DOCUMENT RESUME

ED 260 691 IR 011 763

AUTHOR Vandendorpe, Mary M.
TITLE The k-d Tree: A Hierarchical Model for Human Cognition.
PUB DATE 3 May 85
NOTE 19p.; Paper presented at the Annual Meeting of the Midwestern Psychological Association (57th, Chicago, IL, May 3, 1985).
PUB TYPE Viewpoints (120) -- Reports - Research/Technical (143) -- Speeches/Conference Papers (150)

EDRS PRICE MF01/PC01 Plus Postage.
DESCRIPTORS *Artificial Intelligence; Cognitive Development; *Cognitive Processes; Comparative Analysis; *Computers; *Computer Software; *Information Processing; *Mathematical Models; Research Tools
IDENTIFIERS Piagetian Theory

ABSTRACT

This paper discusses a model of information storage and retrieval, the k-d tree (Bentley, 1975), a binary, hierarchical tree with multiple associate terms, which has been explored in computer research, and it is suggested that this model could be useful for describing human cognition. Included are two models of human long-term memory--networks and hierarchies--and reasons are given for the higher efficiency of hierarchical theories, including the k-d tree. A description of the k-d tree includes its structure, computation rates, and balancing (branching); its applications to human cognition, including a comparison with Piaget's notions of equilibrium and cognitive stages of development; application to memory and forgetting theories; covergent and divergent thinking processes; logic paths and decision-making; and the function of sleep. A concluding discussion compares human and computer processing of information, and raises questions related to the hierarchical structure of brain activity. It suggests that the k-d model from information science may have a strong relevance to the study of human cognition, particularly regarding the study of memory and sleep, while at the same time allowing for vast differences between a computer and the human mind. (JB)

The k-d tree: A hierarchical model for human cognition

Mary M. Vandendorpe
Lewis University
Romeoville, IL 60441

Presented at the fifty-seventh annual meeting
of the
Midwestern Psychological Association
Chicago, IL May 3, 1985

The k-d Tree: A Hierarchical Model for Human Cognition

Information processing, whether in humans or machines, comprises two separate fields: hardware and software. Hardware refers to the physical storage processes and the interconnections between areas. In psychobiology, this means the physical interconnections of neurons and intraneuronal changes that are affected by learning. Software refers to the ways in which data are referenced, accessed and manipulated. Hardware determines what kind of computations can be done; software determines how computations are in fact done.

This paper considers only software questions. It proposes a model of information storage and retrieval, the k-d tree (Bentley 1975), that has been explored in computer research and that should be a useful model for describing human cognition.

Basically two models of human long-term memory retrieval have been put forth, networks and hierarchies. Each addresses one of the two basic information problems that must be considered. Each has advantages and disadvantages.

The network theory is thought to be best in describing the multiplicity of connotations that any one image, work or concept evokes in human thinking. Quillian's Teachable Language Comprenender (1966) was one of the first theories. Anderson's ACT theory (1976) is probably the most well known and discussed. Network theories are generally inadequate in their explanations of the speed of human information processins; nor do they necessarily explain why some concepts are more readily accessed than others. ACT theory, for example, asserts that both speed of retrieval as well as accessability can be explained by the strength of the links between nodes. Strength is directly related to the number of times a link has been executed or used.

In computer science terms, network models invariably generate classes of problems that are NP complete. NP completeness refers to the length of time it would take to recover information. To say that network models are NP complete means that a non-deterministic machine will compute an answer in P(n) time. P(n) is a polynomial dependent on the number of nodes used in the network. Although the non-determinism constraint can be removed by the parallelism of function in brain-cells, computations bound to run within polynomial time do pose troubling questions about the limiting speeds at which a brain would function. Because of the existence of closed loops within the network, some concern must also be raised about the mechanism used to terminate a thought process once it has begun.

Hierarchical theories, on the other hand, are far more efficient data structures for retrieving and storing data. Computer studies of k-d trees have been limited to the hardware search assumption of sequential processing. Even so, the worst retrieval time bound is proportional to n raised to a fraction power, orders of magnitude faster than the network model. Only one of the computational processes used in a k-d tree is relatively computationally inefficient in that it is proportional

BEST COPY AVAILABLE

to n log n. This process is tree re-organization, also referred to as balancing in this paper. Re-organization is a time consuming process. This paper sugests that it occurs over significant periods of time, possibly during sleep. The hierarchical model also provides a natural bounding mechanism, the depth of a tree, that would serve to limit the amount of computational resources needed for a given problem. Thus, the k-d tree is an efficient model.

The k-d tree model

The k-d tree is a binary tree. Although a similar structure called a Quad tree (Bentley, Stanat, 1975) allows for a multiway branching, it is more difficult to program and less general computer science claims have been applied to it to date.

Structure. The k-d tree is composed of decision points that represent stored information. The information is first quantified into k-tuples. Each of the k cross-product domains is a different dimensional axis along which the data has been collected. Information stored within these trees is categorized by the context of preceeding information in a most interesting way: every node of a k-d tree orders the information stored below it in the tree on the basis of one of its k dimensions of information.

This produces a tree that does not cluster concepts together; rather it tends to spread them throughout the tree in paths that are determined by the order in which the information was originally assimilated. These learning dependent associations can account for "insight" or "intuition" since the search process is enriched by the chance association of concepts that preceeded the assimilation of new data and by the tree reorganization process. In this model concepts would continue to be influenced by initial associations long after the concept had been learned.

This model can provide more concise descriptions of human cognitive processing than have been previously available. Previous hierarchical theories were not able to explain parsimoniously the multiplicity of associations that are attached to any one word or concept. The k-d tree structure solves this problem, because of the k dimensions associated with each node.

Computation rates. The k-d tree is the most effective and general software mechanism known for solving associative queries. No other software data structure that has been suggested applies to every category of associative query: exact match search; range search; partial match search; and nearest neighbor search. The performance characterisitic of k-d trees on sequential process machines which are far less efficient than the brain, is surprising.

To categorize the relative speed of computational processes, an approximation notation $O(f(n))$ is used where $f(n)$ is a function of the positive integers n; $O(f(n))$ stands for a quantity that is not explicitly known except that its magnitude is at least as large as f(n). More precisely, there is a positive constant m such that the number x[n] represented by O(f(n)) satisfies the condition |x[n]| < m|f(n)|, for all n >= i. The constants m and i are not specified and will differ for each approximation.

The average running time for sequential k-d tree algorithms with k dimensions, has been shown to be (Bentley, 1975): insertion, O(log n); deletion of the root, $O(n^{(k-1)/k})$; deletion of a random node, O(log n); and optimization (guarantees logarithmic performance of searches), O(n log n). For nearest neighbor searches the empirically observed average running time is O(log n). For partial match queries with t keys specified has a maximum running time is $O(n^{(k-t)/k})$. All of these performances were presented in Jon Bentley's original paper on k-d trees and all of them either surpass or equal all other known algorithms for these Tasks. No other computer stored structure has been found that is either as versatile or as efficient for all forms of associative query.

<u>Balancing.</u> The k-d tree is most efficient when it is balanced; that is, when it is most bushy. This is formally described as a tree with no more than one level difference between the bottom most nodes. Only in this case are the excellent performance characteristics strictly true. As the tree becomes increasingly less balanced, the performance characteristics deteriorate and the response time needed to

answer associative queries eventually becomes O(n). Long before this point is reached, it is advantageous to rebalance at least part of the tree. An algorithm which has optimal O(n log n) characteristics is known that will rebuild a balanced tree. For large trees, however, the need for a complete rebalancing will be infrequent.

Applications to human cognition

Thus we have in the k-d tree, an extremely fast, organized and associative data structure. Certain aspectts of this model seem particularly relevant to research and theory about human learning.

Piagetian theory. The balanced hierarchical tree is certainly relevant to Piaget's notion of equilibration. While Piaget sometimes seems to emphasize the congruence aspect of equilibration, he also included in his notion that of hierarchical order and categorization. Assimilation could then be defined as the adding of information to the tree, without an attempt to rebalance it. However, as the tree becomes more and more unbalanced, disequilibration is reached. Finally, the tree will be rebalanced, at the expense of time and energy, and accomodation is achieved. For the k-d tree this is a computational process requiring an average running time of O(n log n).

It should not be expected that the entire tree will usually be rebalanced. Typically, accomodation will take place in only one part of the tree. This would account for decalage, the uneven cognitive development found in most children. However, occasionally partial rebalancing will not be a satisfactory solution, and the whole tree must be resorted and the data recategorized. Piaget's conception of cognitive stages of development seems logically to parallel the idea of rebalancing the entire tree. The concrete operational stage seems particularly to represent the completely rebalanced tree that results in an integrated hierarchical structure. In that period, children are able to understand sets and subset memberships. Formal operations as a stage may not necessitate a total rebalancing; rather it might be that formal operations require the addition of a level of hypothesizing at the top of the tree rather than a reorganization at the bottom of the tree.

Similar comparisons may be made for the first two of Piaget's stages. The beginnings of representational thought which occur at the end of the sensorymotor stage can usefully be thought of as the formation of the first hierarchies. Initially, schema are discrete memories, but then they are organized into related concepts. Preoperational thought, like formal operations, would be marked by the addition of conceptual levels at the top of the tree.

This model can provide a basis for exploring the hypotheses of Piaget in both computers and children. A computer simulation could be designed to accept new information without rebalancing for a particular time period. Then it would be allowed to rebalance, and we could trace the changes in cognitive structure. Depending on the particular time period, we could expect either the development of superordinate concepts or realignment of cognitive categories.

In children, decalage should be most apparent in rather unrelated concept areas. Furthermore, accomodation, as well as stage transitions could be studied to see if new information alone creates growth or rather if a new way of conceiving what is already known is necessary. For example, a new area of information could be presented. If this model is correct, it should be expected that initially the learner will try to incorporate the information with what he already knows. We should expects gaps in information as well as misinformation. However, as the learning process progresses, there should be detectable moments when the information has been recategorized, less information is lost, and misinformation is minimized. Finally, if enough partial balancing is made necessary, we would expect a total rebalancing resulting in an integration of the newly organized information with older, more established concepts. Incongruities would be noticed and resolved. It should be expected that this total rebalancing would manifest itself suddenly as a consequence of a median k-tuple (concept) achieving its place near the root of the knowledge tree.

Memory and Forgetting. It is semantic memory which seems most suitable for exploration with the k-d tree model. Because semantic memory is dependent upon context and multiple references, its structure should parallel fairly closely the structure of the tree.

Work by Collins and Quillian (1969) suggests that highly similar concepts are retrieved and compared more quickly than less similar ones. These studies offer the beginnings of support to this model.

It is unclear whether episodic memory can also be explained by this model. However, some evidence suggests that the tree structure might be a useful model, if we hypothesize a separate tree or subtree whose key indices are contextual cues. Each cue would be linked to all the relevant data associated with the remembered event. When a cueing stimulus is perceived, it would elicit the remembered event, or one which is perceived by the individual as a deja vu experience.

The k-d tree also can give an explanation for why mnemonics, such as the method of loci, work. The method of loci requires the individual to associate terms or concepts with a well known physical terrain. He then can recall concepts more readily if he "walks through" the terrain in his imagination, and remembers the associates of the physical characteristics he is "seeing". In k-d terms, he has added an extra characteristic to the nodes in a subtree he has already established. Because this subtree is already well established, it is easy to retrieve, along with the associated concepts or terms.

Inhibition, both retroactive and proactive, can also be described in terms of the tree. First, we assume that partial rebalancing goes on fairly frequently. We also assume that interruptions of the rebalancing process take place fairly often. J. Vandendorpe (1980) has shown that an interruption of balancing results in lost information; inhibition occurs when such an interruption happens. When it is new information which is being placed in its proper node, the new information may be lost. When the tree is being rebalanced, links to old information may be lost. It must be remembered that the data records are not destroyed, but that the ability to retrieve the records is lost. This could be why seemingly forgotten memories can be elicited by direct neuronal stimulation. Furthermore, depending upon whether or not there are many pointers to the missing information, the person may or may not realize that he has forgotten anything.

The k-d model is also supported by the research which generally supports the retrieval-failure theory of forgetting, most often used to describe short-term memory failure. Dillon's (1973) research on the accessibility of items supports the idea that the difficulties in retrieving infomation lie not in deciding whether the information is pertinent, but in finding the information.

The relatively greater impact of proactive inhibition as described by Keppel and Underwood (1962) can also be understood in terms of this model. New information is more vulnerable to forgetting because fewer associates have been developed, and this is especially so when the associates used are similar or even identical. Proactive inhibition can be diminished, however, if we make use of a totally different class of associates. Wickens (1972) reported a demonstration of the "release from proactive inhibition" effect. Furthermore, research by Dillon and Bittner (1975), Gardiner, Craik and Birtwistle (1972) and O'Neill, Sutcliffe and Tulving (1976) all suggest that presenting a new subcategory to the subject reduces proactive inhibition.

Retroactive inhibition is less powerful because old information is likely to have many referents, and loss is likely to be more often detected, if not always corrected. If this is true, it should be found that material forgotten because of proactive inhibition is more often totally lost or inaccessible. The individual should more often not even know he has forgotten something. Material forgotten because of retroactive inhibition should be more easily restored--or at least the subject should realize he has forgotten something!

Convergent and Divergent Thinking. Convergent thinking, or the process which results in the single correct answer, is likely to be a rather simple process in terms of k-d trees. Convergent thinking would occur when the tree is searched in the typical, top-down manner. If the correct decisions are made at all the nodes, the correct answer is retrieved.

Divergent thinking could be defined in either of three ways with this model. The person could access the tree in the normal manner, but when she has reached one correct conclusion, she could re-access the tree, making different decisions at some of the nodes. A second approach would be to use the referent nodes found along the path to the first correct conclusion to access the tree in a horizontal reference pattern. This approach would allow for bottom-up as well as top-down searching. A final way that divergent thinking might be explained is in the concept of rebalancing. Divergent thinking would be the product of resorted and recategorized data organizations. The need for an incubation period in the creative solution of problems would suggest that the last definition is a most appropriate one.

Logic paths and decision making. The structure of the k-d tree, as determined by the order of the presentation of information, produces some interesting ideas. In accessing a datum, the path taken provides associations, but it also is in itself a structure or logic path. This logic path could constitute the formation of implicit rules which have been examined in many recent learning studies.

This same logic path structure can model the effects of instructional methods and examples that have been found to be strongly influential in decision making speed and accuracy. In the k-d structure, instructions and examples affect path choices at the root of the tree, and thus can determine what information is accessable, and what kinds of decisions are possible.

The function of sleep. Some researchers have identified the REM state with information processing, such as sorting, coding and referencing. Recently, Crick (1983) has taken the opposite view that dreaming is actually a forgetting process, or at least one in which irrelevant images are discarded. If we conceive as the waking state as one in which information is assimilated, creating more and more unbalanced trees, then REM sleep could be the time in which major rebalancing efforts are made. If this is true, then the currently opposing views can be reconciled. Sorting and coding does take place, as well as inhibition and actual removal of information. There might be a loosely affiliated subtree which retains daily memories, and which is resorted and integrated into the main conceptual tree. If this is true, proactive inhibition should be stronger for material that is followed by a sleep period.

Does the remembered dream have a relation to this rebalancing process? The dream might be a reflection of the material in the temporary storage area which is waiting to be re-inserted in the tree. Data in the temporary storage area might not be organized meaningfully, and this could be one reason why dreams often have little internal logic.

Discussion

This paper does not simply assume that humans and computers actually process information in an identical manner. Most importantly, computers have a single central processor that essentially only does one thing at a time. It seems most likely, however, that humans process many items at the same time. Furthermore, computers typically operate by way of algorithms, and only laboriously and in a very limited fashion can they develop new algorithms. Humans, on the other hand, easily generate new rules and discard nonworking ones with almost as much ease.

Nor does the model proposed in this paper necessarily imply a particular hardware, a particular neural anatomy. While some organizations of neurons might make k-d trees more straightforward there does not seem at the present to be any pattern that would preclude the existence of k-d trees. A relevant physiological question is whether or not brain activity propagates in a roughly hierarchical way, from one area to another area and then back again to the point of origin.

While this paper has taken the position of describing human information storage in terms of a single main tree, that is not necessarily the case. It would seem reasonable that semantic, episodic and daily memory trees are rather unrelated, and may only share the initial root. They may even be entirely unrelated. Whatever the degree of relationship, the method of storage and retrieval of data are thought to be similar.

This paper has offered a model from information science that may have strong relevance to the study of human cognition. The k-d model is efficient, ordered, and has the capacity for associative retrieval. It seems particularly relevant to the study of memory and forgetting.

References

Anderson, J. R. Language, memory and thought. Hillsdale, N.J., Erlbaum, 1976.

Bentley, J. L. Multidimensional binary search trees used for associative searching. Communications of the ACM, 1975, 18, 509-517.

Crick, F., & Mitchison, G. The function of dream sleep. Nature, July 14, 1983.

Dillon, R. F. Locus of proactive interference effects in short-term memory. Journal of experimental psychology, 1973, 99, 75-81.

Dillon, R. F., & Bittner, L.A. Analysis of retrieval cues and release from PI. Journal of verbal learning and verbal behavior, 1972, 11, 778-783.

Gardiner, J. M., Craik, F. I., & Birtwistle, J. Retrieval cues and release from proactive inhibition. Journal of verbal learning and verbal behavior, 1972, 11, 778-783.

Keppel, G., & B. J. Underwood. Proactive inhibition in short-term retention of single items. Journal of verbal learning and verbal behavior. 1962, 1, 153-161.

O'Neill, M. E., Sutcliffe, J. A., & Tulving, E. Retrieval cues and release from proactive inhibition. American journal of psychology, 1976, 89, 535-543.

Rosch, E., Mervis, C. B., Gray, W., Johnson, D. & Boyes, P. Basic objects in matural categories. Cognitive psychology. 1976, 8, 382-439.

Rosch, E. & C. B. Mervis. Family resemblances: Studies in the internal structure of categories. Cognitive psychology: 1975, 7, 573-605.

Quillian, M. R. Semantic memory. In M. Minsky (Ed), Semantic information processing. Cambridge, Mass., MIT Press, 1968.

Vamdendorpe, J. E. A crash tolerant B-tree data structure for data base retrieval systems. Ph.D. Thesis, Illinois Institute of Technology, Chicago, Il. 1980.

Wickens, D. D. Characteristics of word encoding. In A. W. Melton & E. Martin (Eds.), Coding processes in human memory. New York: V. H. Winston and Sons, 1972.