A review of the literature is presented on developmental mathematics courses in two- and four-year colleges and universities. The paper is organized within the categories of status studies, placement, program evaluation, class management, student characteristics, and thought processes. Highlights of the report include: (1) an average of 2.0 remedial mathematics courses were offered in a carefully constructed sample of all colleges and universities in the United States; (2) the format in which courses were most often presented was the lecture-discussion method; (3) 20% to 40% of the colleges with students in developmental mathematics placed them with an examination, while 71% felt they ought to do so; (4) research indicated that older students benefited more from developmental courses than younger students; and (5) one study indicated that 68% of students in developmental education courses had taken similar courses before. Finally, a critique of the research is presented, indicating that many studies were superficial or methodologically weak; the number of students was small; there was little external funding for research; few studies examined the experience of minority groups in developmental mathematics programs; and a tenuous relationship existed between the studies with little cross referencing or building upon earlier studies. A five-page list of references is included. (HB)
Developmental Mathematics in College:
What the Research Is and
Why There Isn't More

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Coupled with the decline in standardized test scores of high school graduates in the past 15 years has been the phenomenal increase in the number of college students taking developmental or remedial mathematics. While such courses have been an integral part of most two-year college curricula since their beginning, four-year colleges now have increasing numbers of students enrolled in developmental mathematics. Research on these programs and students was reviewed in two separate articles about five years ago. Akst (1978) categorized the research as descriptions of programs at individual colleges, regional surveys, and comparative studies on the relative effectiveness of various modes of instruction. Friedlander (1979) organized his review as answers to questions about course content, placement, credits and grades, course presentation, program effectiveness, and instructor characteristics.

As I tried to organize this review of research reported in the past five years in journal articles or papers filed with ERIC, I found that some of those categories were useful, but that new ones were necessitated by the directions recent research has taken. Familiar topics include surveys or status studies, placement, program evaluation, and course presentation or class management. In addition, this review will include research on characteristics of developmental students and their thought processes.

STATUS STUDIES

Starting with status studies gives an overview of the topic and helps define the limits of the discussion. Friedlander (1979) indicated that by far the most common courses offered were arithmetic and elementary algebra (80-90%) followed by geometry, trigonometry (20-30%) and others. Chang's 1983 nationwide survey confirmed the importance of elementary algebra (82%), but only 68% of the schools offered arithmetic and 53% intermediate algebra. These changes may just be due to differences in samples, but they may also reflect the fact that more four-year institutions are offering developmental math and start with algebra.

A recent survey conducted for the National Center for Educational Statistics (NCES) indicated that in 1983-84 an average of 2.0 remedial mathematics courses were offered in a carefully constructed sample of all colleges and universities in the U.S. (Wright & Cahalan, 1985). Overall 25% of all college freshman took a remedial math class that year, but the percentages varied by type of school: 27% in public versus 15% in private colleges, 28% in two-year versus 19% in four-year colleges, 30% in open admissions colleges versus 13% in selective and traditional admissions colleges. Remedial math courses were mandatory if indicated in 59% of the colleges offering them.

The format in which the courses were most often presented was the traditional lecture-discussion method according to both Chang and Friedlander, although various modifications included self-pacing, laboratory settings, mastery learning, and programmed or computer-assisted instruction. Chang found that most teachers used pencil-and-paper exams and most colleges provided tutorial services. Evaluations of program effectiveness in terms of completion rate ranged from 40% to 50%. The percentages of students who
completed a college-level math class after developmental work varied so widely that comparisons were difficult. Sometimes developmental math was taught in a separate department or program, but sometimes it was housed in the mathematics department. Faculty were often randomly assigned to these courses or rotated in; use of part time faculty was common. According to Chang's survey, 33% of the developmental math faculty had PhDs in mathematics or mathematics education; many had masters degrees in those fields. That a growing number of faculty and students are involved in developmental work was documented by the NCES study and by Whitesitt (1982), who offered reasons for this growth.

**PLACEMENT**

While Friedlander's review indicated that only 20% to 40% of the colleges with students in developmental mathematics placed them with an examination, a much larger percent (71%) felt they ought to. Some studies reported recently deal with placement exams. Bridgeman (1982) compared the usefulness of the SAT-M and the Descriptive Tests of Mathematics Skills (DTMS), also from ETS, for predicting grades in freshman level mathematics courses. While the two were equally effective in predicting grades in elementary functions and calculus, the DTMS was clearly superior for placement in remedial algebra. Dwinell (1985) studied the placement effectiveness of those same tests plus the Basic Skills Examination designed and mandated for use by the Georgia University System Board of Regents. The state test was the best predictor of placement and number of quarters needed to reach an exit level and close to the best predictor of course grade.

Other states are constructing and validating their own placement tests. A particularly careful job of doing that is described for the New Jersey College Basic Skills Placement Test used statewide (Dass & Pine, 1981).
Among 30 community colleges in New York state, 17 used locally developed placement tests; the next highest number, four, used the DTMS (Fadale & Winter, 1985). A compromise presented by Palmer (1983), is to construct one's own exam tailored to the specific courses at an institution but using items from a commercially prepared item bank which were already pretested for validity, reliability, and effectiveness of distractors. In an interesting and apparently cost-effective use of a placement exam at Ohio State (Adcock, Leitzel, & Waits, 1981; Leitzel, 1983) high school juniors in Ohio were tested and then advised of the consequences of their probable placement in time to choose appropriate mathematics courses for senior year or to alter their career goals.

PROGRAM EVALUATION

A chapter by Akst and Hecht (1980) on program evaluation was helpful in reviewing the recent research in this area. They point out that objectives of developmental programs should determine methods of evaluation. If the objective of the course or program is to increase student knowledge of mathematics, then analysis of completion rates or posttest scores is appropriate. If the objectives include preparing students for future mathematics courses or college in general, then these course grades or GPAs should be studied. The former method of evaluation is often flattering, while the latter is generally discouraging. For example, Eisenberg (1981) reported that of the students at Northern Michigan University who took an elementary algebra course, about 75% passed. However, less than 40% of those who passed went on to take one of the two next higher mathematics courses, even though most disciplines at NMU required one or the other. Of this 40% only 55% actually pass the next course. The NCES study found that overall, 68% of the
freshmen in remedial math passed the course, but only 35% of the schools kept retention data on remedial students separate so that success in college level work could be evaluated (Wright & Cahalan, 1985).

Dumont and others (1982) evaluated a remedial mathematics course at a regional public university using three of the research designs suggested by Akst and Hecht. Using the single-group, pretest-posttest design, Dumont's group found that 40% more students passed the posttest and the gains from the pretests were marked and statistically significant, an indication of a successful course. However, when the Remediated-Exempted Comparison design was used, the results were not as positive. Many fewer remediated than exempted students went on to take the next math course, although those who did performed as well. A Cross-Program Comparison was also done with students from a nearby community college, with the result that the community college course seemed more effective. The authors concluded that the research design chosen affected the results more than the type of grouping statistic or length of time after the course that measures were taken. Barcus and Kleinstein (1980-1981) and Wepner (1985) also describe program evaluations which used several different methods of assessing the value of a computational skills course.

Fadale and Winter (1985) reported on the development of an evaluation model for the two-year colleges in New York state. Four criteria were chosen: achievement of exit criteria for developmental courses; achievement of individual student goals; program completion or graduation; and academic eligibility to continue (including success in following courses). Standards and methods of measurement were also established. Flexibility was an important goal because the model will be used by many programs.
Akst and Hecht (1980) also described the biases of each of the research designs discussed, especially in terms of the remedial population, such as the effects of dropout rate and regression toward the mean. A study of developmental mathematics courses for adults by Yanosko (1981a) illustrates these points. The first course for the lowest students included arithmetic and algebra; the second course only algebra. It appeared that the students in the first course improved more from pretest to posttest. However, they may have tested below their ability on the pretest, and the dropout rate for the lower group was 48% compared to 29% in the upper group.

CLASS MANAGEMENT

Research focusing on an individual course or specific topics within a course has explored different uses of technology, methods of staffing, and modes of instruction. Although research surveys cited in the section on status studies indicated that the lecture-discussion method was used more often than more individualized modes of instruction, researchers are still experimenting with the latter. Steele, Legg, and Miles (1980) investigated the effects of requiring students in self-paced remedial algebra classes to attend classes staffed by tutors and concluded that the attendance policy was especially beneficial for the less well prepared students.

Thompson and McCoy (1979) investigated the effects on performance in and completion of courses in elementary, intermediate, and college algebra courses of imposing teacher pacing on a mastery-unit testing mode of instruction. They concluded that self-pacing was best for the elementary algebra class and teacher pacing for the college algebra class with the results for the intermediate class falling in between. In a later study Thompson (1983) discussed the advantages of competency-unit testing (67% correct), with limited
repetitions over mastery-unit testing (80% correct) with unlimited retesting and concluded the former had the pedagogical advantages of mastery learning but lacked its inefficiencies. Combining mastery learning with the lecture-discussion format was successful in raising students’ final exam scores and their awareness of the importance of mathematics in a small-scale study by Blackburn and Nelson (1985).

Uses of technology have also generated research. A computer-assisted instructional (CAI) system for bringing engineering students up to the calculus level was compared to two conventionally taught classes (Flower & Craft, 1981). Fewer students survived the CAI course and went on to calculus, but those who did were more successful. The authors considered this early weeding out to be productive.

Another technological innovation whose impact on developmental education has been studied is the hand-held calculator. Hector and Frandsen (1981) compared three self-instructional treatments of fractions: one with only conventional algorithms, one with conventional algorithms and limited calculator use, and the last with calculator algorithms. There were no significant differences among the groups in attitudes or understanding and computing with fractions. Koop (1982) studied two sets of traditionally taught arithmetic classes; one group was taught the conventional algorithms, and the other used calculator algorithms and spent the additional time on problem solving. Surprisingly there were no significant differences between treatment groups on tests of computation or problem solving, on any of the three attitude measures, on retention, or on course completion. There were some interesting interactions, however, with students 30 or older doing better than average without calculators and worse with them; the opposite was true for females.
The writing-across-the-curriculum movement has also influenced research in developmental mathematics. Pallman (1983) described a coordinated offering of freshman composition and an arithmetic/algebra course in which the instructors made explicit the similarities between organizational patterns in writing and in mathematics. Students in the math class were asked to give written descriptions of solutions to problems involving stumbling blocks such as borrowing and order of operations. This treatment resulted in higher retention, less absenteeism, and somewhat higher gain between pretest and posttest scores. Hirsch and King (1983) reported an experiment with writing in an elementary algebra course which was less successful, probably because the writing was not an integral part of the course. Another program which integrated developmental writing and mathematics also involved counselors participating in classroom activities (Reynolds, 1981).

STUDENT CHARACTERISTICS

A wide range of student characteristics have been studied in order to design better instruction in developmental mathematics. One group of factors includes gender, age, and ethnicity or race. Another includes personal mathematics histories and present reasons for taking developmental mathematics. A third group explores affective characteristics such as attitudes toward math, math self-concept, and math anxiety. The fourth, and possibly most important, includes cognitive styles and abilities.

Although data on male/female ratios in developmental courses vary so much by institution that they are not useful, other studies of gender are noteworthy. Brunson (1983) reported that an all-female section of an elementary algebra course started out with a significantly lower SAT-M average than the women in mixed sex sections but ended the course with significantly
higher achievement. Women's pass rate in a basic math course was higher than men's in a study by Goldston (1983). On the other hand, Frerichs and Eldersveld (1981) found that the percentages of male and female students who succeeded in developmental math courses was exactly the same as the percentage enrolled. Males were somewhat better at solving algebra word problems than females in Schonberger's 1981 study; the difference appeared to be more closely linked to Piagetian developmental level of reasoning than to spatial abilities. Moore (1980) also noted better problem solving by males than females.

Building on the literature on causal attribution of success and failure in mathematics by secondary students, Shea and Llabre (1985), looked for gender differences in community college students' attributions for success in two subject areas. Although the math courses students were taking were not explicitly labeled developmental, they were all first-level courses. The authors hypothesized that there would be gender differences in attribution in math in which women do not participate equally later in college, but none in English or social sciences in which they do participate equally. They found differences due to subject matter but not to gender.

Although older students may enter with less knowledge of mathematics above arithmetic (Frederick, Mishler, & Hogan, 1984), there are indications that they benefit more from developmental courses than their younger classmates. Schonberger (1982) studied three groups of elementary algebra students: those whose final exams showed that they were high in problem solving and low in skills, those low on problem solving and high on skills, and those high on both. One characteristic which distinguished the last from the other two was that they were older. Frerichs and Eldersveld (1981) also found successful students as a group to be two years older than the unsuccessful ones.
Studies using race or ethnicity as a variable were curiously absent from this search. One exception was a paper by Brod and Brod (1982). They found that Native American students entered with less math, were more likely to need remedial math, and took less of it. Those who progressed through all the remedial courses did as well in the college level math courses as the other remediated and unremediated students. Yanasko (1981b) also reported percentages of ethnic groups in her program.

Personal mathematics histories of students in developmental courses indicated that 68% had taken similar courses before (Frerichs & Eldersveld, 1981), but others indicated that their high school backgrounds were poor in math (Michigan Council of Teachers of Mathematics, 1979; Yanasko, 1981b). Their high schools required one, or at most two, years of math, their counselors did not encourage them to continue in math, and they had not realized that they would need math for their careers. According to Frerichs and Eldersveld, a sizable percent were uncertain why they were presently in developmental math.

Such previous history with mathematics is probably a source of the affective characteristics of developmental students in mathematics. The Frerichs-Eldersveld study found that 70% of their subjects expressed a negative or neutral attitude toward mathematics and over 90% said their mathematical ability was average or lower. These two variables were among the five significant discriminators of success in their developmental math course. Easton (1983) found that students in basic mathematics who met their criteria for success improved two measures of math self-concept, while those who did not meet their criteria had an unchanged self-concept and adjusted their criteria for success. Math anxiety appeared to be greater with a heuristic method of teaching problem solving than with an algorithmic method and greater
in a group setting than in an individual setting, according to Moore (1980).

Both the Moore study and the Frerichs and Eldersveld study also investigated the cognitive style of field independence/dependence. Moore noted that the field independent students did better with the heuristic method than the algorithmic method; for the field dependent students, the reverse was true. However, according to Frerichs and Eldersveld, cognitive style was not a significant discriminator between successful and unsuccessful students; numerical ability was.

Finally, in two studies the relative usefulness of cognitive style, personality characteristics, and mathematical abilities are evident. Resek and Rupley (1980) explored characteristics of students in a Math Without Fear course which could predict which students were "concept oriented" or became so during the course and which would remain "rule oriented." Several dimensions from the California Psychological Inventory and the Myers-Briggs Type Indicator predicted with probability of error less than .05, but field independence/dependence was as good and considered less intrusive. Better yet were measures of arithmetic skills and problem solving. In Schonberger's 1982 study both groups high on problem solving were more field independent than the low problem solving-high skills group which entered with the best algebra skills. The group high in both entered with the best arithmetic skills.

THOUGHT PROCESSES

In this final category of research, the author relaxed the requirement that the study be done with developmental students. If the study used post-secondary students and involved content taught in developmental mathematics, it was included. The author also verified that errors in thought processes being researched had been observed in her own developmental math students.
Some studies indicated that even arithmetic content proved difficult for postsecondary students. Wheeler and Feghali (1983) document student misunderstandings regarding division of or by zero, recognizing zero as a number, and classifying or partitioning sets involving the empty set. Pollatsek, Lima, and Well (1981) found that statistics students able to compute simple means showed by their approach to several types of weighted-mean problems that their understanding was computational rather than conceptual. In a small-scale, thinking-aloud study of remedial students' retention of the ability to solve addition and subtraction problems with integers, Chaiklen and Greeno (1981) identified five principles which explained students' use of verbal rules in these tasks. They thought that students relied on rules when the problems were only partially familiar and that verbal rules must be correctly chosen and then elaborated for successful operation.

The whole area of problem solving is a dominant one in the research on thought processes, as one might expect. Horwitz (1981) investigated the possibility that developmental students' difficulty with word problems might be partly due to limitations of short term memory. She found that such students made fewer errors on problems using small numbers and discrete quantities (presumably easier to visualize) than on structurally identical problems with larger numbers and continuous quantities. However, this effect was not found with more expert problem solvers who presumably could process bigger chunks of the problems' information at one time.

In another study which involved both remembering and constructing word problems, Mayer (1980) found that students were better at remembering and more likely to construct word problems which assigned values to quantities ($1.70 per pound) than ones which used relational statements (12 mph faster). He
related his results to problem difficulty and learning hierarchies. Threadgill-Sowder (1983) explored the possibility that question placement might affect problem difficulty, especially if prequestions made problems with extraneous information easier by focusing attention on relevant data. However, no difference was found. Using the theoretical framework of information processing, Travis (1981) analyzed errors on each of ten typical elementary algebra problems from a written test. Her suggestions for instruction based on this analysis are mainly related to recognizing problem types and using auxiliary pictures, diagrams, and flowcharts.

As is true with most of the research reported in this paper, the studies just described were unrelated to each other. However, there is a body of problem-solving research using postsecondary students and elementary algebra problems in which one study has built on the results of another. The tasks involved in the study were to write equations for ratio-type problems such as "There are six times as many students as professors at this university." Clement, Lochhead, and Monk (1981) found that 37% of their subjects who were engineering students taking calculus could not model that statement correctly; in a sample of nonscience majors taking college algebra the error rate was 57%. Most students making errors wrote $6S = P$ instead of $S = 6P$.

Thinking-aloud procedures suggested that these were not careless errors, but fundamental misconceptions about the meaning of the variables used. This was confirmed by Rosnick (1981) who presented the correct equation and asked what $S$ and $P$ stood for. This type of error persisted in other problems with the same structure including those in which data were presented in a table (Clement, Narode, & Rosnick, 1981). Tutoring strategies to correct such misconceptions were successful only on the surface (Rosnick & Clement, 1980).

Focusing on the reversal error, $6S = P$, Clement (1982) identified two
types of thought processes: some students mechanically matched the words of
the sentence to algebraic sentences; others made a static comparison that was
like a many-to-one correspondence. Clement characterized the thought
processes which generated the correct equation as operative—the subject had
to invent an operation to make the number of professors equal the number of
students. He also demonstrated a way of modeling the thought processes for
this problem evidenced by the thinking-aloud protocols.

In a follow-up study using teacher education majors as subjects, Wollman
(1983) found that the students making the word match error were more easily
led to the correct equation than those making the static comparison error. He
also investigated the effectiveness of cueing questions involving computations
related to the sentences to be translated, comparisons, and explicit checking
questions. These were effective in eradicating errors in translation.

SUMMARY

Looking back over the research reported in this review, one sees a
mixture of strengths and weaknesses. Status studies seem to have samples more
carefully drawn. More attention seems to be paid to placement mechanisms, and
some good instruments have been developed. In the field of program evaluation
some attention has been paid to developing a theoretical framework for evalua-
tion and a comprehensive model for doing it, as well as to the particular
pitfalls of evaluating developmental programs. Some of the current topics in
mathematics instruction have made their way into the research on classroom
management. Studies of student characteristics and thought processes seem to
be more numerous.

On the other hand, there is cause for concern. A number of studies are
superficial or methodologically weak. Most deal only with students or programs
at one institution or in one year or semester. Numbers of students are often small. Most authors appear only once in the reference list and research programs in which one study builds on another are very rare. Very little of the research appears to be externally funded. Although one might suspect that minority populations are heavily enrolled in developmental mathematics courses, one would assume from reading the studies that all such students were colored Caucasian. Research studies on developmental students' characteristics and thought processes often seem to be stepchildren of established programs of research on secondary school students.

There are a lot of probable causes for these weaknesses. As Chang (1983) noted, only 33% of the developmental mathematics faculty had PhDs in mathematics or mathematics education. Heavy teaching loads make finding time for research a problem. Even those faculty who do have the necessary time, research skills, and interests seldom have graduate assistants or colleagues with whom to share research ideas and problems. Those not at research-oriented universities have little access to computing facilities or statistical consultants. All these factors make it difficult to compete for external funding. Because developmental math is essentially high school content taught to college students, it often falls between the cracks in funding agencies.

Student characteristics also militate against research in developmental mathematics. College students are not the captive audience that secondary and elementary school students are. Especially older students with jobs and family responsibilities may not be willing to spend extra time taking tests or filling out questionnaires. Furthermore, the high attrition rate from developmental studies courses makes research over even an entire semester difficult.
Finally, of course, there is the question of academic and governmental priorities. There is still a pronounced tendency to sweep remediation under the collegiate rug, to grudgingly provide courses and services, but not to regard research on developmental students as generalizable or worthwhile.

REFERENCES


