Briefly summarized are results from a report on mathematics assessments in 32 states. The general trend in mathematics achievement is upward, since the mid-1970's in some states and the early 1980's in others. Patterns are noted for addition, subtraction, multiplication, and division with whole numbers; fractions; decimals; numeration; geometry; and problem solving. Major implications are noted, particularly the point that students are scoring relatively well on items dealing with computation with whole numbers, but scores on concepts and problem solving are not as high. Topics on which scores were below the 50 percent level are listed, with some comments on the five topics which occasioned the greatest difficulty. (MNS)
Achievement in Mathematics Education

How well do students achieve in mathematics? Recently, data from mathematics assessments in 32 states were studied, and the results are reported in an ERIC/SMEAC publication, Assessing Achievement Across the States: Mathematical Strengths and Weaknesses. Some of the tests were administered in 1984; most are from prior years, with some in the mid-1970s. Thus, it is impossible to claim comparability on this factor and on many others, including the fact that different test items were being used, to test differing objectives. Nevertheless, it is possible to make some broad generalizations:

1. The general trend in achievement in mathematics is upward, since the mid-1970s in some states and the early 1980s in others. The pattern appears to occur whether norm-referenced, standardized tests or criterion-referenced, state-developed tests were used.

2. Where sufficient data were available, it was possible to note some patterns for some broad mathematical topics.

Addition with whole numbers: By grade 3, almost all scores are above the 80 percent level, with the mode above 90 percent. Even though items may test seemingly more difficult, multidigit content, proficiency with addition remains high across the grades.

Subtraction with whole numbers: Greater variability is evidenced in subtraction, especially at earlier grade levels. By grades 8 and 9, however, most students have reached about the same level of proficiency with subtraction as they attained earlier with addition.

Multiplication with whole numbers: The range of scores is rather broad in the intermediate grades, but by grade 9, the scores of most of the students indicate mastery above the 90% level.

Division with whole numbers: Not unexpectedly, the ranges of scores are broad and tend to remain so at each grade level. Although the upper limit is in the 80s and 90s, the modal score is in the 70s in grades 4, 5, and 6 and in the 80s at grade 9. Thus, division is of continuing difficulty for many students. Scores do tend to improve in grades 10-12, but they do not reach the high levels attained by almost all students on the other operations.

Fractions: Scores across grades are rather consistent. The lower limit of scores at most grade levels is around 50 percent, while the upper limit is in or near the 90s. The band where most scores lie is between 75 percent and 95 percent only in grade 11. Achievement with fractions for most students is clearly below their level of attainment with whole numbers.

Decimals: The range of scores is broad, indicating that difficulties persist for many students. However, except for grade 11, the modal score is in the 80s.

Numeration: Attainment on most items is acceptable, with the modal score in the 90s in grades 3, 4, and 5 and in the 80s in grade 8.

Geometry: The range of content may account for variability in scores, or the scores may reflect a lack of emphasis in the instructional program. At both grade 8 and for students than are items dealing with metric measurement. By grade 12, the modal score is in the 70s and 80s.

Problem solving: Scores are characterized by wide variability at almost all grade levels, reflecting the difficulty of varying content topics as well as the difficulty of problem solving. Scores on problems with each topic are generally lower than scores on the topics alone. The modal point for scores on items dealing with problem-solving strategies (such as lock for a pattern or find relevant data) cluster around the 60 percent level, while the modal point on consumer or career application items is in the 80s.
Topics Needing Improvement

Items on which scores were below the 50 percent level were compiled from the various state reports on mathematics assessments. This level was arbitrarily chosen as a point below which lack of success was clearly non-acceptable, it does not imply that scores of 50 percent, 60 percent, or 70 percent are satisfactory. The task was then one of determining the mathematical focus that sets of these items had in common. Among the items missed most frequently were ones on the following topics:

- Subtraction with renaming
- Subtraction with three- and four-digit numbers
- Multiplication with two or more digits
- Division by a multiple of 10
- Division by numbers with two or more digits
- Equivalent fractions
- Operations with fractions or mixed numbers, especially with unlike denominators
- Multiplication of decimals
- Division of decimals
- Equivalent fractions
- Operations with fractions or mixed numbers
- Division of decimals
- Percent one number is of another
- Percent of a number
- Conversion of a decimal to a percent, or vice versa
- Exponential notation
- Parallel/perpendicular lines
- Metric measurement
- Determining diameter, radius, and circumference
- Perimeter and area
- Reading and interpreting graphs
- Probability
- Estimating with numbers and with measures
- Problem solving

The five topics which occasioned the greatest difficulty, usually across several grade levels, seemed to be equivalent fractions, operations with fractions or mixed numbers, division of decimals, perimeter and area, and problem solving.

The relationship between the first two seems evident: students who do not understand the meaning of equivalence of fractions have a difficult time determining what to do when faced with the need to perform addition or another operation with fractions. It may be that many children have had little experience with manipulative materials to help them build concepts of equivalence; frequently, work with symbols alone replaces the meaningful development of the concept. It seems to take longer to teach a mathematics topic when concrete materials (or even pictures) are used, but there is much evidence that such materials do have a payoff in increased understanding and achievement. There is also evidence that better "bridges" need to be built between the materials and the symbols, and between related topics. Thus, children need to learn that finding equivalent fractions is not an end in itself, but a tool that they will use when they add or subtract fractions.

Perimeter and area are, like the other topics on the list, known to be points of difficulty. In particular, confusion of the two continues. Perhaps, again, concrete materials are not used to develop the concept of each, perhaps the distinction between the two is not made clearly in the instructional program. There is also evidence that items testing the ability to calculate the perimeter are more difficult when only two dimensions of, say, a rectangle are given, than when all four dimensions are given. The numbers the child sees are the ones he or she uses, without sufficient thought to the meaning of, in this case, "perimeter." "Symbol pushing," without thought or attention to the reasonableness of the answer, is unfortunately a characteristic of much mathematics instruction.

A number of the interpretive sections of the state reports addressed teaching strategies as well as content concerns. An emphasis on the importance of meaningful instruction, promoted through the use of manipulative materials and real-life situations, threads its way through these reports. The way mathematics is taught is as much a concern as what mathematics is taught. Furthermore, the data from any assessment are not an end in themselves. They need to be used to improve instruction.

REFERENCES


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