
Lane Community Coll., Eugene, Oreg.

Oregon State Dept. of Education, Salem.

164p.; For other apprenticeship documents related to this trade, see CE 040 962-969. For pre-apprenticeship documents covering math (using many of the same modules), see ED 217 284-294. Many of the modules are duplicated in CE 040 977 and CE 041 006.

Guides - Classroom Use - Materials (For Learner) (051)

This packet of 14 learning modules on trade math is one of 8 such packets developed for apprenticeship training for low voltage alarm. Introductory materials are a complete listing of all available modules and a supplementary reference list. Each module contains some or all of these components: goal, performance indicators, study guide (a check list of steps the student should complete), a vocabulary list, an introduction, information sheets, assignment sheet, job sheet, self-assessment, self-assessment answers, post-assessment, instructor post-assessment answers, and a list of supplementary references. Supplementary reference material may be provided. The 14 training modules cover linear measurement; whole numbers; addition, subtraction, multiplication, and division of common fractions and mixed numbers; compound numbers; percent; mathematical formulas; ratio and proportion; perimeters, areas, and volumes; circumference and area of circles; areas of plane figures and volumes of solid figures; graphs; basic trigonometry; and metrics. (YLB)
APPRENTICESHIP

LOW VOLTAGE ALARM

RELATED TRAINING MODULES

1.1 - 1.14 TRADE MATH
STATEMENT OF ASSURANCE

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STATEMENT OF DEVELOPMENT

This project was developed and produced under a sub-contract for the Oregon Department of Education by Lane Community College, Apprenticeship Division, Eugene, Oregon, 1984. Lane Community College is an affirmative action/equal opportunity institution.
APPRENTICESHIP

LOW VOLTAGE ALARM
RELATED TRAINING MODULES

0.1 History of Alarms

TRADE MATH

1.1 Linear - Measurement
1.2 Whole Numbers
1.3 Addition and Subtraction of Common Fractions and Mixed Numbers
1.4 Multiplication and Division of Common Fractions and Mixed Numbers
1.5 Compound Numbers
1.6 Percent
1.7 Mathematical Formulas
1.8 Ratio and Proportion
1.9 Perimeters, Areas and Volumes
1.10 Circumference and Area of Circles
1.11 Areas of Planes, Figures, and Volumes of Solid Figures
1.12 Graphs
1.13 Basic Trigonometry
1.14 Metrics

ELECTRICITY/ELECTRONICS

2.1 Basics of Energy
2.2 Atomic Theory
2.3 Electrical Conduction
2.4 Basics of Direct Current
2.5 Introduction to Circuits
2.6 Reading Scales
2.7 Using a V.O.M.
2.8 OHM'S Law
2.9 Power and Watt's Law
2.10 Kirchoff's Current Law
2.11 Kirchoff's Voltage Law
2.12 Series Resistive Circuits
2.13 Parallel Resistive Circuits
2.14 Series - Parallel Resistive Circuits
2.15 Switches and Relays
2.16 Basics of Alternating Currents
2.17 Magnetism
3.1 Electrical Symbols
3.2 Circuit Diagrams and Schematics
3.3 Schematics and Alarm Design
4.1 Solid State Power Supply System
4.2 Charging Circuits
4.3 Selecting the Power Size of Power Supply
4.4 Fuse and Circuit Breaker Protection
4.5 Battery Standby Capacity
4.6 Batteries
5.1 Troubleshooting - Electrical Tracing
5.2 Troubleshooting - Environmental Factors
5.3 Documentation of Design
SAFETY

6.1 General Safety
6.2 Hand Tool Safety
6.3 Power Tool Safety
6.4 Fire Safety
6.5 Hygiene Safety
6.6 Safety and Electricity

ALARM BASICS

7.1 Theory of Diodes
7.2 Theory of Bi-polar Devices
7.3 Theory of Integrated Circuits
8.1 Binary Numbering Systems
8.2 Logic Gates
8.3 Dialers
9.1 Blueprint Reading, Building Materials and Symbols
9.2 Design of Alarm Systems
10.1 Types and Applications of Alarm Systems
10.2 Burglar Systems
10.3 Fire Alarms
10.4 Hold-up Alarm Systems
10.5 Bank Alarm Systems
10.6 Wireless Alarm Systems
11.1 Hand and Power Tools
11.2 Maintain Hand and Power Tools
11.3 Safety Practices
12.1 Photoelectric Space Detectors
12.2 Passive Infrared Motion Detectors
12.3 Microwave Detectors (Radar)
12.4 Stress Detectors in Space and Volumetric Applications
12.5 Capacitance Detectors
12.6 Sound Discrimination
12.7 Ultrasonic Motion Detectors
12.8 Gas Detectors
12.9 Airborne and Structural Problems
12.10 Audio Detection Systems
13.1 Trade Terms
14.1 Invisible Beam Detectors
14.2 Fence Disturbance Sensors
14.3 Electric - Field Sensors
14.4 Seismic Sensors
14.5 Car Annunciators
15.1 Annunciators
15.2 Fire Extinguishing Systems
15.3 Signal Reporting Systems
16.1 Detection Devices
16.2 Contacts
16.3 Volumetric and Space Devices
16.4 Problems and Applications of Devices
17.1 Key Stations
17.2 Keyless Control Stations
17.3 Types of Annunciation
17.4 Shunt Switches
18.1 Red Tape Procedures
19.1 Builder Board Requirements
19.2 Licensing
20.1 Central Stations
20.2 Fire Department Monitoring
20.3 Police Department Monitoring
20.4 Telephone Answering Service Monitoring
21.1 Fire/Police/Emergency Responses
21.2 Card Access Control
21.3 Telephone Access Control
22.1 Computerized Controls and Interfaces
22.2 Key Access Control
22.3 Vehicular Access Control
23.1 Telephone Services
24.1 Basic Sound Systems
25.1 Business Letters
26.1 Video Surveillance Systems
26.2 CCTV Cameras
26.3 CCTV Cables
26.4 CCTV Monitors and Recorders
26.5 Time - lapse Video Recorders and Videotape
26.6 CCTV Camera Lens
26.7 CCTV Computer Interface Control
26.8 Video Transmission
26.9 CCTV Enclosures
26.10 CCTV Control Equipment

COMPUTER USAGE

27.1 Digital Language
27.2 Digital Logic
27.3 Computer Overview
27.4 Computer Software

HUMAN RELATIONS

28.1 Communication Skills
28.2 Feedback
28.3 Individual Strengths
28.4 Interpersonal Conflicts
28.5 Group Problem Solving, Goal-setting and Decision-making
28.6 Worksite Visits
28.7 Resumes
28.8 Interviews
28.9 Work Habits and Attitudes
28.10 Expectations
28.11 Wider Influences and Responsibilities
28.12 Personal Finance

DRAWING

29.1 Types of Drawings and Views
29.2 Blueprint Reading/Working Drawings
29.3 Scaling and Dimensioning
29.4 Sketching
29.5 Machine and Welding Symbols
LOW VOLTAGE ALARM

SUPPLEMENTARY REFERENCE MATERIAL

Intrusion Detection Systems: Principles of Operation and Application

Author: Robert L. Barnard
Edition: 1981

Understanding and Servicing Alarm Systems

Author: H. William Trimmer
Edition: 1981

In the event additional copies are needed, they may be purchased through:

Butterworth Publishers
10 Tower Office Park
Woburn, Ma. 01801
RECOMMENDATIONS FOR USING TRAINING MODULES

The following pages list modules and their corresponding numbers for this particular apprenticeship trade. As related training classroom hours vary for different reasons throughout the state, we recommend that the individual apprenticeship committees divide the total packets to fit their individual class schedules.

There are over 130 modules available. Apprentices can complete the whole set by the end of their indentured apprenticeships. Some apprentices may already have knowledge and skills that are covered in particular modules. In those cases, perhaps credit could be granted for those subjects, allowing apprentices to advance to the remaining modules.

We suggest the apprenticeship instructors assign the modules in numerical order to make this learning tool most effective.
Goal:
The apprentice will be able to use the concepts of linear measurement.

Performance Indicators:
1. Read linear measurement to 1/32".
Introduction

MATH
LINEAR MEASUREMENT

Fundamental to any industrial vocation is the measurement of linear or straight line distances. These measurements may be expressed in one of two systems. Apprentices for the most part still use the more familiar British system (of which the yard is the standard unit of length) although the metric is rapidly gaining popularity in the United States. The problems in this module will assume the use of the British system.
This study guide is designed to help you successfully complete this module. Check off the following steps to completion as you finish them.

STEPS TO COMPLETION

1. Familiarize yourself with the Goal and Performance Indicators on the title page of this module.

2. Read the Introduction and study the Information section of the module. It is intended to provide you with the math skills necessary to successfully complete the assessment portions.

3. Complete the Self Assessment section of the module. You may refer to the Information section for help.

4. Compare your Self Assessment answers with the correct answers on the Self Assessment Answer Sheet immediately following the Self Assessment exam. If you missed more than one of the Self Assessment exam questions, go back and re-study the necessary portions of the Information section, or ask your instructor for help. If you missed one or none of these problems, go on to step 5.

5. Complete the Post Assessment section of the module. Show your answers to the instructor. It is recommended that you score 90% or better on those Post Assessment exams with 10 or more problems, or miss no more than one problem on those with fewer than 10 problems, before being allowed to go on the next math module.
TABLE OF LINEAR MEASURE

12 inches = 1 foot
3 feet = 1 yard
5 1/2 yards = 1 rod
40 rods = 1 furlong
8 furlongs = 1 mile

Apprentices have as a basic tool, a steel rule that measures to the nearest one-thirty-second (1/32") of an inch. In most shops a tolerance of 1/32" is allowed in most measurements.

To read measurements, merely calculate where on the rule the mark falls,
Read the distance from the start of the ruler to the letters A through O to the nearest 1/32" and place your answers in the assigned space below.

A =  
B =  
C =  
D =  
E =  
F =  
G =  
H =  
I =  
J =  
K =  
L =  
M =  
N =  
O =  

13
Self Assessment Answers

A. $\frac{6}{8} = \frac{3}{4}$
B. $1 \frac{1}{2}$
C. $2 \frac{1}{8}$
D. $3 \frac{1}{16}$
E. $3 \frac{3}{8}$
F. $3 \frac{3}{4}$
G. $4 \frac{5}{16}$
H. $3 \frac{3}{32}$
I. $17 \frac{1}{32}$
J. $1 \frac{15}{32}$
K. $2 \frac{15}{32}$
L. $2 \frac{31}{32}$
M. $3 \frac{5}{8}$
N. $3 \frac{9}{16}$
O. $4 \frac{1}{32}$
Find the length of each of the following line segments to the nearest 1/32".  
(Always measure from the inside of end mark on the line segments.)

A)  

B)  

C)  

D)  

E)  

A =  

B =  

C =  

D =  

E =  

15
Draw a line segment equal to each of the following lengths to the nearest 1/32". Use the given end mark as the left end mark for the segment.

Example: 3"

A) 3 1/2"
B) 6 1/8"
C) 4 3/32"
D) 1 5/8"
E) 5 5/16"
F) 7/16"
G) 1/2"
H) 3/4"
I) 4/36"
J) 6/8"
Goal:
The apprentice will be able to compute with whole numbers.

Performance Indicators:
1. Add whole numbers.
2. Subtract whole numbers.
3. Multiply whole numbers.
4. Divide whole numbers.
Introduction

If an apprentice in any of today's skilled trades is to achieve his or her goal of becoming a top-flight journeyman, he or she must have a good working knowledge of basic mathematics. Problems involving common and decimal fractions, percent, ratio and proportion, compound numbers, and areas and volumes are regularly encountered in the trades. Because of their importance to the apprentice, these basic concepts are taken up in turn in subsequent modules of this unit. The present module provides a review of the addition, subtraction, multiplication and division of whole numbers—numbers that do not contain fractions and that are not in themselves fractions.
This study guide is designed to help you successfully complete this module. Check off the following steps to completion as you finish them.

**STEPS TO COMPLETION**

1. ___ Familiarize yourself with the Goal and Performance Indicators on the title page of this module.

2. ___ Read the Introduction and study the Information section of the module. It is intended to provide you with the math skills necessary to successfully complete the assessment portions.

3. ___ Complete the Self Assessment section of the module. You may refer to the Information section for help.

4. ___ Compare your Self Assessment answers with the correct answers on the Self Assessment Answer Sheet immediately following the Self Assessment exam. If you missed more than one of the Self Assessment exam questions, go back and re-study the necessary portions of the Information section, or ask your instructor for help. If you missed one or none of these problems, go on to step 5.

5. ___ Complete the Post Assessment section of the module. Show your answers to the instructor. It is recommended that you score 90% or better on those Post Assessment exams with 10 or more problems, or miss no more than one problem on those with fewer than 10 problems, before being allowed to go on to the next math module.
WHOLE NUMBERS
A whole number is any one of the natural numbers such as 1, 2, 5, etc. Numbers represent quantities of anything. They can be added, subtracted, multiplied or divided.

ADDITION
Addition is the process of combining two or more quantities (numbers) to find a total. The total is called the sum. Addition is indicated by the plus (+) sign and may be written as 2 + 2. The sum may be indicated by using the equal (=) sign.
Example: 2 + 2 = 4. Another way of writing the same thing showing the sum of 4 is:

\[
\begin{array}{c}
\phantom{+}2 \\
+ \phantom{+}2 \\
\hline
\phantom{+}4
\end{array}
\]

The following problem is included to refresh your memory of basic addition in trade terms.

ADDITION PROBLEM
Three bricklayers working together on a job each laid the following number of brick in one day. First bricklayer laid 887, second bricklayer laid 1123, and the third bricklayer laid 1053 brick. How many brick did all three lay that day?
Answer: 887 + 1123 + 1053 = 3063 brick

SUBTRACTION
Subtraction is the process of taking something away from the total. The portion which is left after taking some away is called the difference. The sign which indicates that one quantity (number) is to be subtracted from another is the minus (-) sign. Example: 6 - 4. In this example, 4 is being subtracted from 6. The difference is 2 or 6 - 4 = 2. Another way of writing the same thing is:
SUBTRACTION PROBLEM
A mason ordered 75 bags of cement and used 68 bags on the job. How many bags of cement were left?
Answer: $75 - 68 = 7$ bags

MULTIPLICATION
Multiplication is the process of repeated addition using the same numbers. For example, if $2 + 2 + 2 + 2 + 2$ were to be summed, the shortest method would be to multiply $5 \times 2$ to get the total of 10. The sign used to indicated multiplication is the times ($\times$) sign. In the previous example, $5 \times 2$ equals 10, would be written $5 \times 2 = 10$. This may also be written as:

\[
\begin{array}{c}
2 \\
\times 5 \\
10
\end{array}
\]

MULTIPLICATION PROBLEMS
If a bricklayer can lay 170 brick an hour, how many brick would be laid in four hours?
Answer: $170 \times 4 = 680$ brick

One type of brick cost $9 per hundred. If 14,000 brick were ordered, how much would they cost?
Answer: $9 \times 140 = 1260$. Note: The brick were 9¢ each, $9 per hundred or $90 per thousand. Therefore, the answer could have been determined by multiplying $9\times 14,000, \$90 \times 14 \text{ or } \$9 \times 140$.

DIVISION
Division is the process of finding how many times one number is contained within another number. The division symbol is ($\div$). For example, when we wish to find how many times 3 is contained in 9, we say 9 divided by 3 equals 3 or $9 \div 3 = 3$. The answer is called the quotient. If a number is not contained in another number an equal number of times, the amount left over is called the remainder. The following
problem illustrates such a situation: $9 \div 4 = 2$ with 1 left over. For purposes of calculation, the problem is generally written this way:

$$4 \frac{2}{9} \text{ or } 2 \frac{1}{4}$$

DIVISION PROBLEMS

If a set of steps had 8 risers and the total height of all the steps (total rise) was 56 in., what would the height of each step be?

Answer: $\frac{7}{8}$ or 7 in.

If a brick veneer wall requires five brick to lay up 1 sq. ft., how many square feet would 587 cover?

Answer: $\frac{117}{587}$ sq. ft. of wall
Listed below each problem are four possible answers. Decide which of the four is correct, or most nearly correct; then write the letter for that answer in the blank space to the left of the problem.

1. The estimated cost of a roof on a small building was $1,553. The actual cost was $1,395. What was the amount saved?
   a. $146       c. $168
   b. $158       d. $185

2. A contractor buys 637 ft. of eaves trough for a four-family apartment. On completion of the job, he finds he has 48 ft. of the trough left. How many feet of the material has been used?
   a. 569       c. 589
   b. 578       d. 598

3. A contractor buys 400 sacks of rock for three different jobs. On the first job he uses 78 sacks; on the second, 85 sacks; and on the third, 205 sacks. How many sacks are left?
   a. 30       c. 32
   b. 31       d. 33

4. A contractor's bid on a school building is $78,265. When one wing is omitted to cut costs, he is able to cut his bid by $16,228. What is the new figure?
   a. $60,039       c. $62,037
   b. $61,038       d. $63,063

5. If a dealer gets a shipment of 24,000 lbs. of tile, how many tons does he receive?
   a. 12       c. 120
   b. 24       d. 240

6. A roofer works 40 hours at $3.00 per hour and 10 hours at $4.00 per hour. How much does the roofer earn?
   a. $140       c. $160
   b. $150       d. $170
7. If a bundle of rock lath weighs 35 lbs. and it is permissible to place 700 lbs. on any one area on a floor, how many bundles can be placed on any one area?

a. 20
b. 22
c. 24
d. 28

8. If 5 lbs. of putty are required to install one light of glass, how many lights can be installed with 85 lbs?

a. 16
b. 17
c. 18
d. 19
Self Assessment Answers

1. b
2. c
3. c
4. c
5. a
6. c
7. a
8. b
Listed below each problem are four possible answers. Decide which of the four is correct, or most nearly correct; then write the letter for that answer in the blank space to the left of the problem.

1. \[686 + 240 + 1320 + 16 + 400 = \]
   a. 2,452  
   b. 2,653  
   c. 2,662  
   d. 2,762

2. \[16 + 480 + 26 + 15 + 6000 = \]
   a. 6,436  
   b. 6,437  
   c. 6,536  
   d. 6,537

3. \[29 + 15 + 24 + 13 + 10 = \]
   a. 90  
   b. 91  
   c. 92  
   d. 93

4. \[280 - 116 = \]
   a. 154  
   b. 163  
   c. 164  
   d. 174

5. \[40 - 16 = \]
   a. 21  
   b. 22  
   c. 23  
   d. 24

6. \[220 - 38 = \]
   a. 172  
   b. 173  
   c. 181  
   d. 182

7. \[292 \times 16 = \]
   a. 3,573  
   b. 3,772  
   c. 4,672  
   d. 4,772

8. \[460 \times 15 = \]
   a. 5,900  
   b. 6,900  
   c. 7,900  
   d. 8,900
9. $24 \div 6 =$
   a. 2   c. 6
   b. 4   d. 8

10. $180 \div 5 =$
   a. 32   c. 36
   b. 34   d. 38
Goal:

The apprentice will be able to add and subtract common fractions and mixed numbers.

Performance Indicators:

1. Add fractions and mixed numbers.
2. Subtract fractions and mixed numbers.
Introduction

In solving the many kinds of mathematical problems that are encountered in the skilled trades, the mechanic will often find it necessary to work with fractions as well as whole numbers. The Information section for this topic introduces common fractions--fractions in which both the numerator and the denominator are expressed, as in 1/4, 3/8, or 11/32--and includes practice problems in the addition and subtraction of common fractions and mixed numbers (numbers that consist of whole numbers and fractions).
This study guide is designed to help you successfully complete this module. Check off the following steps to completion as you finish them.

STEPS TO COMPLETION

1. ___ Familiarize yourself with the Goal and Performance Indicators on the title page of this module.

2. ___ Read the Introduction and study the Information section of the module. It is intended to provide you with the math skills necessary to successfully complete the assessment portions.

3. ___ Complete the Self Assessment section of this module. You may refer to the Information section for help.

4. ___ Compare your Self Assessment answers with the correct answers on the Self Assessment Answer Sheet immediately following the Self Assessment exam. If you missed more than one of the Self Assessment exam questions, go back and re-study the necessary portions of the Information section, or ask your instructor for help. If you missed one or none of these problems, go on to step 5.

5. ___ Complete the Post Assessment section of the module. Show your answers to the instructor. It is recommended that you score 90% or better on those Post Assessment exams with 10 or more problems, or miss no more than one problem on those with fewer than 10 problems, before being allowed to go on the next math module.
FRACTIONS

A fraction is one or more parts of a whole. Fractions are written with one number over the other (1/2 or 1/4 or 3/4).

The top number is called the NUMERATOR and the bottom number is called the DENOMINATOR. The denominator identifies the number of parts into which the whole is divided. The numerator indicates the number of parts of the whole which is of concern. In reading a fraction, the top number is always read first. For example, 1/2 would be read "one half"; and 3/4 would be read "three fourths" and 3/8 would be read "three eighths."

A fraction should always be reduced to its lowest denominator. For instance, 3/2 is not in correct form. It should be 1 1/2 because 2/2 = 1 and 1 + 1/2 = 1 1/2. The 1 1/2 is called a MIXED NUMBER. Always when the numerator and denominator are the same number as 1/1, 2/2, 3/3, etc. they are equal to 1.

ADDING FRACTIONS

The easiest fractions to add are those whose denominators (bottom numbers) are the same, as 1/8 + 3/8. Simply add the numerators (top numbers) together and keep the same denominator. For example, 1/8 + 3/8 = 4/8 or 1/2. (Reducing the fraction to its lowest denominator is preferred.) Another example of reducing to the lowest denominator is 8/24 = 1/3, because 24 may be divided by 8 three times.

When fractions to be added have different denominators (bottom numbers), multiply both numerator and denominator of each fraction by a number that will make the denominators equal. For example: 1/3 + 3/5 = 5/15 + 9/15. Observation indicated that 15 was the smallest number that could be divided evenly by both denominators. To complete the example, 5/15 + 9/15 = 14/15. Therefore, the sum of 1/3 and 3/5 is 14/15.
PROBLEMS IN ADDING FRACTIONS
What is the height of one stretcher course of brick if the brick are 2 \( \frac{3}{4} \) in. high and the mortar joint is \( \frac{3}{8} \) in?

Answers: \( 2 \frac{1}{4} + \frac{3}{8} = 2 \frac{2}{8} + \frac{3}{8} = 2 \frac{5}{8} \) in. height for one course

A mason estimated the following amounts of mortar required for a job: 5 \( \frac{1}{2} \) cu. yd., 11 \( \frac{1}{3} \) cu. yd. and 6 \( \frac{1}{4} \) cu. yd. What is total amount of mortar required for job?

Answer: \( 5 \frac{1}{2} + 11 \frac{1}{2} + 6 \frac{1}{4} \)

\[
= 5 \frac{6}{12} + 11 \frac{4}{12} + 6 \frac{3}{12}
\]

\[= 22 \frac{13}{12} = 23 \frac{1}{12} \] cu. yd. of mortar

SUBTRACTING FRACTIONS
Change all fractions to the same common denominator as was done for adding fractions. When the denominators are the same, subtract the numerators.
INDIVIDUALIZED LEARNING SYSTEMS

Self Assessment

Note: The value of a fraction is not changed when both the numerator and denominator are multiplied or divided by the same number.

Reduce to halves. (A denominator of 2)

\[
\frac{4}{8} = \quad \frac{8}{16} = \quad \frac{16}{8} =
\]

Reduce to 8ths.

\[
\frac{4}{16} = \quad \frac{16}{32} = \quad \frac{32}{64} =
\]

Note: Divide the numerator and denominator by the same number. When both the numerator and the denominator cannot be divided further by the same number, the fraction is expressed in its lowest terms.

Reduce to lowest terms:

\[
\frac{4}{16} = \quad \frac{14}{16} = \quad \frac{28}{64} = \quad \frac{16}{32} = \quad \frac{12}{16} = \quad \frac{24}{12} =
\]

Note: To reduce an improper fraction (where the numerator is larger than the denominator) to its lowest terms, divide the numerator (above the line) by the denominator (below the line).

Reduce the resulting fraction to its lowest terms.

\[
\frac{5}{2} = \quad \frac{10}{3} = \quad \frac{10}{5} =
\]

Note: To change a mixed fraction to an improper fraction, multiply the denominator by the whole number and add the numerator. Place the result over the denominator.
Change to improper fractions:

1 \( \frac{3}{4} = \) \_ \_ \_ \_ \_ 8 \( \frac{7}{8} = \) \_ \_ \_ \_ \_ 3 \( \frac{1}{4} = \) \_ \_ \_ \_ \_ 10 \( \frac{2}{3} = \) \_ \_ \_ \_ \_

How many eights of an inch are there in each of the following lengths of steel?

1 \( \frac{3}{8}" = \) \_ \_ \_ \_ \_ 4 \( \frac{3}{8}" = \) \_ \_ \_ \_ \_ 7 \( \frac{3}{8}" = \) \_ \_ \_ \_ \_

Note: The smallest number that can be divided by all the denominators is called the LOWEST COMMON DENOMINATOR.
To reduce fractions to the lowest common denominator, divide the number selected as the lowest common denominator by the denominator of each given fraction.
Multiply both the numerator and denominator by this quotient.

Note: To add fractions, change to fractions having a least common denominator.
Add the numerators. Write the sum over the common denominator. Reduce the result to its lowest terms.

Addition of common fractions:

\( \frac{1}{6} + \frac{5}{6} = \) \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_
\( \frac{1}{3} + \frac{1}{16} = \) \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_
\( \frac{5}{8} + \frac{3}{4} + \frac{3}{8} = \) \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_

Addition of common fractions and mixed numbers:

\( 1 + 7 \frac{5}{12} = \) \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_
\( 1 \frac{17}{64} + 1 \frac{13}{64} + \frac{9}{32} = \) \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_ \_

Note: To subtract a fraction from a whole number, take one unit from the whole number and change it into a fraction having the same denominator as the fraction which is to be subtracted. Subtract the numerators of the original fraction from the one unit that was changed to its fractional value. Reduce the resulting fraction to its lowest terms. Place the whole number next to the fraction.

\[ \frac{4}{3/4} - \frac{7}{15/16} \]

Note: To subtract a mixed number from a whole number, borrow one unit from the whole number and change it to a fraction which has the same denominator as the mixed number. Subtract the fraction part of the mixed number from the fraction part of the whole number. Subtract the whole numbers and reduce
the resulting mixed number to lowest terms.

\[
\begin{array}{ccc}
2 & 3 & 27 \\
-1 1/3 & -1 3/8 & -1 5/16 \\
\end{array}
\]

**Note:** To subtract two mixed numbers, change the fractional part of each mixed number to the least common denominator. Borrow one unit, when necessary, to make up a larger fraction than the one being subtracted. Subtract the fractions first, the whole numbers next, and reduce the result to lowest terms.

**Note:** To add and subtract fractions in the same problem, change all fractions to the least common denominator. Add or subtract the numerators as required. Reduce the result to lowest terms.

\[
\begin{array}{ccc}
1 3/5 & 7 5/6 & 18 7/8 \\
-1 1/5 & -2 1/6 & -9 3/8 \\
\end{array}
\]
• Self Assessment Answers

Reduce to halves: 1/2 1/2 4/2

Reduce to 8ths: -2/8 4/8 4/8

Reduce to lowest terms: 1/4 7/8 7/16 1/2 3/4 2/1

Reduce the resulting fraction to its lowest terms: 2 1/2 3 1/3 2

Change to improper fractions: 7/4 71/8 13/4 32/3

How many eights of an inch are there in each of the following lengths of steel:
11 35 59

Addition of common fractions and mixed numbers: 128 5/12 2 48/64
Listed below each problem are four possible answers. Decide which of the four is correct, or most nearly correct; then write the letter of that answer in the blank space to the left of the problem.

1. ___ The improper fraction 48/32 expressed as a mixed number is:
   a. 1 15/32   c. 1 5/8
   b. 1 1/2      d. 2 1/32

2. ___ The mixed number 4 3/16 expressed as an improper fraction is:
   a. 16/8      c. 67/16
   b. 43/16     d. 35/8

3. ___ What is the least common denominator for the following group of fractions:
    1/8, 1/2, 1/4 and 1/12
   a. 12       c. 24
   b. 18       d. 48

4. ___ What is the sum of the following fractions: 1/2, 1/3, 1/8 and 1/12?
   a. 1 3/12   c. 1 1/24
   b. 1 7/12   d. 1 1/48

5. ___ If 1/2 is subtracted from 7/8, the difference is:
   a. 3/8  c. 1 1/8
   b. 5/8  d. 1 3/8

6. ___ The sum of 1 1/2, 5/6, 14, and 20 2/3 is:
   a. 36 2/3   c. 37
   b. 36 17/18   d. 37 2/9

7. ___ One roof is 1/3 larger in area than another. The smaller roof takes 24 squares of roofing material. How many squares of roofing material will the larger roof take?
   a. 32  c. 36
   b. 34  d. 37
8. One-third of a box of glass is needed to glaze the north elevation of a building; 2/3 of a box is needed to glaze the south elevation; 1/6 of a box is needed to glaze the east elevation; and 1/2 of a box is needed to glaze the west elevation. How many boxes are needed to glaze all four elevations?
   a. 1 1/6  c. 1 1/2
   b. 1 1/3  d. 1 2/3

9. From a bundle containing 101 linear feet of molding, a cabinetmaker uses the following amounts: 11 1/2', 8 3/4', 12 1/8' and 9 5/8'. How many linear feet of molding does he use in all?
   a. 38 1/2  c. 39 3/4
   b. 39 1/4  d. 41 5/6

10. How many linear feet of molding remain in the bundle in problem 9?
    a. 59 1/6  c. 61 3/4
    b. 61 1/4  d. 62 1/2

11. From a roll of hanger wire weighing 100 lbs., a lather uses the following amounts: 6 lbs., 18 1/2 lbs., 9 1/8 lbs., and 22 1/4 lbs. How many pounds of the wire does he use in all?
    a. 54 1/4  c. 55 1/4
    b. 54 3/4  d. 55 7/8
Goal:
The apprentice will be able to multiply and divide common fractions and whole and mixed numbers.

Performance Indicators:

1. Multiply fractions.
2. Divide fractions.
3. Multiply and divide problems that contains both fractions and whole and mixed numbers.
The previous module reviewed the rules and procedures for some fundamental operations with common fractions: reduction of fractions, finding the lowest common denominator, and adding and subtracting fractions and mixed numbers. The study assignment for the present module concludes the review of common fractions, covering the rules and procedures for multiplying and dividing common fractions and common fractions in combination with whole numbers and mixed numbers.
This study guide is designed to help you successfully complete this module. Check off the following steps to completion as you finish them.

**STEPS TO COMPLETION**

1. ___ Familiarize yourself with the Goal and Performance Indicators on the title page of this module.

2. ___ Read the Introduction and study the Information section of this module. It is intended to provide you with the math skills necessary to successfully complete the assessment portions.

3. ___ Complete the Self Assessment section of the module. You may refer to the Information section for help.

4. ___ Compare the Self Assessment answers with the correct answers on the Self Assessment Answer Sheet immediately following the Self Assessment exam. If you missed more than one of the Self Assessment exam questions, go back and re-study the necessary portions of the Information section, or ask your instructor for help. If you missed one or none of these problems, go on to step 5.

5. ___ Complete the Post Assessment section of the module. Show your answers to the instructor. It is recommended that you score 90% or better on those Post Assessment exams with 10 or more problems, or miss no more than one problem on those with fewer than 10 problems, before being allowed to go on the next math module.
MULTIPLYING FRACTIONS
The procedure for multiplying fractions is to multiply the numerators together to find the numerator for the answer. Then, multiply the denominators together to find the denominator for the answer. The answer is called a PRODUCT and the fraction is reduced to its lowest form. Example: $4 \times \frac{5}{8} = \frac{4}{1} \times \frac{5}{8} = \frac{20}{8} = \frac{2\ 4}{8} = 2\ 1/2$.

PROBLEMS IN MULTIPLYING FRACTIONS
If standard bricks are used which are 2 1/4 in. thick to lay a wall with 3/8 in. mortar joints, what will the height of the wall be after nine courses?
Answer: First, add the thickness of one mortar joint to the thickness of one brick (2 1/4" + 3/8" = 2 5/8"). Then multiply 2 5/8" times 9 to find the height. 2 5/8" x 9 = 21/8 x 9/1 = 189/8 = 23 5/8 in.

If a set of steps are five risers high and each riser is 7 1/4 in., what is the total rise of the steps?
Answer: 7 1/4 x 5/1 = 29/4 x 5/1 = 145/4 = 36 1/4 in.

What is the length of a 28 stretcher wall if each stretcher is 7 1/2 in. and the mortar joint is 1/2 in.?
Answer: 7 1/2" = 1/2" = 8"; 8" x 28" = 224"; 224" ÷ 12 = 18 2/3' (2/3 x 121 = 24/3 = 8 in.) Therefore the length is 18'8".

DIVIDING FRACTIONS
The process of dividing fractions is accomplished by inverting (turning up side down) the divisor and then multiplying. For example, $\frac{3}{8} ÷ \frac{3}{4}$ is solved by changing the $\frac{3}{4}$ to $\frac{4}{3}$. Therefore, $\frac{3}{8} ÷ \frac{3}{4} = \frac{3}{8} x \frac{4}{3} = 12/24 = 1/2$.
PROBLEMS IN DIVIDING FRACTIONS

How many risers 7 1/2 in. high would be required to construct a flight of concrete steps 3' 1 1/2" high?
Answer: Change 3' 1 1/2" to 37 1/2"; Divide 37 1/2" by 7 1/2; 75/2 ÷ 15/2 = 75/2 x 2/15 = 150/30 = 5 risers

If a brick mantel is corbeled out 4 1/2 in. in six courses, how much does each course project past the previous course?
Answer: 4 1/2 ÷ 6/1 = 9/2 x 1/6 = 9/12 = 3/4 in.

If a story pole was 8' 11 1/2" long and divided into 39 equal spaces, what is the length of each space?
Answer: 8' 11 1/4" ÷ 39 = 107 1/4' ÷ 39/1 = 429/4 x 1/39 = 429/156 = 2 117/156 = 2 3/4 in.
How many pieces of 10' 5/16" flat bar may be cut from a 12-foot piece of stock if you allow 3/16" for the kerf?

How many pieces of stock 7/8" long can be cut from a 30" bar of drill rod if 1/16" is allowed on each piece for kerf?

Determine center distance A.

A = ________
Self Assessment

Answers

13 pieces of flat bar

32 pieces of stock

\[ A = 1.25/64 \]
Listed below each problem are four possible answers. Decide which of the four is correct, or most nearly correct; then write the letter for that answer in the blank space to the left of the problem.

1. ___ The product of $\frac{1}{2} \times \frac{7}{8}$ is:
   a. $\frac{1}{8}$
   b. $\frac{5}{16}$
   c. $\frac{7}{16}$
   d. $\frac{11}{8}$

2. ___ The product of $\frac{3}{4} \times \frac{2}{3}$ is:
   a. $\frac{5}{12}$
   b. $\frac{1}{2}$
   c. $\frac{5}{7}$
   d. $\frac{8}{9}$

3. ___ The quotient of $\frac{1}{2} \div \frac{1}{4}$ is:
   a. $\frac{1}{8}$
   b. $\frac{3}{4}$
   c. 1
   d. 2

4. ___ The quotient of $\frac{1}{4} \div \frac{1}{2}$ is:
   a. $\frac{1}{2}$
   b. $\frac{4}{6}$
   c. $\frac{5}{6}$
   d. $\frac{13}{18}$

5. ___ The quotient of $\frac{1}{4} \div \frac{1}{3}$ is:
   a. $\frac{1}{9}$
   b. $\frac{1}{6}$
   c. $\frac{3}{4}$
   d. $\frac{11}{3}$

6. ___ If a roll of carpet weighs 467 1/2 lbs. and a running foot of the carpet weighs 2 1/8 lbs., how many running feet are in the roll?
   a. 200
   b. 220
   c. 375
   d. 935

7. ___ A type of linoleum weighs 1 5/6 lbs. per running foot. How many pounds does a roll containing 59 2/3 running feet weigh?
   a. 103 1/6
   b. 109 2/3
   c. 109 7/8
   d. 116 7/18

8. ___ A piece of pipe must be cut to 3/8 the length of another pipe, which is 9' long. How long a piece must be cut?
   a. 3 1/4'
   b. 3 3/8'
   c. 4 1/4'
   d. 4 3/8'.
9. What is the height of the second floor above the first if the stairway connecting the floors has 16 risers and each riser is 7 1/4" high?
   a. 8'10"
   b. 9'0"
   c. 9'6"
   d. 9'8"

10. A truck rated at 1 1/2 tons is to be used to pick up surplus gravel at five local job sites and return it to the yard. The amount of surplus gravel at each site is as follows: Job A, 3/4 ton; Job B, 3/8 ton; Job C, 1 7/8 tons; Job D, 1 1/2 tons, and Job E, 2 5/8 tons. How many trips to the yard must the truck make to return all the gravel?
   a. 3
   b. 4
   c. 5
   d. 6
Goal:
The apprentice will be able to compute compound numbers.

Performance Indicators:

1. Reduce compound numbers.
2. Add compound numbers.
3. Subtract compound numbers.
4. Multiply compound numbers by whole numbers.
5. Divide compound numbers by whole numbers.
6. Add and subtract compound mixed numbers.
Workers in the skilled trades frequently must solve problems involving the addition, subtraction, multiplication, and division of compound numbers, which are expressions containing two or more unlike but related units of measure, such as 6 ft. 2 in. or 4 lb. 3 oz. Each of the two or more parts of a compound number is called a denominate number. In the examples given above, 6 ft., 2 in., 4 lb., and 3 oz. are all denominate numbers.
This study guide is designed to help you successfully complete this module. Check off the following steps to completion as you finish them.

STEPS TO COMPLETION

1. ___ Familiarize yourself with the Goal and Performance Indicators on the title page of this module.

2. ___ Read the Introduction and study the Information section of the module. It is intended to provide you with the math skills necessary to successfully complete the assessment portions.

3. ___ Complete the Self Assessment section of the module. You may refer to the Information section for help.

4. ___ Compare your Self Assessment answers with the correct answers on the Self Assessment Answer Sheet immediately following the Self Assessment exam. If you missed more than one of the Self Assessment exam questions, go back and re-study the necessary portions of the Information section, or ask your instructor for help. If you missed one or none of these problems, go on to step 5.

5. ___ Complete the Post Assessment section of the module. Show your answers to the instructor. It is recommended that you score 90% or better on those Post Assessment exams with 10 or more problems, or miss no more than one problem on those with fewer than 10 problems, before being allowed to go on to the next math module.
REDUCTION OF COMPOUND NUMBERS

The principles of adding, subtracting, multiplying, and dividing compound numbers are outlined in the illustrative problems presented in this topic. Each problem is accompanied by its step-by-step solution. The units of measure chosen for the problems are feet and inches, but the principles demonstrated apply equally to compound numbers involving pounds and ounces, hours and minutes, and the like. Except in the case of the simplest addition and subtraction problems, the reduction (changing) of related but unlike units is an essential step in working with compound numbers. This is so because only like units can be combined in an arithmetical operation. After this reduction has been accomplished, operations involving compound numbers can be performed in the conventional way.

Reduction from higher to lower denomination units

Problem: Reduce 13 feet to inches
Step 1. 1' = 12"
Step 2. 13 x 12 = 156"

Reduction from lower to higher denomination units

Problem: Reduce 216 inches to feet
Step 1. 12" = 1'
Step 2. 216" ÷ 12 = 18'

ADDITION OF COMPOUND NUMBERS

Problem: Add 2'7" and 8'10"
Step 1. Add the inch column.
7" + 10" = 17"
Step 2. Reduce the inches to feet and inches
17" = 1'5"
Write the 5" in the sum, and carry the remaining 1' to the foot column
**SUBTRACTION OF COMPOUND NUMBERS**

Problem: Subtract 3'4" from 9'2"

- **Step 1.** Since 4" cannot be subtracted from 2", borrow 12" from the 9' and add to the 2", thus changing 9'2" to 8'14"

- **Step 2.** Subtract both columns
  - 14" - 4" = 10"
  - 8' - 3' = 5'

**MULTIPLICATION OF COMPOUND NUMBERS BY WHOLE NUMBERS**

Problem: Multiply 3'7" by 8

- **Step 1.** Multiply the inches by 8.
  - 7" x 8 = 56"

- **Step 2.** Reduce the product to feet.
  - 56" = 4'8"

- **Step 3.** Multiply the number of feet in the multiplicand by 8

- **Step 4.** Add the results of Steps 2 and 3.
  - 4'8" + 24'8" = 28'8"

**DIVISION OF COMPOUND NUMBERS BY WHOLE NUMBERS**

Problem: Divide 31'3" by 15.

- **Step 1.** Reduce the feet to inches. 31" = 372"

- **Step 2.** Add the total number of inches. 3" + 372" = 375"

- **Step 3.** Divide the sum by 15. 375" ÷ 15 = 25'

- **Step 4.** Reduce the quotient to feet. 25" = 2'1"

**ADDITION AND SUBTRACTION OF COMPOUND MIXED NUMBERS**

If the lowest-denomination units in an addition or subtraction problem involving compound numbers are expressed in fractions, we must first reduce the fractions to the lowest common denominator before proceeding with the calculation. The following addition problem illustrates this point.

Problem: Add 12'8-1/2", 17'4-3/8", 5'5-1/4", and 2'10-5/8".

- **Step 1.** Reduce the fractions to terms of the lowest common denominator
  - LCD = 8
  - 1/2 = 4/8
  - 1/4 = 2/8

- **Step 2.** Add the fraction column and reduce the sum to inches.
  - 4/8" + 3/8" + 2/8" + 5/8" = 14/8"
  - 14/8" = 1-6/8" = 1-3/4". Write the fraction 12'8-4/8"
  - 3/4" in the sum and carry the 1" to the inch column.

  - 12'8-4/8"
  - 17'4-3/8"
  - 5'5-2/8"
  - 2'10-5/8"

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53
Step 3. Add the inch column and reduce the sum to feet and inches. 

\[ 1" + 8" + 4" + 5" + 10" = 28". \]

Write the 4" in the sum and carry the 2' to the foot column.

\[ 17' 4' -3/8" + 10" = 28". \]

Step 4. Add the foot column.

\[ 2' + 12' + 17' + 5' + 2' = 38'. \]

**MULTIPLICATION OF COMPOUND NUMBERS BY COMPOUND NUMBERS**

To find an area for which both the length and width are expressed in compound numbers, one can multiply the compound numbers, but this can be time consuming, especially if fractions are involved. It is often sufficiently accurate to reduce the compound numbers to the nearest mixed denominate numbers to simplify multiplication. For example, to multiply 2'6" by 8' 3-3/4" to find the area of a panel, change the 7' to 1/2' and 3-3/4" to 1/3'; then multiply 2-1/2' by 8-1/3'. In fact, for estimating purposes it would probably be sufficiently accurate to multiply 2-1/2' by 8-1/2'. If a more accurate answer is essential, reduce both compound numbers to feet and twelfths of a foot, then multiply the resulting denominate numbers; or reduce both compound numbers to inches, then multiply. The result will be square feet or square inches, depending upon the method used. (Remember that a square foot contains 144 square inches.)

**DIVISION OF COMPOUND NUMBERS BY COMPOUND NUMBERS**

Occasionally the need arises to divide one compound number by another compound number, for example to find out how many times one shorter length is included in another longer length, as in the problem that follows:

**Problem:** Divide 12'8" by 3'2".

Step 1. Reduce the feet to inches in each compound number. 12' = 144"; 3' = 36".

Step 2. Add the inches in each reduced compound number. 144" + 8" = 152"; 36" + 2" = 38".

Step 3. Divide the resulting denominate number. 152" ÷ 38" = 4. 4 x 3'2" = 12'8".

**Note:** Any remainder in such a problem will be in inches. For example, if the divisor in the above problem were 3'6" instead of 3'2", the answer would be 3 plus a remainder of 26".
Write the answer to each problem in the corresponding space at the right.

1. Change 372" to feet. 
   ____________

2. Change 16'8" to inches. 
   ____________

3. Add 4'8", 17'3", 11'5", 44'2", and 32'10". 
   ____________

4. Subtract 23'8" from 57'2". 
   ____________

5. Subtract 28'11" from 32'10". 
   ____________

6. Multiply 3'8" by 9. 
   ____________

7. Multiply 22'4" by 37'11". 
   ____________

8. Divide 11'6" by 3. 
   ____________

9. Divide 19'2" by 3'10". 
   ____________

10. Add 7 hr. 18 min. and 3 hr. 47 min. 
    ____________
• Self Assessment Answers

1. 31'
2. 200"  
3. 110'4"
4. 33'6"
5. 3'11"
6. 33'
7. approximately 5.9 sq. ft.
8. 3'10"
9. approximately 4'9"
10. 11 hrs. 5 min.
Listed below each problem are four possible answers. Decide which of the four is correct, or most nearly correct; then write the letter of that answer in the space to the left of the problem.

1. \[9'6" + 3'6" =\]
   a. 13'0"
   b. 13'6"
   c. 14'0"
   d. 14'6"

2. \[6'3" + 6'8" + 5'1" =\]
   a. 17'0"
   b. 17'6"
   c. 17'9"
   d. 18'0"

3. If the height of a ceiling above the floor is 9'6" and the distance from the floor to the top of the window casing is 6'11", what is the distance from the top of the casing to the ceiling?
   a. 2'6"
   b. 2'7"
   c. 2'9"
   d. 2'11"

4. Three identical metal frames are needed to complete a glazing job. The following pieces of metal extrusion are required to make these frames: 8 pieces 10'7" long; 9 pieces 8'4" long; and 3 pieces 3'9" long. How many inches of the metal will be required for each frame?
   a. 572
   b. 614
   c. 681
   d. 724

5. How many 16" lengths of hanger wire can be cut from a roll containing 97'4" of the wire?
   a. 73
   b. 75
   c. 77
   d. 80

6. Four boards, each 12'9" in length, are laid end to end. What is their total length?
   a. 42'6"
   b. 45'0"
   c. 49'3"
   d. 51'0"
7. The following pieces of material are cut from a stock of 10 pieces, each 21' long: 2 pieces 4' long; 3 pieces 6 1/3' long; and 4 pieces 54" long. How many feet of the material remain in stock?

   a. 164  c. 166
   b. 165  d. 167

8. Metal trim for a job was purchased from two different suppliers. Company A supplied the following: 4 pieces 5'11" long; 9 pieces 12'2" long; and 18 pieces 6'9" long. Company B supplied the following: 19 pieces 1'3" long; 18 pieces 9'4" long; 2 pieces 1'10" long; 10 pieces 5'5" long; and 4 pieces 1'3" long. How much more trim was supplied by Company A than by Company B?

   a. 1"  c. 10"
   b. 2"  d. 20"

9. A glass shop receives an order to replace the tops on 6 showcases. Each of these showcases requires a new piece of green felt 2" wide and 6'3" long under the rear edge of the glass. How many square inches of green felt will be needed to do the entire job?

   a. 850  c. 950
   b. 900  d. 1,000

10. What is the total length in feet and inches of the following pieces of flashing: 2 pieces 18" long; 10 pieces 78" long; 1 piece 29" long; and 6 pieces 10" long?

   a. 69'9"  c. 84'7"
   b. 75'5"  d. 88'3"

11. In making a batch of mortar, a workman used lime an an amount equal to 12 percent of the cement. How many pounds of lime are necessary if 995 lbs. of cement are used?

   a. 119.4  c. 123.5
   b. 121.8  d. 130.2
Goal:
The apprentice will be able to compute percentage problems.

Performance Indicators:

1. Change percent to decimal.
2. Change decimal to percent.
3. Change fractions to decimals.
4. Compute problems with percent.
The word "percent," an abbreviation of the Latin "per centum," literally means "for each hundred" or "by the hundred." "Percentage" means the methods of expressing a part of a whole as hundredths of the whole. Thus, 12 percent means 12 parts of a whole that is thought of as consisting of 100 such parts; 100 percent means all 100 parts of the whole taken together; and 108 percent means all 100 parts of the whole plus 8 more such parts.

Since percents are expressions of the parts of a whole, they can be converted to common fractions or decimals: 12 percent is equivalent to 12/100 or 0.12; 100 percent is equivalent to 100/100 or 1.0; and 108 percent is equivalent to 108/100 (1-8/100) or 1.08. It can be seen that percents greater than 100 become mixed numbers in such conversions.

Skill in working with percents is necessary for estimating costs, discounts, and profit margins, and it is very useful in calculating proportions, for example in determining the relative amounts of materials needed for fluid mixture of a given composition.
This study guide is designed to help you successfully complete this module. Check off the following steps to completion as you finish them.

STEPS TO COMPLETION

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To change a percent to a decimal, remove the % sign, then place a decimal point two digits to the left of the number for the given percent. (If the percent is a mixed number, change the fraction to a decimal and place this value after the whole number.)

Use the decimal value for the given percent the same as any other decimals to perform the required mathematical operations.

To change a decimal to a percent, move the decimal point two digits to the right. Place the percent sign after this number.
The bill for a certain job is $332.20. If the customer wishes to pay 15% on the original bill, what should she pay?

During the first four days of a work week, the total daily output reached 276, 320, 342, and 286 welds of a certain type. The rejects each day of these totals were 5%, 4.5%, 6% and 5%, respectively. The weekly quota to meet a contract is 325 perfect welds per day. How many welds must be produced the fifth day to meet the schedule? (Assume that the rejects on the fifth day is the average percent of the other four days.)

Write the letter of the correct, or most nearly correct, answer in the blank at the left of each problem.

---

The fraction $\frac{9}{16}$ is equivalent to what percent?

a. 9.16  
   b. 56  
   c. 56 1/4  
   d. 565

The fraction $\frac{3}{32}$ is equivalent to what percent?

a. 31/32  
   b. 30/32  
   c. 9 3/8  
   d. 93 1/8

The fraction $\frac{9}{32}$ is equivalent to what percent?

a. 28  
   b. 28 1/32  
   c. 28 3/32  
   d. 28 1/8

---
Self Assessment Answers

She should pay 332.20 x .15 or $49.83

490 welds

9/16 = c

3/32 = c

9/32 = d
Listed below each problem are four possible answers. Decide which of the four is correct, or most nearly correct; then write the letter for that answer in the space to the left of the problem.

1. ___ Expressed as a fraction in lowest terms, 43 3/4 percent is:
   a. 7/32  
   b. 7/16  
   c. 43/40  
   d. 46/4

2. ___ Expressed as a fraction in lowest terms, 62 1/2 percent is:
   a. 5/8  
   b. 6/8  
   c. 62/80  
   d. 62 1/2

3. ___ Expressed as a fraction in lowest terms, 83 1/3 percent is:
   a. 5/6  
   b. 10/16  
   c. 8/9  
   d. 1 4/3

4. ___ A certain type of glass is composed of 63 percent silica sand, 23 percent soda ash, and 14 percent lime. The total batch of glass weighs 1,600 lbs. How many pounds of soda ash are in the batch?
   a. 224  
   b. 368  
   c. 472  
   d. 592

5. ___ Two glaziers install 2,100 lights of glass, but 84 lights turned down by the inspectors have to be repurified. What percent of the job has to be done over?
   a. 2  
   b. 4  
   c. 20  
   d. 40

6. ___ The finished width of a certain shiplap sheathing board is 1 5/8". What is this width in decimal form?
   a. 1.525"  
   b. 1.575"  
   c. 1.580"  
   d. 1.625"

7. ___ A roof has an area requiring 476 running feet of a certain kind of insulating material. If 28 percent is to be added for cutting and waste, how many running feet of the material should be ordered, to the nearest foot?
8. A tilesetter purchases a table saw at $475 less separate discounts of 15 percent and 3 percent. What is his actual cost?
   a. $389.65   c. $392.74
   b. $391.64   d. $394.46

9. A portable electric circular saw has a speed of 4,000 rpm under full load. Under no-load conditions, the saw's speed increases 15 percent. What is the no-load speed?
   a. 4,250 rpm   c. 4,550 rpm
   b. 4,400 rpm   d. 4,600 rpm

10. The total cost of a new building is $35,450. If the cost of the roof is 2 percent of this total amount and if the roofing materials represent 27 percent of the cost of the roof, what is the cost of the roofing materials?
    a. $152.97    c. $175.21
    b. $167.50    d. $191.43
Goal:

The apprentice will be able to use formulas in electrical and electronic calculations.

Performance Indicators:

1. Describe signs of operation for addition, subtraction, multiplication, division and equality.
2. Convert rules into formulas.
3. Describe order of operation for solving a formula problem.
4. Describe use of signed numbers in formula.
5. Calculate formulas with exponents.
6. Calculate formulas that use powers of ten.
7. Describe rules for working with equations.
Study Guide

- Read the goal and performance indicators for this package to determine what you are expected to learn from the package.
- Study the examples and rules for the use of equations and formulas in the information sheets.
- Complete the problems on the assignment sheet (see reference) to get practice in using formulas.
- Complete self assessment and check answers.
- Complete post assessment and have instructor score the answers.
Vocabulary

- Equation
- Exponents
- Formula
- Negative signs
- Order of operations
- Positive signs
- Powers of ten
- Rules
- Signs of operation
- Substitution
Introduction

Formulas are a form of shorthand that are used in making calculations in electrical work. Those working with electricity and electronics must know the basic formulas and how to transpose information into a formula. They must also be able to correctly calculate the figures that have been transposed into the basic formulas.

This package reviews some basic rules of mathematics as they apply to the use of formulas.
Many formulas are used in making calculations in electricity and electronics. A technician must know how to work with formulas on an everyday basis. Formulas are a method of shorthand used to express rules.

**SIGNS OF OPERATION**

All formulas are held together by one or more of the following:

- Add (Sum of)
- Subtract (Difference between)
- Multiply (Product of)
- Divide (Quotient of)
- Equality (Equal to)

Some of these operations may be written in different ways:

* Multiplication may be shown as $3 \times 2$ or with a dot between numbers $3 \cdot 2$ or, if it is a number and a letter, nothing between the number and letter, such as $3X$.

* Division may be shown as $3 \div 2$ or separated by fraction bar $3/2$ or 

**CONVERTING RULES TO FORMULAS**

In order to change rules into formulas, replace each quantity with a letter. Letters are substituted for words in the rule. Signs of operation are placed between letters.

**EXAMPLE:** The current is equal to the voltage divided by the resistance.

Replace underlined words with letters:

Current = voltage : resistance

$I = \frac{E}{R}$

and add the signs of operation.
ORDER OF OPERATIONS

Numbers can be substituted into formulas in place of letters. To work with formulas, an order of operations should be followed:

1) Substitute numbers for letters
   \[3X + 4(X+2) = 12\]

2) Find value of all expressions in parentheses
   \[+4X + 8 = \]

3) Do all multiplications from left to right
   \[3X + 4X + 8 = 12\]
   (In this case, the multiplications was completed in step 2).

4) Do all divisions in order from left to right.

5) Do all additions and subtractions in order from left to right.
   \[3X + 4X = 7X\]
   \[7X + 8 = 12\]
   \[7X = 12 - 8\]
   \[X = \frac{4}{7}\]

POSITIVE AND NEGATIVE NUMBERS

Positive signs (+) show gains, increases, directions to the right and direction upward. Negative signs show losses, decreases and directions to the left and downward. There are some simple rules for working with signed numbers:

1) Adding (+) and (+) = Positive
2) Adding (-) and (-) = Negative
3) Adding (+) and (-); (Subtract and use sign of larger)
4) Subtracting (Change sign of number being subtracted and add.)
5) Multiplying and dividing

\[ (-) \times (-) = (+) \]
\[ (+) \times (+) = (+) \]
\[ (+) \times (-) = (-) \]
\[ (-) \div (-) = (+) \]
\[ (+) \div (+) = (+) \]
\[ (+) \div (-) = (-) \]
\[ (-) \div (+) = (-) \]

FORMULAS WITH EXPONENTS

Exponents are often found in electrical formulas. For example, the small number to the right of this number \(5^2\) tells us that the number must be squared. If we take \(5 \times 5\) to square the number, we find that \(5^2 = 25\). If the exponent is \(5^3\), we must take \(5 \times 5 \times 5\) to get the answer of 125. Whatever the exponent, it means that the number must be multiplied against itself that number of times. For example,

\[ 10^6 = 10 \times 10 \times 10 \times 10 \times 10 \times 10 \]

Exponents are shorthand expressions that saves us from lengthy formulas.

POWERS OF TEN

Powers of ten are used in electronics to express numbers that are in very large or very small units.

\[ 10^1 = 10 \]
\[ 100 = 10^2 \]
\[ 1000 = 10^3 \]
\[ 10000 = 10^4 \]

In order to multiply by numbers with powers of ten, move the decimal to the right as many places as shown in the power (exponent).

EXAMPLE: \(10^6 \times 20 = 20,000,000\)

In order to divide numbers by power of ten, the decimal point will be moved to the left by the same number of places as shown in the exponent.

EXAMPLE: \(20 \div 10^6 = 0.000002\)
Information

EQUATIONS

An equation must be kept equal on both sides of the equality sign. If we perform an operation on one side of the equation, we must do the same on the other side. The same numbers may be added or subtracted on both sides without wrecking the equation. This also holds true with multiplication and division. In an equation that shows $5E = 500$ we must solve by:

\[
\begin{align*}
5E &= 500 \\
E &= 500 + 5 \\
E &= 100
\end{align*}
\]

or divide both sides by the same multiplier:

\[
\begin{align*}
\frac{5E}{5} &= \frac{500}{5} \\
\frac{1BE}{\beta} &= \frac{1800}{\beta} \\
E &= 100
\end{align*}
\]
**Assignment**

- Read all information in package.
- Complete self-assessment sheet and check answers.
- Complete post-assessment and have instructor check answers.
Use the following basic formulas to solve the problems listed below:

a) \( E = IR \) (Ohm's Law)

b) \( P = \frac{E}{R} \) (Watt's Law)

c) \( E_1 + E_2 + E_3 = IR_1 + IR_2 + IR_3 \) (Kirchoff's Voltage Law)

1. A 12-volt automobile battery operates a cigarette lighter of 6 Ohms of resistance. How much current is used?

2. A dryer operates at 240 volts and 12 Ohms. How much current is used?

3. What is the resistance of a toaster that draws 6 amps of 120 volt electricity?

4. Write the formula for Kirchoff's Current Law. The algebraic sum of the currents entering any point and leaving any point must equal zero.

5. A lamp operating on 120 volts has a resistance of 2 amps. How many watts of power is used by the lamp?

6. Six batteries supply 2 volts each to make up a 12 volt system. This system supplies two camper lights that have a resistance of 1 Ohm each and pull 6 amps at each light. Show how this fits with Kirchoff's Voltage Law.

7. Write a formula to fit this rule. The volume of a cylinder is equal to the area of the base times the length of the cylinder.

8. \( 10 = \)

9. Complete the following equation: \(-6x + 3 = 15\)

Self Assessment Answers

1. 2 amps
2. 20 amps
3. 20 Ohms
4. \( I_1 + I_2 + I_3 = I_a + I_b + I_c = 0 \)
5. 60 watts
6. \( E_1 + E_2 + E_3 + E_4 + E_5 + E_6 = IR_1 + IR_2 \)
   \[ 2 + 2 + 2 + 2 + 2 + 2 = (6 \times 1)^2 + (6 \times 1) \]
7. \( V = \pi r^2 \times L \)
8. 1,000,000,000
9. \( X = -2 \)
10. \( +\)
Using the basic information and rules from the information sheet, complete the following problems using the prescribed order of operation.

1. Divide Powers of Ten
   a) \( \frac{10^8}{10^3} \) 
   b) \( 0.00015 \div 3 \times 10^{-2} \)

2. Multiply Powers of Ten
   a) \( 0.005 \times 5 \times 10^{-3} \times 0.02 \)
   b) \( 3 \times 10^{-5} \times 4 \times 10^6 \)

3. Signed numbers:
   Add: 
   a) +16.43 and -64.86 
   b) -36 and -43 
   c) +82 and +14

   Subtract:
   d) -16 minus +38 
   e) 63 minus -71

   Multiply:
   f) -3R times +3 
   g) -2R times -6

   Divide:
   h) \( 6R^2 \) by -2R 
   i) -18 by -6

4. Find value of \( P \) if \( E = 100 \) and \( R = 50 \). Use formula \( P = E^2/R \).

5. Write a rule for the following formula: \( I = E/R \)

6. Write a formula for the following rule:
   The voltage of a circuit is equal to the current multiplied by the resistance.

7. Solve the following equations for value of unknown letter:
   a) \( 20 = 100 R \)
   b) \( .5T = 12 \)
   c) \( W = 20 \)
   d) \( 3S + 5 = 20 \)
   e) \( Z - 2 = 10 \)
8. Find total resistance of a coil that draws .010 amps from a 6 volt battery. Show operational steps.

Write the formula: ____________________________

Substitute numbers: __________________________

Solve for R __________________________

79
1. a) $10^5$
   b) $5 \times 10^{-3}$
2. a) $5 \times 10^{-7}$
   b) $2 \times 10^{-7}$
3. a) -48.43
   b) -79
   c) +96
   d) -54
   e) +134
   f) -9R
   g) +12R
   h) -3R
   i) +3
4. 200
5. The current of a circuit is equal to the voltage divided by resistance.
6. $E = IR$
7. a) $R = \frac{1}{5}$
   b) $T = 24$
   c) $W = 40$
   d) $S = 5$
   e) $Z = 8$
8. $E = IR$
   \[ R = \frac{6}{0.010} \]
   \[ 6 = 0.010 \times R \]
   \[ R = 600 \]
Supplementary References

Goal:
The apprentice will be able to compute ratio and proportion.

Performance Indicators:
1. Solve problems involving ratio and proportion.
Problems in ratio and proportion are frequently encountered in the skilled trades. For example, a machinist employs the concepts of simple and compound ratio in solving problems relating to gearing, and a carpenter employs the concepts of ratio and proportion in working from blueprints or other scale drawings.
This study guide is designed to help you successfully complete this module. Check off the following steps to completion as you finish them.

**STEPS TO COMPLETION**

1. ___ Familiarize yourself with the Goal and Performance Indicators on the title page of this module.

2. ___ Read the Introduction and study the Information section of the module. It is intended to provide you with the math skills necessary to successfully complete the assessment portions.

3. ___ Complete the Self Assessment section of this module. You may refer to the Information section for help.

4. ___ Compare your Self Assessment answers with the correct answers on the Self Assessment Answer Sheet immediately following the Self Assessment exam. If you missed more than one of the Self Assessment exam questions, go back and re-study the necessary portions of the Information section, or ask your instructor for help. If you missed one or none of these problems, go on to step 5.

5. ___ Complete the Post Assessment section of the module. Show your answers to the instructor. It is recommended that you score 90% or better on those Post Assessment exams with 10 or more problems, or miss no more than one problem on those with fewer than 10 problems, before being allowed to go on to the next math module.
Ratio is a means of expressing a relationship between two or more things mathematically. A ratio is the quotient of two numbers, and it can therefore be expressed as a fraction. The fraction $\frac{3}{4}$ expresses the ratio of three to four, which may also be written 3:4. When a ratio is expressed in words, the things being related and the numerical terms of the ratio are listed in the same order; for example, if a worker is told to mix sand and cement for a concrete batch in the ratio of three to one, he or she will know that the mixture must include three sacks of sand for every sack of cement, not the reverse.

Proportion is an expression of equality between two ratios. The fraction $\frac{3}{4}$ is equal to the fraction $\frac{6}{8}$; this is a statement of proportion. The relationship between these equivalents can also be written 3:4 = 6:8, which is read "three is to four as six is to eight." This simply means that three bears the same relationship to four that six does to eight. If all but one of the terms of a proportion equation are known, the remaining term can be found. This makes possible a useful short method for solving problems like those in which an object must be proportionally increased or reduced in size but where one of the needed dimensions is not known.
Listed below each problem are four possible answers. Decide which of the four is correct, or most nearly correct; then write the letter for that answer in the space to the left of the problem.

1. The ratio of the height of a building to the length of its shadow is 5 to 9. What is the height of the building if it casts a shadow 90' long?
   a. 50'  
   b. 55'  
   c. 60'  
   d. 65'

2. An architect indicates a 1/8" = 1'0" scale in the drawing of a swimming pool. What is this scale expressed as a ratio?
   a. 1:58  
   b. 1:75  
   c. 1:85  
   d. 1:96

3. A tile subcontractor prepares a shop drawing to a scale of 1" = 1'0". What is this scale expressed as a ratio?
   a. 1:10  
   b. 1:12  
   c. 1:14  
   d. 1:16

4. A contractor estimates that 10 cents of every dollar of his bid will be required for exterior and interior glazing of a building. What is the ratio of the glazing cost to the total building cost?
   a. 1:1  
   b. 1:10  
   c. 1:100  
   d. 1:110

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Self Assessment
Self Assessment Answers

1. a
2. d
3. b
4. b
Listed below each problem are four possible answers. Decide which of the four is correct, or most nearly correct; then write the letter for that answer in the space to the left of the problem.

1. On a tile job in which fireclay is to be used, a tilesetter tells his helper to mix mortar according to the following formula: 6 buckets of river sand, 1 bucket of fireclay, and 2 buckets of cement. What is the ratio of sand to fireclay in the mixture?
   a. 1:6  
   b. 1:2  
   c. 3:1  
   d. 6:1

2. Referring again to the above problem, what is the ratio of cement to sand in the mixture?
   a. 1:2  
   b. 1:3  
   c. 1:6  
   d. 1:8

3. What is the missing term in the proportion 46:30::92:x?
   a. 20  
   b. 40  
   c. 60  
   d. 80

4. What is the missing term in the proportion 42:x::30:2.5?
   a. 1.75  
   b. 3.5  
   c. 4.25  
   d. 5.75

5. If 5 cu. yd. of concrete cost $60, what will 3 cu. yd. cost?
   a. $36  
   b. $42  
   c. $48  
   d. $54

6. If ten cement masons can place and finish 6,400 sq. ft. of concrete sidewalk in four days, how many cement masons will be needed to place and finish 3,200 sq. ft. of concrete sidewalk in the same amount of time?
   a. three  
   b. five  
   c. seven  
   d. nine
Goal:
The apprentice will be able to compute areas and volumes of regular and irregular shaped objects.

Performance Indicators:
1. Compute area of a rectangle.
2. Compute area of a triangle.
3. Compute areas of irregular shaped objects.
4. Measure volumes of regular and irregular shaped objects.
Problems involving the measurement of perimeters, areas, and volumes are frequently encountered on the job. A skilled worker in the construction trades, for example, may need to know not only the length and width of a room but also its perimeter and the areas of its floor, walls, and ceiling for estimating material and labor costs for interior finish work. He or she may also need to know the volume of air space of the room for heating and ventilating calculations. Measurements of perimeters, areas, and volumes are basic to every craft, and the apprentice must therefore become thoroughly familiar with the rules and procedures for making them.
This study guide is designed to help you successfully complete this module. Check off the following steps to completion as you finish them.

**STEPS TO COMPLETION**

1. Familiarize yourself with the Goal and Performance Indicators on the title page of this module.

2. Read the Introduction and study the Information section of the module. It is intended to provide you with the math skills necessary to successfully complete the assessment portions.

3. Complete the Self Assessment section of the module. You may refer to the Information section for help.

4. Compare your Self Assessment answers with the correct answers on the Self Assessment Answer Sheet immediately following the Self Assessment exam. If you missed more than one of the Self Assessment exam questions, go back and re-study the necessary portions of the Information section, or ask your instructor for help. If you missed one or none of these problems, go on to step 5.

5. Complete the Post Assessment section of the module. Show your answers to the instructor. It is recommended that you score 90% or better on those Post Assessment exams with 10 or more problems, or miss no more than one problem on those with fewer than 10 problems, before being allowed to go on to the next math module.
MEASURING PERIMETERS
The perimeter of an object—the distance around it—is found by adding the lengths of all its sides; the perimeter of a building lot 60' x 180' is therefore 60' + 180' + 60' + 180', or 480'. The perimeter of the irregularly shaped structure in the plan view, Fig. D-1, will be found to be 68' if the dimensions of all its sides are added.

MEASURING AREAS
Measurements of areas are expressed in units of square measure—square inches, square feet, square yards, and the like. The area of a square or other rectangle is found by multiplying its length by its width. The result will always be in units of square measure. For example, the area of a plywood panel 4' wide by 8' long is 32 square feet.

Since a linear foot is equal to 12", a square foot (1-foot each way) contains 12" x 12", or 144 square inches. (See Fig. D-2.) Expressions of square measure must be read carefully if mistakes are to be avoided: note that 10-inch-square (one square measuring 10" x 10") is not the same as 10-square inches (ten squares, each measuring 1" x 1").

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Fig. D-1 Perimeter-measurement

Fig. D-2 A 12-inch square (one square foot) contains 144 square inches
AREA OF A RECTANGLE

Multiplying two adjacent sides gives the area of a square or other rectangle. In a square, all four sides are of equal length and all four corners are right angles; other rectangles differ from the square in that their sides and ends are of unequal length. (See Figs. D-3A and D-3B.) A rectangle that is not a square is commonly called an oblong. Since all sides of a square are of equal length, the area of a square is found by multiplying any side by itself; the area of an oblong is found by multiplying its length by its height.

Any four-sided plane figure whose opposite sides are straight and parallel is a parallelogram. Squares and oblongs meet this definition, but the word parallelogram usually applies specifically to a four-sided plane figure whose opposite sides are parallel but whose corners are not right angles. A parallelogram can be thought of as a rectangle with a triangle removed from one end and tacked onto the other end. (See Fig. D-3C.) To compute the area of a parallelogram, multiply base X height (altitude). The base of the parallelogram in Fig. D-3C is 14", and its altitude is 10"; therefore its area is 10" x 14", or 140 square inches.

Fig. D-3. Four-sided plane figures: (A) square; (B) oblong; (C) parallelogram

AREA OF A TRIANGLE

A triangle is a plane figure with three sides, each side being a straight line. A square-cornered or right triangle has one right angle (Fig. D-4A). In an acute triangle, each of the three angles is less than a right angle (Fig. D-4B). An obtuse triangle has one angle that is greater than a right angle (Fig. D-4C).

Any triangle is really one-half of a rectangle (or one-half of a parallelogram, in the case of an acute or an obtuse triangle). This can be seen clearly in Fig. D-4A, where an identical but inverted right triangle is drawn above the shaded right triangle, making a rectangle. Similarly, "mirror-image" triangles could be joined to the acute and obtuse angles in Figs. D-4B and D-4C to make parallelograms.
The area of a rectangle or a parallelogram is equal to its length (base) times its height (altitude). Since a rectangle or a parallelogram can be made by joining two identical triangles, it follows that the area of any triangle is equal to one-half the product of its base and its altitude. The area of the right triangle in Fig. D-4A is therefore 70 square inches; the area of the acute triangle in Fig. D-4B is 70 square inches; and the area of the obtuse angle in Fig. D-4C is 60 square inches.

AREAS OF IRREGULAR SHAPES

Any skilled worker may occasionally find it necessary to determine the area of an irregularly shaped surface. For a practical problem of this kind, assume that a worker needs to determine the area of the floor in a room having a number of projections and recesses. He or she can compute the total floor area in either of two ways: he or she can divide the irregular floor shape into smaller rectangular shapes, then compute the areas of these rectangles and take their sum; or square out the irregular floor shape, compute the area of the resulting square, then subtract from that the areas of the cutouts. (See Fig. D-5.)

Method 1. Divide the floor area into rectangular units (A, B, C, and D); then compute the area of each unit and add the unit areas.

- A) 7′ x 2′ = 14 sq. ft.
- B) 6′ x 2′ = 12 sq. ft.
- C) 7′ x 11′ = 77 sq. ft.
- D) 9′ x 18′ = 162 sq. ft.

\[ \text{Total Area} = 265 \text{ sq. ft.} \]

Fig. D-5, 1

Method 1: \[ A + B + C + D = 265 \text{ sq. ft.} \]
Method 2. Enclose the floor area in a square; find the area of the square, then subtract the areas of the cutouts (units (A and B).

- \(18' \times 18' = 324 \text{ sq. ft.}\)
- \(A \) \(2' \times 5' = 10 \text{ sq. ft.}\)
- \(B \) \(7' \times 7' = 49 \text{ sq. ft.}\)

\[59 \text{ sq. ft.}\]

\[324 \text{ sq. ft.} - 59 \text{ sq. ft.} = 265 \text{ sq. ft.}\]

Fig. D-5, 2. Finding the area of an irregularly shaped floor.

MEASURING VOLUMES

The plane figures described thus far in this topic have the dimensions of length and width only. Because solid objects have thickness as well as length and width, they occupy or enclose space. The amount of space taken by a solid object is its volume. Volume is commonly expressed in cubic measure—cubic yards, cubic feet, or cubic inches, for example—but it can also be expressed in liquid measure (gallons, quarts, pints or ounces) or dry measure (bushels or pecks). Volumes expressed in one kind of measure can be changed to volumes expressed in another measure by means of conversion constants. For example, a cubic foot is equal to 7.48 U.S. gallons, and a bushel is equal to 1.244 cubic feet.

To find the cubic measure of a body such as a cube or a box, where all the corner angles are right angles, multiply length times width times thickness. The result is expressed in cubic units. The dimensions of the box in Fig. D-6 are 2" x 2" x 1". The box therefore encloses (has a volume of) 4 cubic inches. As in the case of square measure, care must be taken in expressing cubic measure if mistakes are to be avoided; a 10-inch cube is not equivalent in volume to 10 inches.

If the shape of an object is such that its ends (or its top and bottom) are identical, parallel, and exactly opposite each other, and if the straight lines bounding the sides of the object are all parallel (as in the shapes shown in Fig. D-7), the volume of the object can be found by multiplying the area of one end (or of the top or bottom) by the length (or height) of the object. If for example the
the area of one end of the prism shown at the left in Fig. D-7 is 10 square inches and the length of prism is 15 inches, the volume of the prism will be 10 square inches x 15 inches, or 150 cubic inches.

Fig. D-6
Cubic measure

Fig. D-7. Solids with identical ends and straight sides

The volume of an irregularly shaped object can best be found by thinking of the object as being made up of a number of smaller solid shapes. (See Fig. D-8.) The separate volumes of these smaller shapes can then be computed and added to find the total volume.
Fig. D-8. Finding the volume of an irregularly shaped object.
Write the answer to each problem in the corresponding space at the left.

1. What is the perimeter of a room 20' wide and 30' long?

2. What is the perimeter of a room 16' square?

3. What is the area, in square feet, of a floor 42' by 42'?

4. What is the area, in square inches, of a 9'' square floor tile?

5. What is the floor area, in square feet, of a room 15' long and 12' wide?

6. What is the area, in square yards, of a rectangle 20' long and 9' wide?

7. What is the area, in square inches, of a right triangle with a base of 8 1/2'' and an altitude of 11 1/4''?

8. What is the area, in square inches, of an acute triangle with a base of '8 1/2'' and an altitude of 11 1/4''?

9. What is the area, in square feet, of the floor shown below?
Self Assessment Answers

1. 100 ft.
2. 64 ft.
3. 1,764 sq. ft.
4. 81 sq. inches
5. 180 sq. ft.
6. 6 2/3 sq. yards
7. 47.8 sq. inches
8. 47.8 sq. inches
9. 294 sq. ft.
Listed below each problem are four possible answers. Decide which of the four is correct, or most nearly correct; then write the letter for that answer in the blank space to the left of the problem.

1. What is the perimeter of a rectangle 8' wide and 12' long?
   a. 32'  
   b. 34'  
   c. 37 1/2'  
   d. 40'  

2. What is the perimeter of a rectangle 17 1/2' wide and 12 1/2' long?
   a. 40'  
   b. 60'  
   c. 80'  
   d. 100'  

3. What is the perimeter of a rectangle 67 7/8' wide and 96 4/5' long?
   a. 237 10/16'  
   b. 297 10/16'  
   c. 327 10/16'  
   d. 377 10/16'  

4. What is the area in square feet of a rectangle 32 9/16' wide and 52 6/16' long?
   a. 1,709.0  
   b. 1,719.375  
   c. 1,729.875  
   d. 1,740.0  

5. An excavation for a basement is to be 40' long, 27' wide, and 8' deep. After 210 cu. yd. of dirt have been removed, how many cubic yards remain to be excavated?
   a. 90  
   b. 110  
   c. 115  
   d. 120  

6. How many cubic feet of concrete are in a slab 12' long, 4' wide, and 1' thick?
   a. 40  
   b. 42 1/2  
   c. 44 1/2  
   d. 48  

7. What is the volume in cubic inches of a 25'' cube?
   a. 625  
   b. 975  
   c. 12,380  
   d. 15,625
8. What is the area in square feet of a room 14' square?
   a. 56    c. 196
   b. 112   d. 208

9. How many cubic yards of concrete will be needed for a garage floor 20' x 32' x 4", allowing 3 cu. yd. extra for foundation walls and footings?
   a. 4.9    c. 7.9
   b. 6.9    d. 10.9

10. How many cubic yards of concrete will be needed for the foundation walls and footings in the plan below if the walls are 6" thick and 18" deep, and if the footings (shown in dotted lines) will require 2 5/27 cu. yd. of concrete?
    a. 6       c. 7
    b. 6 2/3   d. 7 1/6
Goal:

The apprentice will be able to compute problems involving circumference and area of circles.

Performance Indicators:

1. Find circumference of circle.
2. Find area of a circle.
Introduction

A knowledge of the rules and procedures for finding the circumference and area of a circle is important for workers in the skilled trades. A construction worker, for instance, must make computations involving circular areas as well as straight-sided areas when working with structures like circular buildings, silos, or tanks. In a typical problem, he or she might find it necessary to determine the number of feet of insulating material needed for covering a cylindrical hot-water storage tank of a given diameter and height. The first step in solving this problem would be the calculation of the tank's circumference. The present module gives the information needed for finding the area and the circumference of a circle.
This study guide is designed to help you successfully complete this module. Check off the following steps to completion as you finish them:

**STEPS TO COMPLETION**

1. Familiarize yourself with the Goal and Performance Indicators on the title page of this module.

2. Read the Introduction and study the Information section of the module. It is intended to provide you with the math skills necessary to successfully complete the assessment portions.

3. Complete the Self Assessment section of the module. You may refer to the Information section for help.

4. Compare your Self Assessment answers with the correct answers on the Self Assessment Answer Sheet immediately following the Self Assessment exam. If you missed more than one of the Self Assessment exam questions, go back and re-study the necessary portions of the Information section, or ask your instructor for help. If you missed one or none of these problems, go on to step 5.

5. Complete the Post Assessment section of the module. Show your answers to the instructor. It is recommended that you score 90% or better on those Post Assessment exams with 10 or more problems, or miss no more than one problem on those with fewer than 10 problems, before being allowed to go on to the next math module.
FINDING THE CIRCUMFERENCE OF A CIRCLE

The perimeter of an object has been defined as the distance around it; circumference is the term employed for the perimeter of a circle or circular object. Any continuous part of a circumference is called an arc. The diameter of a circle is a straight line passing through the center of the circle and terminating at the circumference. The radius of a circle is a straight line drawn from the center of the circle to any point on the circumference; it is therefore equal to one-half the diameter. (See Fig. D-9).

Regardless of the size of the circle, its circumference bears a constant relationship to its diameter. This ratio is 3.1416 to 1, or roughly 3 1/7 to 1. The number 3.1416 is a "constant" in mathematics; it has been given the symbol \( \pi \) (the Greek letter "pi"). If the diameter of a circle is known, the circumference can be computed by the following rule: Circumference = \( \pi \times \text{diameter} \) (or, in short form, \( C = \pi \times D \)).

The following example shows how the rule would be put to work in solving a practical problem:

Problem: Find the circumference of a circle whose radius is 10 feet.
Rule: \( C = \pi \times D \)

Step 1: Find the diameter
\[ D = 2 \times \text{Radius (R)} \]
\[ 2 \times R = 20' \]

Step 2: Multiply the diameter by \( \pi \)
\[ 20' \times 3.1416 \]

Answer: \( C = 62.832' \)

By applying the rule for the circumference of a circle in another way, we can find the diameter or the radius of a circle if only the circumference is known. Since \( C = \pi \times D \), it is also true that \( D = C \div \pi \). The steps to be followed in solving a typical problem of this type are shown below:

Problem: Find the radius of a circle whose circumference is 34 inches.

Step 1: Find the diameter
\[ D = C \div \pi \]
so \( D = 34'' \div 3.1416 \), or 10.82''

Step 2: Find the radius
\[ R = 1/2 \times D \]
\[ R = 10.82'' \div 2 \]

Answer: \( R = 5.41 \)

FINDING THE AREA OF A CIRCLE

To find the area of a circle, multiply the radius by itself, then multiply the resulting product by 3.1416 (\( \pi \)). The result, of course, will be in square measure. A number multiplied by itself is said to be squared; the symbol for squaring is a 2 following and slightly above the number to be squared. Thus \( 5^2 \) means \( 5 \times 5 \), or 5 squared. The rule for finding the area of a circle, then, is:

\[ \text{Area} = \pi \times R^2 \]

The application of this rule is illustrated in the following problem:

Problem: Find the area (A) of a circle whose radius is 20 feet.

Rule: \( A = \pi \times R^2 \)

Step 1: Find the square of the radius.
\[ R^2 = 20' \times 20' = 400 \text{ sq. ft.} \]

Step 2: Multiply \( R^2 \) by \( \pi \)
\[ 3.1416 \times 400 \text{ sq. ft.} \]

Answer: \( A = 1256.64 \text{ sq. ft.} \)
Determine the word that belongs in each blank and write the word in.

1. The distance around the rim of a wheel is called the _______ of the wheel.

2. The diameter of a circle is a line passing through the _______ of the circle and terminating at the _______.

3. The symbol \( \pi \), which is the Greek letter _______, stands for a mathematical constant having the numerical value _______.

4. The circumference of a circle is equal to \( \pi \) times the circle's _______.

5. The _______ of a circle is equal to one-half the circle's diameter.

6. The area of a circle is found by the following formula: \( A = \pi \times \) _______.

7. The area of a circle is given in units of _______ measure.

8. If the radius of a circle is 5 inches, the circumference of the circle is _______ inches.

9. If the circumference of a circle is 95 inches, the diameter of the circle is _______ (to the nearest inch).

10. The area of a circle having a radius of 10 inches is _______.

---

**ER**
Self Assessment Answers

1. circumference
2. center, edges
3. pi, 3.1416
4. diameter
5. radius
6. radius squared or \( R^2 \)
7. square
8. 31.14 inches
9. 30 inches
10. 314 sq. inches
Listed below each problem are four possible answers. Decide which of the four is correct, or most nearly correct; then write the letter for that answer in the blank space to the left of the problem.

1. ___ The circumference of a hole 14" in diameter is how many inches?
   a. 43.98+ c. 58.39+
b. 49.38+ d. 59.98+

2. ___ What is the area in square inches of a circular vent hole 30" in diameter?
   a. 607.58+ c. 807.58+
b. 706.860 d. 857.850

3. ___ The area of a circular ceiling with a radius of 12' is how many square feet?
   a. 425.930 c. 493.390
   b. 452.39+ d. 857.850

4. ___ The area of a circular putting green with a radius of 17' is how many square feet?
   a. 907.92+ c. 1,002.720
   b. 909.72+ d. 1,007.92+

5. ___ A pole-hole in the second-story floor of a firehouse has a radius of 22". What is its circumference in inches?
   a. 123.230 c. 138.23+
   b. 132.32+ d. 148.320

6. ___ The area of a circular swimming pool with a radius of 10' is how many square feet?
   a. 304.16+ c. 341.46+
   b. 314.16+ d. 364.16+

7. ___ The area of a circular skating rink with a radius of 40' is how many square feet?
   a. 5,026.56+ c. 5,206.560
   b. 5,062.650 d. 5,506.26+
8. A merry-go-round at an amusement park has a radius of 33'. What is its circumference in feet?
   a. 179.04+  
   b. 197.34+  
   c. 206.34+  
   d. 237.04+

9. A water tank has a diameter of 8'6". What is its circumference in feet?
   a. 20.70  
   b. 23.33+  
   c. 25.250  
   d. 26.70+

10. What is the area of a circular floor with a diameter of 10'6", to the nearest square foot?
    a. 85+  
    b. 86+  
    c. 87  
    d. 88
Goal:

The apprentice will be able to compute problems involving areas of plane figures and volumes of solid figures.

Performance Indicators:

1. Compute area of parallelograms, trapezoids, triangles, polygons, circles and ellipses.

2. Compute volumes of cubes, prisms, cylinders, cones, pyramids and spheres.
Introduction

The previous modules, specifically the last two, have demonstrated the importance of math and its application in solving problems which apprentices are faced with daily. Some types of mathematical problems have not been covered in the previous modules. This module introduces several new formulas for determining areas and volumes of "out of the ordinary" or add-shaped figures.
This study guide is designed to help you successfully complete this module. Check off the following steps to completion as you finish them.

STEPS TO COMPLETION

1. ___ Familiarize yourself with the Goal and Performance Indicators on the title page of this module.

2. ___ Read the Introduction and study the Information section of the module. It is intended to provide you with the math skills necessary to successfully complete the assessment portions.

3. ___ Complete the Self Assessment section of the module. You may refer to the Information section for help.

4. ___ Compare your Self Assessment answers with the correct answers on the Self Assessment Answer Sheet immediately following the Self Assessment exam. If you missed more than one of the Self Assessment exam questions, go back and re-study the necessary portions of the Information section, or ask your instructor for help. If you missed one or none of these problems, go on to step 5.

5. ___ Complete the Post Assessment section of the module. Show your answers to the instructor. It is recommended that you score 90% or better on those Post Assessment exams with 10 or more problems, or miss no more than one problem on those with fewer than 10 problems, before being allowed to go on to the next math module.
### AREAS OF PLANE FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Formula</th>
<th>Diagram</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parallelogram</td>
<td>$A = B \times H$</td>
<td><img src="image1" alt="Parallelogram Diagram" /></td>
</tr>
<tr>
<td>Trapezoid</td>
<td>$A = \frac{B + C}{2} \times H$</td>
<td><img src="image2" alt="Trapezoid Diagram" /></td>
</tr>
<tr>
<td>Triangle</td>
<td>$A = \frac{B \times H}{2}$</td>
<td><img src="image3" alt="Triangle Diagram" /></td>
</tr>
<tr>
<td>Regular Polygon</td>
<td>$A = \frac{\text{SUM OF SIDES (S)}}{2}$</td>
<td><img src="image4" alt="Regular Polygon Diagram" /></td>
</tr>
</tbody>
</table>

### VOLUMES OF SOLID FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Formula</th>
<th>Diagram</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangular Solids</td>
<td>$V = L \times W \times H$</td>
<td><img src="image5" alt="Rectangular Solid Diagram" /></td>
</tr>
<tr>
<td>Prisms</td>
<td>$V = \text{AREA OF ENDO} \times H$</td>
<td><img src="image6" alt="Prism Diagram" /></td>
</tr>
<tr>
<td>Cylinder</td>
<td>$V = \pi r^2 \times H$</td>
<td><img src="image7" alt="Cylinder Diagram" /></td>
</tr>
<tr>
<td>Cone</td>
<td>$V = \frac{1}{3} \pi r^2 \times H$</td>
<td><img src="image8" alt="Cone Diagram" /></td>
</tr>
</tbody>
</table>
Formulas for calculating areas or volumes of typical geometric shapes.

CIRCLE

\[ A = \pi r^2 \]

\[ A = 0.7854 \times d^2 \]

\[ A = 0.795 \times c^2 \]

ELLIPSE

\[ A = m \times n \times 0.7854 \]

PYRAMIDS

\[ V = \frac{1}{3} \times \text{area or base} \times h \]

SPHERE

\[ V = \frac{1}{6} \times \pi d^3 \]
Self Assessment

Referring to the Information section, select your own numbers for the various bases, heights, lengths, widths, etc., and work out at least one formula for each of the 12 area and volume figures on the Information sheet.
Self Assessment Answers

The problems completed by students working on this module will be evaluated individually by the instructor.
Referring to the Information section of this module, answer the following questions.

1. What is the volume of the cylinder if the radius (R) is 6 inches and the height is 8 inches?

2. What is the volume of the sphere if D is 11.4 inches?

3. What is the area of the regular polygon if each side is 2.5 inches and the R (radius) is 3.6 inches?

4. What is the total volume of the cylinder and the cone if the height of each is 9 inches, and the R (radius) of each is 4.5 inches?
Goal:

The apprentice will be able to draw and read graphs.

Performance Indicators:

1. Describe independent and dependent variables.
2. Describe linear relationships in graphs.
3. Describe curved relationships in graphs.
4. Draw graphs that show linear relationships and negative values.
Study Guide

- Read the goal and performance indicator to determine what should be learned from the package.
- Study the vocabulary words.
- Read introduction and information sheets.
- Complete self assessment and score using the answer sheet.
- Complete post assessment and ask instructor to score answers.
Vocabulary

- Abscissa
- Base lines
- Curved relationship
- Dependent variable
- Independent variable
- Linear relationship
- Ordinate
- Scale
- Variable
- XX axis
- YY axis
Introduction

Graphs are used in electronics to show the effects of one variable upon another variable. A variable is something that changes its value. Voltage and amperage are variables in electricity. If we change the voltage, the amperage will be changed.

An independent variable is one that is changed so that its effect upon a dependent variable can be observed. In working with electricity, voltage is the independent variable and current is the dependent variable.

Graphs are the easiest way to show the relationship between current and voltage. Graphs are used to show how circuits operate when the variables are changed. Although tables can provide the same information, they become difficult to read when lengthy.
Graph paper is sectioned into little squares. Lines run in both vertical and horizontal directions. The fifth or tenth line is heavier than the others so that the graph is easy to plot and read. Most graph paper has green lines. The heavy vertical line on the left side of the graph paper is called the ordinate. The heavy horizontal line at the bottom is called the abscissa.

The numbers represent the scale of the graph. Scale is determined by the number of units that will need to be shown on a graph. Each of the tiny squares can represent one or 20 units.

**LINEAR RELATIONSHIPS**

When one set of values for voltage and current is calculated and plotted on a graph, other values can be determined by reading the graph. For example:

Using Ohms Law \(E = IR\) calculate the current at 15 volts and 10 ohms of resistance. We find the answer to be 1.5 amperes.
The 1.5 value is used on the ordinate scale and 15 is plotted on the abscissa. If the resistance of this circuit is maintained as voltages are changed, a straight line relationship will exist. This is called a linear relationship, which means straight line relationship. For each increase in voltage, the current will be increased. If the current was calculated at 5, 10 and 20 volts, the values would fall along that line.

The current values at different voltage levels can be taken from a graph once this linear relationship is established. The values of the current would be:

- 5 volts = .5 amps
- 10 volts = 1 amp
- 15 volts = 1.5 amps
- 20 volts = 2 amps

To read these values, move vertically from the voltage value until the linear relationship line is reached. Then move horizontally to the ordinate and read the amperes scale. This is much easier than making calculations at many voltage levels.

CURVED RELATIONSHIPS

Other relationships fall into a curved pattern rather than into a linear arrangement. For instance, if the relationships between current and resistance were calculated at a constant voltage, a curved relationship would be found. The changes in values is not of a linear nature.
Use Ohm's Law to calculate current at:

<table>
<thead>
<tr>
<th>Ohm's</th>
<th>Voltage</th>
<th>Amperes</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>20</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>20</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>20</td>
<td>3</td>
</tr>
<tr>
<td>8</td>
<td>20</td>
<td>2.5</td>
</tr>
<tr>
<td>10</td>
<td>20</td>
<td>2</td>
</tr>
</tbody>
</table>

If the values (amperes) for five levels of resistance are plotted on the graph (shown on the previous page), a curved relationship is found.

POSITIVE AND NEGATIVE VALUES

Many electronic applications involve both positive and negative values. A special graph layout is required to show negative values. The base lines of the graph must pass through the center of the graph. These base lines are called X' axis and Y' axis. See the example shown below:
The $XX'$ axis records values of the independent variable and $YY'$ axis records values of the dependent variable. A negative or positive value can be shown for each variable. For example, the following values will be charted for variables $X$ and $Y$.

<table>
<thead>
<tr>
<th>Point</th>
<th>$X$</th>
<th>$Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>+2</td>
<td>2</td>
</tr>
<tr>
<td>B</td>
<td>+4</td>
<td>2</td>
</tr>
<tr>
<td>C</td>
<td>+6</td>
<td>2</td>
</tr>
</tbody>
</table>

This type of graph layout is not needed when all values are positive.

If the $Y$ variables have negative and positive values, such as the example below, use both positive and negative planes to complete graph.

<table>
<thead>
<tr>
<th>Point</th>
<th>$X$</th>
<th>$Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>+2</td>
<td>+3</td>
</tr>
<tr>
<td>B</td>
<td>-4</td>
<td>+2</td>
</tr>
<tr>
<td>C</td>
<td>+6</td>
<td>-4</td>
</tr>
</tbody>
</table>
Note that point A falls in a positive plane and could have been plotted on a regular graph. Point B has a negative X value which places it to the left of the YY' line. Point C had a negative Y' value which places it below the XX' line.
Assignment

- Complete self assessment and check answers.
- Complete post assessment and have instructor check answers.
1. Compute amperage when voltage is 30 and resistance is 10 ohms and plot it as Point A on the graph. (Use E=IR to complete calculation).

2. Establish a line of linear relationship from 0 to the point on the graph.

3. Plot amperage values for 5, 10, 15, 20 and 25 volts on graph. Show points as Points B, C, D, E and F.

4. Is voltage the independent or dependent variable?

5. This graph shows a ____________ relationship.

Study this graph:
6. Plot the following values for X and Y and label the points A, B, C, etc. when:

<table>
<thead>
<tr>
<th>Point</th>
<th>X value</th>
<th>Y value</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>-8</td>
<td>+6</td>
</tr>
<tr>
<td>B</td>
<td>-4</td>
<td>-2</td>
</tr>
<tr>
<td>C</td>
<td>-2</td>
<td>+2</td>
</tr>
<tr>
<td>D</td>
<td>+2</td>
<td>+4</td>
</tr>
<tr>
<td>E</td>
<td>0</td>
<td>+6</td>
</tr>
</tbody>
</table>

7. What is line XX' called?

8. Does the abscissa run vertically or horizontally on the page?

9. Plot the following X and Y values on the graph below:

```
<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>2</td>
</tr>
<tr>
<td>8</td>
<td>25</td>
</tr>
<tr>
<td>6</td>
<td>33</td>
</tr>
<tr>
<td>4</td>
<td>50</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
</tr>
</tbody>
</table>
```

10. Does current and resistance have a linear relationship when voltage is held constant?
Self Assessment Answers

1. 3 amp.

2. 

3. 

4. Independent

5. Linear

6. 

7. XX' axis

8. Horizontally
9.

10. No
1. Show linear relationship of voltage and current when resistance remains constant.
   
   a) Use $E = IR$ to calculate value at 30 volts. Use 10 ohms of resistance.
   
   b) Use graph to identify other current values at voltages of 5, 10, 15, 20, 25 volts.

2. Plot the X and Y values as points on the graph that follows:

<table>
<thead>
<tr>
<th>Point</th>
<th>X value</th>
<th>Y value</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>+8</td>
<td>+4</td>
</tr>
<tr>
<td>B</td>
<td>+4</td>
<td>+0</td>
</tr>
<tr>
<td>C</td>
<td>0</td>
<td>-4</td>
</tr>
<tr>
<td>D</td>
<td>-4</td>
<td>-4</td>
</tr>
</tbody>
</table>

(Graph is on next page)
Label Problems 3 through 8 on the graph on the following page.

3. Abscissa

4. Ordinate

5. Type of relationship (Linear or Curve)

6. Independent variable

7. Dependent variable

8. Scale (one unit of measurement = ___________ squares)
9. If voltages remain constant, a graph for resistance and current will show a relationship.

10. What advantage does a graph have over a table for showing information?
9. Curve

10. Graph is like a picture. It is more easily understood, especially if the tables are lengthy.
Goal:
The apprentice will be able to calculate and apply basic trigonometry.

Performance Indicators:
1. Describe and label parts of a triangle (angles and side).
2. Calculate tangent, sine and cosine values.
3. Use trigonometric tables.
4. Calculate size of angles from two known sides of a triangle.
5. Calculate value of sides from one known angle and one known side.
6. Calculate values on an impedance triangle.
Study Guide

- Read goal and performance indicators to determine what is to be learned from package.
- Read the information sheet.
- Use reference to find trigonometric function tables.
- Review vocabulary list to make sure that key terms are understood.
- Complete self assessment and score results with answers from answer sheet.
- Complete post assessment and have instructor check answers.
Vocabulary

- Adjacent side
- Cosine
- Hypotenuse
- Impedance triangle
- Opposite side
- Right triangle
- Sine
- Tangent
- Trigonometric function tables
Trigonometry is the mathematics of triangles. Most occupations use trigonometric principles to solve problems.

This package introduces the apprentice to some basic functions of trigonometry. A few applications are used to help the apprentice understand the basic principles.
Trigonometry is used in working with triangles. Through the use of trigonometry, we can solve problems that involve the sides and angles of right triangles. Technicians can use the values of known angles and sides to calculate the value of unknown sides and angles.

A right triangle is made of the following parts:

- **Right Angle** $\angle ABC = 90^\circ$
- **Hypotenuse** $AC$ - Opposite $\angle ABC$
- **Opposite Sides** - $BC$ is Opposite $\angle BAC$
  - $AB$ is Opposite $\angle ACB$
- **Adjacent Sides** - $AB$ is adjacent to $\angle BAC$
  - $BC$ is adjacent to $\angle ACB$

Each angle of the right triangle has an opposite side (one that does not connect to the angle) and an adjacent side (one that is hooked to the angle). Some characteristics of angles never change or remain constant. These constants have been calculated and placed in tables. These tables are called Trigonometric Function Tables.

**TRIGONOMETRIC FUNCTIONS**

The characteristics of angles that are commonly used in trigonometry are:

1. **Tangent** values of angle
2. **Sine** values of angle
3. **Cosine** values of angle

The tangent, sine and cosine values are based on size of angles (in degrees) and will be the same for all angles of that size. Increasing the lengths of the sides of a triangle does not change those values.

$\text{Tangent of angle} = \frac{\text{Opposite side}}{\text{Adjacent side}}$
Sine of angle = \( \frac{\text{Opposite side}}{\text{Hypotenuse}} \)

Cosine of angle = \( \frac{\text{Adjacent side}}{\text{Hypotenuse}} \)

**FINDING SIZE OF ANGLES**

So, if we know the lengths of two sides of a right triangle, we can calculate the tangent, sine or cosine value of that angle. Study the following example:

\[ \text{Tangent } \angle \text{BAC} = \frac{\text{Opposite Side (BC)}}{\text{Adjacent Side (AB)}} \]

\[ = \frac{3'}{4'} \]

\[ \text{Tangent Value} = 0.75 \]

\[ \text{Sine } \angle \text{BAC} = \frac{\text{Opposite side (BC)}}{\text{Hypotenuse (AC)}} \]

\[ = \frac{3'}{5'} \]

\[ \text{Sine value} = 0.60 \]
INSTRUCTIONAL LEARNING SYSTEMS

Information

\[ \cos \angle BAC = \frac{\text{Adjacent side (AB)}}{\text{Hypotenuse (AC)}} \]

\[ \frac{4'}{5'} = 0.80 \]

\text{Cosine value} = 0.80

TRIGONOMETRIC TABLES

When the values from the preceding problem are matched with the table values in a trigonometric table in the supplementary reference, we find:

<table>
<thead>
<tr>
<th>Calculated Values</th>
<th>Table Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tangent</td>
<td>0.75</td>
</tr>
<tr>
<td>Sine</td>
<td>0.60</td>
</tr>
<tr>
<td>Cosine</td>
<td>0.80</td>
</tr>
</tbody>
</table>

These table values show that \( \angle BAC \) is 37\(^\circ\). Either of the three calculated values would have been sufficient for finding the angle. If we know the length of the opposite side and adjacent side, a calculation of the tangent value would be the best choice. Where the opposite side and hypotenuse are known values, calculate a sine value. If only the adjacent side and hypotenuse values are known, calculate a cosine value. Match any one of these values with the trigonometric table to find the size of the angle.

FINDING SIDES OF TRIANGLES

If an angle and one side of a triangle are known, the value of the unknown sides can be calculated.
If we wish to find the value of BC, we must use the sine formula:

\[
\begin{align*}
\sin \angle BAC &= \frac{\text{Opposite side (BC)}}{\text{Hypotenuse (AC)}} \\
&= \frac{.500}{10} \\
&= .050 \\
\Rightarrow BC &= .050 \times 10 \\
&= 5
\end{align*}
\]

If we had elected to calculate side AB, then our choice would have been the cosine formula because the problem involves the adjacent side and the hypotenuse.

\[
\begin{align*}
\cos \angle BAC &= \frac{\text{Adjacent side (AB)}}{\text{Hypotenuse (AC)}} \\
&= \frac{.8660}{10} \\
&= .0866 \\
\Rightarrow AB &= .8660 \times 10 \\
&= 8.66'
\end{align*}
\]
APPLICATIONS OF TRIGONOMETRY

Trigonometry is often used to figure relationships between impedance, resistance and capacitive reactance of AC circuits. Quite often this relationship is referred to as an "impedance triangle".

Either of these values can be calculated from a given angle and one side of the triangle. For example, if we wish to find the R value when Z = 800, we know the value of &angle; BAC and the hypotenuse AC and we wish to find the value of side AB. Since AB is the adjacent side, our choice is the cosine function. The tables show the cosine value of a 30° angle to be .8660. The formula will be:

\[
\cos \angle BAC = \frac{\text{Adjacent side (AB)}}{\text{Hypotenuse (AC)}}
\]

\[
.8660 = \frac{AB}{800}
\]

\[
AB \times .8660 = 800
\]

\[
AB = \frac{800}{.8660}
\]

\[
AB = 922.6 \text{ ohms}
\]

The resistance value (R) is found to be 922.6 ohms.

Another application of trigonometry is to bring "out of phase" current and voltage into phase. With known phase angles and current values, the in-phase condition (I_x) and reactive component (I_y) can be calculated.
The apprentice will find many opportunities to use trigonometric functions in solving practical problems. Remember:

1. The basic formulas for finding tangent, sine and cosine values.
2. The parts of a triangle.
3. How to use trigonometric tables.
4. How to substitute values into a formula.
Assignment

- Check answers on page 670.
- Complete self assessment and check answers.
- Complete post assessment and have instructor check answers.
Label the following drawing of a triangle:

1. Right angle
2. Hypotenuse
3. Opposite side $\angle ABC$
4. Adjacent side $\angle ABC$

5. Find values of $\angle ABC$ in trigonometric table for:
   a) Tangent 
   b) Sine
   c) Cosine

6. Calculate value of AB.

7. Calculate value of AC.

8. Calculate the size of $\angle ABC$ if this triangle:

9. Show formula selected for calculation in Problem 8 above.

10. Draw an impedance triangle. (Use back of this page)
5. a) Tangent \(0.5774\)
b) Sine \(0.5000\)
c) Cosine \(0.8660\)

6. 8.66
7. 5
8. 53°

9. Cosine = \frac{\text{Adjacent side}}{\text{Hypotenuse}}

10.
Post Assessment

Study the details of the right triangle below:

1. What is the size of \( \angle BAC \)?
2. Which is the adjacent side to \( \angle ABC \)?
3. Which is the opposite side to \( \angle ABC \)?
4. What is line BC called?
5. The tangent of \( \angle ABC \) can be calculated by \( \tan \angle ABC = \frac{\text{Opposite side}}{\text{Adjacent side}} \)?
6. The sine value of \( \angle ABC = \frac{\text{Opposite side}}{?} \)
7. The cosine value of \( \angle ABC = \frac{?}{\text{Hypotenuse}} \)
8. The trigonometric tables show \( \angle ABC \) to have a cosine value of .80. If side BC is 10, what is the value of side AB?
9. An impedance triangle shows:

Calculate the impedance (Z) if R is equal to 200 ohms. Use trigonometric tables to find values.

10. Calculate the R value of triangle in question 9 if Z is equal to 300 ohms.
Instructor Post Assessment Answers

1. 90°
2. AB
3. AC
4. Hypotenuse
5. Opposite,
6. Hypotenuse
7. Adjacent
8. 8
9. 230.9
10. 259.8
Supplementary References

## Trigonometric Functions Table

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<th>COS</th>
<th>TAN</th>
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**Note:**
- The table provides values for the sine (SIN), cosine (COS), and tangent (TAN) of angles in degrees.
- The values are rounded to four decimal places.
- The angles range from 0° to 45° and 45° to 90°.
Goal:

The apprentice will be able to make conversions between the English and metric systems of measurement.

Performance Indicators:

1. Convert English to metric measurements.
2. Convert metric to English measurements.
Introduction

Through the years more and more countries have begun using the metric system. The United States is changing from the English FPS (Foot-Pound-Second) system to SI metrics. It is therefore important that we become familiar with the metric units and their relationship to the familiar English units.
This study guide is designed to help you successfully complete this module. Check off the following steps to completion as you finish them.

**STEPS TO COMPLETION**

1. ___ Familiarize yourself with the Goal and Performance Indicators on the title page of this module.

2. ___ Read the Introduction and study the Information section of the module. It is intended to provide you with the math skills necessary to successfully complete the assessment portions.

3. ___ Complete the Self Assessment section of the module. You may refer to the Information section for help.

4. ___ Compare your Self Assessment answers with the correct answers on the Self Assessment Answer Sheet immediately following the Self Assessment exam. If you missed more than one of the Self Assessment exam questions, go back and re-study the necessary portions of the Information section, or ask your instructor for help. If you missed one or none of these problems, go on to step 5.

5. ___ Complete the Post Assessment section of the module. Show your answers to the instructor. It is recommended that you score 90% or better on those Post Assessment exams with 10 or more problems, or miss no more than one problem on those with fewer than 10 problems, before being allowed to go on to the next math module.
The official name of the new metric system is "System International de Unite." Its abbreviation is "SI."

Although this module will not cover all of it, the following seven areas are those in which metrics come into play:

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<th>SI Unit</th>
<th>SI Symbol</th>
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<tr>
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<tr>
<td>Amount of substance</td>
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The area of measurement of length and distance is our primary concern here. Here are a few fundamentals of the metric system:

- 1 inch = 25.4 millimeters
- = 2.54 centimeters
- 1 foot = 30.48 centimeters
- = 3.048 decimeters
- = 0.3048 meters
- 10 millimeters = 1 centimeter
- 10 centimeters = 1 decimeter
- 10 decimeters = 1 meter
- 10 meters = 1 decameter
- 10 decameters = 1 hectometer
- 10 hectometers = 1 kilometer
CONVERSIONS

The following information provides us with all we need to know about converting our system of inches, feet, yards, etc. to metric values:

inch x 25.4 = mm
inch x 2.5 = cm
inch x .025 = m
foot x 30.5 = cm
foot x 0.305 = m
yard x 0.91 = m
mile x 1.6 = km

The following information enables us to convert metric values to inches, feet, yards, etc.:

millimeters (mm) x 0.039 = inches
centimeters (cm) x 0.39 = inches
meters (m) x 39.4 = inches
centimeters x 0.33 = feet
meters x 3.28 = feet
meters x 1.09 = yards
kilometers (km) x 0.62 = miles

Example: A board is 46 inches long. How many centimeters long is it?
The table tells us that if we want to convert inches to centimeters, we multiply the number of inches by the conversion factor of 2.5.
Answer: 46 inches x 2.5 = 115 cm

Example: A Swiss watch measures 21 millimeters across its face. How many inches is it?
The table tells us that if we want to convert millimeters to inches, we multiply the number of millimeters by the conversion factor of .039.
Answer: 21 mm x .039 = .819 inches
Complete the phrases below, referring to the Information section as necessary.

1. To determine how many millimeters are in an inch, you multiply by _____.

2. There are _______ centimeters in a meter.

3. ______ cm equals one inch.

4. A centimeter is ______ times as large as a mm.

5. A mm is ______ the size of a cm.

6. To determine how many cm are in a foot you would multiply ______ x ______ inches.

7. To determine how many millimeters are in a centimeter you would ________ by 10.

8. A meter consists of _______ feet.

9. A meter consists of ______ inches.

10. There are _________ mm in a meter.
Self Assessment Answers

1. 25.4
2. 100
3. 2.54
4. 10
5. one-tenth \((1/10)\)
6. 2.5, 12
7. multiply
8. 3.28
9. 39.4
10. 1,000
Compute the answers to the following problems and write the answers in the blanks.

1. 3 inches = _______ cm

2. 6.5 yards = _______ meters

3. 6.5 yards = _______ cm

4. 12.7 cm = _______ inches

5. 7 feet = _______ meters

6. 1 inch = _______ cm

7. 1 cm = _______ mm

8. 1 mm = _______ cm

9. 1 foot = _______ cm

10. 1500 cm = _______ ft.