sequence charts and concept cluster checklists) to locate the sequence of curricular objectives or competency test objectives.

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II. Teacher's knowledge and use of diagnosis as the first step in remediation.

- A. Uses a variety of survey and analytic tests as well as clinical procedures (error analysis, structured and unstructured student interviews) to diagnose learning problems in mathematics and to evaluate the results of instruction.

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