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ABSTRACT

The techniques for calculating marginal costs in higher education are examined in detail. Marginal costs, as defined in economics, is the change in total cost associated with producing one additional unit of output. In higher education, the most frequently selected unit of output is a full-time-equivalent student or, alternatively, a student credit hour. After reviewing several aspects of the microeconomic theory of costs, detailed analysis is provided of the three basic methods of estimating marginal costs: the regression method, the fixed- and variable-cost method, and the incremental-cost method. For each method, definitions and examples of cost calculations are provided, along with information on data and analytical requirements and strengths/weaknesses. In addition, a literature review is included that indicates the following: the use of statistical costs analysis in which marginal costs are estimated is essentially the same sort of undertaking in higher education as it is in other enterprises, such as business and industry, hospitals, and primary and secondary education; the typical approach to developing specific costs functions has been pragmatic; and basic elements of the microeconomic model are seldom realized in applied work. (SW)

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Marginal Costing Techniques

For Higher Education

Richard Allen
and
Paul Brinkman

1983

National Center for Higher Education Management Systems
P.O. Drawer P Boulder, Colorado 80302

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Contents (Continued)

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1

Introduction

Because of the difficulties currently faced by higher education and consequent emphasis on more effective management, there is reason for taking a fresh look at costing—one of the most widely used management techniques in higher education. Costing has been defined many ways, but no one has improved on the 1923 formulation by Maurice Clark that there are different costs for different purposes. In other words, there is no single right answer to the question, "How much did this cost?" The appropriate answer depends on the context that gave rise to the question. A summary of the many possibilities for calculating costs can be condensed from Adams, Hankins, and Schroeder (1978), who pointed out that costs can be defined by

1. Cost objectives (input, output, activity, organizational unit)
2. Cost basis (historical, projected, standard, imputed, replacement)
3. Cost assignability (direct, indirect, full)
4. Cost variability (fixed, variable, semivariable)
5. Cost-activity relationship (total, average, marginal)
6. Cost-determination method (specific service, continuous service)
7. Cost-time relationship (time period, accrual or cash, deflated)

The elements of these categories can be combined to produce a very long list, indeed, of alternative ways of calculating cost.

In spite of the wide variety of ways to look at cost, the overwhelming bulk of all costing work done in higher education until very recently has been of one type: the calculation of average historical, full or direct, annual cost of outputs or activities. The major costing systems, such as IEP, CAMPUS, RRPM, and SEARCH, that were implemented at numerous campuses and state agencies were all of this type. In the case of the last three, they were modeling tools based on the same underlying costing technology.

In recent years, many persons have become more cautious about the use of average historical unit cost and are choosing a more flexible approach. There is a decreased emphasis on cost calculation (the determination of a single cost for a "unit" of higher education) and an increased emphasis on cost analysis (the understanding of cost behavior and patterns from a variety of different perspectives). Most of the recent work in the field has focused on different aspects of costs rather than on traditional, average historical unit costs. Higher education seems to be heeding Clark's dictum: "different costs for different purposes."

This document also approaches higher-education costing from a different direction: it focuses on techniques for calculating marginal costs. Marginal cost, as defined in economics, is the change in total cost associated with producing one additional unit of output. In higher education, the most frequently selected unit of output is an FTE student or, alternatively, a student credit hour, although both are perhaps more properly viewed as activity measures. Since the bulk of the unrestricted revenue of higher education is tied to students (either through state appropriations or tuition and fees), the cost of an additional student is probably the most widely used type of marginal cost in higher education. It will generally be used throughout this discussion in illustrating marginal-costing techniques. Other types of marginal cost are also viable and important for particular uses.

The marginal cost of an additional student is a particularly important cost to consider at this time. The environment in which higher education will operate during the 1980s and early 1990s will be radically different from the environment in which it operated during the immediate post-World War II era. From 1946 until enrollment stabilization in the mid-1970s, higher education experienced rapid, almost explosive growth. Although enrollment drops did occur during this period—notably when World War II and Korean War GI benefits expired—these were passing phenomena that did not affect the overall pattern of growth.

The current situation, however, is significantly different. Higher education faces annual enrollment declines for the next 15 years beginning in the early 1980s. Two recent and authoritative national studies of enrollments—studies done by the American Council on Education (ACE) and the Western Interstate Commission for Higher Education (WICHE)—show essentially the same pattern. The WICHE study (McConnell 1979) focuses on high-school graduates and shows a serious problem from that perspective. The industrial northeast and upper midwest will potentially be devastated with enrollment declines reaching possibly 40-50 percent in some states. Most of the south and the northern plains will show some loss while some sun-belt and energy-development states will show increases (McConnell 1979). Within these state-by-state changes, enrollments will probably shift away [illegible] rural campuses to urban ones, from residential schools to commuter-oriented schools, and from the institutions that began as teachers' colleges to research universities and community colleges. Many observers believe that the shift from the liberal arts to more vocationally and professionally oriented curricula will also continue (Centra 1978).

ACE has, in part, reversed its gloomy enrollment projections. It has identified a series of events that, if they occurred, would ameliorate enrollment decline. Indeed, if all of the events occurred (and if ACE estimates are accurate), enrollments would remain stable throughout the 1980s. Among the events considered are: (1) an increased high-school graduation rate; (2) an increased percentage of high-school dropouts taking GED tests; (3) an increased participation rate of low- and middle-income persons; (4) an increased high-school graduation rate of minority students; (5) a general increase in the participation rate; (6) reduced attrition; (7) an increase in numbers of nontraditional students; (8) an increased participation rate for younger women and older men; (9) an increase in the numbers of graduate students; and (10) an increase in the numbers of foreign students (Francis 1980).

Considering the probability that all nine of these conditions are unlikely to be met, it does appear that some enrollment decline will occur and that it will be substantial in some states. There will certainly be major shifts in enrollment patterns within states, and it is likely that enrollment figures will be volatile and relatively unpredictable during the 1980s. This instability will serve to create special problems both in educational and financial terms for institutions.

The implications of declining enrollments for costing are most likely to be found in two areas: funding (such as state appropriations and indirect-cost recovery) and pricing (such as tuition levels and internal charges). Costing in these two areas has typically focused on average unit costs. There is growing concern, however, about relying entirely on average costs in a declining-enrollment situation. The results of using average cost figures to determine state formula-funding values and tuition levels will serve to illustrate this concern.

Similar to most other productive activities, the evidence suggests that higher education is probably characterized by economies of scale.[1] In other words, the cost of producing an additional unit—the marginal cost—is probably less than the average cost per unit, at least over some range of output. During the time of rapidly increasing enrollments of the 1950s and 1960s (when most of the existing budget formulas and tuition-setting policies were developed), this cost relationship led to a favorable state of affairs for higher education. Each year, as enrollments rose, many institutions were funded on the basis of the historical average cost of enrolling a student. Since this sum was presumably greater than the marginal cost of educating the increased numbers of students in existing programs and institutions, resources were available for other purposes. Postsecondary institutions were able to use these resources to improve the quality of programs, develop new programs, provide access to new clientele, and improve the relative economic position of faculty and other employees.

Now that enrollments are likely to decline, many analysts are questioning the appropriateness of average-cost appropriation and tuition formulas. Their concern is that the actual, marginal savings attributable to enrolling fewer students will be

1. The evidence for economies of scale is discussed in the section on higher education in Appendix A.

less than the estimated savings calculated on the basis of average historical costs. If so, then reducing resources at the average-cost rate may induce financial stress on the institution. At the very least, it does seem likely, given the way higher education has changed in the last generation, that expenditures cannot be reduced at the same rate as enrollments without reversing the building process of previous decades. Such a reversal could entail reducing quality, eliminating programs, decreasing access, and degrading the economic position of postsecondary-education employees. It is conceivable as well that such a reversal could lead to the closing of some institutions. While some people argue that one or more of these actions should be taken, few persons would support the proposition that they should occur as the unexamined consequence of the mechanical application of average-cost calculations to declining enrollments.

Funding and pricing schemes based on marginal-costing principles can directly address the weakness of techniques that are based on average costs. By focusing directly on the cost implications of changing enrollment levels, marginal costing allows the state or federal government and the institutions to base their actions on estimates of actual cost behavior rather than on a static calculation of costs at a particular enrollment level that is no longer applicable.

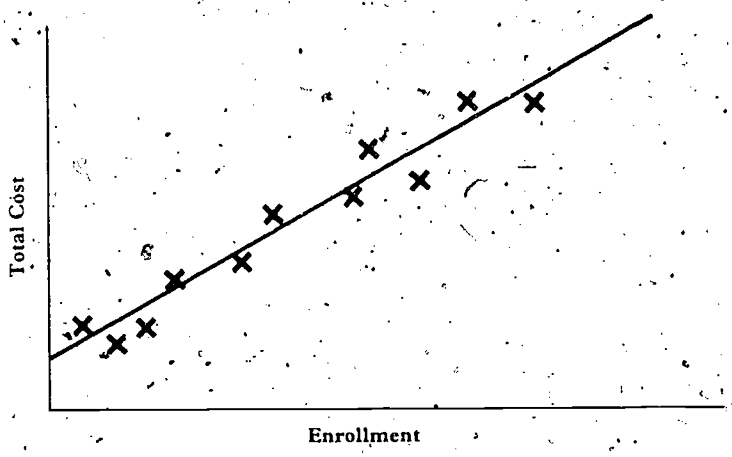
Another reason for looking carefully at marginal costing is its difficulty as a technique. Even though marginal cost is simply defined, its calculation is typically quite complicated and usually subject to a large number of potential errors. The complications are greater because higher education lacks a known or standard set of production relationships. Even if one agrees that a student credit hour in lower-division psychology is an appropriate output measure (by no means a foregone conclusion), we know very little about the technology that produced that credit hour except that it can vary (lectures, television, seminars, and CLEP can all produce credit hours). In addition, the quality and emphasis of that credit hour can and does change in ways that cannot be detected by a costing system. Because of these difficulties and others, an extended investigation of marginal-costing techniques should be useful to practicing administrators.

Three basic methods that show promise in the estimation of marginal costs will be discussed briefly. Each may be used at several levels of detail and for a variety of purposes. The three basic methods are the *regression method*, the *fixed- and variable-cost method*, and the *incremental-cost method*. An overview of each of the methods will be provided here, with more extensive treatments provided in later chapters.

The Regression Method

Regression is a statistical technique that can be used to estimate relationships between costs and output. For example, let the "x's" in figure 1.1 represent paired observations of total cost and total enrollment for a group of institutions.

FIGURE 1.1



The regression technique can be used to fit a line through the "x's." The slope of the line mathematically expresses the relationship between total costs and enrollment. Thus the slope is an estimate of the marginal cost of an additional student.

While regression applications can be as simple as that shown in figure 1.1, they can also be a great deal more complex. It may be appropriate to control for intervening variables, that is, for influences on costs other than enrollment. It may also be appropriate to use the regression technique to estimate nonconstant marginal costs, that is, marginal costs that vary with the size of the institution, department, and so forth.

The regression technique is widely used for estimating marginal costs—especially in industrial and business settings. It is adaptable to various units of analysis, can handle a large number of observations, and can readily provide a regional or national perspective on cost behavior. In principle, the technique is relatively simple. In practice, using actual data, subtleties of interpretation and representation abound, and a variety of estimation problems may occur. It is difficult to eliminate all of the cost analyst's subjectivity when using the technique. Regression-based marginal-cost estimates generally do not warrant unquestioning acceptance. Nonetheless, regression can be a powerful and relatively efficient means for estimating the general shape and position of marginal-cost curves.

The Fixed- and Variable-Cost Method

The fixed- and variable-cost method involves classifying all of the expenditures of an institution or other organizational units as either fixed or variable. Once costs have been classified this way, the marginal-cost analysis is relatively simple: the estimate of marginal cost is simply the average *variable* cost. Fixed costs are excluded from the analysis.

In the fixed- and variable-cost method, the classification of costs is primarily determined by policy. Empirical data are useful to support this effort and to measure the implications of particular policy decisions, but the fixed or variable nature of costs is not determined empirically.

The procedure used to determine the fixed and variable nature of costs begins with the disaggregation of expenditures to the point where each cost element can be classified as either fixed or variable. The most convenient disaggregation takes the form of relatively simple budget or activity categories for which financial data are available in each category. For example, object-of-expenditure data by academic department will almost always be available. In this case, administrative salaries (the department chairperson) and equipment (maintenance of the inventory needed to support the programs being offered) could be classified as fixed costs, while faculty and support salaries and supplies could be considered variable costs. While this does not match actual expenditure patterns, it may estimate them closely enough to be viable for a cost analysis.

In a variation that requires more empirical support, particular activities are designated as representing fixed costs. In this case, a certain set of courses or programs might be designated as fixed courses or programs, and expenses associated with them as fixed costs. This approach requires additional cost studies in order to allocate expenditures to courses or programs and thus requires more data and manipulation of data than the object-of-expenditure, organizational-unit approach.

The fixed- and variable-cost method has a number of important practical advantages. First, it explicitly requires consideration of the decision factors that affect costs. Second, it is an intuitive, understandable process. Institutional operations are broken down into easily understood components and are dealt with individually. Decisionmakers can either deal with the aggregate estimates of fixed and variable costs or, if appropriate, with the more detailed analyses that lie behind the final policy decisions. A third advantage is the ease of adjusting the cost analysis as experience accumulates. Actual costs for the different categories used can be monitored, compared to enrollment changes, and reevaluated relative to their fixed and variable nature.

The fixed- and variable-cost method also has a number of inherent disadvantages. Chief among these is that it requires a great deal of financial data, often including a cost-allocation study. It also requires that administrators and political decisionmakers pay special attention to the necessary political arrangements. In addition, it requires central decisionmakers to evaluate subunit operations at a lower level of detail than is common or appropriate, given the largely decentralized nature of higher education. Finally, this method typically lacks an empirical base. Although a great deal of financial data is necessary to translate this concept into a cost analysis, little data is applicable to the initial classification of fixed and variable costs. This step is almost entirely political and judgmental.

The Incremental-Cost Method

The incremental method estimates directly the cost behavior related to changes in volume at a single institution or its subunit. Each annual change in total costs is assessed to determine if it is most appropriately associated with changes in volume, with changes in the environment, or with specific decision factors. Cost differentials associated with environmental or decision factors are removed from the analysis, and the residual is divided by the change in volume (that is, in the number of students, credit hours, and so on). The result of this calculation can be used as an estimate of the marginal costs of additional students within that range of volume changes. For example, it might be observed that the costs of operating an academic department rose by $100,000 from one year to the next, while the number of student FTEs generated fell by six. Further analysis could show that $90,000 of the increased cost was attributable to inflation (an environmental factor) and $19,000 was attributable to the decision to add a subspecialty and a new faculty member to the department (a decision factor). The residual of −$9,000 can be considered the effect of volume changes. Dividing this amount by the change in enrollment (−6), the analysis yields a marginal cost of $1,500 per student for this range of enrollment change. The accuracy of the estimate could be improved by making the calculation over several years or by making calculations for several institutions.

The incremental approach to marginal costing has several advantages. It is a theoretically simple method: it is related directly to the highly regarded classification of institutional cost behavior developed by Robinson, Ray, and Turk (1977), and it can be readily applied to a variety of uses. It also has a number of significant disadvantages. Foremost among these is the difficulty of actually calculating marginal costs in this manner. Separating environmental and decision factors from volume factors is an extremely complex task. Although some progress in this task has been made in the industrial sector, the situation there is generally simpler than in higher education. This is especially true in comparison with the instruction function in which complications such as joint products, nonstandard and unknown production methods, and a craft-industry approach ("workers" produce a whole product as they see fit) confuse the issue.

This completes a brief overview of the three marginal-costing methods that are the chief concern of this document. In the next chapter, several aspects of the microeconomic theory of costs are discussed. Chapters 3, 4, and 5 deal at length with the regression, fixed and variable, and incremental methods, respectively. In the appendix, we have included a literature review covering marginal-cost studies in industry, hospitals, primary and secondary schools, and higher education.

2

Cost Theory

The term *marginal cost* is becoming increasingly familiar among higher-education administrators. It is unlikely, however, that the term has a common meaning among its many users, particularly regarding its details. The basic concepts of marginal cost were developed as part of the economic theory of the firm. A review of certain of those concepts in their original setting establishes a point of departure; they are subsequently interpreted within the milieu of colleges and universities.

The Microeconomic Framework

The theory of the firm usually begins with a discussion of the technical relationships of production, that is, the relationship between inputs and outputs. A firm's technology is summarized and given mathematical expression in what is called a *production function*. In the simple case where a firm produces one output from two inputs, the production function can be represented as

$$Q = f(X_1, X_2) \tag{2.1}$$

in which Q is the maximum level of output obtainable from any possible combination of input levels X_1 and X_2. In other words, a production function is similar to a completely efficient production plan in which it is not possible to produce more output with the same input, or to produce the same output with fewer inputs (Varian 1978). Occasionally, the production function will be referred to as the *production frontier*, an expression that perhaps better reflects the underlying assumption of output maximization.

Note that the production function as represented in equation 2.1 is stated in *implicit* fashion. An *explicit* version, or model, is required if a firm's production structure is to be estimated using a production function. This means that we must be able to specify not only the variables that influence output, as is done in equation 2.1, but also the particular way in which they relate to output and to each other.

Turning now to costs, and using our same simple example, we start by stating the cost equation

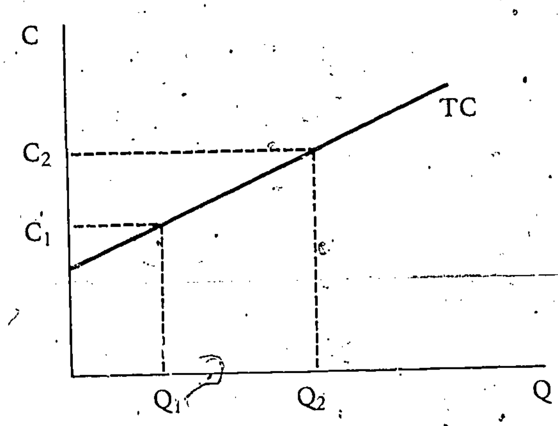
$$C = W_1X_1 + W_2X_2 \tag{2.2}$$

which says that total costs, C, are the sum of the quantity of each input, X_1, times its price (or wage), W_1. This relationship is an accounting identity. We also can say, though, that

$$C = f(Q) \tag{2.3}$$

that is, that total cost is some function of output. If f(Q) minimizes $W_1X_1 + W_2X_2$, subject to the production function $Q = f(X_1,X_2)$, then equation 2.3 is a cost function. In other words, a cost function yields a set of values that represent the minimum cost of production at each level of output. We can represent the cost function geometrically by plotting these values against output levels, as in figure 2.1. The line TC can be referred to as the firm's *total cost curve.* As the firm expands its level of output from Q_1 to Q_2, its total cost of production increases from C_1 to C_2.

FIGURE 2.1



As was true for the production function, a cost function must be stated in an explicit form if it is to be used to estimate the structure of production. Or, in other words, if we are to estimate what a particular cost curve looks like for a given firm or industry, we have to specify a particular form for the cost function. This can be done directly, or the form can be derived mathematically from the production function, providing the latter is available in explicit form. According to economic theory, cost and production functions are equivalent ways of representing a production structure. Thus, the production function can be derived from the cost function, and the converse is also true. The point for our purposes, however, is that we must have a thorough grasp of the underlying production process if we are to correctly specify either of the two functions.

Before extending our discussion of cost functions and cost curves, it may be useful to again emphasize that notions of optimization and the availability of adequate knowledge are at the heart of the classical theory of the firm. The model firm is not only profit-seeking, but profit-maximizing. The firm has enough knowledge of its productive activities to enable it to best utilize any particular input or combination of inputs. It can and does select the best, that is, profit-maximizing, input combination for the production of a particular output level (on the basis of input and output prices). In short, the traditional assumption in economics is that the managers of a firm have the intent and the ability to optimize, that is, either to obtain the greatest possible output for a given cost outlay or to minimize the cost of producing a prescribed level of output. The cost functions and cost curves under discussion here are a product of that optimizing behavior so far as the economic model is concerned, and are interpreted accordingly.

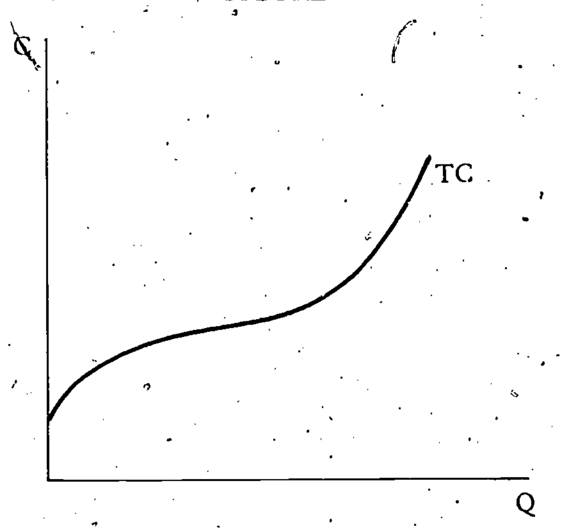
Additional aspects of cost-function theory can best be discussed using an explicit functional form. Suppose the total cost function was specified as

$$C = a + b_1Q - b_2Q^2 + b_3Q^3 \qquad (2.4)$$

in which C is total cost, a is estimated fixed cost (the cost of fixed inputs), Q is level of output, and b_1, b_2, and b_3 are a set of estimated parameters relating output levels to total cost. Figure 2.2 shows the general form of this cost curve. Average total cost (ATC) is just total cost divided by Q, or

$$ATC = \frac{C}{Q} = \frac{a}{Q} + b_1 - b_2Q + b_3Q^2 \qquad (2.5)$$

FIGURE 2.2

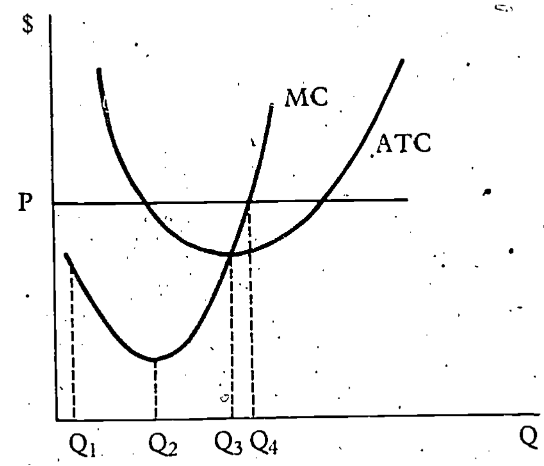


Marginal cost (MC) is the change in total cost associated with an additional unit of output, which can be expressed as the first derivative of total cost with respect to output, or

$$MC = \frac{dC}{dQ} = b_1 - 2b_2Q + 3b_3Q^2 \qquad (2.6)$$

Equations 2.5 and 2.6 will yield U-shaped cost curves such as those shown in figure 2.3.

FIGURE 2.3



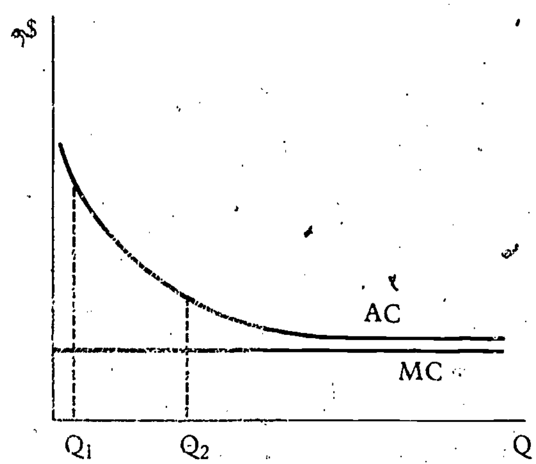
The cost curves shown in figures 2.2 and 2.3 meet the theoretical expectations for a model firm. At relatively low levels of output, Q_1 in figure 2.3, the firm finds it cost-effective to increase output. The cost of each additional unit of output, that is, the marginal cost, is less than the average cost per unit; thus, additional output will pull down average costs. At some point, Q_2, marginal costs begin to increase. Eventually, they equal and then exceed average costs (Q_3). As the level of output continues to increase, average costs increase along with marginal cost. The model firm continues to expand production until Q_4, where MC equals P, that is, where the cost of an additional unit of output equals the price charged by the firm for a unit of output.

Costs can be either short-run or long-run. Short-run costs are said to be incurred when one or more factors of production (inputs) are taken as fixed. Costs that refer to a period of time within which no factor of production is fixed are referred to as long-run costs. In the theory of the firm, it is assumed that most fixed costs relate to the physical plant, so we can think of a short-run cost curve as representing cost behavior when the firm produces different levels of output with a given plant size. A long-run cost curve, on the other hand, represents cost behavior when the firm produces different levels of output with different plant sizes.

Theoretically, it is expected that both the short- and long-run average and marginal cost curves will be U-shaped. In the short-run case, diminishing marginal returns from the variable inputs are thought to eventually counterbalance and then overcome the advantages of spreading out fixed costs over an increasing level of output. In the long run, it is thought that although increases in scale may lead initially to various technical efficiencies, which would lower unit costs, eventually increases in scale are likely to result in management inefficiencies, which would drive up unit costs.

Empirically estimated cost curves have taken various shapes. While some have turned out as expected, the predominant results have been closer to those shown in figure 2.4 for both short- and long-run cost curves (Mansfield 1979).

FIGURE 2.4



Note that the meaning of a horizontal marginal-cost curve such as that shown in figure 2.4 is that an *additional* unit of output could be produced at the same cost at any existing firm no matter what its size. Or, in terms of higher education, assuming that enrollment was the output measure, such a finding would mean that any existing institution could accommodate additional students as cheaply as any other existing institution. As Thompson (1980) points out, this should be kept in mind so that the analyst will not be misled by the declining average-cost curve. A particular institution with enrollment Q_1 in figure 2.4 will indeed experience lower per-student costs, or *economies of scale,* if its enrollment goes to Q_2; but for a system of institutions, it would make absolutely no difference in the addition to total system costs whether the additional students enrolled at a small institution or at a large one.

Interpreting Marginal-Cost Estimates in Higher Education

In estimating marginal costs in higher education, it seems reasonable to make use of the concepts developed by economists. At the same time, it makes little sense to use those concepts in an unquestioning manner. Their meaning is quite precise and highly dependent upon a particular view of organizational behavior. To the extent that the behavior of colleges and universities does not conform to that view, the concepts are likely to require reinterpretation if they are not to be misleading.

The first task, then, is to consider some of the ways in which colleges and universities operate that have a bearing on how we are to understand marginal-cost estimates in higher education. The items that follow appear to be influential from both a short- and long-run perspective, although some may be more important in one of those dimensions than another.

1. The *goals* of higher education do not include cost minimization as such. Virtually all colleges and universities value quality and excellence; many value prestige, influence, and large enrollments (Bowen 1980). In this milieu, high costs per student are not necessarily undesirable. They are quite likely to be viewed as an indication of success by those within the higher-education community, and perhaps by others as well. It is not that waste and profligacy are desirable ends; rather, high costs are viewed favorably because they are often associated with high-quality operations, prestigious programs, and advanced levels of instruction.

   There is some evidence to suggest that colleges and universities do become more cost conscious in periods of financial stress (for example, see Freeman, 1975, on the practice of hiring less-expensive faculty during such periods). Indeed, after the inflation-driven difficulties of the 1970s, and with the prospects for a decrease in demand during the 1980s, most institutions have probably done some belt tightening and intend to do more. Cost reduction thus motivated is one thing, however, while cost minimization as a basic goal of an organization is quite another.

   The latter point is worth pursuing further. We have good reason to believe that the typical profit-making organization is not minimizing costs either. Organizational slack, for example, is thought to be present in virtually all organizations, regardless of whether or not monetary profit is their goal (Simon 1957). The point, though, is that when profit is a goal, there is at least a basic reason why cost minimization (at a given level of output) could be considered inherently valuable. No such fundamental link exists between cost minimization and the goals of higher-education institutions. Its absence must be taken into account when interpreting marginal-cost estimates.

2. The *production process* in higher education is complex and not well understood—and apparently rather flexible. It is complex in that multiple products and services are provided, and these products and services are often generated by joint processes. The typical faculty member advises, teaches, researches, writes, and often administers as well. The library serves students, teachers, and researchers. This kind of arrangement just does not look at all like the single-output firm that is the paradigm in most economic models.

   The production process is not well understood in two senses. First, the outcomes of the process are neither clear, nor agreed upon, nor easily measured. Secondly, the connection between inputs and outcomes remains mysterious. There is plenty of conventional wisdom on both topics, and controversy too, but not much hard data, or even a well-articulated theory. Under these circumstances, discovery of a comprehensive, meaningful production function, as economists use the term, is simply out of the question.

At a gross level of analysis, there clearly seems to be flexibility in the production process. That is, it appears that roughly the same products and services can be provided in a variety of ways. For example, student-faculty ratios and average class sizes can differ considerably with little apparent effect. It is difficult, however, to establish the limits of this apparent flexibility because there is a lack of a thorough understanding of the production process.

What we do know points to severe problems of definition and assessment. How, for example, do we go about assessing the differential impact of substituting one teacher for another? The credit hours produced will be the same, but will the output along other dimensions necessarily be the same? Probably not, but how can we be sure when many of the other possible outcomes are difficult to agree upon and assess, much less measure, in a fashion suitable for a cost analysis?

In view of the characteristics of the production process just discussed, it seems obvious that it would be impossible to trace all of the effects of tinkering with the cost structure of a college or university. To put it another way, even if cost minimization were a goal for higher-education institutions, the goal would be virtually impossible to pursue in a rational manner because of computational problems. This too must be kept in mind when considering marginal-cost estimates in this context.

3. Because of the nature of the goals and production process in higher education, it is to be expected that per-student costs are, at least in part, a function of the amount of *revenue* available. That is, higher unit revenues will tend to be accompanied by higher unit costs, and lower unit revenues by lower unit costs, other things being equal. However, the extent of the influence of revenues on costs is not obvious. It has been argued by some, most recently by Bowen (1980), that in the short run, the major, if not sole, determinant of unit costs is the amount of available revenue (granted a given enrollment level). That assessment may or may not be entirely accurate. It presumes that in the short run output typically remains constant in the face of revenue changes. There is no easy way to tell whether that is in fact true in the absence of a clear understanding of the relationship between inputs and outputs in higher education. Intuitively, it does seem likely that there is a range within which input levels (such as number of support staff) can be altered with little effect on output. To put it another way, it seems reasonable to assume that for modest changes in revenue the extent of organizational slack is more likely to change than either product quality or product mix. One might also argue, however, that there must be some limit, or break point, at which outputs typically begin to change in response to revenue availability, even in the short run.

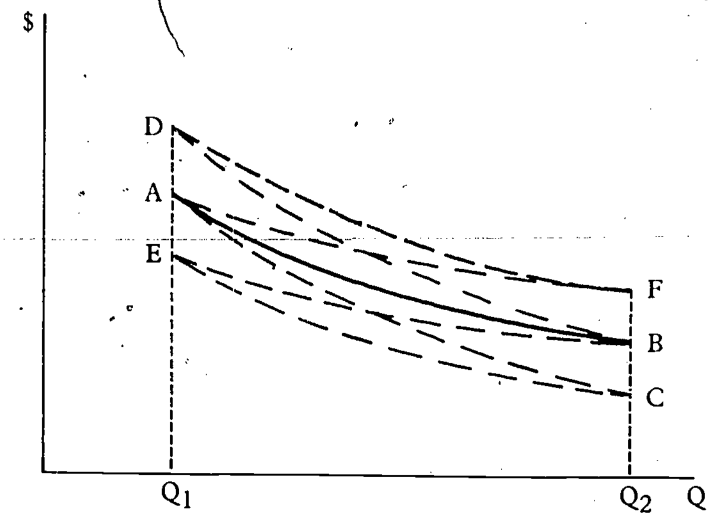
Although a more precise expression of the relationships among cost, revenue, and output would be welcome, it is not required here. Rather, it is sufficient to note that differences in the extent of available revenue—for a given institution over time or between institutions at a point in time—are likely to confound the relationship between cost and output. Such differences are likely to be especially troublesome for estimates of short-run marginal costs.

On the basis of the three factors discussed above—goals, production complexities, and the influence of revenue—it is clear that the typical higher-education institution does not closely resemble the ideal firm of economic theory. This lack of congruence between theory and reality makes it imperative that we reinterpret the key economic concepts—particularly that of marginal cost—that are at stake here. The remainder of this chapter is devoted to that task.

Ideally, from the standpoint of economic theory, one would begin an analysis of marginal costs by specifying the explicit form of the production function. Then, from the production function, a cost function could be derived using the cost minimization assumption. Estimating that cost function by some statistical technique such as regression would yield marginal-cost estimates in the full, economic sense of the term. As we have seen, however, it is unrealistic to expect to be able to specify an appropriate production function for higher education. The production processes are too complex and our knowledge is too limited. Furthermore, the optimizing behavior that is part of the very definition of both production and cost functions in the economic model is not likely to be an accurate representation of what goes on in colleges and universities. Thus, strictly speaking, the terms "production function" and "cost function" ought not to be used in reference to specifications of input-output and cost-output relationships in higher education. Since, however, the terms are used, the practical approach is to label such functions as "approximate" (Cohn 1979).

What does approximation mean for interpreting cost functions and marginal-cost estimates in higher education? Under the optimization assumptions, the ideal firm is *bound* to its cost curves. All such firms in an industry are on the curves somewhere, with their precise location being determined by the level of their respective outputs. If a firm should change its level of output, it could be expected to move along the curve in a predictable way (in either direction). By contrast, because of flexibility in the production process, computational difficulties, and so on, the relationship between the behavior of higher-education institutions and an estimated-cost curve may be relatively weak. The term *approximate* is a way of acknowledging the weakness of that relationship. It is a way of acknowledging that in terms of the underlying fundamentals a given institution need not be on the curve; that is, it need not have a particular marginal cost at a particular enrollment level, because the estimated curve typically represents only average behavior rather than the definitive, least-cost solution of the economic model. In other words, the predictive power of an approximate-cost function is less than that of a true-cost function, and it is so with respect to current costs at a given institution, future costs

FIGURE 2.5



at a given institution, and even future costs for the industry as a whole.

Consider the cost curves displayed in figure 2.5. Suppose that the solid curve on which A and B lie has been estimated from empirical data on instructional costs and enrollment for a group of institutions. It is important to realize that the curve represents something other than a necessary relationship between cost and enrollment. For example, it is conceivable that the curve could have gone from A to C instead of from A to B had the institutions been less interested in expanding their curricula or having more graduate education, and so forth, as their enrollments grew. Conceivably, AC could also have resulted had the funders been less generous in supplying revenues. Similarly, if we use a cost curve to help us envision the future, there is little reason to think that an institution at B will end up at A should its enrollment go from Q_2 to Q_1. If we look at curve AB as representing an approximate expansion path for an institution, then we could say that the contraction path may not, indeed probably will not, resemble the expansion path. The institution's response to enrollment decline will tend to move costs up toward D. The funders, especially for public higher education, are likely to find E more appealing. The vertical distance from E to D or from C to F could be said to represent the amount of flexibility and slack in the production process. Advocates of the revenue theory of cost will argue that ED and CF are large, based on their judgment that the unit cost of producing the same output is quite variable. Others will insist that the area of flexibility is a rather narrow band along AB, based on the assumption that only modest changes in unit costs can occur before output begins to change. Since we lack a thorough understanding of the production process, it is apparently not possible to assess accurately the width of the band. The number of students, as represented along the horizontal axis, is in reality an enormous simplification of the output from the instructional process. We simply are unable, at this point, to grasp fully the difference in *value added* between a student in an institution whose unit costs are at F, for example, and a student in an institution whose unit costs are at C.

Figure 2.5 has been drawn in such a way that AB looks as though it represents a kind of average behavior, which, as noted earlier, is what estimated-cost curves in higher education typically do represent. This is also true with respect to hospital-cost functions (see, for example, Pauly 1978) and perhaps also for most other industries (see appendix). The chapter that follows on the regression technique will discuss a standard approach to estimating average institutional behavior. For now, imagine that the band between DF and EC contains various cost-output data points, and that EC was constructed by connecting the lowest data point at each observed level of enrollment. EC would then represent a "frontier" that could be estimated using frontier-analysis techniques. Examples of the procedure are few. In higher education, the best examples are a study by Carlson (1972) in which he estimated average and marginal costs at the frontier for various types of colleges and universities, and a study by Gray (1977) in which he estimated the production frontier for a group of chemistry departments. Truehart and Weathersby (1975) estimated production functions using traditional averaging methods, but did so for a sample of black institutions that had been previously determined to be operating at the production frontier.[2]

Which is to be preferred, an estimate of average marginal cost or an estimate of marginal cost for institutions at the frontier? It is not clear whether such a question can be answered definitively in view of the present state of our knowledge about the production process in higher education. Some analysts will see line EC (figure 2.5) as representing something desirable, specifically the behavior of the most *efficient* institutions in the group. Others might see EC as representing the behavior of those institutions that have been the most *underfunded*, an undesirable condition. And some analysts simply believe that average estimates are the most appropriate for policy analysis (Verry and Davies 1976). These opposing views must remain unresolved, it would seem, in the absence of a reasonably thorough grasp of how the educational process actually works.

Of the methodologies discussed in this document, only the regression technique is explicitly geared toward estimating average behavior. The incremental and fixed-variable techniques have no inherent relationship to the average-versus-frontier distinction. There is a version of regression called *constrained-residuals regression* that does force the estimated cost function to the frontier of observed behavior. The absence of sampling theory for this sort of regression analysis, and the general lack of familiarity with it, limit its value for present purposes. Those who are interested in estimates at the frontier might well consider the previously mentioned procedures used by Truehart and Weathersby (1975). The crux of the matter is finding a group of institutions or departments that have been somehow shown to be relatively efficient in some agreed-upon sense. Then their behavior can be statistically analyzed using the standard regression technique that is discussed in the following chapter. The net result should not be markedly different from a more direct approach to estimating frontier behavior.

2. See Carlson (1975) for a discussion of linear programming techniques that can be used to determine the production frontier.

3

The Regression Method

In this chapter, we explore the use of regression analysis as a means of estimating marginal costs. Using regression for this purpose is not new. It has been used to some degree in higher education, and it has been used extensively in other sectors of the economy. A discussion of cost studies that have used statistical techniques such as regression can be found in the literature-review section (appendix). The intent of the present chapter is to discuss several of the most basic aspects of regression analysis in relation to marginal costs, and to weigh the merits of using regression-based marginal-cost estimates as part of the process of understanding cost patterns in higher education.

Definition

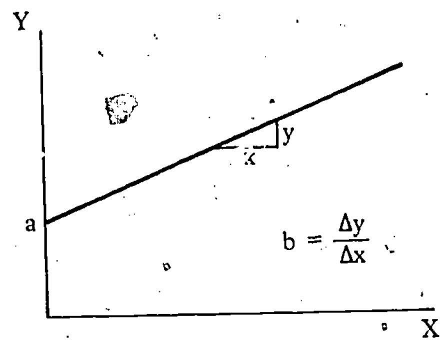
A variety of regression-type techniques have been developed. The discussion here is limited to the basic form known as "ordinary least-squares regression" (OLS). The discussion is also limited to the most direct use of regression in estimating marginal costs, namely, as a technique for estimating the parameters of total cost functions.

As noted in the previous chapter, the term *cost function* denotes cost expressed as a function of output. We can write

$$C = a + bQ \tag{3.1}$$

in which C is total cost; a is fixed cost (that is, the cost of fixed inputs, if there are any); b is a coefficient relating Q to C; and Q is the level of output (which can be expressed in terms of units produced, units of service, or units of activity). Using regression to estimate marginal cost is simply using that particular statistical technique to estimate the parameter b, as will be shown in what follows.

FIGURE 3.1



Note that equation 3.1 has the same *form* as the equation of any straight line. The latter equation is usu lly expressed as

$$Y = a + bX$$

where Y and X are values on the Y and X axis, a is a constant indicating where the line intercepts the Y axis, and b is the slope of the line. These relationships are displayed in figure 3.1. If a and b are known, then the location of the line is known, and we can calculate the value of Y for any value of X. In figure 3.2, we replace Y and X by C and Q. Then b, the slope of the line, represents the change in total costs associated with a change of one unit in the level of output; that is, b represents the marginal cost of a unit of output.

Of course, b, and a as well, are usually not known. They have to be estimated. If we make paired observations of total costs and levels of output in a given industry, and plot those paired observations as a series of points on graph paper, we will typically end up with a scatter of points such as that in figure 3.3. OLS is a technique that is used to fit a line through a set of points in such a manner that the line represents a kind of average relationship between C and Q. The regression line, or line of *best fit*, is found by the method of least squares. This method locates the line in a position such that the sum of squares of distances from the points to the line taken parallel to the Y axis is a minimum. Equivalently, if we think of the regression line as representing predicted values of C for corresponding values of Q, then we can say that the method of least squares minimizes the sum of the squares of the *residuals*, that is, the differences between the predicted and the actual values of C. The parameters a and b are simply the solution to the minimization problem.

Ideally, the spread of the scatter plot, that is, the fact that the points are typically not on the regression line, will be due entirely to random error. The wider the spread, the less accurately will the function predict the value of C for a given value of Q. Note, however, that the slope of the line, and thus the marginal-cost estimate, is not affected (not *biased*, technically speaking) by random error if there is a sufficiently large number of data points.

FIGURE 3.2

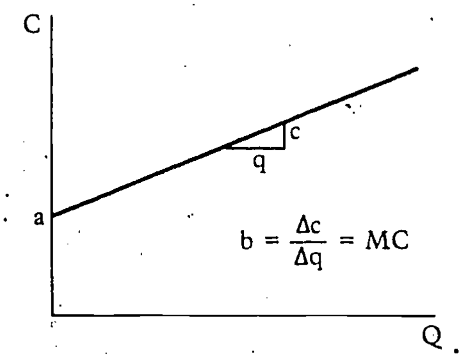
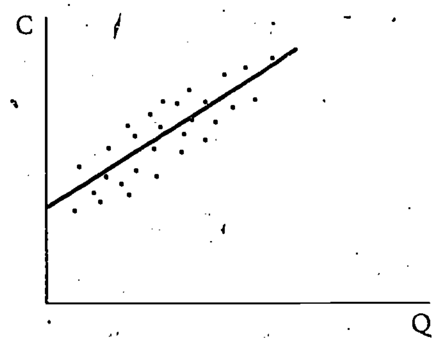


FIGURE 3.3



The presence of error is explicitly acknowledged when the function to be estimated is written in stochastic form. Thus, properly speaking, the estimated function is not

$$C = a + bQ \tag{3.1}$$

but rather

$$C_i = a + bQ_i + u_i \tag{3.2}$$

where, for the ith unit of analysis (for example, a firm, plant, department), u is an error, or disturbance, term. It is assumed that the error terms have a mean of zero, have the same variance throughout the range of C values, and behave randomly relative to the independent variables in the regression equation and relative to one another (that is, from one observation to another). Analysis of the residuals will reveal when one or other of the assumptions is not met, in which case corrective steps are in order.

Up to this point, we have been describing *simple* regression, that is, the estimation of the relationship between a dependent variable and one independent, or explanatory, variable. In equation 3.2, those variables are C and Q respectively. Ordinary least-squares regression allows only a single dependent variable within an estimating equation. There are no restrictions on the number of independent variables, though, other than the restrictions imposed by sample size (that is, the need to retain at least some degrees of freedom). *Multiple* regression is a technique for estimating the magnitude of the variance of the dependent variable that is shared with several independent variables. The method of least squares is employed in fundamentally the same manner as it is in simple regression. More things can go wrong in multiple regression, statistically speaking, essentially because of possible interrelationships among the independent variables. This will be discussed at greater length later. For now, consider the following cost function:

$$C = a + b_1Q + b_2Q^2 + b_3Q^3 \tag{3.3}$$

There are several points to be made about estimating this function by OLS. First, multiple regression will generate estimates of a, b_1, b_2, and b_3. Where is the marginal-cost estimate? Recall that marginal cost is the change in C associated with a change of one unit in Q. Once we have the estimates of the parameters, we can find the relationship between the change in C and the change in Q by taking the first derivative of C with respect to Q. For equation 3.3, the value of that derivative, and thus the marginal-cost estimate, is

$$\frac{dC}{dQ} = MC = b_1 + 2b_2Q + 3b_3Q^2 \tag{3.4}$$

Second, note that in the case of a simple linear regression of cost on output as in equation 3.1, the marginal-cost estimate is constant because the first derivative of equation 3.1 with respect to Q is simply b. By contrast, when a cost function such as equation 3.3 is estimated, the resulting marginal-cost estimate *varies* because the value of the estimate is a function of the variable Q.

Third, although ordinary least-squares regression is a linear estimation technique, it can be used to estimate nonlinear relationships provided that the estimating equation is linear in the parameters. Indeed, that is what is happening when equation 3.3 is estimated by OLS. Equation 3.3 allows for curvature, that is, nonlinearity, in the estimated-cost curve. Nonlinearity is introduced by using powers of the variable Q, while retaining a linear form for the parameters a, b_1, b_2, and b_3. If the signs on the estimated parameters in equation 3.3 are appropriate, that particular cost function will generate the U-shaped marginal-cost curve discussed in chapter 2. In short, OLS is quite versatile with respect to the kinds of relationships it can estimate.

Data and Analytical Requirements

The two basic pieces that come together in the methodology described in this chapter—total cost functions and regression—are both quite general. So long as we can identify costs and their related output, we have the basic data needed for estimating a cost function. Whatever that cost function might be, it can be estimated by OLS so long as the linearity restriction and several assumptions are somehow met. Having said that, we need to consider the kind of data and the kind of relationships that are truly appropriate to the methodology in question.

At a minimum, data are needed on costs and output. Ideally, the data used would be direct measures of the variables in the analysis; however, indirect measures and substitutes of one kind or another are frequently quite acceptable. Indeed, much of the ingenuity needed to do applied social science has to do with finding acceptable proxies for measurements that are not directly obtainable. In regard to the marginal-cost question, a typical approach in higher education is to regress expenditures on numbers of students. From a formula-funding perspective, those variables do reflect the matter at issue. A much more fundamental question could be addressed by regressing opportunity costs on units of value added; unfortunately, widely accepted and available measures for those variables have yet to be developed.

The more precisely that cost can be related to outputs the better. For example, if one were interested in the marginal cost of a graduate student in the humanities, it would be preferable to work with cost data for graduate instruction in the humanities rather than cost data for all graduate instruction, or worse still, cost data for all instruction at all levels, and likewise for data on numbers of students. The difficulties of relating costs and outputs should not be underestimated.[3] The choice of the unit of analysis, such as departments, institutions, and so on, depends primarily on the issue to be addressed and on the availability of data.

Regression analysis requires repeated observations—generally, the more observations the better. The observations can be of the same unit of analysis at different times, or of similar units at the same time, or a combination of the two, that is, a combination of time-series and cross-sectional data.

The data need to be compatible. For example, the cost of instruction should cover the same items in both instances. At least some analysts (for example, Dean 1976) argue that a cross-sectional analysis is usually less hazardous in this regard than is a time-series analysis, although one might expect the relative degree of hazard to differ from one industry to another. It is clear that a cross section does have the advantage of being more likely to include a wide range of behavior, which, as a rule, is desirable in a regression analysis.

As indicated at the beginning of this section, data on costs and output are the minimum data requirements. It is likely that more data will be needed. In order to estimate correctly the relationship between cost and output in a cost function, the relationship must be isolated; that is, any influences that might distort the estimate must be removed. Following the standard microeconomic approach, this involves controlling, or neutralizing, differences in the prices of inputs and in the quality of outputs (Johnston 1960). This is usually done by adding suitable independent

3. See Thomas (1982) for an illustration of the problem as it relates to faculty activity.

variables to the cost function and then performing a multiple-regression analysis. In other words, the cost function, or model, takes on the general form

$$C = a + bQ + c_j X_j \tag{3.5}$$

where X is a vector representing whatever control variables are added to the equation. The regression procedure neutralizes the influence of these variables by holding them constant at their mean values while estimating the coefficients on the other variables in the equation. In higher education, finding variables that can legitimately be used to control for price differences over time or across regions is usually not a problem. One can choose among measures such as the Consumer Price Index, the Higher Education Price Index, average faculty salaries, and so on, depending on the specific function being estimated. Of course, just the opposite is true for indexes of quality. Lawrence and Green (1980), for example, recently reviewed a broad range of attempts to measure quality and concluded that an acceptable index of quality has yet to be developed.

Textbooks on the economic theory of the firm generally deal with simple output models. By contrast, colleges and universities are multiproduct institutions, even when the analysis is restricted to a single function such as instruction. This fact cannot be ignored in putting together a cost function. In the case of instruction, for example, differences in instructional level and in program emphasis would likely distort the marginal-cost estimates if the differences were not controlled in some manner.

As is true of any technique for estimating marginal costs, the data should be reasonably accurate and reliable. Obtaining such data can be a problem, of course. Different persons may count things differently, and other sorts of measurement errors can be expected as well. So long as such errors are random, however, they typically will be averaged out by the regression procedure.

It should be obvious at this point that a particular cost function for higher education may contain a large number of independent variables. Note, however, that it is possible to gain control over at least some of the differences among the units of analysis by appropriate sampling. In a cross section, for example, one might pick just those institutions that are predominantly engineering schools, or that are located in a region of the country where similar prices obtain.

Selecting the appropriate variables for the cost function is not the only analytical task. The function has to be given the proper form. Consider the following cost functions:

$$C = a + bQ + c_j X_j \tag{3.5}$$

and

$$C = a + b_1 Q + b_2 Q^2 + b_3 Q^3 + c_j X_j \tag{3.6}$$

The difference between them is one of functional form. The variables are the same in both equations, but the way in which the variable Q is represented is obviously different. Estimating different functional forms amounts to fitting different sorts of curves through the same scatter of points. Picking the right form is neither easy nor unimportant. The list of possible functional forms is virtually endless, and there is no strong theoretical guidance for selecting any particular form as being the most appropriate. The selection will make a difference—possibly a great difference—in the marginal-cost estimates. Both the dollar value of the estimate at a given enrollment level and the general shape of the marginal-cost curve can be expected to vary with the choice of functional form.

Fortunately, choosing a particular form for the cost function need not be an arbitrary decision. There are curve-fitting techniques available to determine empirically which of the possible curves actually fit the data best. Criteria such as the statistical significance of the marginal-cost estimates and the overall explanatory power of the model (R^2) are typically used to select the best model. Several caveats, however, need to be made. It cannot be assumed, going into a curve-fitting process, that the results will not be ambiguous or not be too close to call—particularly from the perspective of having to make important decisions based on the estimates. In addition, since curve fitting is a way of making the most out of the sample being analyzed, standard statistical tests (t scores and F statistics) take on a different meaning in the process. Ideally one ought to have another sample on which to test the functional form chosen by a curve-fitting technique so as not be be misled by sample idiosyncrasies.[4] Comparing R^2 values can also be misleading; if the nature of the error terms in the alternative models is not the same, the meaning of the respective R^2 values for the models is not the same either, and thus those values cannot be compared in a straightforward manner. For example, it is inappropriate to compare directly the R^2 values for a logarithmically transformed model and the original model. Accounting for the variation in the logarithm of cost is quite different from accounting for the variation in cost. Of course, one can regress cost on the antilog of the predicted values for cost in the logarithmic model, and then compare the resulting R^2 value with the R^2 value in the original untransformed model.

The analytical task, then, can be summarized as follows: (1) selecting a unit of analysis that is appropriate for the issues to be resolved; (2) finding a sufficiently large sample of comparable units of analysis; (3) selecting and developing variables that are suitable for a total cost function; and (4) determining which functional form or model best represents the cost-output behavior under investigation. The data-related issues are as follows: (1) the minimum requirement is data on related costs and output; (2) the more precisely that costs and output are related the better; (3) control variables are likely to be needed to enhance comparability between the units of analysis; (4) the more accurate and reliable the data the better; (5) it must be reasonable to assume that any measurement or reporting errors are random errors; and (6) direct measures should be used when available, but indirect measures can be valid as well.

4. See Klitgaard et al. 1981 for an excellent discussion of the issue within the context of evaluating regression-based studies of public policy.

Strengths and Weaknesses

The regression technique for estimating marginal costs in higher education has numerous strengths and weaknesses. Positive features of the technique will be discussed first, but it is perhaps worth noting from the beginning that many of these features have corresponding negative qualities.

Although most of the regression-based marginal-cost studies have occurred in sectors other than higher education, the extensive prior use of the regression technique for estimating marginal costs has several advantages: (1) the statistical properties of the procedure are well understood; (2) the user group is large; (3) comparative results are relatively easy to find; and (4) there has been ample opportunity to demonstrate the effectiveness of the technique for at least the general sort of economic analysis under investigation here.

Regression also has the advantage of being relatively easy to do. The basic principles are not difficult to understand. The required data are likely to have been collected previously for other purposes, although the degree of likelihood will vary in relation to the unit of analysis. Institutional data, for example, are readily available through the National Center for Education Statistics, whereas departmental data are much more difficult to obtain on a wide-ranging basis. Another reason why a regression analysis is easy to perform is that the proliferation of computers and statistical software packages has eliminated what would otherwise have been rather tedious calculations.

An important advantage of regression is that it readily handles a large number of observations. Practically speaking, there is little difference between processing a large versus a small number of observations. This capability makes it easy to take advantage of the law of large numbers, that is, to overcome idiosyncrasies or errors in the data through averaging.

The generally good prospects for data availability along with the capacity for handling large amounts of data make regression well suited as a means for developing a regional or national perspective on cost behavior. In the absence of hard and fast production relationships, such perspectives would seem to be valuable for policy analysis. In addition, the inclusion of a regional or national perspective is one way of diluting the circularity of estimating cost functions within a single state in which common funding patterns have been rigorously applied.

For the same reasons, regression is well suited for estimating long-run marginal costs. The cost curve for a cross section of institutions that are quite different in size has typically been interpreted as the expansion path of a single institution over the long run. This concept would be quite difficult, or at least tedious, to address using the incremental approach. Any advantage here for the regression technique depends, of course, on the extent to which a long-run marginal-cost perspective is useful for a particular cost analysis.

Another useful feature of the regression technique is that it contains built-in indicators of how well the particular regression analysis works in a given situation.

Performance indicators for the marginal-cost estimates (t scores, standard errors) and for the model as a whole (R^2, sum of squared residuals, behavior of residuals) are part of the standard statistical output for a regression analysis.

Finally, a cost function that is developed to estimate marginal costs conceivably could be used as a kind of funding formula. It is more likely and more appropriate, though, that such estimates would be incorporated within a funding formula as a way of adjusting a base level of funding rather than for generating the entire package.

The regression technique also has several disadvantages with respect to estimating marginal costs in higher education. They stem from both the technique itself and the educational process to which the technique is applied.

While it is true that regression is widely used, familiarity with the technique is clearly no guarantee that it will not be misused. Indeed, the ease with which a regression analysis can be run is an open invitation to its use by those whose understanding is at best superficial. The computer cannot tell the difference between a solid regression model and something less than that. Thus the chances that a regression-based cost study will be misinterpreted are certainly significant. There are opportunities for misrepresentation as well, inadvertant or otherwise. For example, regression coefficients are often quite volatile in response to changes in the sample or in the estimating equation. If the extent of the volatility is not made known in a particular instance, it is likely that the reliability of the model will be oversold. In the absence of strong theoretical support for a given type of estimating equation, the analyst will frequently feel obliged to engage in a curve-fitting procedure. The chances for arriving at a clearly superior model through such a process should not be overestimated.

There are many problems that can beset a regression model, and each makes the estimation of marginal costs more hazardous. Without going into the technical details of these problems, we need to at least mention the major ones. First, it is possible to omit a variable that belongs in the cost function, either inadvertantly or because neither a direct measure nor a decent proxy is available. The result may be a biased estimate of marginal costs, depending on the relationship between the omitted variable and the output variable(s).

Biased estimates can also result if institutions systematically fail to estimate correctly their future enrollments. Large institutions, for example, may tend to underestimate their future enrollments, which would bias downwards their estimated marginal costs. This particular tendency, along with the resulting bias, is referred to as the "regression fallacy" in the literature. It is generally taken as a fairly serious threat to the integrity of statistical cost estimates, but the dimensions of the problem with respect to higher education cannot be readily determined.

Another possible problem is due to simultaneous-equation bias. Ordinary least-squares regression is appropriate only when there is but one variable in the system that is endogenous, that is, when all variables except the dependent variable are determined by factors outside the system being modeled in the regression equation.

For example, with respect to the cost functions discussed above, we must assume that cost is determined by, or is a function of, enrollment, *and* that enrollment is not a function of cost. If this assumption fails, marginal-cost estimates will be biased, perhaps severely. Some analysts (Thompson 1980 and Carlson 1972) are more worried about simultaneity effects than are others (Verry and Davies 1976).

Several other common problems need mentioning. The error terms, that is, the residuals or differences between predicted and actual values in the regression analysis, may "misbehave." For example, they may systematically change with the magnitude of one of the independent variables, a problem known as heteroscedasticity. When the condition is present, measures of statistical significance (t tests for individual parameters and F tests for the regression model as a whole) are no longer valid. The residuals may also be correlated with each other, a problem known as serial correlation (or auto correlation). When this condition is present, the conventional tests of significance are again invalidated. Both problems can usually be treated, but often at the expense of making interpretation more difficult. The independent variables can behave in unfortunate ways too, such as when they move together, that is, are linearly related to one another. This problem, known as multicollinearity, is "one of the most ubiquitous, significant, and difficult problems in applied econometrics" (Intriligator 1978, p.152). When this condition is present, the estimates in the regression model are imprecise and unstable. The problem is often intractable. Fortunately, there are readily available means for detecting multicollinearity as well as heteroscedasticity and serial correlation.

The evaluation to this point can be summarized as follows. The regression technique is widely used for estimating marginal costs in a variety of settings. It is adaptable to various units of analysis, handles a large number of observations, and can readily provide a regional or national perspective. On the other hand, regression is much more complex than it appears. In principle, the technique is simple enough. So is the concept of a cost function. In practice, subtleties of interpretation and representation abound and a variety of estimation problems may occur. It is quite difficult to remove all possible sources of bias in the data and all arbitrariness on the part of the analyst. Regression, then, is not a technique that is likely to generate unassailable marginal-cost estimates. In the next section, both the strengths and weaknesses of the technique will be demonstrated in a calculation using actual data.

Cost Calculation—An Example

In practice, the selection of institutions to be included in the analysis, the unit of analysis, the type of data, and so on will usually flow from the questions that are to be addressed and the data that are available. For our purposes here, we chose a situation or task that would relate to funding-formula issues, represent a typical degree of complexity, and make use of actual data from existing sources. Table 3.1 specifies the task selected for the demonstration.

TABLE 3.1

Activity to be analyzed	: Instruction at three levels (lower division, upper division, graduate)
Type of institution	: public research universities (using the NCHEMS taxonomy)
Unit of analysis	: institution
Type of data	: cross section for one year
Meaning of "cost"	: expenditures for instruction (the standard accounting category)

With that task in mind, the following total cost function was estimated by ordinary least-squares regression

$$\hat{C}_i = a + b_1L_i + b_2U_i + b_3G_i + d_jX_{ji} + e_i \qquad (3.7)$$

where, for the ith institution, C is total expenditures for instruction; L, U, and G are full-time-equivalent enrollments at the lower division, upper division, and graduate levels, respectively; X is a vector of control variables; a is a constant; b_1, b_2, and b_3 are the marginal costs to be estimated; d is a vector of coefficients to be estimated; and e is an error term. If allocated cost data were available, that is, data in which the costs for each level of instruction were allocated to the number of students enrolled, three separate cost functions could have been estimated (see Verry and Davies 1976). Such data were not available in this instance, so all three enrollment variables are entered in the same cost function. The procedure is legitimate, but it can create a multicollinearity problem.

The vector of control variables, X, consisted of the following variables: average faculty compensation, a state price index, research emphasis (expenditures per full-time faculty for separately budgeted research), location in a formula funding state, the total number of degree programs offered, and the proportions of degrees earned in agriculture, biological sciences, business administration, education, engineering, fine arts, health, languages, law, medicine (medicine, osteopathy, dentistry, veterinary), physical sciences, and social sciences, respectively. Faculty compensation played a dual role, as it was both the major control with respect to differences in input prices and a proxy for differences in output quality. The various proportions of degrees earned were intended to provide some means of control over differences in program emphasis. It would be possible, at least theoretically, to refashion equation 3.7 in such a way as to estimate the cost of an additional degree in one or more of the program areas.

The NCHEMS institutional typology classifies 51 publicly controlled institutions as major research universities. Of that number, we were able to get complete information on 50 institutions. The Higher Education General Information Survey (HEGIS) for 1977-78 was the source for the data, except for the state price index

(McMahon and Melton 1978) and the list of formula-funding states (Gross 1979).

Note that the particular form of equation 3.7 constrains the three marginal-cost estimates to be constant with respect to numbers of students. The model also assumes that the number of students at one level of instruction has no bearing on the marginal costs at the other levels of instruction, an assumption that is at least debatable.

The results of estimating equation 3.7 by OLS are shown in table 3.2. Only the results of primary interest, the marginal-cost estimates themselves, are displayed.

TABLE 3.2

MARGINAL-COST ESTIMATES FOR INSTRUCTION AT PUBLIC RESEARCH UNIVERSITIES USING A LINEAR MODEL

	L	U	G	X
Regression Coefficient	−90	2,555	6,550	—
(t-score)	(.141)	(3.02)*	(5.27)*	

R^2 (adjusted for degree of freedom) = .94

*$p < .01$

Clearly the results are mixed. The model as a whole explains 94 percent of the variation in total costs. The marginal-cost estimates, which in this model can be read directly from the regression coefficients, are plausible for upper-division instruction, $2,555, and for graduate instruction, $6,550. Both of the estimates are statistically significant (.01 level) as well. On the other hand, the estimate for lower-division instruction, $−90, is neither plausible nor statistically significant. We might expect lower-division costs to be low, because of the widespread use of teaching assistants at these institutions, but a negative marginal cost, at least over the entire enrollment range, makes no sense. It is possible that multicollinearity, that is, one or more systematic relationships among the independent variables, has interfered with the cost estimate for lower division. Lower-division enrollment is, not surprisingly, highly correlated with upper-division enrollment. This relationship may well be the source of the problem.

Even if all of the estimates had been plausible and significant we would likely want to continue the estimation procedure. After all, the assumption that marginal costs do not change with enrollment size is one that ought to be tested. Another possibility worth investigating is that the marginal costs at one level of instruction might somehow be related to the number of students at another level. In short, even if the linear model had worked well in every way, it would still be rather simplistic and restrictive in comparison with what might in fact be happening at research universities.

A number of additional models were tested in a series of total cost functions using quadratic, cubic, interaction, and logarithmic terms for the enrollment variables. A quadratic term was statistically significant only for upper-division instruction. No significant cubic or interaction terms were found. A multiplicative model worked rather well, everything considered. It was developed in the following way: starting with the notion that marginal costs at one level of instruction are related to enrollment at another level,[5] we can write

$$C_i = a L_i^{b_1} U_i^{b_2} G_i^{b_3} e^{d_j X_{ji} + u_i} \tag{3.8}$$

As such, equation 3.8 could not be estimated using linear regression. The model can be readily transformed, however, so as to permit the use of a linear estimation technique. Taking the natural logarithm of both sides of equation 3.8, we have

$$\ln C_i = \ln a + b_1 \ln L_i + b_2 \ln U_i + b_3 \ln G_i + d_j X_{ji} + u_i \tag{3.9}$$

Equation 3.9 was estimated by ordinary least-squares regression and yielded the results shown in table 3.3.

TABLE 3.3

MARGINAL-COST ESTIMATES FOR INSTRUCTION AT PUBLIC RESEARCH UNIVERSITIES USING A MULTIPLICATIVE MODEL

	L	U	G	X
Regression coefficient	.104287	.406836	.486749	—
(t score)	(.975)	(3.01)*	(3.99)*	
R^2 (adjusted for degrees of freedom) = .96				
Estimated marginal cost	\$651**	\$2,755**	\$6,009**	

*$p < .01$

**At mean enrollment; decreasing at a decreasing rate as enrollment increases.

At mean enrollment, the marginal-cost estimates for upper-division and graduate instruction are similar to those in the linear model; however, as the note in table 3.2 indicates, the transformed multiplicative model estimates that marginal costs change as enrollment changes. Specifically, they decrease at a decreasing rate as enrollment increases. Figure 3.4 displays the marginal-cost curves derived from the regression coefficients in table 3.3.

Estimated lower-division marginal cost, \$651 at mean enrollment, is plausible, although still not statistically significant. Is the multiplicative model (equation 3.9 and table 3.3), *better than* the linear model (equation 3.7 and table 3.2)? We would probably want to answer in the affirmative, if only because restrictions imposed by the linear model are absent from the multiplicative version. In other words, we might be willing to invoke a general rule to the effect that a higher-order function, a

5. Technically, the multiplicative model shown in equation 3.8 implies that if the number of students at one level of instruction increases, then the marginal costs of the other levels of enrollment must also increase. This implication may not be appropriate. In the present case, no evidence of the opposite pattern, that is, an increase at one enrollment level being associated with decreases in marginal costs at the other enrollment levels, could be found. In addition, a so-called transcendental function, a multiplicative model that does not carry the implication of equation 3.8, was also tested, and it yielded results very similar to those obtained using equation 3.8.

FIGURE 3.4

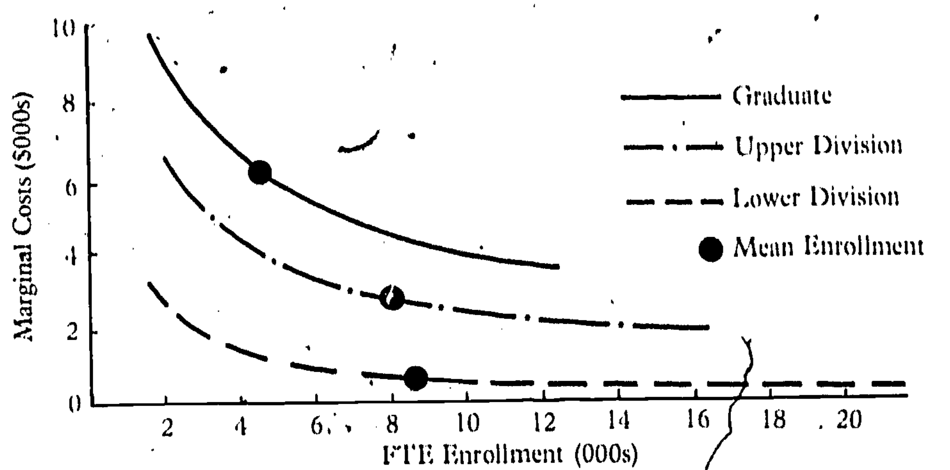
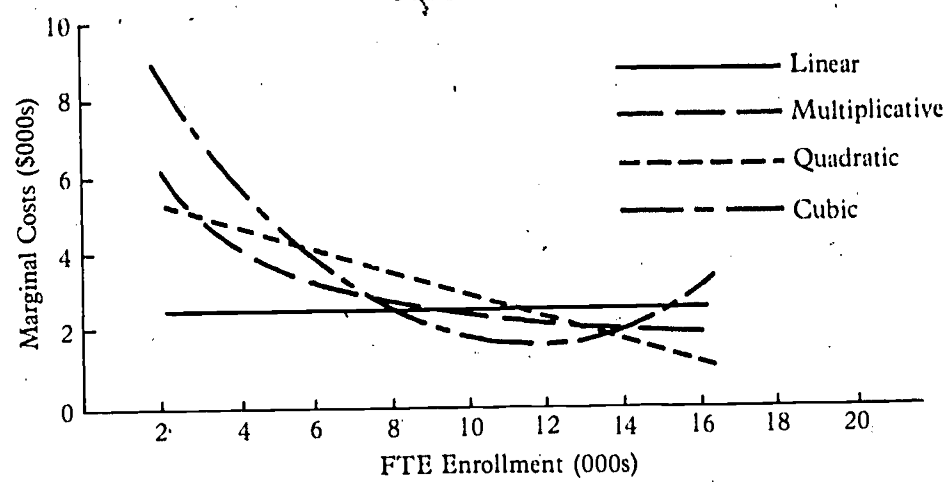


FIGURE 3.5



function allowing more inflection points in the associated cost curve, is usually preferable, other things being roughly equal. Since we do not know beforehand what the curve looks like, the less we impose a particular shape on the curve the better.

The higher-order rule may not always be a definitive criterion. Consider the situation displayed in figure 3.5. Four estimated curves are shown for upper-division marginal costs at public research universities. Invoking the higher-order rule, we can say that line B, the multiplicative model, is preferable to both line A, the linear model, and line C, the quadratic model. The relative status of line B if it is compared to line D is less clear. Line D results from estimating the highest-order function of the four, a cubic version of the total cost function. In contrast to the other models, the marginal-cost estimates in the cubic function were not statistically significant ($p < .05$); however, the estimates did have enough statistical support ($t > 1$) to merit at least some consideration in view of the small sample size ($N = 50$).

Choosing the most appropriate functional form in this case probably cannot be done without establishing some additional criteria regarding levels of statistical significance. There is likely to be some degree of arbitrariness involved in such criteria. The unsettled character of the situation is further compounded because there may be other functional forms that are equally as good or better than the forms represented in figure 3.5.

Assessment

It could perhaps be argued that the most appropriate role for regression-based marginal-cost estimates is *not* to provide specific coefficients for a cost analysis or funding formula. To be an unassailable source for such coefficients, the regression-cost function procedure would have to be more reliable than it currently appears to be. What the procedure can do in many instances, however, is to reveal general cost patterns. For example, the results shown in figure 3.5 make a strong case that over a substantial portion of the observed enrollment range, upper-division marginal costs decline when enrollment increases. Roughly two-thirds of the institutions in the sample fall within that range (that is, where all higher-order curves are declining). In addition, while no one estimate clearly may be superior, the regression technique does provide a reasonably tight range for estimated upper-division marginal costs at mean enrollment ($2;555 to $2,755 across the four models). A regression analysis, then, can be a source of both background information on marginal-cost behavior and data that could be used more directly within a decisionmaking process. While regression-based marginal-cost estimates generally do not warrant unquestioning acceptance, regression can be a powerful and relatively efficient technique for estimating the general shape and position of marginal-cost curves. In other words, at least within the context of higher-education cost analysis, the most appropriate use of the regression technique may be in searching for general cost patterns rather than in providing specific, very precise cost estimates. Reasonably reliable estimates of general cost patterns could be useful in their own right, or for informing the judgments and agreements about costs that are essential to the fixed- and variable-cost method discussed in the next chapter.

4

The Fixed- and Variable-Cost Method

In this chapter, we explore the use of the fixed and variable method for estimating marginal costs. This method is unique because of its explicit dependence on a political process, as well as on empirical studies. In other words, the notion of cost analysis takes on a different meaning in this method. As in the chapter on regression, the intent of the discussion is to examine the basic concepts of the method, to discuss pertinent data and analytical requirements, to evaluate the strengths and weaknesses of the method, and to illustrate how the method has been used in an actual situation.

Definition

The fixed- and variable-cost method for estimating marginal costs requires the classification of all of the different categories of expenditures of an institution as either fixed or variable costs. When this process is complete, the costs that do vary with enrollment can be summed and defined as total variable cost. Average variable cost, that is, total variable cost divided by output, can then be used as an estimate of marginal cost.

No particular type of categories need be used for classifying expenditures. The most convenient types are relatively simple budget or activity categories for which financial data are available in each category. For example, object-of-expenditure data by academic department will almost always be available because it is linked to budget reporting categories. To take this example further, administrative salaries (the department chairperson) and equipment (maintenance of the inventory needed to support the programs being offered) could be considered fixed costs, while faculty salaries, support-staff salaries, and supplies could be considered variable costs.

A variation that requires more empirical support is the designation of particular activities as fixed costs. In this case, a certain set of courses or programs and their expenses are designated as fixed costs. All other costs are defined as variable costs. This approach requires some kind of cost study to allocate expenditures to courses or programs and does, therefore, require more data and data manipulation than the object-of-expenditure, organizational-unit approach.

It is important to note that "fixed and variable" means fixed and variable *with respect to changes in enrollment levels.* Fixed and variable are *not* synonyms for uncontrollable and controllable costs. A fixed cost remains constant, in the short run, when enrollment changes, while a variable cost changes when enrollment changes. Some authorities also use the concept of semivariable costs. When this concept is used, variable costs are defined as those costs that change in proportion to enrollment change, while semivariable costs change in the same direction as enrollment change but not in direct proportion. Or, alternatively, semivariable costs are defined as having a fixed component and a variable component. This alternative definition leads to the same overall effect. By contrast, controllable costs are those that can be affected by management action while uncontrollable costs cannot be. The distinction between controllabililty and variability can best be understood by several examples. When language requirements were rescinded for an undergraduate degree, enrollments in language courses dropped sharply at many institutions. The size of the faculty in these disciplines could have been reduced substantially and still have been sufficient to teach majors and those nonmajors who wished to study a foreign language. Thus, at least a portion of the costs were variable. However, due to tenure and faculty with minimum notice periods, these costs were largely uncontrollable for a year or two. Another kind of situation is illustrated by a decision to establish a new doctoral program. This is a controllable cost (there is no requirement to establish the program) but once the decision is made, the costs associated with a core faculty, library, and equipment for the program are fixed. It can be seen, then, that the concept of variability relates primarily to the technical nature of the production of output while the concept of controllability relates primarily to the scope for management action. In any practical approach to the use of cost analysis, uncontrollable costs (including inflation) need to be dealt with in some way, but the issue of controllability needs to be kept separate from variability. There are, of course, many cases when fixed costs are also uncontrollable; the requirement to pay an institution's debt service is an example.

The flexibility of the production function and the lack of knowledge about it limits the use of empirical analysis in the fixed- and variable-cost method. While the regression method can be used to analyze a number of cases of the relationship between costs and enrollments, and the incremental method can be used to analyze particular changes in cost-enrollment relationships, the fixed- and variable-cost method is used to determine directly the technical relationships of the production function. Thus the separation of costs into fixed and variable components is partially the task of policy determination. However, empirical data are useful to support this effort and to measure the implications of particular policy decisions, as will be discussed further in the next section.

Data and Analytical Requirements

It is difficult to talk about the data and analytical requirements of the fixed- and variable-cost method of estimating marginal costs. The heart of the requirements for the fixed and variable method is the design of a process to decide *as a policy matter* the variability of individual categories of costs. Essentially, the analysis involves the determination of standard costs for the activities being studied. Standard costs, defined as what a particular unit or output *should* cost based on the technical relationships of production insofar as they are known, are widely used in industry as a production control mechanism. Although it is not proposed that higher-education standard costs be used as a control mechanism or even as a device for analyzing variance, many of the principles of defining standard costs apply here.

First, it is necessary to determine what cost categories will be used. For example, it is possible to focus on programs of study, on individual courses, on faculty or other positions, on objects of expenditure, on academic departments or colleges, on legal categories (such as tenured faculty), or on some combination of the above. A focus on academic departments could easily be combined with a focus on legal categories or objects of expenditure. Once cost categories are defined, it is necessary to decide which cost categories vary with enrollments (in what proportion) and which do not. It might be decided, for example, that salaries for tenured faculty and equipment are fixed costs, while salaries for nontenured faculty, classified salaries, and expenses for supplies and travel are variable costs.

Since so much of the fixed- and variable-cost method for estimating marginal costs is based on the determination of policy regarding cost categories, it is important that decisions about variability be made at the policy level. Although technical support may be used, the determination of standard fixed and variable costs is not a technical decision. Therefore, representatives of all groups with a vital stake in the calculation of costs should be included in the decision process. For example, if the purpose of the analysis is to design a budget formula, the decisionmaking group should include, at a minimum, representatives of all institutions or institutional sectors, state governing or coordinating agencies, and the executive and legislative branches.

This is not to say that analysis cannot or should not inform policy decisions. Individual analytical studies may be very useful for determining the variability of a particular category of costs. Examples of analyses (usually done for another purpose) that may prove useful in determining the technical relationships between costs and enrollments are (1) studies that attempt to define a minimum core faculty when opening an institution; (2) accreditation self-studies, especially those that deal with particular programs in depth; (3) program reviews; and (4) special budget studies that specifically address the variability of costs in a particular area. While all of the above types of analyses may be helpful in determining the variability of costs, a complete analysis is impossible because of the extreme complexity of the situation and the lack of knowledge about the production function. One must accept the fact that any analysis done will address only a portion of the costs.

Strengths and Weaknesses

There are a number of important advantages of using the fixed- and variable-cost method to estimate marginal cost. Foremost among these is the advantage of focusing directly on the key cost variables, including those that can be manipulated by management and those that are externally induced. This characterization allows a marginal-cost analysis based on the fixed- and variable-cost method to easily reflect the impact of factors other than those related to volume. If, for example, the cost analysis was being performed to establish an internal price for computing services, this method would serve to protect the computer service unit against environmental factors (such as an increase in hardware rental rates) and to protect computer users against costly, unilateral decisions by the computer center. A determination of marginal costs using the fixed- and variable-cost method is essentially a political agreement as to the kind of decision and environmental factors that will be considered legitimate by the participants in the process. If the values of some of the factors change, the cost per unit can be adjusted, but the factors themselves cannot be changed without renegotiating the political agreement.

The nature of the political process and the inclusion of key decisionmakers in the determination of the status of various costs is another strength of the fixed- and variable-cost method. A cost analysis based on this method constitutes its own political agreement. Any application, then, of the cost factors will already have the necessary support behind it, assuming that all parties involved are willing participants in the process.

A third advantage of the fixed- and variable-cost method is its intuitive nature. The notion of fixed and variable costs has face validity even to nonspecialists. The importance of this factor in a political environment cannot be overestimated. It is relatively easy to convince key decisionmakers that cost elements such as department chairpersons represent fixed costs of operating an institution and cannot be changed as enrollment changes without affecting the nature of the institution. Face validity is even more important when noninstructional functions are addressed, because there are many noninstructional cost elements that obviously do not vary with enrollment (such as debt service and utilities). The intuitive nature of the fixed- and variable-cost method is enhanced further by the method's applicability to cost factors at varying levels of detail. For example, it is possible to perform a micro-level fixed- and variable-cost analysis focusing on individual courses and then present the results as showing that the instruction function as a whole includes some percentage of fixed cost. As experience accumulates, as additional analytic studies are performed, and as political perceptions change, it is technically easy to change the evaluation of cost variability of one cost element and to include that change in the overall cost analysis, although this may require the renegotiation of the entire political agreement. In this way, the face validity of the cost analysis can be maintained under changing conditions. This flexibility is important both in its own right and as a way to preserve face validity.

Balancing these three advantages are four major drawbacks. The first of these is the lack of a systematic empirical cost-calculation methodology. It is a weakness that has, however, a positive side to it. The fixed- and variable-cost method of estimating marginal costs does not depend on a particular analytical technique. Instead, some individual cost elements may be supported by individual analytical studies and others may not. On the other hand, the lack of empirical support should make one cautious about using the results of the method since the marginal-cost estimates are based largely on the judgments of participants as supported by whatever analytical studies they can muster. Such judgments may be weakened by being too close to the issue, by being based only on local conditions, or by being arbitrary. On the other hand, by lacking empirical support and historical base, this method avoids many of the data and analytical problems of both the incremental and regression methods. These two approaches, with their somewhat mechanical analytical techniques applied to potentially faulty or ambiguous data, can lead to erroneous conclusions that camouflage the error until policy mistakes are made. The fixed- and variable-cost method, with its emphasis on judgment, may be better at avoiding this pitfall. More importantly, the empirical methods, and particularly the incremental method, tend to reflect the existing fiscal situation, while this judgment-oriented method has the advantage of focusing more on what costs should be (that is, standard costs).

Complexity, the second weakness of the fixed- and variable-cost method, has both technical and political implications. At the technical level, this method—in particular the fixed-activity variant—requires the detailed consideration of many issues related to cost. This requirement imposes a substantial administrative burden and concentrates that burden on key decisionmakers. The possibilities for overload or for staff usurpation of political decisions are evident. The political implications are even more serious, especially when the cost analysis is intended to support resource-allocation decisions. The method requires central decisionmakers to pass judgment on operations at a level of detail far lower than is commonly done or than is appropriate, given the value placed on decentralized management of higher education in the United States. This criticism is especially telling when critical mass concepts are incorporated into the cost analysis. A dilemma is created between central control of detailed decisions or writing a blank check for the subunits by supporting whatever activities the subunits consider fixed. This problem is particularly severe when the subunits are autonomous, as is often the case when the cost estimates are used to design a state-level budget formula. The cost-calculation example presented in the next section will show this clearly.

The third weakness of the fixed- and variable-cost method concerns the distinction between long-run marginal cost and short-run marginal cost. This distinction suggests the importance of carefully defining the purpose of the analysis and ensuring that the time frame selected corresponds to that purpose. The technical definition of "long run" is the time frame in which all factors of production are variable. For higher education, 50 years would definitely be considered the long run. Buildings will have been recycled or demolished, tenured faculty will have retired or died, debt will have been retired, and so on. In the long run, fixed costs will disappear. The "short run" is defined as that time period in which at least one factor of production is fixed. Obviously, however, there are many varieties of the short run. As the time period gets shorter and shorter, the number of factors of production that are fixed will increase until at some point the entire expenditure base can be considered a fixed cost. The problem then is selecting a planning period that is consistent with the purpose of the cost analysis.

This is not as easy as it sounds. If, for example, the cost analysis is being conducted to support budgetary decisionmaking, a decision needs to be made regarding the budget planning period. This will not necessarily be identical to the normal (annual or biennial) budget period and will probably not be consistent from cost element to cost element. The decision as to time period will have to be made on the basis of the need to pay for fixed costs as opposed to the reponsibility to convert fixed costs into variable costs over time. One solution is to adopt the normal budget period as the planning period. Since this option would have the effect of moving most costs into the fixed-cost category, very little budget adjustment would occur as a result of enrollment growth or decline. Should a longer-run planning period be adopted, more costs are likely to be considered variable. Under this approach, subunits may be left without the capacity to respond effectively to short-run problems. Fiscal adjustments may be necessary to allow subunits to meet their obligations. The long run, of course, consists of a collection of short runs, so actions taken or not taken for short-term reasons can dramatically affect long-run considerations. These concerns must be balanced to determine the proper time period.

The final weakness of the fixed- and variable-cost method is shared with the regression and incremental methods: the lack of knowledge about the higher-education production function. The production technology and the outputs of higher education are not standardized. A credit hour of upper-division psychology may be produced with a lecture, a seminar, a laboratory, an independent study, by correspondence, or by several other modes of instruction. All of these technologies have dramatically different cost implications. The situation becomes even more complex when it is realized that higher education does not produce standard outputs. Not only do the purposes of instructional activities of ostensibly the same type vary, but the quality of these activities may vary as well.

Since the fixed- and variable-cost method focuses directly on the technical relationships of production (that is, the production function) and since we know little about these relationships, in effect we are forced to invent a production function based on our best judgment. This judgment may or may not be accurate. On the positive side, at least the technical relationships of the invented production function are explicit and may be changed if they are found to be unworkable.

Cost Calculation—An Example

Since the fixed- and variable-cost method is dependent on a basically political process of determining the status of various costs, it is rather difficult to simulate the calculation of costs in a research mode. For this reason, the example of a cost calculation described here is an actual case. It is based on the experience of the University of Wisconsin System in designing a budget formula based on marginal cost. The information for the case study has been largely drawn from University of Wisconsin documents and from interviews with key persons in Wisconsin. The basic case study was presented in *Cost Information and Formula Funding: New Approaches* (Allen and Topping 1979). This material was updated by a May 5, 1980, agenda item for the University of Wisconsin board of regents meeting and by additional contacts with individuals in Wisconsin.

Higher education has been funded on a formula basis in Wisconsin since 1953. At that time, the higher-education community was under the stress of an enrollment slump accompanying the Korean War, and it sought to regularize fiscal relationships with the state by means of a dollars-per-FTE-student formula. This formula was based on average historical cost by function (instruction, student services, academic support, and so on), information that had been compiled by the University of Wisconsin since the 1930s. As the formula developed in ensuing years, differentiation by student level was added to the formula (with lower-division students being funded at a base rate, upper-division students at twice the base rate, and graduate students at 3.5 times the base rate). Finally, in 1971, differentiation by discipline was added to the formula.

The fiscal relationship between the University of Wisconsin and the state was unstable during the 1970s. Fluctuations in state policy combined with enrollment fluctuations to produce relative fiscal stringency at the university throughout the decade. By the 1971–73 biennium, the historical-average-cost formula had reached a high level of development, and most participants in the state budget process found it acceptable. This was also a biennium of enrollment downturn, which resulted in a decline in appropriations for the university and prompted a number of second thoughts about the formula. By 1973-75, enrollments were back up. But economic difficulties being experienced by the state led to a 5 percent productivity cut and to other fiscally restrictive measures such as the failure to fund increased graduate enrollments in the university system. These restrictions offset the increased appropriations for additional undergraduate enrollments. In 1975-77, Wisconsin's enrollment-driven formula was suspended pending the development of a new formula that could accommodate a phasedown in university operations during the period of expected enrollment decline ahead. Since 1975-77 was another period of enrollment increase, the system again accommodated a large number of additional students (3,500 head count) without additional resources. Subsequently, a great deal of effort has been devoted to developing a budget formula that can cope with the expected enrollment decline without creating severe dislocation. As a part of the development process for this new formula, the system adopted the fixed-activity variant of the fixed- and variable-cost method of estimating marginal costs in order to adjust historical unit costs to accommodate economies of scale (and thereby ameliorate the difficulties of small institutions). The attempt to apply the method was to take place in two phases, the first of which was implemented during the 1977-79 biennium.

Among the activities covered by the funding formula, judgments about instructional costs were the most difficult to make. Adding a given number of students may impose no additional direct resource requirements (more class sections or more faculty or both), provided that present class sizes are at less than the optimal capacity as determined by campus- and program-specific educational standards. However, it is probable that a variety of classes are operating at or above their individual capacities for quality education and that small enrollment increases could require resources costing even more than the historical average cost per student. The net fiscal effect of enrollment increases is very difficult to identify specifically. Therefore, in the instructional area, it was initially assumed that all *direct* classroom teaching costs are variable at the system level (although not necessarily at lower levels of aggregation) and should be treated as such in the funding formula.

Certain *indirect* college and departmental activities in the instructional function were defined as being independent of enrollments (that is, as representing fixed costs). The salaries of deans, associate and assistant deans, directors, and various related staff were defined as fixed, because their functions were, to a large extent, essential regardless of enrollment variations. In addition, an amount equal to 100 percent of dean and director salaries was identified for salary and nonsalary support of those fixed functions. Further, at least part-time departmental chairpersons were required. After a survey of smaller campuses, 30 percent of the average associate professor's salary was defined as a fixed administrative cost. According to these calculations, about 10 percent of the total system instruction budget was considered fixed instructional administrative costs.

Although it was widely assumed that there really were fixed costs associated with direct classroom instruction, such costs were not considered during the first phase of formula revision. (The second phase of the revision was not completed.) The determination of the fixed and variable costs of instruction described below is included as part of a proposal that was submitted to the political authorities in Wisconsin for the 1981-83 biennium.

The University of Wisconsin system analysis began with a fundamental policy determination: the academic planning process should drive the budget formula rather than vice versa. This did not mean that academic planning was not subject to resource constraints, but rather that the structure of the budget formula should be congruent with the structure of the academic plan. Therefore, the current array of programs and courses offered by the system (as defined by the academic plan) was used as a starting point. Since it would be impossible to drop a program or course without changing the academic plan, the first section of each course was considered to be a fixed cost. The second and subsequent sections would be considered variable costs. The one exception to this rule was a number of courses and programs catering to nontraditional students. It was held that if enrollment levels dictated the reduction of a course to a single section and that section were held during regular working hours, many nontraditional students would be excluded because of their job or family obligations. Therefore, for the courses identified as serving part-time students, a second, off-hour section was considered to be a fixed cost. Once this decision was made, it remained only to count the number of sections that were the first (or eligible second) section of each course. These were defined as fixed activities. The system cost study could then assign costs to these sections to arrive at a fixed-cost calculation. Variable costs would be the remainder. The result of this calculation was that 51.7 percent of all instructional costs (systemwide) were considered fixed (including the 10 percent attributed to instructional administration costs identified earlier).

It will be observed immediately that a formula such as that proposed for Wisconsin encourages the loading of credit hours and expenditures into the first section of a course. This can take two forms—very large sections or course and program proliferation. Wisconsin had relatively rigorous program-approval processes that served to prevent the wholesale establishment of new programs. However, the establishment or size of individual courses was not usually controlled by the campus administration, much less by the system administration. In order to check course proliferation or excessive size increases, the system administration established a number of "Threshholds of Concern," indicators of campus efforts to control the number of courses (and thus the percentage of fixed costs). Conditions designated as causes for concern included the following: (1) the actual or projected composite support index (CSI—a weighted cost per credit hour) exceeded the target CSI by 6 or more percent; (2) the actual or projected enrollment decline exceeded 5 percent in one year or 12 percent over the six-year planning period; (3) fixed instructional expenditures exceeded 65 percent of total instructional expenditures or increased by more than 10 percent in two years; (4) the projected percentage of faculty and related instructional staff whose contracts terminated in any one year was less than twice the percentage of enrollment decline; (5) 35 percent of instructional staff were in departments with 80 percent of their faculty on tenure status and where enrollment decline in more than half of those departments was more than 5 percent in one year or more than 12 percent during the six-year planning period; and (6) the CSI exceeded the cluster average CSI (for doctoral universities, universities, or community colleges) by more than 30 percent. Exceeding a threshold would lead to special planning for a campus at the system level. The actions taken by the system were variable but included assessment of the long-term viability of an institution, given its current mission and course array.

Even with the central program-review process and the "Thresholds of Concern," the University of Wisconsin system proposal raised concerns at the state level. The 51.7 percent fixed cost factor was treated very cautiously by the legislative and executive fiscal staffs. Although they did not take a position on the proposal prior to its submittal, they also declined to participate in the analytical process.

Assessment

The fixed and variable method appears to be worth serious consideration as a means of bringing a marginal-costing dimension to higher-education funding and resource allocation. Because the method depends so heavily on a political process, it is clear that its appropriateness in a given situation is a function of the internal political climate. That is, the officials who are responsible for funding and resource allocation must be willing to assume the further responsibility of creating what amounts to standard costs for higher education. Granted such a willingness, the fixed and variable method appears to be a workable alternative. This alternative readily responds to the lessons of experience—including those lessons that might flow from the application of other marginal-costing methods.

5

The Incremental-Cost Method

In this chapter, we explore the use of the so-called incremental method of estimating marginal costs. The method depends neither on statistical analysis, as in the regression method, nor on political agreements, as in the fixed and variable method, but rather on the analyst's ability to isolate related increments in costs and output. This ability allows a direct calculation of a type of marginal cost. As in the previous chapters, the discussion focuses on the basic concepts, data requirements, and merits of the method. It concludes with two illustrations using a[illegible] data.

Definition

The incremental method of calculating marginal costs is conceptually simple. It involves the use of accounting data and output data (for example, student credit hours) related to the particular organizational or activity unit for which costs are being calculated. Two years of cost and output data are needed to make a single observation of marginal cost. Essentially, the incremental method calculates marginal costs using *changes* in expenditure and outputs.

By subtracting one year's expenditure from the next year's, a gross change in expenditure levels can be calculated. This result can be either positive or negative without affecting the method. In a stable and simple world, the figure for gross change in expenditures could simply be divided by the change in the number of units. The result of that operation would be the average cost of the units in that particular increment of production. While it cannot be said that this average incremental cost will exactly equal marginal cost (except in the extreme case of the change in output being only one unit), it can be expected that average incremental cost will more closely approximate marginal cost than will average total cost. In other words, this method provides an *estimate* of marginal cost for a particular increment or decrement in the size of operations. The estimate will be particularly accurate if the observed increments or decrements are small relative to the total volume of operations.

Since, however, the world is neither this simple nor this stable, additional analysis is required. Robinson, Ray, and Turk (1977) postulated that cost is affected by volume factors, environmental factors, and decision factors. These distinctions are particularly useful in estimating marginal costs by the incremental method. A calculation of marginal cost, if it is to be derived from expenditure records, requires that the exogeneous environmental factors that affect expenditures be controlled. In today's economy, the most obvious environmental factor is inflation, but many other factors also play a role. Changes in the regulatory environment (notably including increased social-security taxes), abnormal winter weather, natural or man-made disasters, and additional factors too many to mention can all have a significant impact on expenditures. In addition, variations in expenditures attributable to specific decisions that changed either the nature of what is being produced or the technology of producing the output should also be controlled in the estimation of marginal costs. Examples of changes of this type would include the decision to offer a medical degree (producing a different kind of product) or to use lectures instead of seminars (changing the production technology and conceivably the nature of the product as well). The process of determining the cost implications of specific decisions is extremely difficult in terms of both theory and measurement.

Both environmental factors and decision factors are intervening variables that can have a large and unpredictable effect on expenditure levels. Any attempt to use expenditure data to calculate marginal costs should in some way segregate these two types of factors from volume factors. It will be recalled that the regression method, by using a large data base and statistical techniques, does not eliminate environmental and decision factors but rather controls for any systematic relationships between costs and enviromental and decision factors. In contrast, the fixed and variable method deals with the problem by avoiding the use of historical expenditure data altogether (in theory). The incremental method has still another approach, which is the removal of environmental and decision factors by means of a micro-level cost analysis.

When using the incremental method, every major and minor factor that caused expenditures to change from one year to the next must be separately considered and categorized as an environmental factor, a decision factor, or a volume factor. The changes in expenditures attributable to volume (only) factors are then summed and divided by the change in output. This yields an adjusted average incremental cost that can serve as an estimate for marginal cost for that level of operations. It is essentially the method outlined originally but performed upon an adjusted expenditure base that explicitly excludes variations that are not related to changes in the number of units produced.

To extend the range of the analysis or increase the confidence that could be placed in the analysis, it would be necessary to look at several two-year intervals (longitudinal analysis), or at a number of similar departments of different size cross-sectional analysis). In effect, the incremental method of estimating marginal cost yields only one point on a cost curve. Additional calculations are necessary to determine additional points and the shape of the cost curve itself. These additional calculations are necessary (but not necessarily sufficient) if estimated cost figures are to be used for anything other than understanding the operations of one organizational unit at one level.

A simpler method of calculating marginal costs with the incremental method involves removing the effects of only a limited number of environmental or decision factors from the expenditure changes. Since inflation is the single largest factor contributing to changed expenditure levels (excepting only the design of a new or radically different program), using inflation as the sole *deduct* from expenditure changes may provide a reasonably accurate estimate. Other factors could also be considered without attempting to capture the full complexity of the situation. The simplified incremental method substantially reduces the amount of data required for the analysis, as well as ameliorating many serious measurement problems. The simplified method also has certain theoretical advantages.

Data and Analytical Requirements

At first, the information requirements for the incremental method seem easy to satisfy. At a minimum, two years of expenditure and output data are needed for one institution. This data should be disaggregated until expenditures and outputs are known for each organizational activity unit that is producing one or more distinguishable products. While these requirements obviously involve a large volume of data, they do not initially appear complex. Since only one institution's data is needed, none of the interinstitutional problems of different reporting practices arise. In fact, since the incremental method analyzes cost by activity unit, even many of the intrainstitutional data comparability questions (such as differential charge-back rates, and special credit hour reporting conventions) are not considered. The multiyear data requirements of the incremental method present some difficulty. Organizational structures, financial reporting practices, and academic reporting policies will change over the years. However, adjustments can be made for many of these changes; financial data, in particular, should be easily transferred from old to new formats. Output-measures data will present more of a year-to-year comparability problem: even basic measures such as student credit hours may have different meanings from year to year. For example, one university recently changed its requirements for dissertation credit. The amount of work done by the faculty or the students did not change, only the reported credit hours and revenue generated. In other cases, the administration or extrainstitutional authorities may simply have changed their mind about what output measures are appropriate to collect. Both of these contingencies create significant data problems and require great caution in using the data that is available.

Beyond these data problems, however, come a series of more difficult informational and analytical problems. First, by no means does a consensus exist regarding a definition of the outputs of higher education. This is critical since some output or activity measure is the divisor in the calculation of cost. Uncertainty about the real *products* to be costed and about the nature of the production function for higher education typically leads to the costing of some sort of activity measure, such as student credit hours, for the purpose of designing budget formulas. (These problems are common in many kinds of costing and are not confined to higher education.) While the widespread use of activity measures would seem to make the concern about output measures academic, it can lead to serious conceptual problems. Some of those problems will be discussed later in this chapter.

Another critical aspect of the analysis needed for the incremental method is the separation of volume factors, which must be included in the calculation, from environmental and decision factors, which must be excluded. While the distinction is conceptually simple, the actual separation is very difficult to accomplish. The three types of factors affecting costs are often inextricably linked. Consider, for example, the linkage of two factors in a faculty salary increase. Is a 10 percent faculty pay raise the result of a university policy (and thus a decision factor) or is it forced by general inflationary pressure (an environmental factor)? Fortunately, many questions of this type are a result of interaction between institutional decisions and the environment and may be disregarded. We are concerned only with separating volume factors from the others.

Linkages between volume factors and the other types of factors exist. For example, it is possible that an institution would expand by simply enrolling more students in its existing programs. However, both the historical record and analysis of academic patterns suggest that institutions typically have dealt with enrollment growth by expanding program offerings. This greatly confounds the relationship between volume factors and other factors. Did the institution expand programs in order to grow, grow in order to expand programs, or expand programs in order to accommodate either planned or unplanned growth? Even if this problem can be solved in some way, it still entails a very difficult analytical process. To separate volume factors from environmental and decision factors associated with changes in expenditure levels requires a large knowledge base about the operations of the organizational activity unit being analyzed, and about the fiscal consequences of those changes. These questions cannot be answered in the abstract or at any distance from the operating unit. Probably the appropriate person to make these determinations is a responsible official in each unit being analyzed. Even for a relatively simple institution, these determinations will require the judgment of many people. New problems may arise since judgments tend to vary widely.

Ultimately, then, the informational and analytical requirements of the incremental method can be seen to be large and complex. It is necessary to do three things: (1) define outputs or activities to be costed, (2) collect output and activity data as well as expenditure data for each relevant unit, and (3) determine the *reasons* for changes in expenditure levels.

The simplified incremental method avoids many of the uncertainties of data by using sources that are relatively easy to obtain. It eliminates the analytical problems discussed above by concentrating on one key environmental factor—inflation. For the simplified incremental method, the only data needed are expenditure and output data. It is useful to have the expenditure data broken down by object of expenditure for each organizational unit being considered. The expenditure and output data are subject to all the caveats discussed in the context of the basic incremental approach. However, the process is greatly simplified because the analyst does not have to identify and categorize every factor that affects expenditure levels. This eliminates the need for an intimate knowledge of the cost behavior of the organizational or activity units involved and allows the calculations to be made centrally for a large number of such units. If there is a way of comparing organizational units to like organizational units in other institutions, then marginal cost could be estimated for several institutions of different size as well as for multiple years, and a number of different points on a cost curve could be obtained.

The one new element of data used in the simplified incremental method requires a reliable method of estimating the impact of inflation over time. There are various techniques for estimating inflation, but the *Higher Education Price Index* [HEPI] (Halstead 1980) is one obvious choice for this use. It measures the changes in factor prices (faculty salaries, other professional salaries, supplies, equipment) faced by institutions of higher education, as opposed to the less relevant price changes measured by the Consumer Price Index. As such, HEPI's inflation measures can be directly applied to changes in expenditure levels observed for various objects of expenditures and a calculation made of inflation's effect.

The HEPI is not a perfect tool for measuring the impact of inflation, especially at an individual institution. It is a standard price index calculated by measuring average price changes on a *market basket* of goods and services purchased by institutions of higher education. Applying the index to individual institutions is a process subject to two kinds of distortions. First, the market basket purchased by an individual institution may differ from the national market basket. Second, the price behavior of individual items in the market basket for an individual institution may depart from the national averages. The first effect can be largely controlled by using the subindices (such as for faculty salaries) included in the HEPI; this is the reason why data on an object-of-expenditure basis are so useful. The second problem is more difficult. The local behavior for such factors as faculty salaries is, in the short run, perhaps far more related to local revenue factors than it is to national average-cost factors. It may, therefore, be better to calculate the inflation factor for such large cost elements as salaries and fringe benefits on a local level. These locally calculated values can then be combined with other factors in the HEPI to make a final calculation. But while the injection of local values may enhance the reliability of the inflation factor, this procedure still has the potential for distortion by the intervening decision factors as discussed above.

Strengths and Weaknesses

The incremental method has several important strengths. Foremost is its direct linkage with the actual cost experience of an institution. This makes it the most concrete of the three alternative methods discussed in this book. The factors that lead directly to changes in costs are considered explicitly and assigned values. Indeed, the incremental method not only allows this, it requires it. The unit-by-unit approach of the incremental method theoretically has the capacity to generate much more accurate marginal-cost figures than the broader estimation techniques of the regression or the fixed and variable methods. There are three other advantages of the incremental method: (1) the same method has obvious uses both for intrainstitutional cost analysis and for extrainstitutional use; (2) the result of the calculation, if done for enough time periods or for enough institutions, produces an actual-cost curve; and (3) the method is conceptually simple enough to explain to nonspecialists.

The incremental method also has several weaknesses. First, as a micro-level analysis of typically small changes that occur over a short period of time, the method operates in the domain where the availability of revenue is likely to have its greatest impact on the cost-output relationship. Indeed, the impact of a change in revenue may be sufficient to render a particular estimate of marginal costs all but meaningless.

Second, the incremental method would seem to be especially sensitive to deviations from planned output. Higher education budgets, which are established on the basis of an estimate of future output (or level of activity), typically do not change, as the academic year unfolds, in response to deviations from planned output and budgeted expenditures. If the deviations are large, implausible marginal-cost estimates can be generated by an incremental method based on actual output. Unfortunately, in analyzing one operating unit at a time, the method does not benefit from the averaging effects that characterize some other approaches such as the regression method.

Third, our inadequate knowledge of the production process in higher education makes the results of an incremental analysis difficult to interpret. Balancing budgets from one year to the next may often be accomplished through slight, yet significant, changes in either the product or the means of production. If these changes go undetected, the calculated relationship between changes in expenditures and changes in volume of output will be misleading. This sort of issue—data comparability—is generally brought up in the context of interinstitutional comparisons, but it is also a threat when a single department or institution is looked at over time. We see here one of the major consequences of using an activity measure such as student credit hours in the cost calculations rather than a measure of what is produced. However necessary the substitution may be from a practical standpoint, the result is an abiding indeterminacy regarding what is really going on between costs and outputs. No method of estimating marginal costs can overcome the problem, but the incremental approach appears to be especially vulnerable to its deleterious effects.

Another serious weakness of the incremental method has already been discussed in the section on data and analytical needs. The volume of expenditure and output data needed is large though probably manageable. However, the information needed to attribute changes in expenditure levels to various environmental and decision factors is likely to be quite extensive and extremely complex. This will probably preclude institutionwide, much less statewide, cost analysis on this basis. However, this weakness is largely eliminated by the use of the simplified incremental method.

Cost Calculation—Two Examples

As an example of the incremental method for calculating marginal costs, we investigated the cost behavior of a university engineering department. Cognizant of the data-overload problem mentioned earlier, we analyzed only one department, albeit a complex one. We confined ourselves to a single object of expenditure, faculty salaries. We used student credit hours generated by the department as our activity measure. Although credit hours disaggregated by course level were available, expenditures were not available disaggregated by course level. Therefore, we used only a total student credit hour number that was much less than optimal. However, in the three years of data that we analyzed, no major changes occurred in the student mix being served by the department. Graduate-student credit hours varied only from 17.6 percent to 17.8 percent of the total. It is reasonable to assume that no large perturbation in the cost functions was caused by changing student mix.

The expenditure, output, and average-cost data for the three years follows.

TABLE 1

	1976-77	1977-78	1978-79
Total expenditures	$898,683.00	$948,494.00	$985,777.00
Faculty salaries	$637,155.00	$658,386.00	$690,470.00
Student credit hours (SCH)	11,438.00	11,918.00	11,827.00
Expenditure/SCH (average cost)	$ 78.57	$ 79.58	$ 83.35
Faculty salaries/SCH	$ 55.71	$ 55.24	$ 58.38

Note that average-cost figures both for total expenditures and faculty salaries were relatively stable during this period as was student-credit-hour production. In addition, faculty salaries were a relatively constant proportion of total expenditures.

After generating this basic data, the next step in the incremental method is to examine the year-to-year changes. Here we will concentrate on faculty salaries and SCH production.

TABLE 2

	1976-77 to 1977-78	1977-78 to 1978-79
Change in Faculty Salaries	$21,231	$32,084
Change in SCH	480	(91)

Following the calculation of this basic information, we began conversations with the department chairman and the department fiscal officer (both of whom had had a lengthy tenure in office including all three years under consideration). Initially, there was confusion about the data. The expenditure data that we had reviewed from the university's central administration did not match those in the department's internal budgets. After adjustments for restricted funds, funded vacancies, and so forth, we reduced the discrepancies to manageable, although still important, levels. After much discussion we decided to use central-administration figures since those were drawn from the official accounting records of the university. By discussion with department officials, we discovered that the following events occurred between 1976-77 and 1977-78:

TABLE 3

Event	Amount	Cause
Faculty raise	$25,000	Environmental
Change in position vacancy rate	16,000	Environmental*
Faculty leave	(9,000)	Environmental*
Total	$32,000	Environmental

*These are considered an environmental factor since the changes were caused by independent actions of faculty members and were a deviation from planned expenditures.

The calculation of marginal cost for this range of operation is as follows:

TABLE 4

Total change in expenditures	$21,231	(from Table 2)
–*Environmental/decision factors*	*$32,000*	(from Table 3)
=Volume factors	($10,769)	
–Change in credit hours	480	(from Table 2)
=Marginal cost	($22.44/SCH)	

A similar calculation could be performed for 1977-78/1978-79. The following events occurred during this time period:

TABLE 5

Event	Amount	Cause
Faculty raise	$26,000	Environmental
Change in vacancy rate	(37,000)	Environmental*
Appointment change	700	Decision
Total	($10,300)	Environmental/ Decision

*These are considered an environmental factor since the changes were caused by independent actions of faculty members and were a deviation from planned expenditures.

The calculation of marginal cost for this range of operations is as follows:

TABLE 6

Total change in expenditures	$32,084	(from Table 2)
−Environmental/Decision factors	*($10,300)*	(from Table 5)
=Volume factors	$42,384	
−Change in credit hours	(91)	(from Table 2)
=Marginal cost	($466)	

In both cases the marginal-cost figures are negative. This is certainly an enviable position for the department since it means that educating more students costs them less money (not just less per student). It is, unfortunately, also an unacceptable result. The generation of negative marginal-cost figures is not consistent with the classical theory of marginal cost although it may be consistent with the revenue theory of cost.

There are several possible reasons for the failure to generate more plausible marginal-cost estimates.

1. The problem may lie in the expenditure or output data. As already noted, data discrepancies did exist. While they were not large, they may have been large enough to cause a serious distortion; additionally, the student mix (expressed more subtly than graduate-undergraduate) may have changed or the mix between instruction and departmental research and service may have changed.
2. The departmental officials may have overlooked one or more key factors or may have wrongly estimated the fiscal impact of factors they did identify.
3. The department may have failed to adjust expenditure levels in response to the relatively small changes in output. Advocates of the revenue-theory of cost would argue that such behavior is typical, at least in the short run.

4. The production relationships may have changed, so that the department was producing something different from what it was producing in the prior year. This department did seem to have a relatively strong notion of what its production function was, however, and had taken steps (such as restricting enrollments) to keep the technical production relationships constant. Still, relatively small changes in those relationships could produce the result in question.
5. The discrepancies may have been the result of unplanned factors. In its efforts to maintain its technical production relationships, the department developed an activity plan and a financial plan. Both of these areas showed deviation from the plan as the year progressed. This was particularly true in the financial area since the faculty vacancy rate was large and varied greatly.
6. Other factors of production may have been substituted for a portion of the faculty input.

Any of these developments or some combination of them could have led to the apparent negative marginal costs.

A similar calculation was made using the simplified incremental method. In this case, the example is based on the cost experience of a social-science department at a university. In accord with the simplified method, no attempt was made to identify all the factors that are related to changes in expenditure levels. Instead, only the effects of inflation were deducted from the changes in expenditures. The basic data are shown in table 7.

TABLE 7

Yr.	Professional Salaries	Nonprofessional Salaries	Operating Expense	Total	SCH
75-76	520,111	32,441	10,537	563,089	28,911
76-77	518,320	40,883	12,452	571,655	18,557
77-78	506,719	38,424	11,531	556,674	16,620
78-79	493,357	41,784	11,052	546,193	15,668
79-80	524,384	43,608	13,234	581,226	13,423

Several objects of expenditure were excluded since they were episodic and incomplete. None accounted for a significant amount of funds. The information shown in table 8 was calculated using the data shown in table 7 along with the relevant HEPI data.

TABLE 8

Yr.	Change in Total $s	Change Due to Inflation	Net $s	SCH	Net $s per SCH
75-76/76-77	8,566	27,220	(18,654)	(354)	52.69
76-77/77-78	(14,981)	30,158	(45,139)	(1,937)	23.30
77-78/78-79	(10,481)	17,592	(28,073)	(952)	29.49
77-78/78-79	35,033	40,872	(5,839)	(2,245)	2:60

The amounts attributed to inflation were calculated by applying the appropriate HEPI index to the object-of-expenditure categories listed above. The net change in expenditures is simply the gross change in expenditures minus the inflation amount. The results are the average incremental costs for the increment in question that is the estimate for marginal cost.

Those numbers appear more reasonable than those generated by the basic method. They are all of the correct sign and the middle two are close to the average costs for the department. The first and fourth calculations show large deviations from the others. This is indicative, as in the earlier example, of the volatility of marginal costs over a short time period.

Assessment

The incremental method seems a shaky foundation on which to build a comprehensive marginal-cost analysis. The method is subject to the instability of the production relationships, which, at least over a short time period, can lead to implausible marginal-cost estimates. In its basic version, it is heavily dependent on the judgments and memories of a diverse group of persons, while its simplified version omits potentially important factors. The incremental method, therefore, is not the method of choice for calculating marginal cost. It may be useful for microanalysis by administrators at a given level or for supplementing other methods of marginal costing. It may prove useful in its simplified version as a support to a statewide costing effort, but it cannot stand alone.

Appendix

Literature Review

As background for assessing the techniques for conducting marginal-cost studies in higher education, we reviewed previous marginal-cost studies in both higher education and in three other sectors: business and industry, health care, and primary and secondary education. The theory of marginal cost has been discussed in chapter 2. Here we concentrate on the assumptions and methodology of empirical studies.

The bulk of empirical marginal-cost studies have been in the business and industrial sector, and the theoretical and practical norms for such studies have developed within the context of profit-seeking firms. In recent times, cost analysis in the health-care field has occasionally involved the estimation of marginal costs. Aspects of hospital economics provide interesting parallels with higher-education institutions. The parallels between higher education and primary and secondary education are also obvious. While very few marginal-cost studies can be found in the latter areas, considerable work has been done on estimating average-cost curves, and that literature is sufficiently relevant to warrant review here.

Most of the literature in which empirical-cost studies are discussed is to be found in economics or in subspecialties such as the economics of health care and the economics of education. The cost studies in these areas almost invariably involve statistical estimation—typically some form of regression analysis. Empirical-cost studies based on accounting techniques (that is, using the incremental method) are generally not found in the literature. Almost every text or theoretical work on cost accounting describes the technique (often with examples) but there is no body of literature reporting on empirical studies. We believe that the structure of the accounting discipline with its emphasis on technique, definition, control, and theory rather than on generalizable findings is the reason for this gap in the literature.

In a similar fashion, literature reporting on empirical marginal-cost studies using the fixed and variable technique is sparse. Again, the technique is based on an accounting approach and is described in accounting texts, but empirical-cost studies of this type are not central to the accounting discipline as such. In this case, however, the situation is further complicated by an explicitly normative and political method of cost calculation. A small number of case studies of costing using the fixed and variable method have been reported, but these do not claim to be generalizable for a sector. Rather, they relate to a single organization or small groups of organizations within a sector.

The bulk of the literature review, then, will be devoted to statistical marginal-cost studies. The review will discuss two issues that were alluded to on several occasions earlier in the present work. They are the form of the estimating function, the relationship between cost and production functions, and assumptions regarding optimal economic behavior. The treatment of these issues in the empirical studies will be summarized, as will some of the reported findings.

The explicit form of the estimating function is crucial, and a likely source of difficulty in instances where the underlying physical relationship between output and input is not stable or not well understood, or when multiple outputs, joint production, or joint supply are present. In the literature review, we note what types of functions are estimated, what kinds of functional forms are most frequently employed, and what sort of relationship, if any, is developed between cost and production functions.

Optimization is often assumed in theoretical expositions of the behavior of the firm. Technically speaking, the very meaning of the terms *cost function* and *production function* contain the notion of optimization. The cost function expresses the optimal solution to producing a given level of output at some level of factor prices. These factor prices are subject to the constraints of the production function. The production function expresses the maximum amount of output that can be obtained from a set of inputs. It can be said that the full microeconomic model is being applied when two conditions exist: (1) when it can be assumed that optimization exists, and (2) when the cost and production functions are properly related mathematically. In the literature review, we describe how the empirically oriented cost analysts actually use the microeconomic concepts.

The findings of marginal-cost studies that are of interest here are not the specific cost estimates for an additional ton of steel or kilowatt of electricity. Rather, what is of interest are the shapes of the estimated-cost curves. Theory suggests that marginal-cost curves ought to be U-shaped, where marginal costs first decline as output increases, then eventually increase as output continues to expand beyond the most efficient production level. The literature review will indicate the extent to which that theoretical expectation is met in the empirical studies and the relevance of the findings for the related issue of scale economies.

Empirical-cost studies in business and industry, health care, primary and secondary education, and higher education will be reviewed in turn. The intention of the review is to provide an overview along with specific examples of representative studies in each sector. Greater detail is provided for the marginal-cost studies in higher education. The narrative is followed by a bibliography covering most of the marginal-cost studies in higher education and a representative sample of empirical-cost studies in the other three sectors.

Business and Industry

Economists have conducted a great many studies to estimate cost functions, or cost curves, in various businesses and industries. Mansfield (1979) provides a partial, but representative, list of about 40 major studies, some of which deal with a number of industries. Manufacturing, mining, retailing, distribution, transportation, utilities, and service industries have been analyzed.

The record with respect to the microeconomic model is mixed. According to Uzawa, "It is customary in econometric studies of production structure to specify the form of production functions, up to a certain parametric class (such as Cobb-Douglas or Constant Elasticities of Substitution—CES) and then estimate the parameters, through the cost curves which are usually derived by minimization of total cost" (1964, p. 216). Nerlove's (1963) study of the electric-power industry is an often-cited example of that procedure, as is the original CES-based study by Arrow, Chenery, Minhas, and Solow (1961). On the other hand, in many cost studies in this sector a production function is not even mentioned, much less used, as a basis for deriving a cost function. This is true, for example, in Dean's (1976) series of studies of manufacturing and retail firms and Bengston's (1965) analysis of the banking industry. In his textbook on statistical cost analysis, Johnston (1960) demonstrated the derivation of a (short-run) cost function from an explicit production function, but he apparently did not use derived cost functions in the empirical studies on which he reported, nor is there much evidence of specific production functions at work in the 31 studies he reviewed.

Cost minimization does not appear to be of much concern in cost studies of nonregulated industries. Presumably, it is being taken for granted. Cost studies of regulated industries are more likely to include mention of the least-cost assumption, with some division of opinion on the issue. In reviewing cost studies in the electric-power industry, Galatin (1968) found it convenient to divide the studies into those that did and those that did not assume cost minimization, for example, Nerlove (1963) and Lomax (1952), respectively.

Numerous types of cost functions have been used to estimate marginal costs in business and industry. Table A.1 provides a representative sample of such functions. Walters (1963) in his review of cost functions indicates that the quadratic form (equation 2) was used most often. More recently, Griffen (1979) has commented on the widespread use of the translog cost function (equation 7). The latter is attractive because of its generality. It places no prior restriction on the substitution elasticities of the factors of production, and it allows scale economies to vary with the level of output (Christensen and Greene 1976).

Granted substantial differences between the various cost studies in terms of quality, content, and coverage, the results do seem to converge with respect to two major issues. The most frequently found pattern is one of constant marginal cost and declining average cost in the short run (Johnston 1960 and Walters 1963) and an L-shaped long-run average-cost curve (Johnston 1960 and Mansfield 1979). The issue with respect to long-run average costs is not whether they decline as scale increases for small firms, but whether they eventually rise as large firms continue to expand. Such an eventual upturn has some theoretical support, although less so than for similar behavior in the short run (Walters 1963). Referring to long-run average cost, M. Feldstein (1967, p. 58) spoke of "overwhelming evidence" that such costs do not increase with size. He cited as evidence numerous studies relating to gas supply, electricity supply, road transport, rail transport, and retail distribution. Using a production-function approach, Griliches and Ringstad (1971) found that the returns from factors of production increased at a decreasing rate in 23 of 27 separate industries in Norway, a behavior that implies that the long-run average-cost curve in those 23 industries was not rising due to increases in scale. It may be inferred from the preponderance of L-shaped long-run average-cost curves that the long-run marginal-cost curve must be rather flat over a considerable range of output in many industries.

TABLE A.1

COST FUNCTIONS FOR BUSINESS AND INDUSTRY

Cost Function	Type	Industry	Reference
1[a] $TC = b_o + b_1Y$	linear	steel manufacturing	Yntema (1940)
2[a] $TC = b_o + b_1Y + b_2Y^2$	quadratic	electric power	Nordin (1947)
3[a] $TC = b_o + b_1Y + b_2Y^2 + b_3Y^3$	cubic	retailing	Dean (1976)
4[b] $TC' = b_o + b_1(1/Y) + b_2(1/D)$	reciprocal	railroads	Sidhu, Charney, et al. (1977)
5[c] $TVC = AY^{b_1}e^{b_2V}$	multiplicative	electric power	Johnston (1960)
6[d] $\log E = b_o + b_1 \log N + \beta X$	double log	banking	Bengston (1965)
7[e] $\ln C = b_o + b_Y \ln Y + \frac{1}{2}\gamma_{YY}(\ln Y)^2 + \Sigma b_i \ln P_i + \frac{1}{2}\Sigma_i\Sigma_j\gamma_{ij} \ln P_i \ln P_j + \Sigma\gamma_{Yi} \ln Y \ln P_i$	translog	electric power	Christensen & Greene (1976)

a. TC = total cost; Y = output.

b. D = density of traffic.

c. TVC = total variable cost; A = scalar; V = thermal efficiency.

d. E = annual deficit expenses; N = number of accounts; X = a vector of other variables affecting costs, such as average balance of all accounts or service charges on accounts.

e. C = total cost; P = price of input.

Hospitals

Although microeconomic theory was developed with the competitive, profit-seeking firm in mind, the theory is thought to be relevant to the economic behavior of the nonprofit organization as well. A variety of empirical investigations show that estimating cost curves in the nonprofit sector is fundamentally the same sort of task as it is in the profit sector. Nonprofit hospitals make up a particularly interesting portion of the former because their multiproduct, nonprofit, service-oriented environments resemble those of colleges and universities. The review begins as before by looking at the practical applicability of the microeconomic model.

M. Feldstein's (1967) study of the cost-output structure of hospitals is cited by Mansfield (1979) as an excellent example of how microeconomic concepts can be useful in analyzing nonprofit organizations. Although Feldstein estimated both cost and production functions, the functions are not mathematically related as they are in the microeconomic model, nor does he assume cost minimization. Pauly constructed a production function in the process of estimating a cost function for hospitals, but then went on to say that the cost-minimization assumption, which would permit interpreting the cost function as the dual of the production function, is questionable, especially for nonprofit hospitals. Pauly concluded that "it is probably better to interpret the [cost] function as a behavioral relationship rather than a technical one" (1978, p. 79). Newhouse (1970) argued that the goal of a nonprofit hospital is not to minimize costs but to maximize some combination of quality and quantity. Lave and Lave (1970) developed cost functions without recourse to production functions and did not assume cost minimization although they did incorporate some constraints on hospitals' costs in their estimating equation. P. Feldstein stated in a textbook on health-care economics that "observed hospital behavior diverges from profit-maximizing behavior with regard to the assumption of cost minimization" (1979, p. 188). There is general agreement, then, that the microeconomic model and the operation of nonprofit hospitals are not a good match; nonetheless cost studies purporting to estimate average and marginal costs have been conducted. All that changes, it would seem, is the interpretation given to the estimated-cost curves.

Several examples of cost functions used in estimating hospital cost functions are shown in table A.2. Of particular interest are the case-mix vectors used in all the equations shown in the table except equation 5. One way in which colleges and universities could be said to produce a multiproduct is by educating students in a variety of programs. The parallel with hospitals that treat patients with a variety of illnesses is obvious. The reason why casemix is not included in equation 5 (table A.2) is that Lave and Lave (1970) did a time-series analysis in which they explicitly assumed that casemix remained constant over time. Also of interest are the inclusion of scale and utilization rate variables in two of the estimating equations (equations 1 and 5, table A.2). The inclusion of these variables can substantially alter marginal-cost estimates.

TABLE A.2

COST FUNCTIONS FOR NONPROFIT HOSPITALS

Cost Function	Type	Reference
1[a] $E = a_0 + a_1N + b_jX_j$	linear	M. Feldstein (1967)
2[a] $E = a_0 + a_1N + a_2B + b_jX_j$	linear	M. Feldstein (1967)
3[a] $E = a_0 + a_1N + a_2N^2 + a_3B + a_4B^2 + b_jX_j$	quadratic	M. Feldstein (1967)
4[b] $AC = a_0 + a_iC_i + b_jD_j$	linear	Lave et al. (1972)
5[c] $\log AC = a_0 + a_1t + a_2\log U_t + a_3S_t$	double log	Lave & Lave (1970)
6[d] $\ln(TC/P_1) = a_0 + a_1\ln A + a_2\ln C_1 + a_3P + \sum_{i=2}^{4} b_1 \ln(P_i/P_1) + \Sigma g_i \ln Z_i$	translog (first order)	Pauly (1978)

a. E = total ward costs; N = number patients; B = number beds; X = a vector of casemix variables.

b. AC = average cost per patient; C = a vector of hospital characteristics; D = a vector of casemix variables.

c. U = utilization rate; t = time period; S = number beds.

d. TC = total hospital costs; P_1 = price of reference input; A = equivalent inpatient admissions; C_1 = casemix index; P = proportion of discharges for normal delivery; P_i = price of input i; Z = other hospital and physician staff characteristics.

Three results of the hospital studies are pertinent to the objectives of the present study: the influences of the casemix variables, the shape of the cost curves, and the relative magnitude of marginal costs when compared to average costs. The vector of casemix variables in Martin Feldstein's (1967) study explained 27.5 percent of the variation in overall ward costs, while in Pauly's (1978) study, the casemix vector explained 21 percent of the variance in total hospital costs. Casemix, then, makes a substantial difference, which suggests that a comparable program-mix vector might be a useful addition to a higher-education cost function. As for the shape of the cost curves, recent studies have provided "some evidence that the long-run average-cost curve is U-shaped, but they have not precluded the possibility that the cost curve is actually L-shaped (first declining, then constant average and marginal costs)" (Sorkin 1975, p. 85). At least part of the difficulty in sorting out the conflicting evidence may be due to differing interpretations of the relationship between output and scale, which leads to different, rather than similar, hypotheses being tested in the several studies (Mann and Yett 1968). A related conceptual issue can also lead to confusion in regard to the magnitude of marginal costs. This can be readily seen by comparing the results from Feldstein's (1967) cost functions shown in table A.2. On the basis of equation 1 (table A.2), where scale is free to vary, marginal cost (coefficient a_1) turns out to be about 87 percent as large as average cost. On the basis of equation 2 (table A.2), where scale (number of beds) is being held constant, marginal cost (coefficient a_1) turns out to be only 21 percent as large as average cost. The difference in the ratios is interesting but not surprising, because two different types of marginal costs have been calculated. In any event, his estimate that marginal costs were 87 percent of average costs when scale is free to vary is a little higher than comparable findings in other studies. For example, the data reported by Ingbar and Taylor (1968) suggest a ratio of about 80 percent, whereas Lave, Lave, and Silverman (1972) reported a rate of 70 percent, calculated at mean output levels.

Martin Feldstein referred to the estimate a_1 in equation 1 (table A.2) as average-incremental cost, while retaining "marginal cost" for the estimate a_1 in equation 2 (table A.2). Another way of making the distinction, it would seem, would be to call the former "long-run marginal cost" and the latter "short-run marginal cost." Theoretically, the short-run marginal-cost curve may lie below the long-run marginal-cost curve (Henderson and Quandt 1971), as apparently is the case with respect to the hospitals in Feldstein's study. Many of the hospitals in his study probably had underutilized capacity, which would explain why, when scale (number of beds) was held constant, the addition to total cost for an additional patient was small.

Primary and Secondary Education

Cost studies in primary and secondary education are typically concerned with economies of scale rather than with marginal costs as such. Many of the studies have been prompted by issues related to the consolidation of schools and school districts (such as Hind 1977) or, less frequently, by legal issues related to equitable expenditure patterns among school districts (for example, Michaelson 1972).

There have been cost studies in primary and secondary education such as those by Bieker and Anschel (1973) and Kiesling (1967) that have used a production-function orientation. Most investigators, it appears, have chosen to estimate cost functions such as those shown in table A.3. Rarely are production and cost functions related in the manner stipulated in the microeconomic model, and then usually in only a general way. For example, Cohn (1968) listed the variables that would properly belong in a production function and then used that list to generate variables for a cost function; the functional form of the cost function, which he estimated, is not, however, derived from an explicit production function. The specification of a production function for public schools is thought to be quite difficult, in part because of problems in specifying and measuring output. Even if the output problem could be settled, the subsequent derivation of a cost function by the standard optimization procedure would be questionable in the face of serious doubts as to whether schools are operating efficiently (Levin 1974). In this regard, Cohn and Riew (1974) found only one study, that of Katzman (1968), in which an attempt was made to derive least-cost combinations in education. It should be noted that in their own empirical studies Cohn and Riew (1974) would have been willing to assume cost minimization, but their admitted inability to specify meaningful production functions precluded the use of the full microeconomic model. What they did instead was to provide a substitute for a true economic cost function, that is, they resorted to estimating what Cohn (1979) elsewhere calls a *pragmatically oriented* or *approximate* cost function.

TABLE A.3

COST FUNCTIONS FOR PRIMARY AND SECONDARY EDUCATION

Cost Function	Type	Reference
1[a] $TC = a_0 + a_1E + a_2E^2 + a_3E^3 + a_4S$	cubic	Mitten (1975)
2[b] $AC = a_0 + a_1 \ln E + a_2V + a_3D_1 + a_4D_2 + a_5G$	partial log	Shapiro (1973)
3[c] $AC = a_0 + a_1E + a_2E^2 + a_3C_1 + a_4T + a_5V + a_6L$ $+ a_7D_3 + a_8W^2 + a_9H$	quadratic	Osburn (1970)
4[d] $AC = a_0 + a_1E^{-1} + a_2(CH) + a_3U_1 + a_4W + a_5C_2$ $+ a_6(BV) + a_7(BI) + a_8U_2$	reciprocal	Cohn (1968)
5[e] $AC = a_0 + a_1E + a_2E^2 + a_3W + a_4C_3 + a_5U_3$ $+ a_6G_2 + a_7R$	quadratic	Riew (1966)

a. TC = total cost; E = enrollment; S = square feet of facility.

b. AC = average cost per student; N = equalized assessed property value per pupil; D_1 = dummy for northern and southern Alberta; G = percentage rate of growth from previous year's enrollment; C_1 = curriculum breadth; D_2 = dummy for urban areas.

c. C_1 = curriculum breadth; T = tax levy; V = assessed valuation per pupil; L = median educational level of residents in county; D_3 = dummy for geographical location; W = average teacher salary; H = percentage of students in high school (relevant data on primary and secondary students and costs could not be disaggregated).

d. CH = college hours of teachers; U_1 = assignments per teacher; W = average teacher salary; C_2 = number of units offered; BV = building value; BI = bonded indebtedness; U_2 = class size.

e. C_3 = number of units offered; U_3 = average number of classes taught per teacher; G_2 = percentage changes in enrollment (1957-1960); R = percentage of classrooms built after 1950.

Because returns to scale are often the primary interest, the estimated cost functions typically include average cost as the dependent variable. Unlike most cost functions for business and industry, but like most cost functions for hospitals, the functions shown in table A.3 contain quite a variety of independent variables, reflecting the pragmatic orientation referred to above. The large number of variables in the hospital cost functions was necessary because of variation among hospitals in services provided or in other characteristics. In the case of schools and school districts, most of the variables accompanying the enrollment measures reflect variations in the funding climate (such as indebtedness, tax levy, assessed valuation per pupil), in teacher characteristics (college hours, average salaries), in the deployment of teachers (assignments per teacher, class size), or in the extent of the curriculum (curricular breadth, units offered). The inclusion of funding climate variables would seem to be tacit admission that the cost-minimization assumption is not viable, because the implication is that expenditures per pupil depend to some extent, at least, on the availability of funds rather than on the technical relationships of production.

In regard to findings, Cohn remarked in a recent survey that "the overwhelming conclusion has been that schools of larger size can operate at lower per-pupil costs, other things equal (1979, p. 202)." One interesting exception is a study of Michigan secondary schools (Cohn and Hu 1973), where economies of scale were not found for institutions as a whole but were found in various particular programs such as mathematics and homemaking within the institutions.

The evidence is not conclusive regarding the shape of the long-run average-cost curves as output reaches higher levels. The quadratic form of the average-cost function, as in equation 3 (table A.3), has worked well in a number of studies (see Cohn 1968; Riew 1966; Osburn 1970; Sabulo, Egelston, and Halinski 1979), implying a U-shaped curve (assuming the appropriate signs on the coefficients). The reciprocal form, as in equation 4 (table A.3), has also worked well in some instances (see Cohn 1968; Hettich 1968; Hind 1977), implying an L-shaped curve. Similarly, the semilog form, as in equation 2 (table A.3), has also been effective (Shapiro 1973), implying a curve in which average costs decline continuously at a declining rate as enrollment increases. It would seem reasonable to conclude that over some range of enrollment near the mean, marginal costs are relatively constant and not very different from average costs. Marginal costs at low enrollment levels are clearly less than average costs and perhaps declining as well. Once again, the behavior of marginal costs at higher enrollment levels remains an unsettled issue.

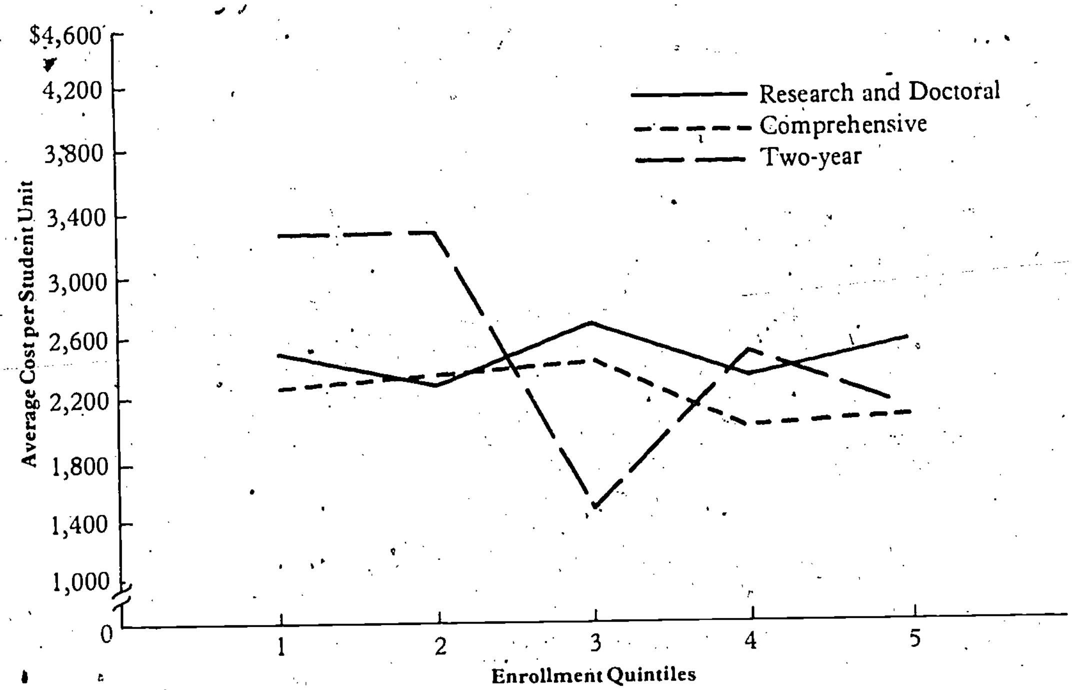
Higher Education

Unit cost studies have a long history in higher education, dating as far back as 1894 (Witmer 1972), but the total number of such studies is small (Cavanaugh 1969). Adams, Hankins, and Schroeder (1978) credit Stevens and Elliot (1925) as being the first to note the difference between marginal and average costs. The Russell and Reeves (1935) analysis of unit costs at 44 colleges is a landmark study in which some of the fundamental relationships between unit costs and size, quality, and program breadth were initially assessed. In contrast to Russell and Reeves, who used data aggregated at the institutional level, the *California and Western Conference Cost and Statistical Study* (Middlebrook 1955) is often cited for its detailed analysis at the departmental level, focusing on technical relationships in the production process at 12 research universities. O'Neill's (1971) study of resource use in higher education from 1930 to 1967 has also received much attention. Substantial bibliographies along with commentaries on cost-analysis literature can be found in the studies by Witmer (1972), Adams et al. (1978), and Cohn (1979). Most recently, Bowen (1980) has provided a summary of a number of previous cost studies along with new empirical work of his own, and a discussion of the findings of both in the context of public policy toward the financing of higher education.

Among better known unit-cost studies since World War II, considerable attention has been given to the effects of enrollment size on average costs per student. Because those effects are closely associated with the behavior of marginal costs, a summary of the findings is in order. According to Bowen, "There can be little doubt that potential and substantial economies of scale in higher education actually exist (1980, p. 193)." Reicherd (1971) came to the same conclusion a decade earlier. But what does the average (variable) cost curve actually look like? Figures A.1 and A.2 were constructed on the basis of tabular data presented in Bowen's (1980) work, the most recent study to address this issue. The data came from a random sample of 268 institutions for the year 1976-77. Educational costs, as defined by Bowen, include outlays for instruction and departmental research, student services, student financial aid paid from institutional funds, and a prorated portion of expenditures for academic-support facilities such as libraries, computers, administration, and plant operations and maintenance. Enrollment was calculated in terms of student units by assigning weights to full-time equivalent (FTE) students at various levels: lower division = 1.0; upper division = 1.5; students in advanced professional programs = 2.5; first-year and unclassified graduate students = 2.1; and graduate students beyond the first year = 3.0 (Bowen 1980). A weighting system was necessary because, as is widely recognized, average costs per student differ by level of enrollment.

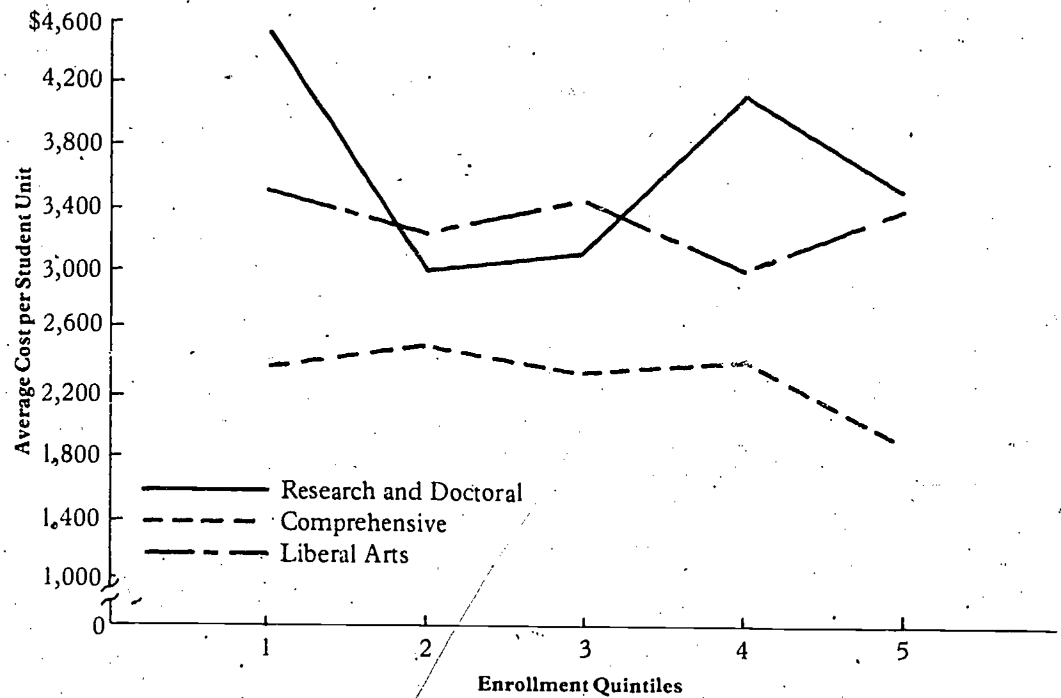
As shown in figures A.1 and A.2, only private research and doctoral-granting universities and public two-year colleges appear to behave in something resembling a U-shaped curve. Other recent studies such as those by Dickmeyer (1980) and Brinkman (1981) lend support to the behavior shown for two-year and research institutions, respectively. Earlier studies, a number of which were reviewed by

FIGURE A.1



AVERAGE INSTRUCTIONAL COSTS AT PUBLIC INSTITUTIONS
Data from Bowen (1980, p. 181)

FIGURE A.2



AVERAGE INSTRUCTIONAL COSTS AT PRIVATE INSTITUTIONS
Data from Bowen (1980, p. 181)

Reicherd (1971), indicate that very small institutions, relative to their purported mission, tend to have disproportionately high unit costs. Indeed, the Carnegie Commmission (1971) argued for minimum levels of enrollment based in part on efficiency criteria. However, as Bowen (1980) points out, only a very small percentage of students attend very small institutions today; if only public institutions are included in the analysis, the percentage is smaller still. It is quite conceivable, then, that with the possible exception of two-year colleges, most public institutions are operating within an enrollment range where unit costs are not particularly sensitive to size. Furthermore, when public two-year colleges are analyzed at the district level, there also appears to be little evidence for either economies or diseconomies of scale (Kress, 1977).

A search of the literature on cost studies in higher education uncovered 13 cost studies that included marginal-cost estimates. All have been published since 1969. Marginal costs are a major focus in 10 of the 13 studies. The extent of unpublished marginal-cost estimates developed by state higher-education coordinating agencies is probably not very large, according to Allen and Topping (1979). As for institutional sources such as offices for institutional research or planning and budgeting, Adams et al. (1978) reported that the availability of marginal-cost data dealing with instructional programs is "almost non-existent," based on a recent survey of 305 institutions of various types.

With respect to the microeconomic model, Southwick (1969), Jenny and Wynn (1969), Brovender (1974), Wing and Williams (1977), and Shymoniak and McIntyre (1980) made no mention of the cost-minimization assumption. Carlson (1972) and Tierney (1980) expressly rejected it, whereas Razin and Campbell expressly sidestepped the issue by stating that "the behavior of colleges of the University is taken as an institutional datum, without broaching the question of whether or not colleges 'minimize costs' (1972, p. 312)." Maynard (1971) acknowledged that, by definition, the cost functions he estimated are based on the cost-minimization assumption; however, his extensive efforts to control for possible differences in institutional-funding environments would seem to be tacit admission that he did not accept the assumption in practice. Sengupta (1975) endorsed the idea that colleges and universities "satisfice" rather than maximize, but that did not prevent him from developing and applying minimization-based techniques. Finally, Verry and Davies (1976) assumed cost minimization as the objective of the university departments in their study, but they denied that their cost estimates were indicative of maximum levels of efficiency.

Only Southwick (1969), Sengupta (1975), and Verry and Davies (1976) estimated production functions. These functions play the major analytical role for Southwick, while providing a complementary approach to cost functions in the other two studies. Only in Sengupta's report are cost functions and production functions mathematically related in the theoretically prescribed fashion.

A selection of the cost functions found in five of the studies is shown in table A.4. All equations were estimated by ordinary least-squares regression. They yield a considerable variety of marginal-cost curves, especially when the possible signs and magnitudes of the coefficients are taken into account. The distinction between graduate and undergraduate students is recognized in all equations except equation 2 (table A.4), where graduate students were judged to be too few to matter. The interaction terms in equation 7 (table A.4) allow for testing joint-supply effects. The use of variable D (number of departments in a subject group) in equations 5 through 7 (table A.4) permits the estimation of departmental set-up costs, with the intercept terms set to zero. Neither Sengupta (1975) nor Maynard (1971), who used institutional-level data, attempted to control for differences in program emphasis, whereas Brovender (1974), Razin and Campbell (1972), and Verry and Davies (1976) ran separate regressions for different programs (departments, groups of like departments, and so forth).

Five of the studies report ratios between marginal costs (MC) and average costs (AC). Brovender (1974) found that the ratio MC/AC was smaller for programs in the humanities and natural sciences than for programs in the social sciences at the University of Pittsburgh. Using equation 4 (table A.4), he calculated the ratios to be 0.492, 0.526, and 0.720, respectively, or a mean of 0.579. On the basis of an alternative cost function, in which undergraduate and graduate enrollments were combined (without weights) prior to estimation, the ratios were 0.658, 0.658, and 0.811, respectively, or a mean of 0.709. Of the six types of departments at universities in Great Britain studied by Verry and Davies (1976), the ratio MC/AC was lowest for mathematics, 0.436, and highest for engineering, 0.665. The ratio for the social sciences was 0.544, slightly below the mean of 0.575, for the six department types. These results were based on a multiplicative-cost function (not shown in table A.4) in which a composite enrollment variable was used. To calculate the composite enrollment, undergraduate and graduate students were weighted on the basis of their respective marginal-cost estimates in equation 5 (table A.4). Razin and Campbell (1972), using data on undergraduate instruction in a cross-sectional study of six colleges at the University of Minnesota, found that the MC/AC ratio was 0.577. There appears to be some convergence, then, in these results, which suggests that marginal costs tend to be 55 percent to 65 percent of average costs at four-year institutions. Tierney (1980), however, found much lower ratios in a study of departmental costs at private liberal-arts colleges. The average ratio across nine departments was 0.38. Shymoniak and McIntyre (1980), on the other hand, in a study of 66 community-college districts in California, reported that during 1978-79 the marginal costs of instruction were 90 percent of average costs, but the marginal costs of student support were only 65 percent of average costs. The authors suggest that the high ratio of marginal to average costs in instruction was probably due to the rigorous application of average-cost funding formulas.

TABLE A.4

COST FUNCTIONS FOR HIGHER EDUCATION

Cost Function	Type	Reference
1[a] $X = a_0U + a_1G + a_2R$	linear	Sengupta (1969)
2[b] $TC_1 = a_0 + a_1E + a_2E^2 + a_3E^3$	cubic	Maynard (1971)
3[c] $\log E_{ijk} = a_0 + a_1D_i + a_2D_k + a_3\log SCH_{ijk}$	double log	Razin & Campbell (1972)
4[d] $TC_2 = a_0 + a_1U_s + a_2G_s$	linear	Brovender (1974)
5[e] $TC_3 = a_0D + a_1U + a_2G + a_3R$	linear	Verry & Davies (1976)
6[e] $TC_3 = a_0D + a_1U + a_2U^2 + a_3G + a_4G^2 + a_5R$	quadratic	Verry & Davies (1976)
7[e] $TC_3 = a_0D + a_1U + a_2G + a_3R + a_4U^{1/2}G^{1/2} + a_5U^{1/2}R^{1/2} + a_6G^{1/2}R^{1/2}$	interactive	Verry & Davies (1976)

a. X = FTE number of senior teaching faculty; U = number of full-time plus one-half of part-time undergraduates; G = headcount of graduate students; R = expenditures for sponsored research (sample: 23 universities).

b. TC_1 = total educational general expenditures; E = FTE enrollment (sample: 123 public four-year colleges.)

c. E_{ijk} = total expenditures at course level i and year j in college k; D_i = dummy for course level; D_k = dummy for college; SCH_{ijk} = student credit hours at course level i and year j in college k (sample: 6 colleges of a public research university).

d. TC_2 = faculty salaries; U_s = undergraduate student credit hours; G_s = graduate student credit hours (sample: departments and programs at a private research university).

e. TC_3 = total departmental costs; D = number of departments; U = number of undergraduates; G = number of graduate students; R = weighted sum of articles and books produced by faculty (sample: departmental data across all but 4 of the universities of Great Britain).

Verry and Davies (1976) reported wide differences when comparing marginal cost between departments. They used a number of estimating equations, with somewhat varying results. In general, engineering had the highest marginal costs at the undergraduate level, while the physical sciences were highest at the graduate level. The arts and social sciences were consistently low at both graduate and undergraduate levels, whereas marginal costs in mathematics were very low for undergraduates but in the middle range for graduate students. Differences in marginal costs between high- and low-cost departments at both the undergraduate and graduate level were on the order of slightly under 4 to 1. The differences reported by Razin and Campbell (1972) were smaller. Estimated only for undergraduate enrollments, the highest marginal costs were recorded by the College of Agriculture. They were about 2.25 times as large as the lowest marginal costs, which belonged to the Colleges of Liberal Arts and Business Administration. The most surprising result, compared to that of the Verry and Davies (1976) study, was that the Institute of Technology had relatively low marginal costs, about half as large as those in the College of Agriculture. Brovender's (1974) data are more highly aggregated, so it might be expected that the differences he reported would be less. This turns out to be the case, as the highest marginal costs, which were in the social sciences, were only 1.55 times as large as the lowest marginal costs, which were in the humanities. It is interesting that the social sciences were found to have somewhat higher marginal costs than the natural sciences. The latter did have slightly higher average costs. In Tierney's (1980) study, marginal costs across nine departments differed by as much as 3.23 to 1. While the evidence is somewhat conflicting on the details, the four studies demonstrate that marginal costs differ substantially with respect to curriculum content. Carlson (1972), in analyzing the behavior of the "most efficient" institutions within various institutional categories, also reported that marginal costs differ by program. In another work, Carlson (1975) reports on cost-estimation efforts at the Esmee Fairbairn Research Centre (1972) that show substantial differences in marginal costs between the social and biological sciences for both undergraduate and graduate students at universities in the United Kingdom.

The average costs of graduate education are thought to differ substantially from those of undergraduate education. Estimates range from 2 or 3 to 1 (Bowen 1980) to as high as 5 or 6 to 1 (James 1978), comparing graduate to lower-division average costs. Similar ratios for marginal costs might be expected. The lowest ratio reported by Verry and Davies (1976) was in engineering, where marginal costs for graduate students were 2.4 times as large as the marginal costs for undergraduates (using mean values across six estimating equations). The highest ratio reported, 9.3 to 1, was in mathematics. Additional evidence on this matter is sparse. In a cross-sectional study of public research universities, Wing and Williams (1977) reported that marginal costs for graduate students are only one-third as large as those for undergraduates. This surprising result is difficult to interpret, however, as Wing and Williams regressed total instructional costs on several variables, including revenues, which would normally not be found in a cost function. The studies by Southwick (1969) and Sengupta (1975) suggest that maximal-cost ratios between the graduate and undergraduate levels are about 1.7 and 1.5, respectively. Southwick, however, did not provide data on significance tests, and the graduate-student coefficient was not significant in Sengupta's estimated function. In the studies at the Esmee Fairbairn Economics Research Centre (1972), marginal costs for graduate students were 3 to 6.8 times as great as those for undergraduate students in the social sciences, and 2 to 4 times as great in the biological sciences, depending on the form of the estimated-cost function. Interestingly, in a study of administrative rather than instructional costs, Pickford (1974) found that the marginal cost of a graduate student was over 5 times that of an undergraduate student for a group of 46 universities in the United Kingdom.

In their extensive study of British universities, Verry and Davies (1976) estimated both aggregated- and allocated-cost functions. For the former functions, such as those shown in table A.4, their overall conclusion was that marginal costs for both graduate and undergraduate students were generally constant. On the other hand, they found considerable evidence for a variety of curves when marginal costs were estimated using allocated-cost functions. An example of the latter would be the regression of faculty salaries allocated to undergraduate instruction on undergraduate enrollment. There was considerable variation in the shape of the cost curves from one department to another, and from one level of instruction to another.

Evidence in regard to the shape of marginal-cost curves at U.S. colleges and universities is far less complete. Maynard (1971) found a U-shaped curve for public four-year colleges. Carlson (1972) reported that the marginal costs at efficient institutions are lower for institutions below the average level of enrollment than for those with above-average enrollments for various levels of enrollment at various types of institutions. Sengupta (1975), using a logarithmic function with a composite-enrollment variable, obtained results that show increasing marginal costs for a small group of universities. Tierney (1980) found U-shaped marginal-cost curves for liberal-arts colleges. Razin and Campbell (1972) found that marginal costs declined at a decreasing rate for six colleges in a large research university. Shymoniak and McIntyre (1980) reported that a linear model was appropriate for all but the very small and very large community-college districts. Only linear functions were discussed or did well statistically in the remaining cost studies.

Summary

The results of the literature review may be summarized as follows:

1. The use of statistical cost analysis is widespread, having been employed in many different industries and sectors of the economy. Methods and procedures for the studies are generally similar from one industry or sector to another.

2. Although statistical cost analyses are typically grounded in the microeconomic theory of the firm, the assumption of optimal behavior is often not met, nor are the prescribed techniques for relating cost and production functions carried through in all, or even most, cost studies. Most estimated cost functions, then, are best thought of as approximations, that is, as functions that describe behavioral rather than technically efficient relationships.
3. The typical approach to developing specific cost functions has been pragmatic because of the absence of a theoretically prescribed form for the cost function along with a frequent lack of adequate information regarding the production process.
4. Studies in numerous industries indicate that long-run average costs decline when output is increased at relatively small firms, but tend to be essentially constant when output is increased at middle- and large-size firms.
5. Similarly, there are studies that suggest that long-run marginal costs in many industries are relatively constant over the observed range of output. No clear pattern has emerged with respect to marginal-cost behavior in higher education.

On a more interpretive level, the review would seem to show that a statistical cost study in which marginal costs are estimated is essentially the same sort of undertaking in higher education as it is in other industries. The evidence is twofold. On the one hand, previous marginal-cost studies have been done in higher education using the same general methods and procedures as in comparable studies in other areas. In addition, although there is little reason to think that colleges and universities are cost minimizers or that their production functions can be properly specified, such characteristics do not make higher education unique. The review has shown that those two basic elements of the microeconomic model are seldom realized in applied work. This is not to say that a statistical cost study in higher education does not involve issues that are peculiar to higher education, or that these issues are easy to handle in a practical sense.

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