Preschoolers 4 and 5 years of age were found to use four strategies differing in temporal characteristics as they solved simple addition problems with sums of 10 or less. Three strategies had visible and/or audible aspects, and one was covert, involving retrieval from memory. The harder the problem, the more often the children used an overt strategy. The use of overt strategies most often on the hardest problems was adaptive and efficient. How children knew when to use overt strategies, though, remained unclear. Addressing this problem, Siegler and Shrager (1984) proposed a distribution of associations model to explain how children arrive at their addition strategies. Their model involves representation and process. In brief, children represent correct and incorrect answers that vary in "strength of association" between problem and answer. Then they retrieve an answer. If the answer is sufficiently strongly associated with the problem, the child advances the retrieved answer. If not, the child elaborates through overt strategies the representation of the problem. Elaboration of representation is followed by retrieval. This alternation between elaboration of representation and retrieval may progress through three phases. The model has implications for explaining parallels between children's and adults' problem-solving behavior. In addition, it suggests that ordering children not to use overt strategies such as counting on their fingers is worse than useless. (RH)
DEVELOPMENT IS DESTINY

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From among all of the ways that children could solve a given problem, how do they decide which one to use? The results of my research on this question suggest that even preschoolers possess sophisticated strategy choice procedures that help them perform both accurately and efficiently. The research also suggests that the strategies children use when they are first acquiring a skill may influence their performance on the task long after they have abandoned the initial approaches. For this reason, analysis of how children initially solve problems can render understandable adult performance that otherwise would be difficult to explain. Hence the rather enigmatic title, "Development is Destiny."

In this brief article, I will touch on several topics: what is involved in the issue of strategy choice, some empirical findings on children's strategy choices, a computer simulation of how strategy choices develop, and some speculation about how early cognitive development leaves its signature on later performance.

The Issue of Strategy Choice

Early in their experience with many tasks, such as reading, spelling,
adding, subtracting, and multiplying, children use a variety of overt (visible and audible) strategies. In adding numbers, for example, they often count on their fingers; in spelling, they often sound out the words. As children gain greater experience with the problems, they progressively abandon the overt strategies, instead relying on internalized solution procedures. The questions of central interest in my recent research are what mechanisms lead children of a given age to use overt strategies on some problems and not others, and what mechanisms lead to changes in the relative frequency of different strategies at different ages?

Cognitive and developmental psychologists often have phrased their models in terms that suggested that all people, or at least all people of a given age, perform a given task in a given way. These models defy the everyday observation that even a single individual may perform a given task in different ways on different occasions. For example, a child one day might write down a word's spelling as retrieved from memory, but the next day might look up the same word in a dictionary. I suspect that the universalistic phrasings stem more from the difficulty of formulating and providing evidence for alternatives than from deep conviction that everyone uses the same approach. The effect, however, has been to shunt attention away from an important issue: how do people decide what to do from among the various things that they might do.

Good reasons exist for people to know and to use multiple strategies for achieving a goal. Strategies differ in their accuracy, in the amounts of time they require, in their memory demands, and in the range of problems to which they apply. Strategy choices involve tradeoffs among these properties so that people can cope with cognitive and situational constraints. The broader the
range of strategies that people know, the more they can shape their approaches to meet these changing circumstances. My colleagues and I have found that even preschoolers can vary their strategies in adaptive ways. But how do they do so?

The main possibility that has been explored to date involves metacognitive knowledge. In this view, children use knowledge of their own cognitive capacities, the demands of the material, and available strategies to explicitly choose which strategy to use. For example, when confronting a complex problem, they might think, "This is a tough problem, too tough to solve unless I do X, I'd better do X."

It almost certainly is the case that people sometimes proceed in this manner. As a general explanation of strategy choices, however, the approach seems less promising than it once appeared. On an empirical level, research has revealed only modest correlations between explicit knowledge about cognitive capabilities and strategy use (Cavanaugh & Perlmutter, 1982). On a theoretical level, there is considerable lack of clarity about how metacognitive knowledge would lead to strategy choices. Do people make explicit judgments about their intellectual capacities, available strategies, and task demands every time they face a task they could perform in two or more ways? If not, how do they decide on which tasks to do so? Do they consider every strategy that they could conceivably use on a task or only a subset of them? If only a subset, how do they decide which ones? How is cognitive capacity estimated, especially cognitive capacity for novel material? In light of these empirical and conceptual uncertainties, it seems worthwhile to at least entertain the possibility that people can arrive at adaptive strategies without explicitly assessing their intellectual abilities,
available strategies, and task requirements. That is, the workings of the cognitive system, rather than metacognitive knowledge, may account for some range of strategy choices.

Empirical Findings on Strategy Choices

The first area in which my colleagues and I examined strategy choices was elementary addition. To investigate children's strategy choices, Siegler and Robinson (1982) videotaped 4- and 5-year-olds as they solved simple addition problems with sums of 10 or less. We found that the preschoolers used four strategies. Sometimes they used the **counting fingers** strategy, in which they put up their fingers and counted them. Sometimes they used the **counting** strategy, in which they counted aloud without any obvious external referent. Sometimes they used the **fingers** strategy, in which they put up fingers but answered without counting them. Finally, sometimes they seemed to simply state an answer that they retrieved from memory (the **retrieval** strategy). The strategies differed substantially in their temporal characteristics. Retrieval was the most rapidly executed strategy, fingers the next most rapidly executed, and counting and counting finger the slowest.

The most interesting finding of the experiment was the discovery of very strong relations among three variables: the number of errors on each problem, the length of solution times on that problem, and the number of uses of the three overt strategies on the problem. The harder the problem (measured either by large numbers of errors or long solution times), the more often children used an overt strategy. The relations were of substantial magnitude; for example, the correlation between the percentage of errors on each problem and the percentage of trials on which children used overt strategies on that problem was r=.91. The correlation between the mean length of solution times
on each problem and the percentage of trials on which children used overt strategies on that problem was $r = .90$.

Using overt strategies most often on the hardest problems was adaptive. It helped children solve the problems. On 24 of the 25 problems, children were more accurate when they used overt strategies than when they did not. It also was efficient. Children required more than twice as much time to solve problems when they used overt strategies as when they retrieved answers. By limiting the overt strategies to the hardest problems, children only expended large amounts of time on problems where the time was truly needed.

Since this initial experiment, similar patterns of data have been found in a variety of domains. Siegler and Shrager (1984) replicated the findings on the initial set of addition problems, and obtained similar findings with two sets of more difficult addition problems and with a set of subtraction problems. Siegler (in press) demonstrated parallel patterns of data in multiplication and spelling. In all of these cases, the correlations among percentage of overt strategy use on each problem, percentage of errors on the problem, and mean solution time on the problem ranged from a minimum of $r = .78$ to a maximum of $r = .92$. Use of the overt strategies just on the most difficult problems has proven to lead to efficient and effective speed/accuracy combinations in all cases as well.

How Do Children Arrive at Their Strategies?

How did children know when to use overt strategies? The most straightforward possibility was that their metacognitive knowledge of the difficulty of the problems led to their choices. However, when children were asked how hard each problem was, their judgments did not correlate highly with how often they earlier had used overt strategies on the problem or with how
many errors they had made on it. The finding opened the possibility that a less straightforward decision process might be at work.

Siegler and Shrager (1984) proposed the distribution of associations model to explain how children arrive at their addition strategies. The model is described in detail in that article; here, only a few key assumptions will be mentioned. At the highest level of generality, the model includes a representation and a process. The representation consists of a set of associations between each problem and various possible answers to the problem. Thus, the representation for the problem "2+2" might include an association of strength .1 between "2+2" and "3," an association of strength .8 between "2+2" and "4," and an association of strength .1 between "2+2" and "5." Note that associations exist between incorrect answers and the problem as well as between the correct answer and the problem. This reflects the assumption that whenever children answer a problem, they increase the strength of the association between that answer and the problem. Since children sometimes state wrong answers to a problem, they will associate those answers with the problem to some degree.

The process that operates on this representation proceeds in three phases. First is the retrieval phase. A child retrieves an answer. The likelihood of any particular answer being retrieved is proportional to the associative strength of that answer relative to all of the associative strengths for the problem. If the answer that is retrieved is sufficiently strongly associated with the problem to exceed the child’s current confidence criterion (his or her criterion for stating a retrieved answer), the child advances the answer that was retrieved. Otherwise, the child proceeds to a second phase in which he or she elaborates the representation of the problem.
The child can do this either by putting up fingers or by forming a mental image of the number of objects being added. The child then again retrieve an answer, and states it if its associative strength exceeds the confidence criterion. If it does not, the child proceeds to the third phase. Here, he or she counts the objects in the elaborated representation and states the value of the last count as the answer.

This model is intended to account for the strategies that children use, for the relative durations of the strategies, and for the close relations among the percentage of overt strategy use, the percentage of errors, and the mean solution times on each problem. First consider how the model accounts for the existence of the four strategies. The retrieval strategy appears if children retrieve an answer whose associative strength exceeds their confidence criterion. The fingers strategy emerges when children fail to retrieve an answer whose associative strength exceeds their confidence criterion, put up their fingers, and then retrieve an answer where the associative strength exceeds their confidence criterion. The counting fingers strategy appears if children fail to retrieve an answer whose associative strength exceeds their confidence criterion, put up their fingers, and finally count their fingers. The counting strategy is observed if children fail to retrieve an answer whose associative strength exceeds their confidence criterion, form a mental image of the objects in the problem, and finally count the objects in the mental image.

The mode also accounts for the relative solution times of the strategies. The retrieval strategy takes the shortest time, because its steps are all included within the other strategies. The fingers strategy is the next fastest, because its steps are included within the two remaining
approaches. The counting and counting fingers strategies are the slowest because they include the steps necessary to execute the other strategies as well as steps unique to themselves.

Most important, the model accounts for the high correlations among percentage of overt strategy use on a problem, percentage of errors on the problem, and mean solution time on the problem. The correlations arise, according to the model, because all three dependent variables are functions of the same independent variable—the degree to which associative strength is concentrated in a single answer, rather than being scattered over several answers. The more that the associative strength is concentrated in a single answer, the more often that retrieval, rather than an overt strategy, will be used (because the greater the concentration of associative strength in one answer, the more often that the answer that is retrieved will have sufficient associative strength to exceed the confidence criterion and be stated). Since the answer with the greatest associative strength ordinarily is the correct answer, the greater the concentration of associative strength in that answer, the more often the answer will be correct. Finally, the greater the concentration of associative strength in one answer, the more likely that solution times will be short, since children will retrieve statable answers early in the retrieval process. Thus, the same problems, the ones with associative strength heavily concentrated in a single answer, will produce frequent use of retrieval, low percentages of errors, and short mean solution times.

This account raises the question of how distributions of associations develop. Why do some problems have their associative strengths concentrated in one answer, while others have theirs distributed over several? As noted
above, the central developmental assumption of the distribution of associations model is that children associate whatever answer they state with the problem on which they state it. Therefore, the issue reduces to why children state certain answers to problems. Three factors seem likely to be influential: preexisting knowledge of the counting string, frequency of exposure to each problem, and the sum of the addends.

One factor is knowledge of the counting string. Gelman and Gallistel (1978), among others, have demonstrated that most 4- and 5-year-olds know the counting string quite well. This knowledge appears to both help and hurt their addition performance. Counting helps by providing a backup strategy to use when children cannot retrieve an answer that they are sufficiently confident of to state. It can also hurt, however, by suggesting incorrect answers. On all 6 problems where the second addend was larger than the first and where the answer one greater than the second addend was not correct (problems such as "2+4" and "4+5"), the most frequent error made by the 4- and 5-year-olds was to say that the answer was the number one greater than the second addend. That is, they answered that "3+4=5" and "3+5=6". The result suggested that such problems triggered associations with children's knowledge of the counting string. The children momentarily forgot that they were adding and reverted to the better known procedure for counting.

A second potential influence on which problems had associative strength concentrated in a single answer was the sum of the addends. Presumably, the more objects children need to represent, and the more objects they then need to count, the more likely they are to err. In line with this view, Gelman and Gallistel (1978) reported that 3- and 4-year-olds' counting errors increased as the sets they counted grew larger.
The third potential influence was amount of exposure to problems. Parents, teachers, and other children may present some problems more often than others. To test this possibility, Siegler and Shrager invited parents of 2- to 4-year-olds to teach their children to solve addition problems, as they might at home. The results helped explain two otherwise anomalous previous findings. First, several previous investigators have noted that ties (problems such as "3+3" and "4+4" are easier than the size of their addends would suggest. Parents presented ties much more often than other problems with large minimum numbers or large sums. Second, addition researchers consistently have found that "+1" problems, such as "3+1" and "4+1," are easier than the corresponding "1+" problems, such as "1+3" and "1+4." Parents were found to present "+1" problems five times as often as the corresponding "1+" problems (for example, they presented "4+1" nine times as often as "1+4"). Amount of presentation is important within the model, because the greater the amount of exposure to a problem, the more that problem's associative strength becomes concentrated in a single answer (see Siegler & Shrager for details).

To examine how well these three factors together accounted for children's addition performance, Siegler and Shrager used them as predictors of 4- and 5-year-olds' percentage of errors on the 25 problems with sums no greater than 10 that were studied by Siegler and Robinson. The results indicated that the three variables together accounted for 85% of the variance in the number of errors on the 25 problems. Further, each variable accounted for significant independent variance when the other two were held constant. Thus, all three seemed likely to play a role in development.

Siegler and Shrager (1984) formalized the distribution of associations
model as a running computer simulation. The purpose of the simulation was to demonstrate that the three processes hypothesized to be important in the development of the distribution of associations could lead to patterns of performance much like those of the 4- and 5-year-olds who had been studied. The results of the simulation were encouraging in a number of ways. The four strategies produced by the simulation--retrieval, fingers, counting, and counting fingers--were the same four used by the children. The relative solution times of the strategies--retrieval fastest, fingers next fastest, and counting and counting fingers slowest--also was identical to the children's pattern. Most important, the simulation produced the same high correlations among its errors, solution times, and overt strategy use as the children had shown. The correlation between the simulation's percentage of errors on each of the 25 problems and the percentage of errors made by the children on each problem was $r = .87$. The corresponding correlations for solution times and overt strategy use were $r = .82$ and $r = .85$ respectively. These similarities were consistent with the view that the mechanisms embodied in the simulation were much like those through which children learn to add.

Relations Between Early and Later Performance

Now we can return to the "development is destiny" theme. A number of researchers have remarked on the parallels between the problems that children and adults find difficult. Groen and Parkman (1972) noted that adults as well as 7-year-olds take longer to solve addition problems with higher minimum numbers. Siegler and Robinson (1982) noted parallels between the frequency of children's errors on magnitude comparison problems and adults' solution times. Jorm and Share (1983) noted a similar close relation between which words young children often misread and which words required long lexical access times in
adults. Parallels also have emerged between which words school children spell incorrectly and which words adults take relatively long times to spell (Frith, 1980).

Why should this occur? Why, for example, should it take adults longer to add "4+3" than "3+2"? Why should it take them longer to decide whether 5 is larger than 3 than to decide whether 8 is larger than 3? After years of experience with numbers, do adults still need to recompute the outcomes of these problems, as many current models would suggest (e.g., Banks, 1977; Baroody, 1984; Groen & Parkman, 1972)?

The distribution of associations model suggests another possibility. Within this model, adults would retrieve answers to these problems. Because of the associative strengths of incorrect answers that were built up years earlier, however, they sometimes would retrieve an incorrect rather than the correct answer. The associative strengths of these incorrect answers would be very low relative to the strengths of the correct answers; thus adults would rarely if ever state incorrect answers. However, the fact that the incorrect answers were retrieved would increase the time that would pass before the correct answer was retrieved and stated. Thus, problems that were difficult for the children's early strategies, such as addition problems with high sums or high minimum numbers, would later take relatively long to solve even though the individuals had long ago stopped using the early strategies. This is the sense in which development would be destiny.

This view has an interesting educational implication: the widespread policy of discouraging schoolchildren from using their fingers to add may be misguided. The policy has a certain logic on its side. One of the goals of education is to make younger children and less good students more like older,
better students. Older, better students do not use their fingers; younger, less good students do. The distribution of associations model, however, implies that ordering children not to use their fingers is worse than useless. Older, better students do not use their fingers because they possess distributions of associations with most associative strength concentrated in the correct answer. These distributions render their using overt strategies unnecessary. Younger and less knowledgable children, however, use overt strategies precisely because they lack such distributions of associations. Forcing them to retrieve answers will lead to many errors, which, since each response adds to the associative strength of that answer, will slow the concentration of associative strength in the correct answer. Thus, paradoxically, pressuring children not to use their fingers may lead to them needing to use them for a longer time than if the children were not pressured.

Parts of this account are borne out in teachers' descriptions of the effects of banning overt strategies. They note that children continue to put up their fingers under their desks and behind their backs even when they have been told not to do this. One teacher responded to my question of how often she had to tell a child in her class not to use his fingers by asking "How many days have there been in the school year?" Teachers also note that children often make many errors after being told to stop using their fingers. Thus, it may be unwise for teachers to discourage children from using overt strategies. The children will spontaneously stop using the strategies when their associative strengths become sufficiently concentrated in a single answer, that is, when they no longer need them.
REFERENCES


