Background information needed to understand the literature on the impact of price on college attendance (i.e. price-response literature) is provided. After briefly introducing price theory and its use in demand studies in higher education, the major expository articles are reviewed, and major analytical methods used by researchers are examined. Examples from studies that typify these approaches are included, and important facts learned from the literature are summarized. Whether the research was done with multiple regression or with conditional logit analysis, price was seen as an important factor influencing student demand; as price goes up, demand falls. The effect of price varies, however, depending on both the type of student and type of institution. Other factors, such as student's sex, can moderate the effect of price on student demand. Price was significant in explaining female full-time-equivalent enrollment. The impact of a price change affects students differentially depending on their income and ability levels. The impact also differs across types of institutions. Attention is directed to use of the following analysis techniques: regression analyses with different demand functions, linear probability models, and logit analysis. (SW)
A Review and Introduction to Higher Education Price Response Studies

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A Review and
Introduction to
Higher Education Price
Response Studies

Mark Chisholm
Bethaviva Cohen
1982

National Center for Higher Education Management Systems
P.O. Drawer P
Boulder, Colorado 80302
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The work upon which this publication is based was performed by NCHEMS pursuant to Contract No. 400-80-0109—Program on Educational Policy and Organization—with the National Institute of Education. It does not necessarily reflect, however, the views of that agency.
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Table

1. Relationship of the Magnitude of Price Elasticity to Changes in Price .................................................. 5
This document was written under the auspices of a grant to the National Center for Higher Education Management Systems (NCHEMS) from the National Institute of Education (NIE) for the 1980 Fiscal Year as part of the Strategic Planning project in the Financing and Planning Program. The central purpose of the Planning and Financing Program is to develop a conceptual foundation for planning and financing in the context of the many different settings that constitute higher education. Within that program, the Strategic Planning project focuses on the interface between an institution and its environment. A review and introduction to the literature that investigates the effects of price on students' decisions, therefore, constitutes a basic building block in NCHEMS research and development of strategic planning concepts for higher education.

This document will serve as a starting point for anyone who desires to explore this literature. As such, the document has several goals:

(a) To provide a basic background in the economic theory used to investigate the relationship between price and student demand

(b) To categorize the various analytical techniques used by researchers in the field and to describe some of the basic properties and assumptions inherent in each methodology

(c) To review some of the major studies and discuss their relationship to the economic theory and analytical methodology that they employ

(d) To summarize some of the findings and conclusions that are commonly reported

(e) To provide a selected bibliography that is the reader to the literature available on this subject.

The document assumes that the reader is familiar with multiple regression and is comfortable with mathematical notation. In particular, functional notation is used throughout and several algebraic and calculus derivations are included. A reader who is unfamiliar with the notation should skip the equations and concentrate on the text—most of the conclusions and summary material are nonmathematical. Nevertheless, the literature reviewed in this document is often more technical than the material herein, so it was not feasible to completely eliminate the technical aspects of the document.

The audience for this document is assumed to include institutional researchers, at either public or private institutions, who may be faced with the problem of determining the effects of price increases on the enrollment at their institutions. One of the conclusions reported here is that such determinations cannot be made precisely. But price does have an effect, which differs by type of student and type of institution, and an understanding of the literature will give some guidance to institutional planners who are trying to address this problem.
The authors wish to thank the following REINS staff members for their contributions: Beverly Jones and Doug Collett, who provided the initial suggestions and subsequent encouragement; John Beller and Susan Hense, who provided editorial services; and Carole Andree, who typed the first draft and prepared the charts and figures.
Chapter I

Introduction

As post-secondary education moves into the 1980s, institutions will be facing increasing demographic and economic pressures. These external forces are well documented (Carnegie Council 1980, Glenn 1980, and Roe 1980), but the appropriate response of institutions to these influences is not as clear. Institutions will be trying many different strategies to cope with the impending decline in traditional students and the increased competition for resources, with only time and experience to determine which strategies are effective.

In the meantime, the institutional managers making these decisions will be looking at whatever literature is available to help them evaluate the probable effect of different courses of action. Since there will be pressures to increase tuition, fees, and other revenue sources as costs continue to increase, while there will also be a need to attract additional students to offset the decline in the size of the traditional college-going cohort, the literature dealing with the impact of price on college attendance (herein referred to as price-response literature) becomes particularly important.

Unfortunately, this price-response literature is of a highly technical nature, and the interpretation of many of the articles requires some knowledge of economic price theory and of techniques such as conditional logit analysis. Many of these studies are exploratory analyses, using questionable data and experimental techniques, and one danger is that the results of some of these studies might be picked up and applied indiscriminately by analysts that do not fully understand the limitations and conditions pertaining to the study. Alternatively, some useful results might not be used because of the uncertainty and confusion of the administrators trying to interpret them.

The purpose of this paper, therefore, is to provide some of the background information necessary for understanding the price-response literature. The intent is not to go through a study-by-study review and comparison. Several authors have already provided this type of literature review (see Chapter III for a summary of these reviews), but even those reviews are sometimes difficult to understand without an initial orientation to the subject. This paper will attempt to provide this orientation and will describe the various research studies only to the extent that their description may be instructive. A brief introduction to price theory and its use in demand studies in higher education will be presented first. This will be followed by a review of the major expository articles that are available, and then the major analytical methods that are used by researchers in this field will be presented, along with examples from studies that typify those approaches. Finally, a summary chapter will discuss some of the common results and the degree to which they might be applicable to institutional planning.
Chapter 11
Use of Price Theory in Higher Education Demand Studies

The techniques used to predict changes in the demand of students for higher education, given changes in price, are derived from economic price theory. Price theory is a large and complex subject, but only a relatively small part of it is needed to understand the theoretical foundations used in demand studies in higher education. This chapter will concentrate on explaining these concepts, giving the reader a foundation for recognizing the price theory ideas that are most often referred to in the student response literature.

The most fundamental concept is that of a demand function: the relationship between the cost of a product and the quantity that will be purchased by consumers. In higher education, this is usually expressed as the relationship between the price of attendance and enrollment, though the actual measurement vary widely. Enrollment is often expressed as a participation rate, the ratio of the number of enrolled students to the eligible population, while price can include a variety of factors (for example, tuition, fees, room and board, transportation costs, other out-of-pocket expenses, a deduction for student aid awards, forgone income, and so forth).

A demand function that accurately represents the real world would need to include many factors other than price-factors such as the quality of a school, the state of the economy, and the availability of jobs. Also, the demand relationship would vary according to characteristics of the student: factors such as ability, income, and parent education greatly influence a student's sensitivity to changes in institutional price and eventual selection of an institution. Nevertheless, this discussion will focus on price variables, since this is a common factor throughout the student demand literature, and because it is the price factor that is most subject to the control of an institution.

The general student demand function is represented by the general equation:

\[ E = f(P) \]  \hspace{1cm} (1)

where \( E \) is enrollment, \( P \) is price, and \( f \) represents the function that relates price to enrollment and that also includes all the other factors that are thought to influence enrollment. Research into student demand requires that a specific functional form be selected, followed by the analysis of historical data to estimate the parameters of the function. There are many possible functions to choose from, but most of the demand studies use one of the following general forms:

\[ E = a + b P \]  \hspace{1cm} (2)

\[ \log E = b \log P \]  \hspace{1cm} (3)

\[ E = a + b \log e^P \]  \hspace{1cm} (4)
where \( E \) is price elasticity and \( \Delta x \) refers to a very small change in enrollment or price. This basic concept describes the general shape of the demand function at different levels of price. In other words, at a given price level of an institution, with a small price increase (or decrease) student enrollment, and still there is a relatively large (small) change. The term \( E \) is calculated in a later chapter of the text. However, the term \( E \) is not constant; it depends on price and is subject to error.

Price response, on the other hand, refers to the absolute change in enrollment that would be caused by a fixed dollar change in price. In general, the relationship between price response and price elasticity is:

\[
E(x) = \frac{\Delta x}{\Delta p} \cdot \frac{x}{p}
\]

Similarly, the small change, \( \Delta p \), in the relationship for price elasticity would be represented by the partial derivative of the demand function with respect to price:

\[
E(x) = \frac{\partial x}{\partial p} \cdot \frac{x}{p}
\]
the other hand, colleges facing inelastic demand will have a proportionately
smaller decline in enrollment due to an increase in the tuition price, such
that the gross tuition revenue will increase despite the enrollment drop.

Ordinarily, price elasticity relates changes in enrollment at an
institution to changes in tuition price at the same institution. This is
sometimes called own-price-elasticity. A closely related concept is
cross-price-elasticity, which relates changes in price at other institutions.
But means cross-price-elasticities are a way of assessing the degree of
composition between institutions while own-price-elasticity focuses on the
relationship between potential students and a single institution.

Price response coefficients, as opposed to elasticities, do not have such
a clear-cut relationship to changes in institutional tuition revenues. The
price response coefficient tells how many students will be affected by an
increase (or decrease) in price, but this information alone does not indicate
whether there will be an overall increase or decrease in revenue. For example,
two institutions could have the same price response coefficient, -.0124/$100,
but one could be inelastic at -0.50 and the other could be elastic at -1.50.
If both institutions raised their tuition by $100, they would both lose 1.24
percent of their enrollment, but the first school would have an increase in
revenue while the second would suffer an overall decrease.

Another important point, true in both the case of price response
coefficients and elasticities, is that after the tuition price has been
changed, the college may well find itself in a new situation. The nature of
students' preferences for enrolling at a particular college may change along
with the tuition price, and both the price response coefficient value and
elasticity value may change. (Note that in equations 5 and 7 elasticity and
price response are functions of both current enrollment and current tuition.)
The exact nature of these changes will vary with the demand function chosen,
and this is one factor that makes comparability across studies so difficult.

Chapter IV discusses each of the major regression demand models and
indicates the general characteristics of the elasticity and price response
coefficients that are generated by the particular model. Some specific studies
are also presented to illustrate the different points. Before proceeding to
that discussion, however, it should be made clear that this paper is not
attempts to provide a thorough review of all the literature. Such an effort
is unnecessary since there are several recent articles that provide excellent
literature reviews and comparisons across studies. These survey articles are
discussed in the next chapter.
Chapter III
Review of the Literature on the Effect of Price on Enrollment

Aside from reports by researchers of their actual study results in the area of student response to the price of higher education, several scholars have undertaken the task of reviewing, synthesizing, and/or critiquing the results of those research studies. These secondary sources of information on price response studies are (in chronological order):

1974 Carlson, Farmer, and Weathersby
1975 Dresch
1975 Jackson and Weathersby
1977 Weinschrott
1978 Cohn and Morgan
1978 Hyde
1978 McPherson
1980 California Postsecondary Education Commission

The nature and emphasis of each of these studies is different—some are really annotated bibliographies, some contain incisive criticisms of research in this area generally, and others attempt to compare the numerical results of the diverse research studies that have been carried out. (Naturally, most research studies also contain some reference to the body of literature from which they stem, but they usually do not provide as comprehensive a discussion of the literature as the reviews discussed here.)

Three of these secondary sources—Carlson et al. (1974), Hyde (1978) and the California Postsecondary Education Commission (1980)—provide very brief summaries of the literature while highlighting the few universally accepted results: (a) the price of higher education does affect enrollment behavior; (b) as the price goes up, enrollment goes down if all other things remain the same; (c) the effect of price seems to decrease as family income goes up. For example, Carlson, Farmer, and Weathersby present short excerpts from five studies (Radner=Miller, Barnes=Erickson-Hill-Winokur, Kohn=Manski=Mundel, Corazzini=Grabowski, and Hoenack=Weller) including tables of results from three of the studies before discussing the overall limitations (pp. 149-59).

Hyde has a slightly longer discussion of the research on the interaction of price variables with attendance at some postsecondary educational institutions. More studies are mentioned here and two tables are presented which compare the basic results of ten research studies. The discussion is substantive but not exhaustive or very technical.

The most recent brief literature review is by the California Postsecondary Education Commission. Forming a foundation for a model to assess the effect of student charges on enrollment and revenues, this review attempts to "convey the strengths and limitations of the data and methods used in these studies, as well as to summarize their findings" (1980, p. 39). The footnotes citing the relevant literature are as valuable as the discussion in the text. This monograph also demonstrates how research results have been incorporated into a statewide model with the understanding that they are imperfect but better than no information at all.
The most comprehensive recent summary of the price response literature in a journal is by Cohn and Morgan (1978a). They summarize each of 19 widely known research studies in a paragraph each and go on to mention nine other studies. They conclude with what seems to be an agreed upon caution in using the results of the studies of price response as a tool for educational planning at this time.

Two of the literature reviews are written in a distinctly critical vein. Weinschrott (1977) singles out eight research studies (Hoenack, Hoenack-Feldman, Hoenack-Weller, Corazzini et al., Bishop, Radner-Miller, Kohn-Hanski-Mundel, and Barnes et al.) to evaluate along five specific dimensions. Each study is discussed in relation to (a) its treatment of higher education as a consumption versus an investment decision on the part of potential students, (b) its identification of alternative choices open to a potential college student, (c) its inclusion of some measure of financial aid, (d) its method of dealing with the possible confusion in a model that arises when supply and demand relationships are both changing during the time of the investigation, and (e) its stratification, if any, of the data before analysis of results. In addition to the critique of specific articles, Weinschrott (1977) also presents an exhaustive annotated bibliography in the area of price response studies. Taken together with the extensive reference list in Cohn and Morgan (1978), these two sources probably cover 99 percent of the literature extant in 1977.

Dresch (1975) is the other work that examines "the adequacy of available knowledge" (p. 251) about the research available for use in specifying concretely "the essential facets of the student and institutional components of the postsecondary educational system" (p. 254). Anyone seriously considering the use of research results in academic decisionmaking for more than the most general purposes should read this article carefully (especially pages 254-71) in order to understand the problems that beset research in the area of modeling the demand for higher education. The references cited here include more on the topic of modeling in general and are not meant to exhaust the price response literature.

The remaining two literature reviews actually attempt to compare the numerical results of various research studies despite their diverse research designs. Jackson and Weathersby compute a single estimate of price responsiveness for each of seven studies. (Campbell-Siegel; Hoenack, Hoenack-Weller-Orvis, Corazzini et al., Spies, Radner-Miller, and Kohn et al.). The bulk of this article is devoted to citing enough information from each study to be able to compare the results across studies by assuming the existence of a 'typical' individual with an income of $12,000 in 1974 facing a college cost of $2,000 per year and by using "the estimate that 26.2 percent of the eligible population is enrolled in some form of postsecondary education" (Jackson and Weathersby 1975, p. 643).

McPherson probably provides the most comprehensive discussion of the demand for higher education, encompassing a summary of the research literature, a critique of the research methods, and a revision of the price response estimates presented by Jackson and Weathersby. Although it is contained in a book about the private or independent sector of higher education, the discussion is not limited to studies of private colleges. The most succinct summary of the state of the art seems to be McPherson's statement that:
There is probably not a single number in the whole enrollment demand literature that should be taken seriously by itself. But a careful review of the literature will show that there are some important qualitative findings and order-of-magnitude estimates on which there is consensus, and which do deserve to be taken seriously. There are also some serious gaps (1978, p. 180).

In the discussion, McPherson attempts to support this statement. The only shortcoming of this review is its length (over fifty pages) and the presentation of all citations as footnotes rather than in a single reference list. Still, significant information is presented for anyone attempting to use the results of research about the effect of educational costs on enrollment.
Chapter IV

Price Response Studies Using Regression as the Analysis Technique

This chapter discusses the price-response studies that estimate the parameters of a demand function using multiple regression. The data used in these regressions represents the attendance behavior of large groups of students, either over time (longitudinal) or across several institutions (cross-sectional). The interpretation and application of the results of these studies depends to a great extent on the form of the demand function (or regression equation) chosen, so each of the forms commonly chosen will be discussed separately.

**Linear Demand Function: \( E = a + bP \)**

Perhaps the simplest form of a demand function is one in which the enrollment is assumed to respond in a linear fashion to changes in price. The general form for such a function is:

\[
E = a + bP + c_1X_1 + c_2X_2 + \ldots + c_kX_k
\]

where the variables \( X_1 \ldots X_k \) describe the institution and its environment. For the purposes of this discussion, the simpler equation, \( E = a + bP \), will be used without loss of generality since the \( X \)'s are held constant when price response or elasticity are computed. Such a function is represented graphically in figure 1. As can be seen in the figure, enrollment is assumed to vary in a negative linear relationship with price. In these studies, enrollment is sometimes represented as a ratio of enrolled students to a population of potential students, and sometimes it represents an actual enrollment number. A price of zero would result in "a" students enrolling (where "a" stands for all the other factors that help determine the likelihood of enrollment) while at some price, \(-a/b\), the enrollment would drop off to zero. (Here "b" is assumed to be a negative number so \(-a/b\) is actually a positive value.)

The price response characteristic of such a demand function is very simple and is constant for all values of price—it is the coefficient "b". This can be verified from examination of the effect of changing the price by a small amount, \( \Delta P \):

\[
\begin{align*}
\text{if} & \quad a + bP = E \\
\text{then} & \quad a + b(P + \Delta P) = E + \Delta E \\
& \quad (a + bP) + b\Delta P = E + \Delta E \\
& \quad a + b\Delta P = E + \Delta E
\end{align*}
\]

This relationship is therefore independent of the current price and enrollment level.\(^3\)

\(^3\)Technically, the price response is the partial derivative of the demand function with respect to price, scaled to a constant change, \( \Delta P \).
Fig. 1. Linear Demand Function, $E = a + bP$
The situation is more complicated, however, for elasticity. In this formulation the elasticity depends on the current price level.

The steps for computing the elasticity follow:

from equation (6),

\[
\frac{\Delta Q}{\Delta P} = \frac{b_0 + b_1 P}{P}
\]

The above formula shows that as the price increases, the elasticity would go from inelastic to elastic. Initial increases in tuition might increase revenue, but eventually such increases would cause an overall drop in revenue (see table 1). Figure 2 graphs this relationship. If one knows the values of "a" and "b", then the elasticity can be computed for any given price.

Of course, the above discussion about the calculation of price response coefficients and elasticity assumes that the linear demand function accurately portrays the reaction of student demand to changes in price. This is probably an unjustifiable assumption--especially over the full range of possible prices. Such a formulation may provide a close approximation to actual underlying demand preferences over a small range of price values, but it is still only one out of many possible models. Nevertheless, this formulation is used in a number of studies and elasticities and/or price response coefficients are reported in some of those analyses. These values should be interpreted according to the definitions in equations (9) and (10). A reported price response value represents the model's estimate for all price levels being considered, but the reported elasticities are completely dependent on the price used to compute it. Therefore, comparisons of elasticities between these studies would be quite difficult.

One of the earliest and most commonly cited studies that used a linear demand function is Corazzini, Dugan, and Grabowski (1972). They used national data from Project Talent, collected in the early 1960s, that followed a sample of 10th graders through their decision to enroll in college. Corazzini et al. (1972) cross-sectionally stratified this data to produce average values for each state. Their regression equation included tuition variables for junior colleges, four-year public universities, teacher's colleges, and four-year
private universities. In addition, they included variables for average hourly wage of production workers, unemployment rate, father's attained educational level, and student performance on achievement tests. Figure 3 gives their results for the total sample. (They also ran the regression with the data divided into four income quartiles.) All the coefficients in figure 3 are significant at the .05 level with the exception of the tuition at teacher's colleges and the average hourly wage of production workers.

Fig. 2. Elasticity of linear Demand Function
The coefficients for the four tuition variables represent their estimate of price response coefficients. As quoted from Corazzini, et al.:

The total enrollment rate is most responsive to tuition changes at four-year public universities, and a decrease of $100 in tuition in 1963 is associated with a 2.65 percent increase in the nation's enrollment based upon these cross-section results. (1972, p.47)

An overall estimate of price response is unavailable since there are four tuition variables, but they are all negative, as expected. Though Jackson and Murphy (1975) reported average elasticity of tuition coefficients from the four income group regression equations of Corazzini et al., to arrive at one number for comparison, this procedure is highly questionable since the coefficients range from -0.055 to +0.029 and not all of the coefficients are significant in those equations.

\[
E_T = 14.43 + (-.037)T_j + (-.027)T_u + (-.005)T_c + (-.009)T_p + (-3.62)W + (.834)U + (2.84)F + (.176)A
\]

Percent of 1960 10th graders enrolling in college

Fig. 3 Linear Regression Results from Corazzini, Dugan, and Grabowski
No estimate of elasticity is possible from this study since the original tuition values are not available and the elasticity of linear regression equations is dependent on price. One researcher that did try to estimate elasticity, however, was Funk (1972). In Funk's study, the data used was longitudinal enrollment and tuition data for a single university over a twelve-year period. The form of the linear equation was simple:

Actual Enrollment = a + bTuition + cTime

Because actual tuition values were available, Funk estimated elasticity values for each time period. His results showed a fairly inelastic value of about 0.3, but this increased with tuition increases as would be expected at an inelastic institution.

Two other studies using a linear formulation are (a) Hopkins (1974) and (b) Cohn and Morgan (1978). Each of these studies used state cross-sectional data in the regression equation and included a large number of possible explanatory variables. Neither study was highly concerned with price as an important factor. For example, tuition is not directly included in the Cohn and Morgan equations (instead they use an aggregate measure of total state support), and Hopkins is more concerned with a student's decision between public and private alternatives than with price response (though price response is reported in the Hopkins study).

One conclusion to draw from these studies is that a linear regression model is not an effective way to determine price elasticities of student demand. It may be useful as a means of measuring important factors influencing college-going behavior, but the coefficients produced would not be reliable indicators of the effect of a price change on the enrollment at a specific institution.

Log-Log Demand Function: \( \log (E) = \log (a) + b \log (P) \)

Another commonly used demand function looks much like the linear model in equation (8), but logarithms are taken of each variable:

\[
\log (E) = \log (a) + \log (P) + c_1 \log (X_1) + \ldots + c_n \log (X_n)
\]

(11)

As in equation (8), each \( X \) stands for some non-price explanatory variable, and for this discussion they will be collapsed into the constant, \( \log (a) \), since they would be held constant when price response and price elasticity are computed. Therefore, the general equation for this discussion is:

\[
\log (E) = \log (a) + \log (P)
\]

(12)

While the log-log form of (11) is the regression equation used to analyze the data, it is derived from the demand function:

\[
E = aP^bX_1^{c_1}X_2^{c_2}\ldots X_n^{c_n}
\]

(13)

Equation (11) is simply the logarithm of equation (13), a transformation that produces a linear equation that can be solved with multiple regression techniques. Similarly, the reduced form that only considers price is:
The graph produced by equation (14) is depicted in figure 4 (note that "b" is negative and "a" is positive).

One reason that (13) is used as a demand function is that the price elasticity is equivalent to the coefficient "b". Therefore, when equation (11) is solved, the coefficient "b" becomes the estimated elasticity. This model is also convenient because the elasticity is independent of current price level. Unfortunately, this function does not behave so nicely for computing the price increase coefficient.

\[ E = aP^b \]  

The definition for elasticity, \( \varepsilon = \frac{dE}{dP} \), is more generally given as:

\[ \varepsilon = \frac{\frac{dE}{dP}}{\frac{E}{P}} = \frac{abP^{b-1}}{E} = \frac{abP^b}{ap^b} = b \]

In fact, the functional from \( E = ap^b \) is the only one where elasticity remains constant over all values of \( P \). To prove this:

assume \( \varepsilon = b = \frac{dE}{dP} \frac{P}{E} \)

then \( \frac{baP}{P} = \frac{dE}{E} \)

and \( b \int \frac{P}{P} = \int \frac{dE}{E} \)

\( \ln(a) + b \ln(P) = \ln(E) \) (where in some constant) \( e(\ln(a) + b \ln(P)) = E \)

\( ap^b = E \)
To compute the price response coefficient for a given price and price change \( p \), the definition, (7), (Jackson and Westhersby 1975, p. 643), can be used as follows:

\[
\text{price response} = \frac{\Delta p}{p}
\]

The price response is defined as the percentage change in price that is produced by a unit price change, and it depends on the current price level. Figure 5 shows the relationship: for a low price level, an increase of $100 will cause a large drop in enrollment, while at high price levels a $100 increase in price is relatively insignificant and will lead to only a small decrease in attendance rate.

Fig. 5 Plot of Price Response Coefficient as a Function of Price for the Exponential Demand Function
As with the linear model discussed in the last section, one can question
the validity of this approach as a model of how the real world operates.
Clearly they are in conflict. In the first case, every $100 change in price
would always cause a constant decrease in enrollment, but as the price goes up,
students will become price-elastic (a larger percentage of students would not
go) and total revenue would decrease. In the log-log model, the opposite is
predicted: a $100 increase in price would cause a varying decrease in
enrollment, depending on the current price, but the magnitude of the elasticity
(that is, whether or not demand was elastic or inelastic) would remain constant
and could be predicted by the $100 change in the original log in price.

The study by Campbell and Siegel was published in 1967 (with 1965-1966 data)
and is usually cited as the first study to try to estimate a demand function for higher education. They estimated their equation in a log-log format:

\[
\log R_t = \beta_0 + \beta_1 \log P_t + \beta_2 \log Y_t + \epsilon_t,
\]

where \( R_t \) is the ratio of enrollments to eligibles in year \( t \), \( P_t \) is the real
disposable income per household in year \( t \), \( Y_t \) is the average real tuition in
year \( t \) and \( \beta_0, \beta_1, \beta_2 \) are the estimated coefficients.

The coefficient \( \beta_1 = -0.44 \) (see p. 26) represents the price elasticity of this
functional form. The value for \( \beta_1 \) as determined by Campbell and Siegel, \( -0.44 \), and this value is often cited in the literature as a typical price
elasticity value, even though they report a fairly large standard error of \( 1.15 \)
and they point out that their small sample size means this estimate may not be
a reliable estimate.

Campbell and Siegel also discuss price responsiveness, using as data of several studies, Jackson and Monlembly compute a price
responsiveness value by assuming a representative price and attendance values
of $2,000 and 26.2% a $100 change in price, and applying these values to
equation (7):

\[
\text{price response} = \frac{-.44 \times 100}{26.2} = -0.58
\]
In the context of college education costs, it is often observed that the price elasticity of demand is negative, suggesting that an increase in the price of education leads to a decrease in the quantity demanded. This elasticity can be understood by considering the ratio of the percentage change in the quantity demanded to the percentage change in the price. It is given by the formula:

\[ \text{Price Elasticity} = \frac{\% \text{ Change in Quantity Demanded}}{\% \text{ Change in Price}} \]

The numerator represents the change in the quantity demanded, and the denominator is the change in the price. The elasticity value can be positive or negative, indicating whether the demand is elastic or inelastic, respectively.

To illustrate, let's consider an example where the price elasticity of demand for college education is -1.47. This value indicates that a 1% increase in the cost of education leads to a 1.47% decrease in the quantity demanded. This elasticity value is relatively high, suggesting that demand for college education is sensitive to price changes.

In summary, the price elasticity of demand for college education is a crucial indicator for understanding how changes in tuition costs affect enrollment. It helps educators and policymakers make informed decisions regarding tuition policies and financial aid programs.
scribed in this document, the specific numbers reported are not as important as the common result that changes in price do have a statistically significant effect on enrollment.
Individual Choice Studies

All the studies of student demand discussed in the last chapter used multiple regression techniques to estimate the responses of students to changes in price. While the form of the particular demand equation often varied across studies, the data always represented the aggregate behavior of students. For example, the dependent variable might represent the percentage of the eligible population of students that attended college in a given year, under a specified set of conditions. There are many cases, however, where the data being used represents the behavior of individual students. While the individual data could be aggregated into group data and analyzed with the techniques described in chapter IV, this would mean giving up much of the richness of information inherent in individual level data. Further, the researcher is usually most concerned with the behavior of individuals in response to changes in price or other environmental conditions, and an aggregation across individuals may obscure some of those relationships.

In fact, many researchers do use individual level data, usually collected from surveys of recently graduated high school students, and there are several research reports of this type. The chief characteristic of this data is that the dependent variable represents discrete values. In the simplest case, a student either attends college or doesn’t. More generally, the student has several discrete choices, either one of several different colleges or of a noncollege alternative.

The objective of an individual choice study is to infer an individual’s rule for making decisions about college attendance. This is usually done by comparing the characteristics of the school chosen by a particular individual, such as price and selectivity, with the student’s personal characteristics, such as ability and family income. The coefficients estimated for individual students are then generalized to provide probability estimates for the entire population of eligible students.

These explanatory variables are also often treated as discrete, rather than continuous, variables. For example, schools may be divided into selectivity quartiles, according to the range of average SAT scores of their enrolled students, or the student data may also be divided into discrete groups, depending on the level of variables such as family income or SAT score. The presence of discrete independent variables, however, does not cause any difficulties in estimating demand relationships; in fact, several of the studies described in chapter IV include categorical independent variables. The problem comes when the dependent variable is discrete. Hanushek and Jackson (1977) provide an excellent discussion of this subject, and much of the following explanation is based on the material presented in chapter 7 of their book. Swafford (1980) also provides a very good review of the techniques used for analysis of individual choice data.

The graph in figure 6 illustrates some hypothetical data with a dichotomous dependent variable. As can be seen from this figure, there is no obvious functional form that fits the data. In particular, any attempt to fit a straight line through such data will result in large errors. If one assumes
A straight line through such data will result in large errors. If one assumes that the underlying function that describes the true probability distribution is an S-shaped function, then the three graphs in Figure 7 show some of the fairly large errors that may be observed in linear probability modeling. Occasionally, the fit may be moderately close, as in graph (a), but it can also produce a very poor fit, as in graph (c).

![Graph of decision to enroll vs. price](image)

Fig. 6. Observed Data With A Dichotomous Dependent Variable

The S-shaped curves in Figure 7 represent a logistic function and logistic models are often used to fit individual choice data. Logistic models will be discussed in the following section, but first, the discussion of linear probability models will be further elaborated, since they are used in this type of research.

**Linear Probability Models**

The general form of a linear probability model, in which there are only two values for the dependent variable, is given in equation (19):

\[ A = a + bP + c_1X_1 + c_2X_2 + \ldots + c_kX_k \]

where \( A = 1 \) if a student enrolls in college
\( = 0 \) otherwise.
\( P \) is price
\( X_1,\ldots,X_k \) represent other variables that may influence enrollment decisions, such as SAT scores, family income, race, sex, religion, etc.

\( a,b,c_1,\ldots,c_k \) are the coefficients to be estimated. (19)
This equation can be solved with individual level data using an ordinary least squares multiple regression analysis, but such an estimate contains certain limitations that should be recognized before the results are interpreted. Bres (1972, 1978) used a model similar to equation (19) to examine the decision rules used by students in their application decision to select colleges, and in an appendix to his 1978 paper, he discusses these limitations and concludes that for his data, the method is justified.

Fig. 7. Examples of the Ability of Linear Probability Modeling to fit Discrete Data
There are three main problems with this method. First, the statistical
property of the error term is that the standard test of significance for
the coefficients (the t-statistic) is inappropriate and cannot be used to
measure their contribution to the model. The error terms are not
asymptotic, in other words, the variances of the error terms are not equal
each observation. Second, the predicted probabilities from equation (16) can be
less than zero or greater than one. This is particularly likely if one tries
to use the equation to predict results when the independent variables take on
values outside the range of the data used in the sample. Such predictors
would be hard to interpret since no probability can be outside the range of
zero to one. Finally, the R-squared value produced by the regression cannot be used
directly to measure the goodness-of-fit of the linear model to the data because the
predicted dependent variable will cause the R-squared value to be abnormally low.
The predicted probability values will normally be between zero and one, so the
squared differences between the observed data (0 or 1) and the predicted values
will be large.

Spies recognizes these limitations to linear probability modeling, but he
determines that the more accurate results that might be produced from other
methods do not justify their increased costs. His tests on some sub-samples of his
data indicate that in this case the results are similar to those in graph (c)
of figure 7.

Spies' results are limited to high-ability students applying to select
colleges. He is interested in the factors that influence a student's decision
to apply to different categories of schools, and as such, he runs a number of
analyses on his data. His dependent variables are dichotomous, being set to 1
if a student applies to a certain class of institutions, which are grouped by
their typical tuition level and by the average SAT test scores of their
students, and 0 otherwise. As independent variables, he includes factors
such as the students' SAT score, annual family income, total family savings,
number of other dependent children in the family, number of years of schooling
of parents, sex, race, and religion.

Price was not an important factor in these regressions because the
(groupings of the schools lumped schools with similar tuition levels together.
Spies' complete study included some regressions on aggregate data where tuition
price was included, but his results indicated that, for select schools, price
was not an important factor. He found that the ability level of students, as
measured by their SAT scores, was the most significant factor influencing their
application decision, and that for high-ability students, price and income were
much less important. A rough estimate of his computed price response
coefficient from his 1976 study is -0.17 percentage points per $100 increase in
price, and Jackson and Watersby report a price responsiveness value of -0.06
for his 1975 data.

These two studies by Spies are among the most informative and well written
studies available. They cannot be generalized beyond high-ability students and
select institutions, but Spies is a very clear writer, and his methodology and
results are easy to understand and interpret. His use of ordinary least
squares multiple regression to estimate linear probability models, however,
should not be indiscriminately applied by other researchers. Spies found that
other methods did not greatly improve his results, but other researchers might
not be so lucky. Data from different samples could be distributed more like
graphs (b) and (c) in figure 7.
Swatford (1980) provides a very good discussion of linear probability modeling and concludes that the use of generalized least squares to estimate the model can produce results that are just as reliable and accurate as a logit analysis. The generalized least squares technique adjusts the error terms in order to compensate for heteroscedasticity and substitutes a chi-squared test of significance for testing the fit of the model to the data. The more general case of individual level data, where the categorical dependent variable takes on more than two values, cannot be analyzed with a linear probability model. There is no way for a linear equation to predict the probabilities of a student applying to several different colleges. This type of data must be analyzed with a non-regression technique, such as conditional logit analysis.

Logit Analysis

The most sophisticated techniques that have been used to study student demand make use of the logistic distribution. A few studies use the normal distribution instead, which results in a probit analysis (see Christensen, Melder, and Weisbrod (1975), but Hanushek and Jackson (1977) show that, except at probability values near 0 or 1, the probit results are almost identical to those produced by a logit analysis (p. 206). The general formula for a logistic distribution is:

\[ P = \frac{1}{1 + e^{-X}} \]  

(20)

where \( P \) stands for a probability value (rather than price, enrollment would be determined as the eligible population times \( P \)) and \( X \) stands for a linear sum of products, such as \( \beta_0 X_1 + \beta_1 X_2 + \ldots + \beta_k X_k \).

The advantage of this function is that \( P \) ranges from 0 to 1 as \( X \) goes from \(-\infty\) to \(+\infty\). Figure 8 graphs a logistic distribution function, which is the basic S function shown in the graphs of figure 7.

The price elasticity and price response values for the logistic function can be computed, as usual, by applying the definitions in equations (6) and (7). While the specific results will depend on the particular formulation used (see the discussion of Radner and Miller's study in the following pages), in both cases they will depend on the current values of all the \( X \) values. For example, if \( X_1 \) stands for price, then taking the derivative of the logistic function in equation (20) with respect to \( X_1 \)--this is equivalent to \( \partial P / \partial X_1 \)--in equation (6)--results in:

\[ \frac{\partial P}{\partial X_1} = \frac{\beta_1 e^{-X}}{1 + e^{-X}} \]

(21)
as a formula for price response. Therefore, in logistic formulations, the
price elasticity and price-response values are not constant across different
types of student, different institutional characteristics, or different price
levels. This is such a limitation on any interpretation of their values in
actual practice that Oum (1979) argues:

that linear logit models are inappropriate to use for ... demand studies
because they impose many rigid a priori restrictions on the parameters of
price responsiveness of demand, ... (p. 374-83)

Fig. 8. The Logistic Distribution Function

\[ P = \frac{1}{1 + e^{-\frac{x}{\beta}}} \]
Nevertheless, logit analysis is a powerful tool for investigating the relationship of continuous and categorical independent variables to categorical dependent variables. Swafford (1980) even if the interaction of the estimated parameters is sometimes difficult to interpret.

One of the main features of logit analysis is the conversion of probabilities to odds, which is simply the ratio of two probabilities. When there are only two alternatives and \( P \) represents the probability of even A occurring (such as deciding to attend college) then \( 1-P \) is the probability of A not occurring, and the odds of A occurring is computed as

\[
\frac{A}{1-A} = \frac{P_A}{1-P_A}
\]

In an example if \( P_A = .40 \), then \( \frac{A}{1-A} = \frac{.40}{.60} = 2 \). More generally, there may be several possibilities, such as options A, B, C, and D, whose probabilities sum to 1.0 (e.g., \( P_A + P_B + P_C + P_D = 1.0 \)). In this case, the odds of A occurring rather than B would simply be the ratio of \( P_A/P_B \). If \( P_A = .30 \) and \( P_B = .10 \), then the odds of A over B are \( .30/.10 = 3 \) to 1. If one starts with an odds value, it is easy to convert it back to a probability. In the dichotomous case, \( P_A = \frac{\Omega_A}{(1 + \Omega_A)} \).

The conversion to an odds measure is particularly convenient with the logistic function since:

\[
\frac{P}{1-P} = \frac{1}{1 + e^{-X\beta}} \quad \text{as in equation (20)},
\]

\[
1-P = \frac{e^{-X\beta}}{1 + e^{-X\beta}}
\]

\[
= \frac{1}{1 + e^{-X\beta}}.
\]

The logarithm of the odds ratio, \( \frac{P}{1-P} \), is called a logit, and is computed as:

\[
L = \log \frac{P}{1-P}
\]

\[
= \log P - \log(1-P)
\]

\[
= \log(1 + e^{-X\beta}) - \log(1 + e^{-X\beta}) - \log(1 + e^{-X\beta})
\]

\[
= X\beta.
\]

Therefore, if one is using aggregate data, and has a probability estimate for the probability of going to college versus not going to college, then a logit transformation can be made and the \( \beta \) values can be estimated with a linear regression (Swafford 1980).

More typically, researchers use individual (data where \( P \) is 1 for college attendance and 0 otherwise) and the ratio \( P/(1-P) \) is undefined for \( P = 1 \). In this case, maximum likelihood estimation is used to solve for the \( \beta \) values. A maximum likelihood estimation technique searches for the values of \( \beta \) that
result in the closest possible fit to the observed data (see Banneshel and Jackson, pp. 200-203). The formulas in equations (20) and (23) are used to specify a function that predicts the number of 1's and 0's observed in the data, and then a computerized procedure is used to find the values of $C$ that "maximize" the function.

When the data is polytomous (the dependent variable indicates which college alternative was chosen by the student) then the logit model is generalized to examine the odds of a student picking a particular college (or type of college) versus the other available college options. Such a model is referred to as a conditional logit model, and when individual data is used, it must be solved with maximum likelihood techniques.

McFadden (1974) developed a procedure for conditional logit analysis, and it was first applied to educational demand research by Radner and Miller (1975) and Kohn, Manski, and Mundel (1976). These studies pioneered the use of this technique and they laid an important groundwork for future users of this method. Unfortunately, there are many serious problems with the data in both studies. The biggest problem is their inability to estimate precisely the choice set of viable college alternatives for each individual student. Also, the data is quite old (from the mid 60's), and it is dangerous to generalize to today's students. Nevertheless, their results were consistent with other findings.

The Radner and Miller price response coefficients have been widely reported and may be, next to Campbell and Siegel's, the most widely reported values in the literature. They were used by the National Commission on the Financing of Postsecondary Education (1973) to forecast the effects of changes in tuition on the enrollment in different segments of postsecondary education, and were also incorporated into a national planning model research project developed at the National Center for Higher Education Management Systems (NCHEMS), Huckfeldt, Weathersby, and Kirschling, 1973. The basic formulation used by Radner and Miller (1975) included data from individual students on:

- $A_{ij}$ = an academic ability score for student $i$
- $Y_j$ = a measure of family income for student $i$
- $S_j$ = a measure of the selectivity or quality of option $j$
- $C_{ij}$ = the out-of-pocket dollar cost to $i$ of option $j$ (net equal to zero for the option "no school")

where $J_i$ stands for the set of options available to $i$.

They then computed a cost-income ratio and academic interaction term, defined respectively by:

$$R_{ij} = \frac{C_{ij}}{Y_i}, \quad Z_{ij} = \frac{A_i S_i}{1000}$$
To construct their logit formulation, they first defined two values, $L_{ij}$ and $L_{ij}$ for each $i$ and $j$ such that

$$L_{ij} = R_{ij} L_{ij}$$

and

$$L_{ij} = R_{ij}$$

where $\gamma$ and $\delta$ are the parameters to be estimated. Then the conditional probability of $P_{ik}$ that student $i$ chooses option $k$ from the set $J_i$ of alternative options was given as

$$P_{ik} = \frac{L_{ik}}{\sum_{j \in J_i} L_{ij}}$$

Radner and Miller's formula for the price elasticity (1975 p. 68) of student $i$ for option $j$ is:

$$\gamma(C_{ij}/Y_i) (1-P_{ij})$$

Radner and Miller also compute cross-price elasticities, the percentage change in probability of $i$ choosing option $k$ after a one percent change in cost of option $j$. This formula is given as

$$-\gamma C_{ij} Y_i P_{ik}$$

for $k \neq j$.

Their price elasticity values are all negative, as expected, and they confirm that:

At all levels of income, at all ability levels, larger percentage changes in demand accompany higher-cost institutions: demand is more elastic as cost rises (P. 66).

Note also that price elasticity depends on the current values of all the independent variables because of the presence of $P_{ij}$ in the formula.

Jackson and Weathersby (1975) also compute a price response coefficient for Radner and Miller (1975); basically, by applying the definition in equation (7) and their assumed values of $2,000 price and 26.2 percent attendance to come up with a price response coefficient of -0.4 percentage points per $100 increase in cost. As usual, however, that estimate is completely dependent on the values of all the other variables in the model.

Two recent studies have taken the conditional logit technique, and applied it to more complete data. Chapman (1979) used student data from a single institution where he surveyed all the applicants to get their choice set. Ghall, Mikilus, and Wada (1977) used Hawaiian data, where the entire possible
Ghall, Millius, and Wada (1977) set out to "provide an estimate of the elasticity and cross-elasticities of demand for higher education facing an individual institution" (p. 478), which in their study includes all the branches of the University of Hawaii. Their sample was a cross-section of 1970 Hawaii high school graduates, and they assumed that each student made a series of choices involving two alternatives: (1) to enroll in postsecondary education or not; (2) if attending, to take academic courses or to go to a vocational program in a Hawaiian community college; (3) if academic, to go to the University of Hawaii or to a mainland college or university; and (4) if the University of Hawaii was chosen, to go to either the Manoa or Hilo campuses or to an academic program in a Hawaiian community college. They modeled each decision point with a conditional logit model, the logarithm of the odds of choosing one alternative over the other. As was shown in equation (24), this can be expressed as a linear function of personal and institutional characteristics. Ghall et al. included family size, family income, sex, ability, type of high school, and cost in their analysis.

They found that cost was a significant factor, but that its overall effect was fairly low. They report a tuition elasticity of only -0.04 (since tuition is only 8.6 percent of total costs, they estimate a total cost elasticity of -4.48). They do not specifically report price response values, but they do claim that a doubling of tuition in 1970, from $232.50 to $465.00, would have reduced enrollment in the university by 151 students, a reduction from a total freshman enrollment (3,684) of 1.76 percent per $100 increase in price.

While Ghall et al. have built a nice model that properly considers the choices available to individual students, their computation of elasticities is of questionable validity. They do not seem to recognize that in a logistic formulation the elasticity values are dependent on all the independent variables, nor do they provide a formula for elasticity as Radner and Miller do. In addition, they estimate elasticity by computing the percentage change in enrollment to a 100 percent change in cost. This seems inappropriate, since the elasticity of a logistic function changes at different parts of the curve. If they choose to estimate elasticity with that type of calculation they should have used a 1 percent tuition change.

Chapman used a methodology very similar to that used by Ghall et al. (1977), but his data consisted of the freshman applicants to a private university in Pennsylvania in Fall 1973 and Fall 1974. He surveyed the admitted students to determine family history, student background, and other schools applied to. He included institutional variables for quality, size, technical orientation, rurality, fine arts orientation, and liberality; and student variables for the amount of scholarship aid awarded, tuition costs, parental income, total financial aid awarded, distance from student's home zip code to campus, and average achievement score. The analysis was done separately for high and low income students and for the three main colleges of engineering, liberal arts, and fine arts at the university.

Chapman's analysis was aimed at measuring the importance of these variables to the college enrollment decision rather than to determine specific
price discrimination across regions. The results indicated that reducing quality was the most important factor in increasing prices for high-quality products. It was not possible to reject the hypothesis that price behavior followed a specific model or equation. As Chapman (1997) notes: "The empirical results are not really new." The value of these empirical results is that they provide a statistical model developed here that can be employed to quantitatively analyze the effects of changing. The specific factors of college and university fees (p. 145) are conclusions that help an appropriate summary for use of the statistical in research. The specific factors that they produce are not as important as the determination of what variables are important and by what magnitudes.
Whether the research was done with multiple regression or logit analysis, price was seen as an important factor influencing student demand. The effect of price varies, however, depending on both the type of student and type of institution. When a group of institutions of similar price levels are being compared, then factors such as the match between student ability and institutional quality becomes more important (Spies 1978). Also, the size of the price effect can become more significant as the overall cost of attending the institution increases (Judson and Miller 1975).

Other factors, such as the student's sex, can moderate the effect of price on student demand. For example, Thrasher (1978) found that in explaining male FTE enrollments from 1964 to 1975, price was not significant in a model that included measures of the military draft, expected benefits of college, parental education and income, and the ability to finance attendance. Price was, however, significant in explaining female FTE enrollment. In an economic-demographic model of the demand for higher education in Michigan, Moer (1976) found that "male participation was, without exception, unrelated to relative price changes and only nontraditional male students were sensitive to real income trends. Older females, however, appear to be more responsive to price stimuli and also exhibit significant, although less dramatic, income responses" (p. 52).

Switching from sex to "nontraditional students," Bishop and Van Eyke (1977) also found that "adult students are more responsive to tuition levels."
As with any study, there are a number of potential benefits and drawbacks. The benefits include the ability to identify trends and patterns in student demand, as well as the potential for institutions to tailor their strategies based on these findings. However, there are also potential drawbacks. For example, if the results are not as expected, it may be difficult to determine whether the findings are due to the methodology or the actual data. Additionally, the data may not be able to capture all relevant factors, such as external market conditions or internal institutional policies.

In this study, two questions were posed: (1) Are the effects of price changes reported in terms of price elasticity or price response, and does the researcher adequately explain how these values were computed? (2) If price elasticity or price response values are reported, are they dependent on current enrollment and price levels, as well as on the values of the other explanatory variables included in the analysis?

Understanding the interplay between these variables is crucial for institutions to make informed decisions about pricing strategies. Additionally, the study highlights the importance of considering both the internal and external factors that may influence student demand.

Overall, the study provides valuable insights into the factors that influence student demand and can serve as a guide for institutions looking to optimize their pricing strategies. However, it is important to remember that these findings may not be applicable to all situations and settings.
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