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ABSTRACT

This study investigates the influence of changes in the wording of simple addition and subtraction problems without affecting their semantic structure on the level of difficulty of those problems for first and second graders and on the nature of their errors. The objective is to contribute to a better understanding of the process of constructing a mental problem representation starting from the verbal text. A quantitative and qualitative analysis of the data produces findings supporting the hypothesis that rewording the problem in such a way that the semantic relations are made more explicit has a facilitating effect on the construction of an appropriate mental representation. (Author)

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THE INFLUENCE OF REWORDING VERBAL PROBLEMS ON CHILDREN'S PROBLEM REPRESENTATIONS AND SOLUTIONS

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The Influence of Rewording Verbal Problems on Children's Problem Representations and Solutions

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Abstract

This study investigates the influence of changes in the wording of simple addition and subtraction problems without affecting their semantic structure on the level of difficulty of those problems for first and second graders and on the nature of their errors. The objective is to contribute to a better understanding of the process of constructing a mental problem representation starting from the verbal text. A quantitative and qualitative analysis of the data produces findings supporting the hypothesis that rewording the problem in such a way that the semantic relations are made more explicit has a facilitating effect on the construction of an appropriate mental representation.

Recent research on simple addition and subtraction word problems has produced convincing evidence that the semantic structure of verbal problems strongly influences the relative difficulty of such problems and the strategies used by first and second graders to solve them. Typical of this kind of research is the work by Riley, Greeno & Heller (1983), and by Carpenter & Moser (1982).

With respect to the level of difficulty it has been found that, on the average, change problems are easier than combine problems, which are themselves easier than compare problems. However, this general finding has to be qualified in the sense that, within each of these three types of problems, there are substantial differences in relative difficulty mainly in function of the identity of the unknown quantity. For example, change problems in which the initial quantity or start set is unknown are more difficult than those with the result set or the change set unknown; change problems with the start set unknown are also consistently more difficult for children than combine problems in which the superset or combined quantity is unknown (Riley et al., 1983).

The relation between the semantic structure of problems and the solution strategies applied by children has been well-demonstrated in a longitudinal study by Carpenter & Moser (1982; see also Carpenter et al., 1981). Their results show that "the strategies that children use represent an attempt to model the semantic structure of the problem" (Carpenter & Moser, 1982, p. 21). As illustrations, we give the three following problems :

(1) Change/Result set unknown	Pete had 8 apples. He gave 3 apples to Ann. How many apples does Pete have now ?
(2) Change/Change set unknown	Pete had 3 apples. Then Ann gave him some more apples. Now Pete has 8 apples. How many apples did Ann gave him ?
(3) Compare/Difference set unknown	Pete has 3 apples. Ann has 8 apples. How many apples does Ann have more than Pete ?

Each of these problems can be solved by subtracting the smallest number from the larger one. However, Carpenter & Moser (1982) found a significant tendency in young children to apply a different solution strategy with manipulatives for each problem:

Problem (1): separating strategy: the child constructs a set of eight blocks, then takes away three blocks, and finally counts the remaining blocks, which yields the answer.

Problem (2): adding on strategy: the child constructs a set of three blocks, then adds blocks until the new set equals the larger given number, and finally counts the number of blocks added.

Problem (3): matching strategy: the child puts out a set of three blocks and a set of eight blocks; then both sets are matched one-to-one, and the child counts the number of blocks in the larger set that are unmatched.

The results of research in our center over the past few years are generally consistent with the findings of the American investigators concerning the influence of the semantic structure of verbal problems on the relative difficulty of these tasks and on the solution strategies used by children (De Corte & Verschaffel, 1982; De Corte, Verschaffel & Verschueren, 1982; Verschaffel, 1984). However, this work suggests at the same time that, in addition to the semantic structure, some other task characteristics also have an important effect on children's problem-solving processes, namely, the sequence of the known elements in the problem text and the degree in which the semantic relations between the given and the unknown quantities of the problem are made explicit in the verbal text. In the present paper we will focus on the second aspect but we will give behorehand a short illustration of the first aspect.

In a recent longitudinal study with thirty first graders (see also De Corte & Verschaffel, 1983a) the following combine problem with one of the subsets unknown was administered: "Pete has 3 apples; Ann has also some apples; Pete and Ann have 9 apples altogether; how many apples does Ann have ? We found that children solved this problem almost exclusively with an indirect additive strategy, either adding on when using blocks or counting up from the smaller given number. Carpenter & Moser (1982; see also Carpenter et al., 1982), on the contrary, report that the majority of the children in their study tended to apply a direct subtractive strategy, either separating from when using blocks, or counting down from the larger given number. Carpenter & Moser's combine problems/subset unknown were stated as follows:

"There are 6 children on the playground; 4 are boys and the rest are girls; how many girls are on the playground?" Comparing this problem with the preceding one reveals a significant difference in the verbal text that may have caused the observed difference in the solution strategies applied by young children. In our verbal problem the given subset is mentioned first in the text, before the superset is given; in Carpenter & Moser's combine problem this sequence is reversed. This suggests the following hypothesis, which should be systematically tested in future research: that the strategy used by children to solve simple addition and subtraction problems depends not only on the semantic structure of the task, but also on the sequence of the known elements in the problem text.

As mentioned above, our research has suggested a second task characteristic which, besides the semantic structure, can have a significant influence on children's solution processes, namely, the degree in which the semantic relations between the quantities in the problem are stated explicitly. In a recent investigation, we studied this aspect more systematically than has previously been done. Research by others has already yielded evidence supporting the hypothesis that rewording simple addition and subtraction problems can affect the relative difficulty of certain types of problems (see also Riley et al., 1983). Lindvall & Ibarra (1980) have reported that traditional combine/subset unknown problems become significantly easier for kindergarten children when they are restated as follows: "Tom and Joe have 8 marbles altogether; 5 of these marbles belong to Tom and the rest belongs to Joe; how many marbles does Joe have?". The usual, more condensed version would be: "Tom and Joe have 8 marbles altogether; Tom has 5 marbles; how many marbles does Joe have?"

In a study with 12 nursery-school, 24 kindergarten, and 28 first-grade children, Hudson (1980) concentrated on compare problems. He presented eight pictures to the children showing, for example, five birds and four worms. With respect to this pictures two different questions were asked with a short interval between them: (1) the usual question in compare problems: "How many more birds than worms are there?"; (2) an alternative question: "Suppose the birds all race over and each one tries to get a worm! How many birds won't get a worm?" Hudson found that the problem was significantly easier when the second question was asked; to obtain their solution children used a matching strategy.

The results of these studies suggest that children's difficulties in solving word problems stated in the traditional form are not primarily due to a lack of quantitative actions or procedures to perform a solution, but rather to the fact that they do not understand these problems well enough. This brings us to the theoretical background of our investigation concerning

the effect of changes in the usual wording of simple addition and subtraction problems.

Theoretical framework

The study on the influence of rewording verbal problems on children's problem representations and solutions was designed within the framework of our competent problem-solving model (De Corte & Verschaffel, 1983a), which is based on work done by Greeno and associates in which semantic processing is considered to be a crucial component in skilled problem solving (Greeno, 1982; Riley et al., 1983). The model consists of five stages:

(1) The first phase is conceived as a complex, goal-oriented text-processing activity: starting from the verbal text the pupil constructs a global, abstract, mental representation of the problem in terms of sets and set relations.

(2) On the basis of this representation, the problem solver then selects an appropriate formal arithmetic operation or an informal counting strategy to find the unknown element in the problem representation.

(3) The execution of the selected action or operation is the next stage in the problem-solving process.

(4) Then the problem solver reactivates the initial problem representation, replaces the unknown element by the result of the action performed, and formulates the answer.

(5) The final stage consists of verification actions to check the correctness of the solution found in the preceding stage.

As stated above, the first stage of the solution process is conceived as a goal-oriented text-processing activity. More specifically, the mental representation constructed in this phase is considered as the result of a complex interaction of bottom-up and top-down analysis, i.e. the processing of the verbal input as well as the activity of the competent problem solver's word problem schema' (De Corte & Verschaffel, 1983a) and semantic schemata (change, combine, and compare schema) contribute to the construction of the representation.

The verbal problems that are usually given to children in schools are most often stated very briefly and sometimes even ambiguously, unless one knows and takes into account various textual presuppositions (see also Nesher & Katriel, 1977; Kintsch & Greeno, in preparation). As an illustration, let us consider the following problem: "Pete has 3 apples; Pete and Ann have 9 apples altogether; how many apples does Ann have?" In this problem text it is not stated explicitly that Pete's three apples mentioned in the first sentence also form at the same time part of the nine apples that Pete and Ann

have altogether. Kintsch & Greeno (in preparation) give another example of a typical presupposition of the "word-problem game": in verbal problems the utterance that someone has "n things" means "exactly n things". However, in natural language the sentence "Pete has 3 apples" says nothing more than that Pete has at least three apples; the sentence would still be true even if he has more than three apples.

Experienced problem solvers have no difficulties in overcoming the indistinctness of the usual word problems and in constructing an appropriate representation because they process the verbal text largely in a top-down way, i.e. the processing is conceptually-driven using the semantic schemata mentioned above. Competent problem solvers' well-developed semantic schemata enable them to compensate for omissions and ambiguities in the problem statement. In less able and inexperienced children, however, the semantic schemata are not yet very well developed, and, therefore, these children depend more on bottom-up or text-driven processing to construct an appropriate problem representation. Therefore, we would suggest that, especially for those children, rewording verbal problems in such a way that the semantic relations are made more explicit without affecting the underlying semantic and mathematical structure will facilitate the construction of a proper problem representation and, by extension, on finding the correct solution.

Method

Materials. Two series of six rather difficult word problems were constructed: Series A and Series B. Each series consisted of two change problems in which the start set was unknown (change 5 in the classification of types of word problems by Riley et al., 1983), two combine problems in which one of the subsets was unknown (combine 2), and two compare problems in which the difference between the referent set and the compared set was unknown (compare 1). In Series A the problems were stated in the usual form in which they normally appear in first graders' textbooks and in the most recent investigations on addition and subtraction word problems. In Series B the same kinds of problems were reformulated in such a way that the semantic relations between the sets were stated more explicitly so that they would be clearer to young children. Table 1 gives an overview of both series of word problems.

In the usual statement of change 5 problems, there is no explicit reference to the unknown startset; for example, in the first problem of Series A in Table 1, it is not mentioned explicitly that Jan had already some marbles before he won three more marbles. The rewording consisted mainly in

adding a sentence to the problem statement in which this unknown startset is identified.

In a traditional combine 2 problem it is not stated explicitly that the given subset is at the same time part of the superset; for example, in the third problem of Series A in Table 1, it is not mentioned that Tom's three nuts, which are introduced in the second sentence, are part of the nine nuts that he and Ann own together. In the preceding section we have already referred to this textual presupposition in combine problems. The reformulation of these tasks was intended to make the part-whole relations more obvious and explicit in the verbal text.

Rewording the compare 1 problems was done in the same way as in the Hudson study (1980) that was reviewed above. However, we did not present pictures to the children but only the verbal text of the problem. The reformulation avoids the expression "more than" and suggests more obviously the matching of the two given quantities to find the solution. In a sense the rewording of the compare problem is more radical than that of the change and combine problems.

Insert Table 1 here

Subjects and procedure. Both series of word problems were collectively administered near the end of the school year to four first-grade classes (6-7 year olds) and four second-grade classes (7-8 year olds), with a total number of 89 and 84 children respectively. In both grades, half of the pupils were given Series A first and Series B one week later; for the other half of the children the order was reversed.

On the basis of our hypothesis, it was predicted that, in the first as well as in the second grade, the results for Series B as a whole and for each of the three problem types separately would be significantly better than for Series A. It was also predicted that in each case the results of the second graders would be significantly higher than those of the first graders. The second prediction is based on the plausible hypothesis that, because of their more extensive experience with word problems, second graders have better developed schemata for top-down processing of the verbal text.

The data collected were subjected to quantitative analysis as well as to error analysis.

Quantitative analysis

Global results. Table 2 gives an overview of the global results for both grades and for the total group on the two series of word problems.

Insert Table 2 here

Table 2 shows that in both grades and in the total group the reworded problems of Series B were solved significantly better than the standard verbal problems of Series A. A t-test of the differences between the means of the first and the second graders revealed that on both series of problems the second graders obtained significantly higher results than the first graders. Those two findings are in accordance with the predictions derived from the hypothesis in the previous section.

A further analysis of the answers of the individual children showed that 90 (50 first graders and 40 second graders) of the 173 children obtained a higher score on Series B than on Series A; for 65 (28 first graders and 37 second graders) both of the scores were equal, and only 18 (11 first graders and only 7 second graders) children solved Series A better than Series B. From the preceding results one can conclude that rewording the verbal problems had a positive effect on the solution processes of more than half of the children who participated in this study.

Results for the three problem types. The results for each of the three problem types are summarized in Table 3.

Insert Table 3 here

For each problem type the following null hypothesis was tested using the X^2-test: the proportions of correct and wrong answers are equal for Series A and Series B. Table 3 shows that this null hypothesis was rejected in all cases. This implies that the general finding that Series B is solved significantly better than Series A holds also for each problem type separately in each grade. The difference between both series is about the same for the change 5 and the compare 1 problems ($\pm$ 20%), but is smaller for the combine 2 problems; for this last type of problem the reformulation seems to have less effect.

Findings that might look surprising at first glance are the low scores of the first graders on the change problems and the relatively high scores on the compare problems, which are solved better than both other problem types. However, one should keep in mind that we used the most difficult type of change problems in this study, namely, those with the start set unknown; on the other hand, the easiest kind of compare problems is represented in our series of tasks, namely, compare/difference unknown problems. Indeed, our results are generally in line with those reported by Riley (see Riley et al., 1983).

Effect of the sequence of presentation of the two series of problems. By comparing the results for both series in the two sequences of presentation (A - B, and B - A), we can check the possible effect of Series A on Series B and vice-versa. As Table 4 shows both series are solved better during the second presentation, which suggests the occurrence of a certain learning effect from the first to the second presentation. More important, however, is that in both sequences Series B has the highest score, but the difference with Series A is much greater in the A - B group than in the B - A group. This finding also supports the hypothesis that the rewording of the problems substantially facilitates children's solution processes. In the B - A sequence, the favorable influence of the rewording is left out during the second presentation. Consequently, the results for Series A is even lower than for the Series B when presented first despite a certain learning effect. Finally the results suggest that by varying the sequence of presentation we have been able to neutralize the learning effect to a large extent, so that the data allow us to make a rather good estimate of the rewording effect. Indeed Table 4 shows that the global results for Series A + B do not differ substantially between the two sequences of presentation.

Insert Table 4 here

Error analysis

Answer categories. To obtain a more detailed analysis of children's responses we have classified their answer on each word problem in the following five categories:

(1) correct answer (CA);

(2) adding error (AE), i.e., adding the two given numbers in the problem

instead of subtracting the smaller number from the larger
problems in our study were subtraction problems, adding alw
an incorrect answer);

(3) given number error (GNE), i.e., answering with one of the give
in the problem, either the first (FGNE) or the second (SGNE);

(4) a miscellaneous category containing low frequency errors, eith
technical errors (i.e. mistakes that occur when a child choos
correct operation but fails in the execution) or errors for w
have no ready explanation (MC);

(5) no answer (NA).

Table 5 gives the distribution of the answers over these
separately for the three types of problems (change, combine, and c
will now review the data for each problem type.

Insert Table 5 here

Change problems. Inspection of Table 5 shows that the great m
errors on the change problems belongs to two categories, namely, a
(AE) and "first given number" error (FGNE). The FGNE outnumbers th
first grade, but not in the second grade. To get a better insigh
origins of children's errors, we asked them to write down on th
sheet how they obtained the solutions of the problems. This techni
yield much interesting data. Nevertheless, we hypothesize that mos
who committed an AE or a FGNE did not construct an appropri
representation, and that semantic top-down processing of the probl
largely lacking in their solution process. There is some evide
protocols and also in questions the children asked that they eith
to cue or key words in the verbal text or guessed which arithmetic
to perform. For example, some children asked: "Should we fill in
number?"; this was probably a reaction to the last words in the q
the problem text: "in the beginning". Other children presumably
the key words "win" and "get" with the adding operation.

Table 5 gives us also an initial picture of the influence
rewording on the children's answers. Obviously, the frequency of
types (AE and FGNE) decreases significantly from Series A to Series
In the first grade the decrease is greater for the AE than for
while the reverse is true for the second grade. However, a decrea

total error percentage on Series B did not result in an equal increase in the percentage of correct answers because some children committed a different error on Series B than on Series A. Therefore, we analyzed the errors on Series A of those children who gave a correct answer on Series B. Eighty-four children fell into this category. Their errors on Series A were distributed as follows: 26 AE, 44 FGNE, 7 SGNE, and 4 errors in the miscellaneous category. These data suggest that our reformulating of the change problems was most effective with respect to the FGNE, and less effective with respect to the AE. In view of the way in which we have reworded the change problems, this is not at all surprising. Indeed, to the traditional problem statement we have added one sentence in which more explicit reference is made the unknown start set. This facilitates an appropriate bottom-up processing of the verbal text, and prevents the child from associating the first given number with the start set.

Combine problems. In Table 5 one can see that again the AE and the FGNE are the most frequently errors. The AE outnumbers the FGNE only on Series A in the first grade. Here too the frequency of both error types is generally significantly lower on Series B than on Series A, although the difference is smaller than on the change problems and almost non-existent for the FGNE for the first graders. However, the total number of errors on the combine problems is also considerably smaller than on the change problems, especially for the first graders.

A difference between the combine problems and the other two problem types is the higher percentage of errors in the miscellaneous category. It is also noteworthy that the rewording of the problems certainly did not influence these errors positively.

To have a better idea of the rewording effect we analyzed the individual errors on Series A of those pupils who solved the problem correctly on Series B. Among the 71 children who were in this case, the distribution of errors was as follows: 28 AE, 21 FGNE, 6 SGNE, and 13 errors in the miscellaneous category. In other words, rewording seems to have a more or less equal effect on both main error categories. However, these data also suggest an explanation for the finding in Table 5 that rewording has less positive influence on the FGNE. A number of children who answered the traditionally formulated problems with the outcome of the wrong operation (AE) probably committed a FGNE on the reformulated problems.

As was the case for the change problems, the answer protocols yielded little relevant data on children's solution processes. However, on the basis of work by others and the results of a longitudinal study in our center, we assume that the AE and especially the FGNE are mainly due to shortcomings in

the children's understanding of the problems that can be ascribed either to a lack of understanding of part-whole relations (Riley et al., 1983), or to misunderstanding isolated words and/or sentences in the verbal text (e.g. misinterpreting a sentence like "Person A and person B have x objects altogether" as follows: "Person A has x objects, and Person B also has x objects") (De Corte & Verschaffel, 1983a). Another source of errors, and especially AE, could be that children process the verbal text only superficially: instead of trying to construct a mental representation of the problem as a whole, they focus on a key word that is associated with a certain operation (e.g. "altogether" is associated with adding); certain data suggest that instructional practice in schools produces, or at least fosters--albeit unwillingly--such a solution procedure (De Corte & Verschaffel, 1983b).

The facilitation effect of the rewording can be attributed either to the circumvention of the need for a combine schema by making the part-whole relation more explicit in the text (Riley et al., 1983), or to the elimination of possible misunderstandings of words or sentences in the problem, or to the breaking of the association between a key word and an arithmetic operation.

Compare problems. Table 5 shows that, on the compare problems, one error type outnumbers all the other categories, namely, the adding error; it represents each time about half or more than half of the total number of errors. In comparison with the combine problems and especially with the change problems the percentage of FGNE is remarkably low. Nevertheless, answering with one of the given numbers, either the first or the second, still remains an major error category.

Rewording the problems had a strong and favorable effect; in this respect our study confirms Hudson's (1980) findings. In particular, the most frequently occurring AE drops significantly on Series B. This is shown not only in Table 5, but also by the analysis of the errors on Series A of those children who answered correctly on Series B. Seventy-nine pupils were in this category. Their errors on Series A were distributed as follows : 51 AE, 20 FGNE, 2 SGNE, and 5 errors in the miscellaneous category.

In the absence of data on children's solution processes, we can again only give some hypothetical interpretations of the observed errors. It is important here to take into account that, in our schools, first and second graders are much less familiar with compare problems (especially in their traditional formulation) than with change and combine problems. Therefore, we

may plausibly assume that they do not yet have available a well-developed compare schema that would facilitate top-down processing of the verbal text. It is not surprising, then, that some children were unable to construct an appropriate mental problem representation, especially when they are given a Series A compare problem. Some of those children may have interpreted such problems in terms of the more familiar change schema (Verschaffel, 1984), or they may have applied the so-called "key-word strategy", i.e. they react to the key word "more than", which is associated with adding (De Corte & Verschaffel, 1983b). It is even possible that some children who did not understand the problem at all, simply used the best known and most familiar arithmetic operation: adding the two given numbers.

There is a ready explanation of the facilitation effect of the rewording of the compare problems. By avoiding the unfamiliar and difficult expression "more than" and by obviously suggesting a matching procedure in the verbal text, we make the problem situation much easier for the children to grasp.

Discussion

The results of the present study support the hypothesis that rewording verbal problems in such a way that the semantic relations are made more explicit without affecting the underlying semantic and mathematical structure facilitates the understanding of word problems for, and the solution of these problems by young elementary school children.

Over the past few years a considerable body of research has yielded evidence that the semantic structure of word problems significantly influences the difficulty level of the problems and children's strategies applied to solve them. The findings of the present study are not in conflict with this well-documented finding but rather complement it. Indeed, our data show that, with respect to young problem solvers, considerable differences in the level of difficulty can occur within a given problem type, depending on the degree to which the semantic relations between the sets in the problem are made explicit, obvious, and unambiguous in the surface structure of the verbal text.

These young and inexperienced problem solvers have difficulties in understanding word problems that are stated in the usual condensed and sometimes even ambiguous form, because they have not yet sufficiently mastered the semantic schemata underlying the problems. Therefore, they cannot, to the same extent

as experienced problem solvers, apply top-down, conceptually-driven semantic processing of the verbal text, but are committed largely to bottom-up or text-driven processing to build up a representation of the problem. Rewording problems by making the semantic relations more explicit compensates for the less developed semantic schemata and facilitates appropriate bottom-up processing. The nature of the main error types and the difference in their frequency on the two series of problems (Series A and Series B) supports this interpretation of the facilitation effect of problem rewording. Although the present study did not yield much data on the solution processes that produce the main error types, their nature and origins have already been well-documented in previous research (De Corte & Verschaffel, 1983 a and b; Riley et al., 1983; Verschaffel, 1984).

Kintsch & Greeno (in preparation; see also Van Dijk & Kintsch, in press) have recently developed a model that can account appropriately for our findings and that, in so doing, provides a refinement of our competent problem-solving model outlined earlier in this paper. In the Kintsch & Greeno model the initial stage of the problem-solving process, namely, the construction of a mental problem representation, is divided in two substages: in the first phase, the problem solver transforms the verbal input into a propositional text base; in the second phase, starting from those propositions, he constructs the internal representation of the problem situation.

This model implies that modifications in the usual problem text (e.g. adding or changing words, expressions, or a sentence) will give rise to a different text base. More specifically, our rewordings, which consist mainly in rendering the semantic relations between the sets in the problem statement more explicit, will result in a more elaborated text base. As a consequence, the construction of an appropriate mental representation[5] of the problem situation starting from this more elaborated text base will be facilitated.

The present study is also relevant in the perspective of educational practice. An important implication relates to the formulation of verbal problems in textbooks for elementary mathematics education. Usually, textbook writers pay more attention to the purely arithmetic aspects of word problems than to the wording of those tasks. Our investigation demonstrates clearly that children are often given problems that they fail to solve not because they lack the necessary arithmetic skills but because they do not succeed in constructing an appropriate problem representation due to their inability to understand correctly the condensed and sometimes ambiguous statement of the problem. The present study also contains suggestions concerning the direction in which one can search for rewordings that are helpful in overcoming some of the difficulties that children experience.

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Table 1. Traditional (Series A) and reworded (Series B) verbal problems

Type of problem	Series A	Series B
Change/start set unknown (change 5)	Joe won 3 marbles. Now he has 5 marbles. How many marbles did Joe have in the beginning?	Joe had some marbles. He won 3 more marbles. Now he has 5 marbles. How many marbles did Joe have in the beginning?
	Bob got 2 cookies. Now he has 5 cookies. How many cookies did Bob have in the beginning?	Bob had some cookies. He got 2 more cookies. Now he has 5 cookies. How many cookies did Bob have in the beginning?
Combine/subset unknown (combine 2)	Tom and Ann have 9 nuts altogether. Tom has 3 nuts. How many nuts does Ann have?	Tom and Ann have 9 nuts altogether. Three of these nuts belong to Tom. The rest belongs to Ann. How many nuts does Ann have?
	Ann and Tom have 8 books altogether. Ann has 5 books. How many books does Tom have?	Ann and Tom have 8 books altogether. Five of these books belong to Ann. How many books does Tom have?
Compare/difference unknown (compare 1)	Pete has 8 apples. Ann has 3 apples. How many apples does Pete have more than Ann?	There are 8 riders, but there are only 3 horses. How many riders won't get a horse?
	Ann has 6 puppies. Sue has 3 puppies. How many puppies does Ann have more than Sue?	There are 6 children, but there are only 3 chairs. How many children won't get a chair?

Table 2. Mean scores and standard deviations for Series A and Series B

Group	List A		List B		t-test of significance
	$\bar{X}$	SD	$\bar{X}$	SD	$(H_o: \mu_A = \mu_B)$
First grade (N=89)	1.96*	1.87	3.05	2.21	$p < .01$
Second grade (N=84)	4.15	2.11	4.34	1.51	$p < .01$
Total group (N=173)	3.03	2.28	3.98	2.44	$p < .01$

* Maximum score on each list = 6.00

Table 3. Proportions of correct responses for the three types of word problems

Problem type	Group	Series A	Series B	X^2-test of significance
Change 5	First grade (N=89)	.13	.33	$p < .01$
	Second grade (N=84)	.61	.79	$p < .01$
	Total group (N=173)	.36	.55	$p < .01$
Combine 2	First grade (N=89)	.43	.57	$p < .01$
	Second grade (N=84)	.71	.83	$p < .01$
	Total group (N=173)	.56	.70	$p < .01$
Compare 1	First grade (N=89)	.47	.70	$p < .01$
	Second grade (N=84)	.76	.90	$p < .01$
	Total group (N=173)	.61	.80	$p < .01$

Table 4. Proportion of correct responses on Series A, Series B, and Series A + B for the two sequences of presentation

Sequence of presentation	Series A	Series B	Series A+B
A - B	.43	.72	.58
B - A	.59	.64	.62

Table 5. Distribution (in %) of the answers over the different answer categories for each type of problems

Problem type	Answer categories	Group of pupils					
		First grade		Second grade		Total group	
		Series A	Series B	Series A	Series B	Series A	Series B
Change 5	CA*	13	33	61	79	36	55
	AE	30	17	14	8	22	12
	FGNE	46	36	14	5	30	21
	SGNE	2	5	5	3	4	4
	MC	4	8	3	5	4	7
	NA	5	2	3	0	4	1
Combine 2	CA	43	57	71	83	56	70
	AE	23	14	11	3	17	9
	FGNE	15	14	9	4	13	9
	SGNE	5	2	4	3	4	2
	MC	11	10	4	7	7	9
	NA	3	3	1	0	3	1
Compare 1	CA	47	70	76	90	61	80
	AE	34	14	16	4	25	9
	FGNE	11	6	3	1	7	4
	SGNE	2	2	1	1	1	2
	MC	5	7	4	4	5	5
	NA	1	1	0	0	0	0

* CA = correct answer
AE = adding error
FGNE = "first given number" error
SGNE = "second given number" error
MC = miscellaneous category
NA = no answer