The use of a step-by-step procedure for sequencing materials from concrete objects to paper and pencil can provide kindergarten teachers with a defensible means for showing that children are progressing in their mathematical competencies even though they may not be doing paper-and-pencil activities. Developmental characteristics of young children are first described; of particular importance is the fact that they have developed many mathematical concepts through informal learning before they come to school. They deal best with concepts through activities involving concrete materials and need opportunities to continue to actively construct their own knowledge through involvement with things in their environment. A six-step guide is then outlined which provides a sequence for moving from concrete materials to paper and pencil: manipulation of real objects, real objects and pictorial representations used in combination, manipulation of pictorial representations, manipulation of card sets or cut-up workbook pages, laminated workbook pages or other activities, paper-and-pencil activities. A specific example is given to illustrate this procedure. A number of references are included. (MNS)
Kindergarten Mathematics: Step-by-step from Concrete Materials to Paper and Pencil

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Kindergarten Mathematics: Step-by-step from Concrete Materials to Paper and Pencil

Today responding to pressure to reach specific levels of student competence, many kindergarten teachers plunge children immediately into workbooks and other paper and pencil materials before students have the developmental prerequisites for these kinds of activities. Taking this approach with mathematics can be especially devastating for the young child. Inability to deal with paper and pencil may squelch the natural development of mathematics concepts and skills which has begun before school entrance (Charlesworth, 1981; Charlesworth and Radeloff, 1978). This lack of attention to child development appears to have increased as, in response to criticisms of the quality of basic education, elementary education has become more and more geared to specific competencies or objectives as guides to teaching. The expectations for academic readiness for entering first graders have become more stringent. Increased pressure has been placed on kindergarten teachers to prepare students for the demands of the elementary grades. Many teachers feel that they can only demonstrate children's increasing competencies through paper and pencil materials which can be used as concrete evidence of progress. While this evidence may satisfy administrators and uninformed parents it ignores the dictates of child development. The use of a step by step procedure for sequencing materials from concrete to paper and pencil can provide the kindergarten teacher with a defensible means for showing that children are progressing in their mathematical competencies even though they may not be doing paper and pencil activities. A step-by-step procedure will be described after a brief look at the developmental characteristics of the young mathematician which support its use.

Characteristics of the Young Mathematician

A growing body of research (Ginsburg, 1980; Starkey, 1983; Lam, 1979; Suydam and Weaver, 1975) is documenting that children's mathematical concepts
begin to emerge from the time they first perceive their environment. By the
time they enter school at age four or five they have already developed many
math concepts through informal learning. For example, five-year-olds usually
have some concept of "more" and "less," that given two equal groups of objects
adding one item to one of the groups makes that group contain "more." and that
a group of two objects and a group of three objects combined makes a group of
five objects. Many five-year-olds, when they enter school, can count ten
objects, can recite the number names up to ten in order, know the meaning of
first, can recognize some numerals, understand the basic concepts underlying
addition and subtraction and know something about money, time, quantity, parts
and wholes and geometric shapes. Most of the basic ideas which underlie these
concepts have been constructed by children through their normal everyday
activities and through informal instruction by adults and/or older children.
When children enter school at age four or five, most are still at the
preoperational stage of development. They deal best with concepts through
activities involving concrete materials and need opportunities to continue to
actively construct their own knowledge through involvement with things in
their environment (Sigel and Cocking, 1977, 225-234).

Whereas young mathematicians can deal with mathematics with concrete
materials they may not be ready, even at the time of kindergarten entrance, to
deal with paper and pencil representations. Many kindergarten entrants do not
have the prerequisite language, perceptual and motor skills needed to
integrate paper and pencil with their mathematical knowledge. (Hardy,
Stennett and Smythe, 1974). They may not understand "underline," "draw a line
to," "start at the top of the page," etc. They also may not be able to handle
a pencil or crayon well enough to "underline" or "circle" even though they may
understand the concept shown on the page. A young child may be able to
recognize a numeral and match it to a set of the correct amount before being
able to write the numeral. All the prerequisite skills for writing (Lamme, 1979) may not have developed for even many older five-year-olds and some six-year-olds. In addition young children need opportunities to explore and discover concepts on their own through unstructured manipulation of materials. They begin with concrete materials for exploration and initial instruction and move step by step to paper and pencil.

Six Steps from Concrete Materials to Paper and Pencil

The following six step guide provides a sequence for moving from concrete materials to paper and pencil:

Step 1: Real objects which can be manipulated by the children are used. Some examples would be blocks, sticks or chips for counting, sequencing and comparing; containers and sand and/or water for measuring; attribute blocks for learning about shape; cups and saucers or horses and riders to learn about one to one correspondence; unit blocks for learning about the construction of space. The materials are especially useful for free exploration and discovery but also, should be used for direct instruction activities.

Step 2: Real objects and pictorial representations are used in combination. The Workjobs approach (Barrata-Lorton, 1972 and 1979) consists is a good example of activities at this level. For example, matching real objects with outline drawings of each object (1972, p. 51), putting the correct toy car in a garage with a numeral on top (1972, p. 163) or placing kernels of real corn in pictorial dishes to create concrete addition problems (1979, p. 51). Usually these materials are introduced with some instruction as to what the child is to do. However, they can also be placed in the Math Center and children observed as they explore
the materials on their own. These types of materials are especially appropriate for matching and counting activities.

Step 3: Pictorial representations or cut outs which can be motorically manipulated. For example, "Little Larry" (Carson and Dellosa, 1977, 28-31) must sort and classify items into those that are toys, those which are food and those which belong in a laundry basket. The items used are cut-out of toys, food and clothing. In another activity the children put the correct number of cut-out fish into paper fish bowls (Dellosa and Carson, 1981, pp. 31-37). For one to one correspondence the child matches riders to horses (Carson and Dellosa, 1977). With these materials the child moves away from real things to two-dimensional cut-outs which can still be manipulated freely.

Step 4: Commercially available card sets, workbook pages cut up or teacher made picture cards are manipulated. For example, instead of matching sets which have the same amount by connecting them with lines the child physically pairs them up by placing them side by side. The child is free to work with the concept without being impeded by lack of motor control and/or eye-hand coordination. Mailman (Schutte, 1978) is a board game which would fall in this category. To play the game the child matches house numbers on cards to pictures of houses that are numbered to match the cards. Work with a microcomputer could be included at this step.

Step 5: At this level workbook pages or other activities which normally involve the use of a writing implement are mounted on cardboard or in a file folder and laminated or covered with clear Contac. This type of activity introduces the child to the use of writing implements to denote math responses but offers the advantage of being erasable so the child can easily correct mistakes. Difficult activities can be put aside and can be presented again when the child is ready. The teachers' original art work can be used or
inexpensive books can be purchased from the local variety and/or discount store. Some examples listed in the resource section of the references are About ABC/1-2-3, Clark and Hedges, Hargreaves, Match-ups, and 1-2-3.

Step 6: At this level the child is introduced to paper and pencil activities. The teacher observes that the child has mastered a skill or concept with materials at levels one through five and has the needed perceptual motor skills. Workbooks and/or worksheets are introduced. The child is still allowed time and opportunity to work at all the other levels.

With each new concept or skill the kindergarteners begin with level one materials and then go through the other levels as each one's developmental pace permits. Within a classroom children will probably be found working at all six levels. To demonstrate how this procedure works the following is a description for the objective "Places in one to one correspondence."

From Concrete to Paper and Pencil with "Places in one to one correspondence."

Places in one to one correspondence is a basic mathematics concept that begins to develop prior to kindergarten. Prekindergarten children spend long periods during their play time matching one to one: putting peg people in holes, setting animals on blocks, putting dolls in beds, giving each friend a cookie, etc. Through one to one correspondence activities children develop the concept of equivalence: that one group has the same amount as another group. Therefore mastery of this concept is usually included in the list of critical skills objectives for kindergarten. Although children have been working with this concept prior to entering kindergarten they may not have reached the level where they can compare two sets of pictured/objects by connecting each pair with a line as presented in a workbook. The following are examples of materials that can be used as the student moves from concrete materials to paper and pencil:
Step 1:

a. Big Beads and Small Beads

Large beads can be purchased from a craft shop. They are available in a variety of colors and sizes and will stand up on a hard surface. They can be available in the math center for exploration or for structured matching activities, "Put a big bead next to each small bead." (These materials are also good for sorting and for counting.)


Have ten or more small toys and ten or more pennies. Starting with five pennies ask the child to buy one toy with each penny.

Step 2:

The Outline Game: Real objects such as pencil, scissors, paperclip, etc. are outlined on posterboard or in a file folder. The child then matches the real objects to the outlines. (Resource: Baratta-Lorton, 1972)

Step 3:

Cowboys and Horses (10 of each). The cowboys and horses are colored, cut out and laminated. They can be available for exploration and also used for structured activities, "Find a horse for each cowboy." (Resource: Carson and Dellosa, 1977)

Step 4:

a. Match the Cars (Resource: Match-Ups, 1982)

Pictures of cars are cut out of a workbook page, colored with markers, glued on cards and laminated. Cards can be matched and sorted.


Pictures of children, dogs and cats are cut out, glued on cards and laminated. They can be matched, "Does each child get a pet?" (and also
sorted and counted).

**Step 5:**

a. Shape Matching (Resource: Hargreaves, 1982)

Workbook pages are glued into file folders and laminated. Child can draw a line from each shape on the left column to the matching one in the right column. If a mistake is made the line can be wiped off and the child can try again. If the task is too difficult it can be presented again later in the year.


At this level the pictures of the children and pets are glued on the right hand half of a file folder, the children in a column on the left and the pets on the right (four of each will fit), and the folder laminated. The child draws a line from each child to a pet of his choice, "Is there a pet for each child?"

**Step 6:**

The child is given the pages from the school's required kindergarten workbook.

**Accounting for Learning**

The procedure described provides a guide for the teacher to use to keep track of each child's progress. For example, the teacher can show that while Samantha can place two sets of ten items into one to one correspondence at all levels, Jon can accomplish this task with beads, objects and outlines, cut-out cowboys and horses, and card sets but can only match two pictured objects to two other pictured objects using a pencil or crayon to draw a line from item to item. Both children understand the concept but Jon whose motor coordination is immature is unable to demonstrate his knowledge with paper and pencil. By using the materials in sequence the teacher can account for each child's
progress toward each objective required for the children in the kindergarten class.

Some early childhood educators would probably suggest that we refrain from setting specific objectives for kindergarteners and that we throw out the workbooks. However, this is neither realistic nor is it fair to the child who is expected to enter first grade with certain skills attained and with some paper and pencil competency. With the emphasis on accountability and pressures to improve our educational system it seems likely that objectives and paper and pencil activities will not disappear from kindergartens in the near future. It's imperative that kindergarten teachers have some tools for working within the system while maintaining a developmentally sound program. The guidelines suggested can be used to place children with appropriate materials and supply the teacher with a rationale for not planning the whole mathematics program around paper and pencil activities.
Background References


Instructional References


Schutte, B. (1978) Readiness Gameboards. 26616 Indian Peak Road, Palos Verdes Peninsula, California 90274: Frank Schaffer Publications.