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ABSTRACT

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RESEARCH REPORT

IMPLICATIONS OF THEORY FOR INSTRUCTION IN PROBLEM SOLVING

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Implications of Theory for Instruction

in Problem Solving

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Abstract

Cognitive theories of problem solving and suggestions made by cognitive psychologists regarding how to teach problem solving are reviewed. Theories and suggestions from creativity research are also considered. The results are summarized in terms of a description of how high levels of proficiency in problem solving are acquired and how problem solving skills might best be taught, keeping in mind a distinction between well- and ill-structured problems. The need for practice materials is discussed, and some desirable qualities of such materials are suggested. Finally, several unresolved issues regarding instructional methods are considered.

Implications of Theory for Instruction in Problem Solving

The primary missions of educational institutions, from elementary to graduate and professional schools, are to impart knowledge and to teach cognitive skills. One of the most important cognitive skills is no doubt problem solving ability. Problem solving is of course predominantly involved in such basic courses as mathematics and science and in such professional areas as medicine, engineering, and architecture. But problem solving pervades almost all areas of instruction; reading and writing have important problem-solving components, for example. Even such a "rudimentary" process as retrieving information stored in long-term memory can be viewed as a problem-solving activity (Williams & Hollan, 1980).

Instruction in problem solving generally emphasizes well-structured problems--the kind of problem which is clearly presented with all the information needed at hand and with an appropriate algorithm available that guarantees a correct answer, such as long division, areas of triangles, Ohm's law, and linear equations. But many of the problems we face in real life, and all the important social, political, economic, and scientific problems in the world, are ill structured (Simon, 1973). Schools seldom require students to solve such fuzzy problems--problems that are not clearly stated, where the needed information is not all available, there is no algorithm, and there may not be a single answer that can be demonstrated to be correct. Yet, as Simon (1980) points out, teaching generalized procedures for problem solving in new situations is necessary, in view of the enormous changes in the world's knowledge that can take place during a lifetime. According to Simon, ". . . powerful general methods [of problem solving] do exist and . . . they can be taught in such a way that they can be used in new domains where they are relevant [p. 86]." Both kinds of problems will be considered in this review.

Over the last 20 years or so, cognitive scientists have attempted to describe the psychological processes that occur while one reads, plays chess, solves puzzles, or attempts to solve mathematical problems. The result is an information-processing theory of cognition which is seen by some as highly relevant to teaching, as is evidenced by the number of edited volumes that deal with the application of cognitive science to instruction (e.g., Anderson, 1981a; Glaser, 1978; Klahr, 1976; Lesgold, Pellegrino, Fokkema, & Glaser, 1977; Snow, Federico, & Montague, 1980a,b; Tuma & Reif, 1980). Theories of creative thinking and psychometric theories of intellectual abilities may also have implications for the teaching of problem-solving skills. The purpose of this review is to consider some of what is known about problem solving, to identify some possible methods for teaching appropriate skills and strategies for solving problems, and to see what recommendations can be made about how schools might teach students to cope with both well-structured problems and the ill-structured problems of the future.

The following section provides an overview of some salient aspects of cognitive theory and research that are possibly relevant to the development of methods for teaching problem solving. A brief general description of information-processing theory is presented as well as a description of theories specific to problem solving, and suggestions made by cognitive psychologists as to how theory might be applied to instruction in problem solving are reviewed.

Cognitive Theory, Problem Solving, and Instruction

Information Processing Theory

Memory is basic to any theory of cognitive processes. Most cognitive psychologists distinguish at least three kinds of memory: a sensory buffer, a long term memory, and a short-term or working memory.

The sensory buffer (Atkinson & Shiffrin, 1968; Norman & Rumelhart, 1970) registers and maintains very briefly a stimulus event, providing time for it to be recognized, classified, stored in working memory, or ignored. If it is transferred to working memory, it becomes accessible for dealing with whatever is going on at the time in terms of motor or cognitive events.

Long term memory (LTM) is a repository of permanent knowledge and skills; its capacity is thought to be virtually limitless. Information is stored in the form of "nodes" which are interrelated in complex ways through learning (Kintsch, 1972; Rumelhart, Lindsay, & Norman, 1972; Schneider & Shiffrin, 1977). A node represents an item of information, or a cluster or "chunk" of related items; if any of the elements of such a cluster are activated, all are activated. Some nodes contain sensory-perceptual knowledge, and still others store semantic or propositional information consisting of knowledge of facts, word meanings, beliefs, theories, and the like (Gregg, 1974). Still others store procedural information having to do with learned motor or cognitive skills. Information may be highly organized into conceptual networks (Anderson, 1981b; Bobrow & Collins, 1975; Puff, 1979; Rumelhart & Ortony, 1977; Schank & Abelson, 1977) in which nodes may represent concepts, and lines connecting such nodes stand for meaningful associations between concepts. LTM contains thousands of such networks, each with connections to other networks. Because of these interconnections, information other than that which was explicitly stored can be derived (Bower, 1978).

Most nodes in LTM are normally inactive; those that are active at a given moment are contained in working memory (Feigenbaum, 1970). Working memory thus contains the information that is actively being used, and information processing consists of controlling the flow of information into and out of working memory, by such processes as retrieving information from LTM and receiving information

from the sensory buffer; by recognizing, comparing, and manipulating symbols in working memory; and by storing information in LTM (Shiffrin, 1975). Thus working memory maintains an internal representation of what is going on. Its capacity, however, is limited to a small number of items of information--generally no more than six or seven--which may sharply limit the size of problems one can deal with successfully (Miller, 1956). But capacity can be greatly increased by "chunking" (Battig & Bellizza, 1979; Miller, 1956; Tulving, 1962). Complex cognitive structures can be developed that allow a single symbol or concept to represent a collection of related items of information, such as one's telephone number, a familiar pattern of pieces on a chess board (deGroot, 1965), or even a set of formulas for solving problems in mechanics (Chi, Glaser, & Rees, 1981). Chunking helps make it possible to process a great deal of detailed information automatically.

Schneider and Shiffrin (1977; Shiffrin & Schneider, 1977) have shown that there are two kinds of information processing: controlled and automatic. A controlled process involves the activation of a sequence of nodes under the control of the person; since it requires attention, only one sequence can operate at a time. Controlled processing thus quickly uses up the capacity of working memory. Automatic processing also involves activation of a set of nodes, but under control of a particular input to working memory (internal or sensory) rather than by control of the subject. The sequences are thus carried out automatically without requiring the attention of the subject, and therefore they use little or none of the capacity of working memory. A high degree of automaticity can result from a great deal of training and practice.

It has been shown that it is possible to carry out two visual-search tasks concurrently without measurable decrement in performance if one of the tasks

can be performed automatically (Schneider & Fisk, 1982). One important factor in determining whether or not one can learn to process information automatically in such tasks is the consistency with which symbols are used as targets and as distractors. Automatic processing can develop when conditions are less than perfectly consistent, but there is a point where inconsistency makes automatic processing impossible. Another important factor is the number of times the target is detected in relation to the number of trials (Shiffrin & Dumais, 1981).

Thus it is possible that problem-solving capacity can be greatly increased by learning to use automatic processing for the more routine elements of an activity, making available controlled-processing resources for the novel aspects of problem solving. For example, with much practice the basic skills required in reading, such as decoding orthographic forms, translation into speech units, retrieving word meanings, and establishing relationships among semantic propositions, may become automatic, making possible the huge amount of simultaneous information processing that is required of a skilled reader (J. R. Frederiksen, 1980, 1981, 1982; Perfetti & Lesgold, 1977).

Ill-structured Problems

Simon (1973, 1978) distinguishes between well-structured problems, such as puzzles or arithmetic word problems, and the fuzzy problems that are frequently encountered in real life. The former mainly require the information contained in the problem statement and perhaps other information stored in LTM, including procedural knowledge such as knowledge of an algorithm; while ill-structured problems require one to rely more extensively on resources of long-term memory or to go to external sources for additional information. Ill-structured problems

are defined by Simon (1978) as those that (a) are more complex and have less definite criteria for determining when the problem has been solved, (b) do not provide all the information necessary to solve the problem, and (c) have no "legal move generator" for finding all the possibilities at each step. He believes that the processes are basically the same for solving well- and ill-structured problems, but for ill-structured problems one's conception of the problem alters gradually as new elements are evoked from LTM or from outside sources, and a wide repertory of recognition processes is necessary to evaluate whether one is "getting warmer" as a result of each altered state.

There is of course no sharp division between well-structured and ill-structured problems. Simon (1973) concludes that ill-structured problems are often solved by being simplified into a series of small well-structured sub-problems: ". . . the problem is well-structured in the small but ill-structured in the large (p. 190)." Similarly, otherwise well-structured problems often have aspects of ill-structured problems (Greeno, 1976b, 1978); for example, the initial statement of a problem may not completely specify the problem space, as in a geometry problem where construction lines have to be added in order to prove a proposition. Such problems, according to Greeno, require that intermediate indefinite goals be set up which are solved by a pattern-recognition system.

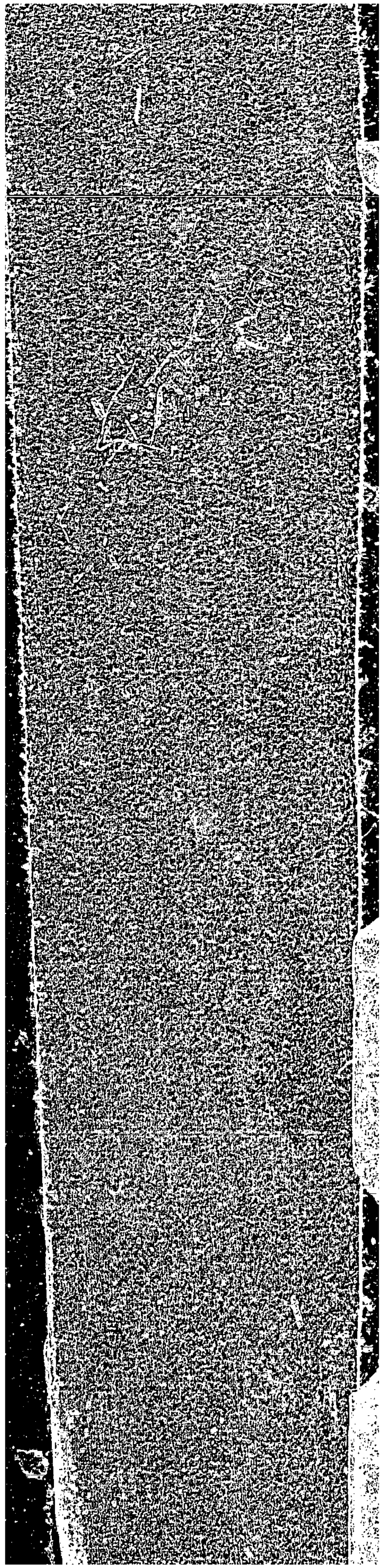
Greeno (1973) distinguishes between productive and reproductive thinking. He describes five stages in solving a problem: reading the text, interpreting the concepts, retrieving relevant information from LTM, constructing a solution plan, and carrying out the operations required to solve the problem. Information

from LTM is used in developing a conception or representation of the problem and a network of relationships among the various variables and features of the problem. Reproductive thinking is involved when the solution plan is an algorithm retrieved from LTM. Productive thinking is required when the problem-solving procedure must be constructed from propositional or semantic elements, or when the structural properties of the problem representation must be reorganized or new features must be added. Thus problems that for Greeno require productive thinking resemble Simon's ill-structured problems.

Whether a problem is well- or ill-structured may of course depend in part on the problem solver. A problem may be well-structured for the problem solver who possesses the requisite knowledge and has practiced the relevant problem-solving procedures, and ill-structured for one who has had little or no experience or training in solving problems of that type.

Some Elements of Problem Solving Theory

Theories of problem solving rest on an information-processing theory of the sort we have described; they are concerned primarily with well-structured problems. According to Newell and Simon (1972), the information-processing system generally operates serially rather than in parallel. The elements of the process require no more than a few hundred milliseconds, and the outputs of the processes are held in working memory. The outputs may be in the form of chunks, which greatly enhances the power of the system, thus compensating somewhat for the limited capacity of working memory. The store of information in LTM is potentially available for the solving of a problem, and the organization of this information into networks may greatly facilitate the search of LTM.



Problem representation. The concepts of "task environment" and "problem space" are important to the Newell-Simon theory of problem solving. The task environment is the structure of facts, concepts, and their interrelationships that make up the problem, and the problem space is the problem solver's mental representation of the task environment. An inaccurate or incomplete problem representation may make it difficult or impossible to solve the problem. The task environment for some problems, such as a puzzle, may be quite simple, but the problem statement may obscure some critical aspect and thus make it difficult to develop an adequate representation of the problem. Other problems, such as might be involved in a scientific investigation, may be very large and extremely complicated, requiring a great deal of information not included in the problem statement. Riley, Greeno, and Heller (1981) define a problem requiring information not included in the problem statement as "a semantic network structure, consisting of elements and relations between these elements (p. 23)." These elements initially may consist of propositions specifying the initial state, related information stored in LTM (including information about the operations that may be legally employed to change the state), the desired goal, and the set of interrelationships among these elements. The quality of the solution to the problem will be determined by the adequacy of this representation of the problem.

Novices and experts in physics were compared with respect to their representations of problems in mechanics by having them sort problems into categories on the basis of similarities in methods of solution (Chi, Feltovich, & Glaser, 1981; Chi, Glaser, & Rees, 1981). It was found that the novices tended to sort problems on the basis of surface features, such as inclined

planes, pulleys, and friction, while the experts categorized problems on the basis of the fundamental principles of physics that were involved, such as conservation of energy and Newton's third law. The greater knowledge and experience of the experts made it possible for them to represent a problem in terms of a schema containing both factual knowledge and procedural knowledge of solution methods (including formulas) for solving problems of its particular kind.

deKleer and Brown (1981) describe how a mechanical device such as a doorbell is represented mentally by an expert, how the mental model is formed, and how the structure and functions included in the model influence its usefulness in solving problems.

Problem-solving procedures. Newell and Simon have found that many problem solvers make use of a heuristic called "means-end analysis," in which the problem solver repeatedly compares his present state with the desired state and asks himself, "What is the difference between where I am now and where I want to be? What can I do to reduce that difference?" Such questions result in a series of subgoals and actions to change the situation to approximate each subgoal. Very little trial-and-error search goes on. The search is sequential, with very little backtracking, presumably because the capacity of working memory makes it too difficult to try to remember the previous steps in the search. The success of the problem solver will depend largely on how well he represents the critical features of the task environment in this problem space.

Bhaskar and Simon (1977) extended the analysis to problem solving in a "semantically rich" domain involving engineering thermodynamics, where much

semantic information is stored in LTM as textbook knowledge and procedures. They describe a subject's problem-solving behavior as closely resembling performance in situations that are much simpler with respect to the knowledge required; the approach was means-end analysis. The principal difference from the less rich domains was mainly in the need for a recognizing or "evoking" mechanism for retrieving relevant information from LTM.

Sweller and Levine (1982) have shown in a series of experiments involving mazes that means-end analysis is more likely to be used if the position of the goal is clearly specified. When the goal position is not specified, means-end analysis cannot be used. They also found that when means-end analysis was used, opportunities for rule induction were decreased and students learned little about the structure of the problem. The results were replicated with numerical problems. Their work demonstrates not only that different problems may require different strategies (Greeno, 1978), but also that some procedures are more likely to be generalizable.

There are many procedures in addition to means-end analysis that can be used in problem solving; they range from algorithms for arithmetic computation to general strategies, heuristics, and plans. Polya (1946), for example, provided a list of heuristics for understanding a problem and devising a plan to solve it, including making sure that the givens, the conditions, and the goal state are understood; reformulating the problem; thinking of known analogous problems; making the problem more general; and breaking the problem into parts.

Simon (1980) describes two procedures in addition to means-end analysis that are frequently used in artificial intelligence programs. They are the

"hypothesize-and-test" method and the "best-first" search. Larkin (1980) describes two additional strategies; one is a kind of planning in which the original problem is replaced with an abstracted version which retains the central features and is used as a guide in solving the original problem. Another involves replacing an unattainable goal by a simpler subgoal which when attained can be used in achieving the original goal.

The hypothesize-and-test method may be particularly relevant for ill-structured problems. The method, however, may be subject to error because of such common tendencies as overemphasizing positive information while failing to give sufficient weight to negative instances (Bruner, Goodnow, & Austin, 1956).

Sacerdoti (1977) has developed a theory of planning in considerable detail. It is summarized by Greeno (1980b) in terms of the organization of knowledge about actions into a procedural network. Each action has a set of preconditions, a set of consequences, and a set of subactions that are necessary in order to accomplish the action. Such an organization facilitates planning, beginning with the larger units and then proceeding to the subactions. Greeno believes that since planning depends upon knowledge, it probably occurs in different ways depending on the problem solver's knowledge of the domain. For the person experienced in the domain, planning may be automatic, while the novice must generate and try out various sequences.

<u>Pattern recognition</u>. deGroot (1965) compared chess grandmasters and masters with ordinary chess players with respect to their ability to reproduce the position of the pieces on a chess board after seeing them in a midgame position for 5 to 10 seconds. The experts were able to reproduce correctly the

positions of the 20 or 25 pieces almost without error, while ordinary players were able to place only a half-dozen pieces correctly. When the experiment was repeated with the pieces randomly arranged, only about 6 pieces were correctly placed both by masters and ordinary players. Apparently the experts had learned to recognize at a glance patterns (chunks) of related pieces on the board and to use such patterns in processing information, rather than the positions of individual pieces. The ability of a chess master to defeat a large number of lesser players in simultaneous play is no doubt attributable to such pattern recognition skills (Chase & Chi, 1980). Such a skill is also involved in learning to recognize functional elements in schematics depicting electronic circuits (Egan & Schwartz, 1979) or to recognize a word or phrase at a glance without using the decoding processes that are necessary at an earlier stage in learning to read.

Simon and Chase (1973; Simon, 1974) timed the placing of each of the pieces by chess masters after seeing the board in midgame position and found that the intervals between placements were relatively short for pieces within a cluster, and that longer intervals defined the boundaries between clusters. The number of chunks so defined turned out to be 5 or 6, which is consistent with what is known about the limitations of working memory. Similar results have been reported for the Japanese game go (Reitman, 1976). It was estimated, using a computer simulation, that between 25,000 and 100,000 clusters constitute the "vocabulary" of chunks in a master's memory, which is comparable to the estimated vocabulary of an educated adult.

The importance of what may perhaps be interpreted as pattern-recognition skills has also been shown in studies of physicians (Norman, Jacoby, Feightner,

& Campbell, Note 1; see also Wortman, 1972). Four written case histories were presented to practicing physicians, third-year residents, first-year residents, and second-year medical students. Two of the histories were based on common diseases and two contained findings not suggestive of any disease. Subjects were asked to read each history and then write as much of the history as they could remember. For histories based on common diseases, experienced physicians recalled the most details, followed by third-year residents, first-year residents, and undergraduates, in that order. For the histories not suggestive of any disease, there was little difference among the physician and resident groups. With training and experience physicians apparently learn to perceive patterns of signs and symptoms that correspond to disease entities.

Theories of Problem-Solving

The three aspects of problem solving that have been discussed--the problem representation, problem-solving procedures, and pattern recognition--are very closely interrelated for the experienced problem solver, and together they determine how the knowledge structure influences the solving of a problem. As Chase and Chi (1980) summarize their analysis of problem-solving skills, ". . . it appears that a large long-term knowledge base underlies skilled performance in several varieties of . . . domains. Further, a very important component of the knowledge base is a fast-action pattern-recognition system . . . that greatly reduces processing load . . . these patterns serve the purpose of retrieval aids for desirable courses of action . . . What is striking among all these domains is the similarity in the hierarchical nature of the organization of knowledge. At the lowest level, the memory representations are very localized, containing "structural" properties, whereas at the highest levels, functional properties are more important [pp. 11-12]."

In discussing how expertise is acquired, Chase and Chi go on to say "The most obvious answer is practice, thousands of hours of practice . . . There may be some as yet undiscovered basic abilities that underlie the attainment of truly exceptional performance, . . . but for the most part practice is by far the best predictor of performance . . . Practice can produce two kinds of knowledge...a storage of patterns or lexicons [and] a set of strategies (or procedures) that can operate on the patterns [p. 12]." They believe that "there appears to be no limit to the extent to which cognitive skills can be developed, except perhaps for physiological processes such as aging [p. 14]." They also comment that the skills so developed are specific to the area of expertise involved.

Anderson (1982; Neves & Anderson, 1981) describes in detail a theory about the acquisition of problem-solving expertise which involves three stages: (1) A declarative stage, during which the learner receives instruction which is encoded as a set of facts about the skill. The information may be used to generate behavior, but the retrieval of the relevant facts must be rehearsed to keep them available. (2) A knowledge compilation stage, during which the knowledge is converted into a set of procedures that can be carried out without any interpretive operations. (3) The procedural stage, during which the activity can be carried out autonomously. There is a gradual increase in speed because of a reduction in the load on working memory, making possible a unitary rather than a piecemeal operation. Anderson believes that "the configuration of learning mechanisms described is involved in the full range of skill acquisition from language acquisition to problem solving to schema abstraction. Another strong claim is that the basic control architecture across these situations is hierarchical, goal structured and basically organized for problem solving

[p. 403]." Thus Anderson presents a comprehensive theory of problem solving that brings together its elements and describes a learning process that may be involved in a wide range of complex human achievements.

Courses in Problem Solving

A number of comprehensive programs for instruction in problem solving based on cognitive theory have been developed for use with college students. Hayes (1976) developed a course in problem solving for students at Carnegie-Mellon that included three sections: (1) A diagnostic section was intended to inform the student about his current levels of skills in problem solving and to teach him procedures for probing for himself the processes he used. (2) A theory-practice section included a skill-improvement project, designed by the student to improve one of his weakest skills, and a skill-teaching project, also designed by the student. (3) The third section included a series of lectures on problem-solving techniques (trial and error, heuristic search, pattern recognition, planning, etc.), representations in problem solving (including procedures for constructing a representation), management of short-term memory (with demonstrations of the limitations of working memory and techniques for avoiding them), the importance of LTM (including techniques for storing information), the nature of rule induction, the use of hypothetical reasoning, techniques for decision making, the nature and importance of planning, perceptual processes and imagery, and the functions of mathematical notation.

Rubenstein (1980) for more than a decade has been giving a course in problem solving at UCLA to classes that include students ranging from freshmen to graduate students. The course covers problem representation, models, problem-solving styles, overcoming conceptual blocks, dealing with uncertainty,

the process of problem solving, decision making, the role of values in problem solving, and the interdisciplinary nature of problem solving. The syllabus has been published under the title Patterns in Problem Solving (Rubenstein, 1975).

Cyert (1980) presents a list of ten heuristics that were drawn from the work of Rubenstein (1975). They may be paraphrased as follows:

1. Get the total picture; don't get lost in detail.
2. Withhold judgment; don't commit yourself too early.
3. Create models to simplify the problem, using words, pictorial representations, symbols, or equations.
4. Try changing the representation of the problem.
5. State questions verbally, varying the form of the question.
6. Be flexible; question the credibility of your premises.
7. Try working backwards.
8. Proceed in a way that permits you to return to partial solutions.
9. Use analogies and metaphors.
10. Talk about the problem.

Larkin and Reif (1976) developed a procedure for teaching introductory physics students how to learn from textbook descriptions the application of quantitative relations in problem solving. The method involved three elements: (1) giving students a statement of the abilities required to "understand" a relation (e.g., ability to give an example, to list properties of the quantities involved, and to use the information in various symbolic relationships), (2) providing practice with feedback, and (3) testing with feedback. The students were required to pass a test on each unit of material before proceeding to the next. The training was found to improve performance in comparison

with a control group, and to provide a general skill potentially useful in various quantitative areas.

Elstein, Shulman, and Sprafka (1978) developed a process model of diagnostic problem solving based on their studies of how physicians go about collecting data and diagnosing the ailments of patients. The model was translated into a set of recommended heuristics that might be used in teaching medical students. The following is an abbreviated version.

Generating a list of alternative hypotheses or actions:

a. Think of a number of diagnostic possibilities consistent with the chief complaint and preliminary findings. Key on symptom clusters; nesting overcomes limits of working memory.

b. Consider the most probable diagnoses first.

c. Consider diagnoses for which effective therapies are available and in which failure to treat would be a serious omission.

Gathering data:

d. Form a reasoned plan for testing each hypothesis. Order tests to rule out first the most common diseases, next diseases most needing treatment. There is no reason to differentiate between hypotheses if there would be no difference in the action taken.

e. Use branching procedures in history taking and the physical examination to make overly detailed examinations unnecessary.

f. Consider cost and possible harm of tests.

g. Strive for an adequate degree of reliability for the decision at hand.

Aggregating data, evaluating hypotheses, and selecting a course of action:

h. Seek evidence to rule out as well as to confirm hypotheses.

i. Don't forget the possibility of multiple diagnoses.

j. Revise estimates of probabilities after collecting data. Give special weight to a diagnostic hypothesis if it is common. Try to weight each finding as at least tending to confirm, disconfirm, or not change each hypothesis.

k. In selecting a course of action, consider both the probability of the diagnosis and the benefits or penalties that would accrue.

Suggestions for Instruction by Cognitive Psychologists

Many cognitive psychologists are confident that their findings will lead to useful applications in teaching. The development of a "cognitive engineering" has even been suggested (Reif, 1980). But there is also some skepticism. Norman (1980) writes, "I do not believe we yet know enough to make strong statements about what ought to be or ought not to be included in a course on general problem solving methods. Although there are some general methods that could be of use . . . , I suspect that in most real situations it is . . . specific knowledge that is most important [p. 101]." Glaser (1976a) worried that when a body of practice is separated from its scientific origin it may be deintellectualized and carried out in a rote fashion. Olson (1976) commented that "much of the knowledge most worth having--making discoveries, speaking convincingly, writing effectively . . . cannot be taught explicitly because the algorithms underlying them (if indeed there are such algorithms) are not known [p. 119]." But as is shown by the various attempts to teach problem solving,

others are confident that generalized problem-solving skills can be taught. Simon (1980) believes that evidence from examination of artificial intelligence programs and from human transfer experiments "indicate both that powerful general methods [of problem solving] do exist and that they can be taught in such a way that they can be used in new domains where they are relevant [p. 86]." The following sections describe the major aspects of problem solving that cognitive psychologists believe can and should be taught, as judged from their writing and their attempts to teach problem-solving skills.

Teach cognitive processes. Since the product of cognitive science is knowledge about cognitive processes, it is not surprising that cognitive scientists suggest that teaching those processes would be beneficial. It is also understandable that we find more suggestions as to what processes should be taught than how to teach them. Resnick (1976) recommends that ". . . we . should perform detailed empirical analyses of skilled performance' . . .', and teach directly the routines uncovered in the course of such analysis [p.72]." Glaser (1976a), however, wonders if knowledge of processes would be useful, or if it would "put the learner in the position of the centipede who analyzed the processes by which he moved his hundred legs, and became incapable of walking . . . . [p. 307]." Teaching cognitive processes has nevertheless been suggested by many researchers, including Glaser (1976b, 1979), Greeno (1976a), Pellegrino and Glaser (1979), Rubenstein (1980), Simon (1980), and Snow (1982). Greeno suggests that "Careful attention to the components of instructional tasks is potentially helpful in at least three ways: First, it aids in the design and evaluation of curriculum materials. Secondly, it constitutes useful knowledge

for teachers . . . Third, it probably would constitute useful information for students . . . [p. 158]." Brown, Collins, and Harris (1978) believe that by explicating the underlying domain-independent cognitive processes, strategies, and knowledge and by finding ways to teach these processes, along with learning strategies, students can be given a foundation for acquiring new knowledge that will decrease their fear of facing problem situations.

There are a number of examples from the area of mathematics of how understanding process can contribute to instruction. Brown and Burton (1978) have developed computer programs that make it possible to discover the erroneous algorithms ("bugs") used by students in solving subtraction problems, such as always subtracting the smaller from the larger number regardless of which one is on top. They find that there are many more bugs in children's problem-solving "programs" than teachers know about, and that discovery of a bug makes it possible for a teacher to give appropriate remediation instead of merely advising the child to try harder or not to be careless. Marshall (1980) has developed a computerized adaptive system for diagnosing errors in fraction problems. The program recognizes the possibility that there may be more than one way to solve a problem. Each subskill required is represented as a node, and various paths through the system of nodes are possible; the diagnostic information identifies missing nodes and erroneous or missing connections between nodes.

DeCorte and Verschaffel (1981) also searched for underlying causes of errors in solving arithmetic problems, using error analysis and interviews rather than computers. They concluded that second-grade children's errors in solving such problems as $x - 7 = 5$ and $x - 4 = 6 - 2$ were attributable to

failures in instruction; the children had not learned how to deal with the tasks when presented in unfamiliar form. A training program was set up to teach more general principles, such as the concepts of equality and part-whole relationships, and how to apply these concepts when problems are unfamiliar in form. The instruction markedly reduced errors in elementary addition and subtraction problems.

Teach development of problem structure. The problem space (Newell & Simon, 1972) is a specific example of the problem structure, and the development and modification of the problem space--the problem solver's conception of the task environment--is central to Newell and Simon's theory of problem solving. Simon and Hayes (1976) recommend that instruction be given in the use of information to improve understanding of the task environment. Practice could be given in asking questions, in attempting solutions to identify ambiguities, and in finding and exploiting redundancies in the available information. Pellegrino and Glaser (1979, 1980) also suggest that instruction should be given in such aspects of problem solving as defining the problem space and using information within the problem space to help in restructuring the problem.

Simon and Newell (1971) listed six sources of information that can be used to develop a problem space: (1) the task instructions, (2) previous experience with the same or a very similar task, (3) previous experience with analogous tasks or with components of the task, (4) general procedures, such as means-end analysis "programs" that are stored in LTM, (5) procedures stored in LTM for combining the task instructions with other information in LTM, and (6) information accumulated while attempting to solve the problem. The list suggests that training ought to stress methods that will transfer, or generalize, to other types of problems.

Egan and Greeno (1973) compared two methods of instruction related to solving problems in binomial probability, one of which required learning by rule and the other learning by discovery. They found that discovery learning resulted in a well-integrated cognitive structure, while teaching rules resulted primarily in additions to the existing structure rather than reorganizing the problem space.

A number of writers and teachers of problem-solving courses (e.g., Elstein, Shulman, & Sprafka, 1978; Greeno, 1976b; Hayes, 1980; Newell & Simon, 1972; Rubenstein, 1975) have commented on the need for flexibility in developing the representation of a problem. The opposite of flexibility--rigidity, or functional fixedness--has been much investigated in relation to problem solving, beginning with Maier (1930) and Duncker (1945). Several investigators have attempted to reduce functional fixedness in Maier's two-string pendulum problem by preliminary treatments that tended to elicit ideas related to the solution (Flavell, Cooper, & Loiselle, 1958; Judson, Cofer, & Gelfand, 1956; Maltzman, Brooks, Bogartz, & Summers, 1958). Such treatments included presenting lists containing words related to the solution, evoking uncommon responses to objects, and having students read lists of unusual uses for objects. Generally positive results were obtained, but there is little reason to believe that such training would have much generality.

Resnick (1976) cites a study by Schadler and Pellegrino which showed that having the problem solver verbalize his goals and strategies before attempting to solve the problem increased the likelihood of inventive approaches.

Creativity theory research, which is of course related to flexibility, will be reviewed in a later section.

Teach pattern recognition. Simon (1980) thinks of problem solving in terms of production systems that involve condition-action pairs. Recognition of a condition (e.g., a configuration of pieces on a chessboard) evokes actions (e.g., moving a pawn). Ability to recognize a condition is based to a large extent on pattern recognition, and for a problem solver who is highly skilled in a particular area, such as chess, the recognition may be prompt and automatic. The perceptual aspects of problem-solving skill, Simon believes, deserve increased emphasis: "We need to help our students improve their skills of recognition . . . , so that if they have learned what to do, they will not be slow in recognizing when to do it . . . a large part of [one's] professional skill resides in his ability to recognize rapidly the situational cues that signal the appropriateness of particular actions [p. 94]."

The only prescription for teaching pattern perception seems to be to provide opportunity for a great deal of practice. Gregg (1974), for example, claims that "Recognition memory depends on the structure of the perceptual data base refined through years of experience with our visual world [p. 16]."

Teach problem-solving procedures. The possibility of teaching general problem-solving procedures, by whatever name--strategies, heuristics, or plans--has been mentioned by many writers. Glaser (1979) believes that "the strategic knowledge needed for problem solving can be learned and should be considered as a form of knowledge, that is, knowing a procedural skill . . . a problem for instructional research is to investigate ways in which these procedural problem-solving skills can be taught . . . [p. 9]." Simon (1980) wrote that "In teaching problem solving, major emphasis needs to be directed towards extracting, making explicit, and practicing problem-solving heuristics--

both general heuristics, like means-end analysis, and more specific heuristics, like applying the energy-conservation principle in physics. It is desirable for students to become aware of how heuristics are organized in memory, as sets of productions that provide . . . a repertory of problem-solving actions . . . and also conditions, associated with these, that serve to index the actions and to evoke them when they need to be used [p. 94]." Greeno (1980b), however, suggests that the acquisition of well-organized procedural networks for specialized domains may be more useful than general planning procedures.

Both Resnick (1976) and Glaser (1976b) think of strategies as approaches that tend to make the learner less dependent on instruction; hence they are similar to general "learning to learn" abilities. Their papers suggest that two of the strategies that could be taught are feature detection, by systematically scanning the task situation for appropriate cues, and verbalization of goals and strategies for solving a problem before making overt moves toward a solution. Pellegrino and Glaser (1980) suggest instruction in strategies for organizing, controlling, and monitoring the analysis of problem features during problem solving; Simon and Hayes (1976) recommend strategies that maximize use of analogy and use of semantic cues; while Shaw and Wilson (1976) suggest the need for direct experience with the exemplary instances of a concept to supplement the learning of facts and principles, in order to promote generalization and abstractness of thinking and to promote transfer of conceptual knowledge from one situation to another.

Greeno's (1980a) theoretical analysis of geometry problem solving identified three components: inference, pattern recognition, and strategic knowledge for planning and setting goals. The first two, he says, are explicit in

conventional instruction, but the third is learned implicitly if it is learned at all. He raises the question of the wisdom of explicitly teaching strategies for planning approaches to such problem solving. The argument for teaching strategies is that strategic knowledge is part of what must be learned in order to solve problems in geometry, and therefore it should be taught. The argument against is that if students learn the strategies for themselves through discovery methods, they will have acquired a more active approach to problem solving that may be generalized to other kinds of tasks.

Research on how students actually learn to solve geometry problems shows that the generation of proofs involves two major stages (Anderson, Greeno, Kline, & Neves, 1981); the first stage is called planning and the second execution. The plan is an outline for action involving a specification of the rules needed to get from the "givens" to the solution. This planning stage appears to be tacit rather than overt, since students rarely mention it in think-aloud problem solving. The authors nevertheless consider planning to be a more significant aspect of problem solving than execution, which is more mechanical, and they urge that development of plans be taken into account in instruction.

The importance of analogical processes in teaching is stressed by Rumelhart and Norman (1981). They believe that much of our knowledge is organized as schemata, which are "packets" of specialized procedures that have been built up through experience and used in dealing with problem situations as they arise. A new schema is created by modeling it on an existing schema and then modifying and refining it on the basis of further experiences. Thus the acquiring of new

schemata is based on analogical processes. Teaching problem solving, according to Rumelhart and Norman's prescription, would involve beginning with a domain that the student is already familiar with and presenting a new "target" domain that differs only in small ways with respect to number of dimensions, attributes, and operations--for example, teaching fractions by beginning with the schema for cutting a pie. Such learning is thought by Rumelhart and Norman to be ubiquitous in real life. Gick and Holyoak (1983) demonstrated analogical transfer by showing that when two stories depicting a single schema were presented, subjects could derive an abstraction of the schema and use it in solving a problem involving the same schema.

A procedure that is used in many areas, including trouble-shooting, medical diagnosis, and experimental science, is the hypothesize-and-test method. Moshman (1979) postulated that hypothesis testing requires (a) understanding of conditional relationships, (b) a realization that to test a hypothesis one must seek information that would falsify it, and (c) a realization that hypotheses are not conclusively verified by supporting data. He showed that there was improvement in all three of these areas from grade 7 to college, but mastery was far from universal even in college students. Mynatt, Doherty, and Tweney (1977) demonstrated what they called a "confirmatory bias" in solving abstract problems that required formulation of hypotheses. When college students were offered choices of experiments to test their hypotheses, they showed strong bias toward experiments that would confirm their hypotheses and against experiments that would test competing hypotheses. Such tendencies to overvalue confirming evidence and to undervalue disconfirming evidence has been reported by many other investigators (e.g., Gollob, Rossman, & Abelson, 1973;

Jenkins & Ward, 1965; Smedslund, 1963). Mynatt et al. (1977) suggest the desirability of teaching students to avoid biased methods. They also showed that students tended not to use base rate (neutral) information. Both findings were confirmed in a later study (Doherty, Mynatt, Tweney, & Schiavo, 1979), which also revealed a strong tendency for students to choose diagnostically useless information and to alter conclusions on the basis of that information.

Christensen-Szalanski and Bushyhead (1981) showed, however, that experienced physicians working in an outpatient clinic were sensitive to the predictive value of symptoms both when present and when absent, and they appeared to use base-rate information correctly in making clinical judgments. It seems possible that long experience in a real-life setting tends to overcome the biased use of information that is often found in laboratory settings.

Teach the knowledge base. As Simon and Hayes (1976) remark, there is no substitute for having the requisite knowledge if one is to solve a problem. For some kinds of problems, such as engineering (Bhaskar & Simon, 1977), the knowledge base is very large, while there are many problems (such as puzzles) that require a restricted knowledge base that could be taught in a short period of time.

In Greeno's (1973) model of problem solving, the primary function of knowledge is to aid in constructing a network of relationships connecting the variables and features given in the problem with the variables and features of the desired solution. Information retrieved from LTM is used to modify the problem structure held in working memory, in order to establish a corrected network among these problem elements. The two kinds of information stored in LTM are in the form of rules, or "algorithmic" knowledge, and relationships

among concepts, or "propositional" knowledge (although the distinction is not rigorous). It seems to be agreed that both should be taught as may be required for a given class of problems and a given group of students.

But there is some disagreement about the generality of the knowledge and procedures that should be taught. Norman (1980) believes that knowledge specific to a class of problems is most important, and Greeno (1980a) also argues that acquisition of problem-solving procedures for a specialized domain may be more useful than generalized procedures. He recommends that a class of problems should first be analyzed to find what knowledge is needed and that that knowledge should be taught. He acknowledges that more general concepts and procedures might lead to greater transfer, "at least for those students who are able to discover how to apply the general knowledge to new situations [p. 12]," but finally expresses the view that "as we learn more about the cognitive processes involved in problem solving, the . . . implications for instruction will involve suggestions for teaching students more about problem solving in specific domains rather than about problem solving in general [pp. 19-20]." Reif (1980), however, believes that the conflict is exaggerated and recommends that knowledge be structured hierarchically by embedding specific knowledge in more generally applicable knowledge so that it can be remembered more easily and more flexibly applied.

Shaw and Wilson (1976) believe that the ability to formulate abstract concepts is an ability that underlies the acquisition of knowledge. One implication is that instruction should provide direct experience with exemplary instances of the core concepts in the field being taught by making heavy use of laboratory or field experience. Such experiences, if sufficiently varied,

generate concepts that are abstract, and the abstractness accounts for generality or transfer to new situations. Such general procedural knowledge is usually tacit, in the sense that the reasoning processes used to reach sound judgments cannot be specified by most experts in the field. The kind of instruction recommended by Shaw and Wilson would lead to acquiring tacit knowledge of plans and heuristics, which they feel may be all that is required to be an expert.

Simon (1980) comments that ". . . we can best think of most skills--both general skills and competence in specific subject matter--as being represented in productions rather than propositions . . . we need to teach our students that this is the form that most professional knowledge takes, so that they . . . [do] not mistake learning propositions for acquiring basic skills. I continue to encounter many students . . . who wonder why, after they have memorized some material with great thoroughness, they cannot pass examinations that require them to solve problems [p. 93]."

Teaching development of knowledge structures. Atkinson (1976), in considering why psychology has not had a more substantial impact on education, answered the question by saying that what is needed are theories not only about how skills and facts are learned but also about how knowledge structures are acquired--"how knowledge is represented in memory, how information is retrieved . . . how new information is added to the structure, and how the system can expand that structure by a self-generative process." The schemata described by Rumelhart and Norman (1981; see also Norman, Gentner, & Stevens, 1976) may constitute such knowledge structures. Each schema contains functional as well as semantic knowledge, and new information is organized in memory in relation to the appropriate schemata, which may in turn be modified and expanded to accommodate the new information and for use in solving analogous problems.

Reif and Heller (1982) stress the importance of structure in acquiring knowledge and present a "prescriptive" analysis of the knowledge and procedures required for effective problem solving in the area of mechanics. The structure is illustrated by an outline of the knowledge and problem-solving procedures they consider necessary. The knowledge is organized hierarchically. The outline begins with an overview of knowledge about mechanics, including elaborations of areas concerned with individual descriptors (e.g., mass, position, velocity), interaction descriptors (e.g., force, torque), interaction laws (e.g., gravitational, string, spring), and motion principles (e.g., Newton's second law of motion). Ancillary to the basic knowledge is the knowledge needed to solve routinely various kinds of primitive problems (e.g., how to find the change in one quantity from a change in another) and commonly occurring problems of greater complexity. General procedures are described that aid in analysis of a problem, the search and decision processes for constructing the solution, and the assessment of the solution to see if it is correct and optimal. The list of procedures represents an attempt to make explicit the tacit knowledge of experts in the field. Reif and Heller also present a plan for instruction that would aid in integrating the accumulating knowledge of a student into a structure that would facilitate flexible applications. They believe that the problem-solving skills should be taught explicitly and that all of the components should be integrated for effective problem solving.

The ideas on teaching the knowledge base that have been reviewed tend to be concerned with problem solving in large, but bounded, domains of knowledge, such as a branch of mathematics, engineering, or physics. In such an area one may continue to learn through years of study and experience to the point where some, and perhaps a large proportion, of the information processing becomes

automatic. But genuinely ill-structured problems may be quite different with respect to the feasibility of teaching a knowledge base. The knowledge potentially required for the ill-structured problems that may arise in real life is so broad that it is unteachable except in the sense of providing broad experience and a general education. For solving such ill-structured problems, the suggestions for teaching for generalization seem particularly relevant. As Greeno remarked (quoted by Norman, 1980), "General problem-solving methods are of primary use when you do not know much [p. 101]."

Teach aptitudes. A recent book entitled How and How Much Can Intelligence Be Increased (Detterman & Sternberg, 1982) is concerned with the application of cognitive psychology to the improvement of intelligence by teaching specific skills, or aptitudes, such as those that according to current psychometric theories comprise the domain of intelligence. The teaching takes the form of instruction in solving problems of the kind found in tests of intelligence or aptitude, such as verbal-analogies items or number-series problems. Such items were originally chosen by the test makers on the basis of their hypotheses about the processes involved in intelligent behavior (Snow, 1982; Thurstone, 1938, 1951). Cognitive psychologists in turn have been attempting to describe aptitudes at the more detailed level of the cognitive processes involved in solving the items, and they propose using what is learned about process to teach aptitudes.

Factor-analytic research based on a variety of aptitude tests has resulted in a distinction between two major kinds of ability called crystallized intelligence and fluid intelligence (Cattell, 1963, 1971; Cattell & Horn, 1978). Snow (1982) describes the distinction as follows: ". . . crystallized intelligence represents previously constructed assemblies of performance processes retrieved as a system and applied anew in . . . situations not unlike

those experienced in the past, while fluid intelligence represents new assemblies . . . of performance processes needed for more extreme adaptations to novel situations . . . it is possible that the crystallized assemblies result from the accumulation of many "fast-process" intentional learning experiences, whereas the facility for fluid assembly and reassembly results more from the accumulation of "slow-process" incidental learning performance . . . with new or unusual instructional methods or content. Crystallized ability will be more relevant in . . . conventional formal instruction [p. 2]." This distinction matches well the distinction between well- and ill-structured problems and suggests the possibility that different instructional methods will be needed to teach the different types of problem-solving skills.

Some aptitudes are regularly taught in school, particularly verbal aptitude (vocabulary and reading comprehension) and arithmetical reasoning. Most other aptitudes identified in factor-analytic studies are not taught systematically, including those requiring perceptual and spatial skills and those involving fluency and flexibility in thinking.

Maltzman (1960; Maltzman, Simon, Raskin, & Licht, 1960) attempted to teach originality, one of the divergent-thinking skills (Guilford, 1967) that would fall in the category of fluid intelligence. The method was to give practice and reinforcement in producing unusual responses to word-association tests. The experimental groups usually showed gains, in comparison with control groups, and the training effect was retained for at least 48 hours.

Jacobs (1977; Jacobs & Vandeventer, 1971, 1972) taught young children how to deal with items in the Raven Progressive Matrices test, whose items require one to solve double-classification problems. He gave practice with problems whose stimulus attributes were, say, color and shape, and found evidence not

only of improvement but also of transfer to problems with different attributes. The effects of training were retained for as long as three months.

A more general approach to the teaching of aptitudes has been developed by Feuerstein (Feuerstein & Jensen, 1980; Feuerstein, Rand, Hoffman, & Miller, 1980; Feuerstein, Rand, Hoffman, Hoffman, & Miller, 1979) for the purpose of modifying the cognitive abilities of disadvantaged adolescents. The approach could presumably be adapted for use in ordinary classrooms. The program makes use of sets of paper-and-pencil exercises that encourage the learner to discover relationships, rules, principles, operations, and strategies in essentially content-free problems. The materials are graded in difficulty, and in some instances they are self corrective. The tasks are concerned with a variety of operations that were derived from an analysis of the processes involved in mental activity. They range from simple recognition to complex tasks such as classification, seeing analogies, and seriation, and they make use of a variety of modalities including numerical, spatial, pictorial, and verbal. Each operation is considered as having input, elaboration, and output phases, and attention is given to identifying the phase that may be responsible for failure on a particular task. There is some evidence of transfer of training based on the program.

Such instructional procedures depend primarily on practice with feedback. ~~Recent studies in which training is based on attempts to modify the processes~~ involved in solving a problem suggest that there may be more efficient methods for teaching some aptitudes. For example, a study of number series problems and numerical analogies of the form $A:B::C:(D_1,D_2)$ was carried out by Holzman, Pellegrino, and Glaser (1982). (See also Sternberg, Ketron, and Powell, 1982.)

Such problems require the induction of a rule. It was found that the most important determinant of item difficulty, especially for children, was the amount of information that had to be managed in working memory. Another source of difficulty, especially for adults, was ambiguity regarding the relationship between the first two numbers in the analogy problem; their greater facility with numbers apparently lead them to assume complex relationships without considering simpler relationships based on addition or subtraction. For children, many errors were attributable to inadequate computational skills.

Several suggestions for instruction in inductive reasoning emerge. One is to teach adults a verification procedure for resolving ambiguities (a "metacognitive" or "executive" skill). Another is to provide more drill to improve computational skills of children; the effect of such practice may be to increase automaticity and thus make available more capacity in working memory for inducing rules. Similar findings are reported by Pellegrino and Glaser (1980).

Training of aptitude at the level of strategies was investigated by Sternberg and Weil (1980), using linear syllogisms (e.g., "John is taller than Bill; Bill is taller than Pete; who is tallest?"). The strategies taught included a spatial strategy in which the names are mentally arranged in a spatial array, and a strategy requiring the use of an algorithm that does not require complex linguistic or spatial procedures. Success was evaluated in terms of response time. Training in the use of the algorithm reduced response time almost by half, while teaching the spatial strategy did not improve average performance. It was found that different strategies involved different aptitudes: the spatial strategy required spatial ability and a linguistic

strategy required verbal ability. The results show not only that students can easily be taught to use different strategies, but also that the best strategy may depend on the aptitudes already developed in a particular student. Similar findings were reported by MacLeod, Hunt, and Mathews (1978) for a task that required subjects to verify a statement after comparing it with a picture.

Snow (1982) believes that ". . . the stage now seems finally set for the development of a cognitive psychology of intelligence and learning in education; and the direct training of cognitive aptitudes for learning becomes a central focus for this work [pp. 16-17]." He concludes from his review of the evidence that "First, . . . attempts to train abilities must go well beyond simply manipulating practice and feedback . . .; they must provide substantial training in the component processes and skills involved in task performance, and they must also train directly the superordinant executive and control strategies involved in guiding performance . . . Second, . . . the best effects of direct training are likely to come from treatments that . . . involve long-term regularized educational programs. Third, abilities and . . . methods of training interact. Attempts to train either component skills or metacognitive strategies must fit training methods to . . . aptitude profiles [p. 29]."

Provide practice with feedback. Olson (1976) suggested that important cognitive skills that cannot be explicitly taught because the processes are not known ". . . may be 'taught' by providing demonstrations and by providing sessions for repeated practice accompanied by appropriate feedback [p. 119]." The idea of giving practice with feedback for purposes of reinforcement and

insight are mentioned frequently by cognitive psychologists. Glaser (1979) states that we are now beginning to appreciate more fully the extent to which training and practice are required to attain high levels of competence in complex cognitive activities.

The work of Simon and Chase (Chase & Simon, 1973; Simon, 1980; Simon & Chase, 1973) indicates that a chess master is able to recognize approximately 50,000 configurations of pieces that are encountered repeatedly on the chess board. A great deal of experience is required to achieve such ability--at least "10 years of intense exposure to the task environment of chess [p. 82]," according to Simon (1980). Thus the necessity of a great deal of practice for developing pattern-recognition skills is recognized. Practice is very likely the only way to acquire such skills (Gregg, 1974; Norman et al., Note 1). Such findings are not inconsistent with Snow's (1982) conjecture that practice with feedback is most appropriate for learners who already have some proficiency.

Simon and Hayes (1976) believe that practice would also be necessary in developing the ability to choose a style of attack in solving a problem, in identifying important information mentioned in a problem, and in noting relationships, constructing a representation of a situation, and identifying operators and conditions. They suggest that skill in obtaining feedback from the task environment could be taught by giving practice in asking questions to clarify instructions, in identifying ambiguities, and by searching for redundancies.

Practice has been shown to improve performance even in solving "insight" problems such as those studied by Maier (1930) and Duncker (1945). Jacobs

and Dominowsky (1981) gave seven such problems in different random orders to each of 56 college students, without instruction except to indicate why incorrect answers were wrong and to provide the solution to those who failed to solve a problem in the 15-minute time limit. They found that solution times improved, but only after several problems had been attempted; solution times were significantly better for the last three problems. What was learned probably was at the level of strategies and heuristics, since the elements of the seven problems were quite different.

Use models in instruction. A number of people propose to use computers to model problem-solving processes and to tutor students (Brown & Burton, 1978; Brown, Collins, & Harris, 1978; Burton & Brown, 1979; Goldstein, 1980; Sleeman & Brown, 1982). Brown, Collins, and Harris, for example, suggest using a computer as an "articulate expert" in teaching cognitive processes. A student might pose a problem for the computer to solve, and the computer would not only solve it but also explain its plan of attack, how it formulated the plan, and why it performed each step. Goldstein describes the development of a computer coaching system in which the computer program observes the performance of a student engaged in solving game-like problems and occasionally intervenes to suggest how the student's performance might be improved; the machine evolves a representation of the problem solver's representation of the problem and uses this model to guide further steps taken by the student. Such tutors are at present quite limited with regard to the variety of problems that can be taught.

There are of course simpler ways of using models in instruction. Simon (1980), for example, describes how a student might, after reading a chapter in a textbook, be able to acquire an algorithm for solving a class of problems merely by studying the worked examples in the book. Shaw and Wilson (1976) emphasize the importance of experience with exemplary instances, suggesting that "every classroom should be a 'laboratory' for first-hand, rather than second-hand experiences [p. 218]." They emphasize, however, that the exemplars should be selected in such a way that the student will generate an abstraction of the concept being taught.

Klein, Frederiksen, and Evans (1969) investigated the effect of model responses on subsequent performance, using a test called Formulating Hypotheses. Each problem consisted of a graph or table from a research investigation and a statement of the major finding based on the information given. The task was to write hypotheses (possible explanations) of the finding. Feedback consisted in giving subjects model responses in the form of a list of "acceptable" hypotheses after they completed each problem; the list contained more ideas than were typically written. The effect was to increase the number, but not the quality, of hypotheses written on subsequent problems. In a later study (Frederiksen & Evans, 1974), a "quality" as well as a "quantity" feedback condition was investigated. It was again found that the quantity treatment increased the number of ideas written, and the quality treatment was found to increase the quality of the ideas. In view of the brevity of the treatment, it was surmised that the increases in quantity or quality were attributable to

changes in standards for evaluating one's own performance rather than a change in ability. The possibility of using models to change a student's subjective standards in judging the quality of his own performance is suggested.

Creativity: Theory and Instruction

There are elements of problem solving that obviously involve the notion of creativity, particularly in the case of ill-structured problems. These elements include the search for ideas (retrieval of information from LTM) and the sudden insight that changes the whole character of the problem and transforms it into one that can be solved (restructuring the problem representation). Theories of creativity are older than cognitive theories of problem solving, and many of them are quite simplistic in comparison. But a brief review of theories of creativity may be worthwhile for the light they may throw on possible instructional procedures.

Theories of creativity seem to fall into three groups: (1) stage theories, (2) theories concerned with the personal and cognitive attributes that characterize creative people, and (3) theories growing out of psychometric research.

Stage Theories

The earliest stage theories were largely based on the introspections of poets, scientists, and mathematicians (Hadamard, 1945,1954; Patrick, 1935, 1938; Polya, 1946). According to Whiting (1958), Helmholtz was one of the first to describe creativity in such terms; his stages were (1) saturation, (2) incubation, and (3) illumination. Saturation involved the gathering of information useful in developing new ideas, incubation was an unconscious reorganizing

of information, and illumination was the realization of the solution to the problem or of a method for producing a solution. Poincaré (1952) described the process in a similar way, calling the first step preparation and adding a fourth step called verification. These stages are almost identical to those later described by Wallas (1926) in his more familiar account of creativity.

Hadamard's (1945) list of stages includes (1) preparation, (2) incubation, (3) illumination, and (4) verification, exposition, and utilization of the results. The unconscious mind considers a large number of combinations and recognizes the one (or the few) that may be useful. He agrees with Poincaré's final conclusion that "to the unconscious belongs not only the complicated task of constructing the bulk of various combinations of ideas, but also the most delicate and essential one of selecting those that satisfy our sense of beauty and, consequently, are likely to be useful [p. 32]" (Hadamard, 1945).

Rossman (1931) produced a more detailed account of the stages in creative performance on the basis of his study of inventors. The steps he defined are (1) observation of a need or difficulty; (2) analysis of the need (problem formulation and definition); (3) survey of available information, (4) formulation of possible solutions; (5) critical examination of the advantages and disadvantages of the possible solutions, probably followed by incubation in the case of complex problems; (6) formulation of new ideas and (7) testing and elaboration of the most promising solution followed by final acceptance of a revised solution.

A psychoanalytic stage theory (Kris, 1952) holds that there is an inspirational phase and an elaborational phase. During the inspirational (incubation)

phase, the ego temporarily loosens its control of the thinking processes to permit a regression to a preconscious level of thinking where the ego is more receptive to drive-related impulses and ideas. This state facilitates associations between ideas related to the problem and other seemingly unrelated but potentially useful ideas. Woodworth (1938) had a simpler explanation; he attributed incubation to the fading of material that previously interfered because of its recency.

Most of the stage theories imply that the mind continues to work on problems subconsciously after conscious efforts to solve them have been abandoned--the hypothesis of autonomous unconscious processing. Read and Bruce (1982) studied this phenomenon by asking subjects to recall information acquired in the past--to recall the names of entertainers of an earlier period, given a picture or a verbal description. Eleven sessions over a 19-day period were devoted to studying retrieval blockages. Few spontaneous retrievals were reported. Read and Bruce concluded that "the case for the hypothesis of autonomous unconscious work . . . is practically nonexistent [p. 298]." They are inclined, along with Ericsson and Simon (1980), to attribute reports of spontaneous retrieval and incubation to "an inability of the individual to report the temporary contents of short-term memory (p. 297)."

Theories Concerned with Characteristics of Creative Individuals

Other theories about the nature of creativity are based on studies of groups "known" to be high in creativity, on comparisons of such groups with other groups of lesser reputation, or on correlational information.

Ann Roe (1946, 1953a, 1953b) was an early investigator of eminent people. She administered personality tests and obtained biographical information from

individuals judged to be highly creative artists and scientists. She concluded that high energy levels, persistence, curiosity, and independence characterized research scientists. Painters and scientists were both found to have high motivation to succeed.

MacKinnon (1962, 1965) and his associates at the Institute for Personality Assessment and Research at Berkeley studied individuals judged to be highly creative, including eminent scientists, writers, mathematicians, and architects. In some studies, subjects who were nominated as "highly creative" were compared with those of less distinction. Sources of data included adjective check lists, Q-sorts, and various tests and inventories. The highly creative architect, for example, turned out to be self-confident, flexible, self-accepting, little concerned with the opinions of others, and strongly motivated to achieve. Barron (1953, 1961, 1969) showed that a preference for complexity characterized crea ive individuals, including artists as well as scientists and architects.

Psychometric Approach to Creativity

Psychometric investigations of creativity began in the early 1950's. In 1950 Thurstone (1951) proposed a number of hypotheses about abilities that might be involved in creativity and how the abilities might be measured and the hypotheses tested. At about the same time Guilford (1950) presented his APA presidential address entitled "Creativity," which marked the beginning of a long series of investigations by Guilford and his students that culminated in his structure of intellect (SI) theory (Guilford, 1967).

The SI theory is graphically represented by a solid figure containing 120 cells that are defined by categories that correspond to the three dimensions

of the solid. One dimension is labeled content, (or input), another represents output, and the third represents cognitive operations: cognition (knowing), memory, divergent production, convergent production, and evaluation. Divergent production involves a broad search of the memory store, while convergent production requires a focused search (Guilford, 1982). Tests have been developed to represent a great many of the cells in the table, and many factor analytic studies have been carried out to verify various parts of the theory.

The slice through the SI solid that corresponds to divergent production is the one most involved in Guilford's theory of creativity (1964, 1967, 1970; Michael, 1977). The most important cells in this slice, according to Michael, include products that reflect verbal fluency, spontaneous flexibility, adaptive flexibility, and elaboration. Other cells are particularly relevant to mathematicians, scientists, and engineers; they represent flexibility of closure and sensitivity to problems.

Michael has shown in detail how the seven-step theory of Rossman can be conceptualized in terms of SI theory. Stage 1, observation of need or difficulty, involves sensory inputs that may catch the attention of the individual and influence him to retrieve and evaluate information from the memory store. Guilford's operations of cognition and evaluation are involved in deciding whether the problem is trivial, or interesting enough to justify further steps. Step 2, analysis and problem formulation, is a restructuring of the problem in relation to additional information from memory or from additional sensory inputs. The result may be to drop the problem or to continue to Steps 3 and

4--survey of available information and formulation of possible solutions. These operations require a plan for search of memory or the outside world for relevant information, and some evaluation of the results. Especially in Step 4, divergent and convergent production operations are used to generate and evaluate ideas, after which Step 5, critical analysis, may begin. The result may be the emergence of a solution or progress toward a solution. Rossman's sixth step, the formulation of a new idea or solution, involves a number of transformations involving the use of both convergent and divergent production and various kinds of flexibility. The last step, testing the solution, like Step 5 requires critical analysis. The model permits much looping and allows the attempt at problem solving to be terminated at any point.

Another psychometric approach to the study of process is one in which the processes required for performance on a test are inferred from correlations of the scores with other variables that are known (or assumed) to measure more specific kinds of cognitive skills or processes (Hunt, Frost, & Lunneborg, 1973; Hunt, Lunneborg, & Lewis, 1975; Lunneborg, 1978; Pellegrino & Glaser, 1979, 1980). Several such studies have been carried out using tests similar to the Formulating Hypotheses test described earlier.

The question of what abilities were involved in taking free-response and multiple-choice versions of problems requiring students to formulate hypotheses to account for findings of behavioral science investigations was investigated (Frederiksen & Ward, 1978; Ward, Frederiksen, & Carlson, 1980) by computing extension loadings of the scores on a set of cognitive factors (Ekstrom, French, & Harman, 1976; French, Ekstrom, & Price, 1963; Guilford, 1967). These

factors included verbal comprehension, reasoning, cognitive flexibility (Scheier & Ferguson, 1952; J. R. Frederiksen, 1967), fluency, and knowledge of psychology (based on subscores of the GRE Advanced Psychology Test). The quality scores were found to be related to reasoning, cognitive flexibility, and knowledge factors for both free-response and multiple-choice forms. The number scores (number of unusual ideas, number of ideas that were both unusual and of high quality, and total number of ideas) were found to be related to fluency, but only for the free-response form of FH. Thus it appears that when the subject has to think of the ideas for himself rather than choose from a list, skill in searching LTM for ideas is necessary. This skill may represent the "evoking mechanisms" postulated by Bhaskar and Simon (1977).

A more elaborate problem-solving test (Frederiksen, Ward, Case, Carlson, & Samph, Note 2) posing ill-structured problems in free-response form was also developed. It required several cycles of formulating hypotheses, asking for additional information, and revising hypotheses on the basis of new information, until a solution was proposed. It was administered to fourth-year medical students along with a test of medical diagnostic problem solving in a similar format. Means and extension loadings of scores on cognitive factors suggest that solving the ill-structured problems primarily required ideational fluency and reasoning. For the medical problems, ideational fluency was unimportant; good performance was associated primarily with knowledge of medicine, and reasoning was involved in the later stages of the problem when more information was available. Apparently the fourth-year students had acquired sufficient skill in automatic processing of medical information to make retrieval skills unimportant.

<u>Courses in Creative Problem Solving</u>

The Productive Thinking Program (Covington, Crutchfield, Davies, & Olton, 1974; Crutchfield, 1966; Crutchfield & Covington, 1965) is a self-instructional program for fifth and sixth graders; it is in part based on the notions of programmed instruction with its emphasis on reinforcement. In a series of 16 booklets, cartoon characters are involved in solving mysterious occurrences while the learner "participates" in the activities under the tutelage of a character known as Uncle John. The stories lead subjects through each problem by presenting information, posing questions at various points, providing further information until the problem is solved. A number of "principles" for effective problem solving are given to the student and are illustrated as the stories unfold, including the following: think of unusual ideas, generate many ideas, be planful, use a tree structure to map the possibilities, assemble the facts, and get the problem clearly in mind. The instruction is aimed at teaching such skills as the ability to generate many ideas, including original ideas; to be adaptive; to evaluate one's own ideas; to reformulate problems when necessary; and to develop self confidence, and independence of thought. Crutchfield speaks of a "master thinking skill" which has to do with the "ability to plan, organize, mobilize, and deploy his repertory of specific skills in an optimal attack on a creative problem [Crutchfield, 1966, p. 38]."

deBono (1967, 1970) also teaches creative problem solving by assigning problems for students to solve, with encouragement to generate new ideas and approaches; he offers material for a course called "CoRT Thinking" comprising five units of ten lessons each, which may be spread over a period as long as three years. Three of the units are concerned especially with "creative"

thinking: Unit 1 emphasizes breadth of thinking by encouraging the learner to consider many factors in solving a problem, the short- and long-term consequences of a proposed solution, and a number of possible objectives, not just one. Unit 2 is intended to help the student direct his attention to a situation systematically, and Unit 4 stresses methods for generating ideas one might not otherwise consider. The other two units deal with reasoning and the role of information and affect in problem solving. The aim of the program is to make the problem solver aware of the range of mental operations available and to use them systematically.

Another program aimed at teaching creative problem solving is conducted as part of the Creative Studies Project at Buffalo State University (Parnes, 1967, 1981; Parnes & Noller, 1972 a, b); it is based on notions of creativity like those of Wallas and others, but stresses a "deliberate and exaggerated use of the imagination [Parnes, 1981, p. 127]." Parnes (1981) credits Osborn (1963) and his "brainstorming" approach for the basic orientation of the program. The methods recommended for instruction are presented in a Guide to Creative Action (Parnes, Noller, & Biondi, 1977); it includes practice exercises and explanations for 225 hours of instruction, including both individual and group involvement, peer tutoring, awareness development sessions, outside projects, and progress-testing exercises. The content deals with objective finding (what changes would you like to make?), fact finding (list all you know about the problem problem finding (ask what is the real problem), idea finding (list as many ideas as possible while deferring judgment), look for analogies, solution finding (list criteria), and acceptance finding (list ideas for getting your idea to work). As the program evolved, greater emphasis was placed on judgment

and evaluation, and the program is reported to produce significant increments in quality of ideas as well as number of ideas (Parnes, 1975).

Torrance (1961, 1962, 1965) attempts to influence creativity by instruction aimed at teachers and school administrators. He emphasizes the importance of teachers' influence on their pupils and the influence of school principals and other school administrators on the climate of the school. Torrance (1981) describes various ways by which schools can foster creative growth such as: provide materials which develop imagination and enrich imagery (e.g., Mother Goose stories), permit time for thinking and daydreaming, encourage children to record their ideas (e.g., publish a magazine containing their stories), give children's productions some concrete embodiment (e.g., by framing their drawings), and encourage children to use analogy and to view things from different viewpoints.

Suggestions for Instruction Based on Creativity Research and Teaching

Scholars who are interested in the theory and training of creativity offer a rather different set of suggestions than that derived from cognitive theory. Some procedures deal directly with instruction, but others are concerned with providing a proper climate for problem solving. Many of the ideas were derived from introspective descriptions of the processes of discovery which stress incubation and the freeing of the mind of influences that tend to inhibit discovery. Training procedures may tend to stress the number, originality, and variety of ideas.

Allow time for incubation. The most obvious application of the stage theories of creativity is to allow time for incubation, which is mentioned in one form or another by many writers (e.g., Kris, 1952, Patrick, 1935, 1938;

Poincaré, 1952; Hadamard, 1945, 1954). The idea, as we have seen, is based on the observation that an idea being sought may appear spontaneously when one temporarily abandons the search and turns his attention to other matters. The theory that passage of time permits unconscious thinking to go on leads to no recommendation except to bide your time. Recommendations of relaxation and encouraging daydreams and reverie seem to be based on the notion of getting rid of competing ideas.

Suspend judgment. A related idea is that one should suspend judgment and actively seek ideas without any attempt at evaluation. Premature evaluation, according to this view, inhibits the production of more ideas, some of which might prove to be useful. Brainstorming (Osborn, 1963) is a specific technique for stimulating creativity in group situations; criticism is ruled out, "free wheeling" is welcomed, and quantity of ideas is encouraged. Clark (1958), Parnes (1967), and others have included brainstorming in programs for teaching creativity.

Establish appropriate climates. Other approaches involve attempts to establish social climates that encourage creative production, particularly in school settings. Torrance (1961), for example, attempted to change teachers' attitudes and teaching methods by teaching them such principles as "treat imaginative ideas with respect" and "show pupils that their ideas have value." The school administration is thought to have an important role in establishing the climate of a school (Stein, 1974; Torrance, 1962).

Wallach and Kogan (1965) stress the importance of an attitude of playfulness, rather than evaluation, when generation of new ideas is to be maximized--an attitude that can presumably be created in a situation where there is freedom

from time pressures and where a game-like rather than a test-like setting exists. They urge the development of classrooms in which "freedom of associative processes can be nourished in a permissive atmosphere [p. 321]." Feldhusen and Hobsen (1972) have shown that the freedom implied by a play situation enhances creative production. There is also some evidence (Frederiksen, Jensen, & Beaton, 1972; Pelz & Andrews, 1966; Pelz, 1976) that climates of organizations may influence the behavior of adults with respect to innovative performance.

Analyze and juxtapose elements. Several methods of training have been suggested that involve analysis and juxtaposition of elements as a way of searching for new ideas. The simplest is merely attribute listing (Crawford, 1954), which consists in identifying all the major characteristics of an object or situation that is central to a problem. Then the problem solver thinks of ways of varying each of the attributes, without evaluation, with a view of finding variations that provide cues to the solution. A more complex version was named morphological analysis (Zewicky, 1957, 1969; Allen, 1962a). It consists in first finding two or more dimensions or aspects of a problem (e.g., size, shape, and material) and listing attributes that pertain to each dimension. Then it is possible to consider combinations of two or more attributes with the view of finding cues leading to a solution to the problem. Allen (1962b) has described a device called The Allen Morphologizer that makes it possible to lay out a problem in such a way that congenial sets of ideas emerge (after half an hour of incubation) that can be examined in various combinations. Whiting (1958) suggests making a generalized checklist of questions that can be asked about each element or attribute in any of a class of problems for the purpose of generating ideas. Osborn (1963) recommends a similar procedure, as do Stein (1974), and Davis (1974).

Teach the underlying cognitive abilities. Michael (1977) considers the implications of the structure of intellect model for the teaching of creativity. He first suggests that teachers should reinforce creative and imaginative ideas, provide ample opportunity for practice, and allow generous amounts of time for examinations and class assignments. He then suggests that formal training in the underlying abilities from the SI model that are involved in creative problem solving be undertaken, on an individual basis. He provides many specific suggestions as to how this training might be done. For example, cognition (knowing) can be taught by showing similarities and differences among units of information and the classes to which they belong, and by illustrating the place of a unit of information in a system. Memory can be taught by classifying and organizing information, while instruction in divergent production might include providing frequent opportunities for the generation of ideas from given information. (Other suggestions for improving aptitude are described in an earlier section.)

Provide practice with feedback. The idea of practice in a permissive atmosphere is implicit, if not stated, in many of the creativity-training programs, and was mentioned specifically by Michael (1977). Categories of hypotheses such as those used in scoring Formulating Hypotheses protocols could be provided as part of a self-scoring feedback system. Such feedback might influence the students' subjective standards--how good is "good enough"--as well as give practice in retrieving and evaluating information through a broad search of LTM.

Theories based on characteristics of creative individuals do not appear to lead to any feasible ideas for teaching, unless one is willing to assume that

he knows how to teach people to be curious, persistent, flexible, and strongly motivated.

Discussion

The Nature of Problem Solving

We have seen that it is possible after much practice to perform remarkable feats of problem solving. Such feats are made possible by an appropriate and well-organized knowledge structure, adequate representations of problems, automatic information processing, and an efficient pattern-recognition system that can trigger appropriate problem-solving procedures. Once a process becomes automatic, it can be carried out rapidly, without attention, and with minimal demands on the limited capacity of working memory, making it possible to use that capacity to deal with more complex or novel aspects of a problem. Such skill is specific to a relatively narrow area of expertise, such as algebra, mechanics, or chess, and there appears to be little if any transfer from one domain to another; being an expert in chess apparently does not transfer to go, and skill in solving physics problems does not transfer to politics or economics. However, a given individual may acquire such skill in more than one domain.

We have also gained a general understanding of how such proficiency may be acquired. The development of problem-solving skill begins by learning a set of propositions relating to an area of expertise and how to generate from those propositions a set of problem-solving procedures. At this stage, the problems may be considered ill structured; problem solving is a slow and laborious task that requires close attention, frequent review of the propositional knowledge,

and search of LTM for relevant ideas and information. With practice, larger procedural units are formed by combining individual units, and an efficient pattern-recognition system is developed that links a more accurate problem representation to more efficient procedures. With a great many more rehearsals, the problems become well structured, and the procedures can be carried out automatically, rapidly, and without attention. The increase in processing speed is accounted for by collapsing procedures into larger units, by rapid pattern perception, and by lack of interference from the parts of the task that have become routine and automatic. There is some risk that automaticity may produce rigidity in the way problems are solved, as is illustrated by the Luchen's water-jar problems, but the risk is far outweighted by the speed and efficiency of the process.

Such a description of problem-solving skill and its development applies only for sets of problems that have the potential for becoming well structured, in the sense that they appear repeatedly in forms that are sufficiently similar to permit the consistencies to be perceived and pattern-recognition skills to be developed. When problems lack such consistency, the problem solver must continue to use slow processes such as those that characterize the first stage of learning.

Since most of the research on problem solving has been based on well-structured problems, the processes involved in solving ill-structured problems cannot be described with much confidence. However, it seems reasonable to guess that the primary method may involve hypothesis generation and testing. The problem solver may begin by encoding the problem statement and constructing some sort of representation of the problem. The formulation of hypotheses

occurs very early in the process, and the problem representation may possibly take the form of a multiple tree structure in which the trunks represent general categories of hypotheses and the branches more specific hypotheses. Hypotheses may be directly suggested by items of information contained in the problem statement, they may come from a broad search of LTM, or they may involve inferences based on information in the problem or drawn from LTM. If a problem is very similar to problems that have previously been encountered, hypotheses may come automatically. The number of hypotheses under consideration at any one time will typically be small; one or more may be tentatively selected, and the next step is to consider how to test each hypothesis and what additional information would be needed. Then steps will be taken to obtain the information by searching LTM, asking questions, consulting external sources, or carrying out logical, mathematical, or experimental procedures. Then each hypothesis will be evaluated in the light of the outcomes and rejected, modified, or retained. A number of cycles of such processes may be carried out in which new hypotheses will be sought and ways to test them devised. Disconfirming as well as confirming evidence may be sought. Eventually that hypothesis will be selected that is judged to be most consistent with the network of facts and relationships that has been assembled.

Such processes are slow and laborious, and in the absence of consistencies in a set of problems there is no possibility of developing pattern-perception or automatic processing skills. But if a succession of similar problems is encountered, such as a series of similar diagnostic problems that might be seen by a first-year resident in a hospital, it may be possible to acquire a pattern-

recognition skill--that of recognizing a syndrome which in turn initiates a set of procedures for testing and evaluating hypotheses.

Teaching Problem Solving

The suggestions for teaching problem-solving skills that have been reviewed imply some instructional goals that are quite different from those ordinarily employed in typical schools and colleges. Any school, of course, attempts to teach the propositional knowledge and algorithms that are commonly used for problem solving, and techniques of instruction commonly include practice, feedback, and use of examples--all of which are suggested in the literature we have reviewed. But many of the other ideas that have been proposed would be new to most educators. Few schools would explicitly set out to teach pattern recognition and automatic processing, how to develop problem representations, and how to deal with the limitations of working memory, and the typical school would probably tend toward an authoritarian, rote-learning climate rather than a climate that encourages idea seeking. Introducing a curriculum based on the suggestions that have been reviewed would require a small revolution in education.

Planning a program of instruction for well-structured problems would have to be done separately for each of a large number of domains that have different knowledge bases and different problem-solving procedures. It would be necessary in each instance to consider separately at least two major phases in the development of expertise: (1) a "slow" phase that requires teaching propositional knowledge and the use of propositions in generating problem-solving procedures, and (2) a "fast" phase during which problem-solving procedures become autonomous. In any one domain, both phases might progress in tandem,

with new propositional knowledge being taught while previously taught skills are being practiced.

The nature of an instructional program is suggested by the description of problem-solving processes and their development. It would obviously be necessary to teach the knowledge base required, and it would be desirable to do so in such a way that it is organized hierarchically and functionally so as to facilitate retrieval. It would be desirable to teach the processes involved in each particular kind of problem solving, including how to develop an appropriate internal representation of a problem and how to develop or select appropriate problem-solving procedures. These procedures might include algorithms that specify in detail the steps required to solve a problem, and, at the other extreme, metacognitive or executive functions that plan and control the set of procedures to be used. More general kinds of skills, or aptitudes, that are relevant to an area, such as rotation of solid figures in space or rule induction, might also be taught. Demonstrations and models as well as explication should be used in instruction, and practice should be required in translating propositional knowledge into problem-solving procedures.

The second phase would primarily involve training the student to move from the laborious translation methods to some degree of automaticity in which pattern recognition activates appropriate procedures. The primary method of instruction would be practice with feedback. Problems with easily recognizable consistent features might be used initially, and the content, format, and settings of problems might gradually be varied in order to facilitate transfer and generalization. The specific methods of instruction would of course depend upon the area of expertise to be taught.

There is no clear dichotomy of well- and ill-structured problems. All but the most routine problems have some novel features that cannot be dealt with automatically and that require some element of idea seeking or hypothesis development. Thus there is a continuum of problems ranging from those that can be dealt with automatically to those that require controlled processing for all but the most routine cognitive operations, such as those involved in reading.

Instruction in solving ill-structured problems might in general deal with the same topics as do well-structured problems, but with rather different emphases. A knowledge base would certainly be required, but since the nature of the ill-structured problems that any given individual might encounter cannot be predicted, the content of the knowledge base cannot be specified. Assuming that one would most often face ill-structured problems that involve fields cognate to his or her own, we might recommend acquiring knowledge of such allied fields.

Instruction on how to develop representations of ill-structured problems would probably be very useful, but we need to know a lot more before any specific methods can be recommended. Problem-solving procedures should be taught, including the basic skills likely to be required in any of a wide variety of problems, and they should be made automatic to the extent possible. Skills specific to ill-structured problems would most likely be those concerned with strategic or heuristic approaches to idea seeking, such as generalization and looking for analogies, as has been proposed by both cognitive psychologists and those interested in creativity.

Instruction aimed at developing general aptitudes may be more useful for ill- than for well-structured problems, since aptitudes would have more general

application than the skills specific to problems in a single area of expertise. Perceptual skills, induction, and skill in searching LTM, for example, might have general value for the less well-structured problems. More broadly, perhaps instruction in the abilities that comprise fluid intelligence would be especially useful in preparing students to deal with the unique aspects of problems that are unlikely to be practiced in the classroom. More research in this area is needed, including investigations of the cognitive processes involved in solving ill-structured problems and in the various manifestations of fluid intelligence.

Methods of instruction might include demonstrations, examples, models of good responses, and practice with feedback; but not much is really known about what to demonstrate or what should be practiced. It would seem that the examples, models, and practice problems used for teaching should vary more widely than for well-structured problems with respect to format, content, and settings. Feedback would be important, since in the present state of our knowledge we may have to rely on learning by discovery. Perhaps much of what is learned will be tacit. Teachers should not lose sight of the need for students to learn to evaluate their own solutions, using methods that to the extent possible are as rigorous as those employed in solving well-structured problems.

In training for problem solving, both for well- and ill-structured problems, it seems wise to be alert for individual differences in problem representations and problem-solving procedures, especially differences that are related to aptitudes, and to learn to adapt the instruction to the student's particular skills and abilities rather than requiring that the student adapt to the teaching.

Development of Practice Material

The emphasis on practice implies a need for large quantities of practice problems if students are to acquire reasonable levels of skill in automatic processing and pattern recognition. The materials should exist in a variety of formats and settings in order to maximize transfer. The more realistic the settings, the greater the likelihood of generalization to real-life problems.

The materials should be scorable in ways that permit prompt feedback, and the scores should provide information about the quality or correctness of responses, not only to reinforce good performance but also to provide a basis for learning by discovery. Scores should also provide information that would have diagnostic value, such as reports of speed in processing certain information or of bugs in an algorithm.

Conventional test formats could be used to provide useful information about the knowledge base, and, perhaps, about its organization. Quite different approaches would be required in order to yield useful feedback to student and teacher about the nature of the problem representation; more work is needed in this area. Measures of speed of responding would be needed to measure progress in acquiring pattern-recognition and automatic-processing skills, since error rates are likely to be insensitive after some degree of skill has been acquired, because of lack of variability. Timed tests might serve, but better diagnostic information would be obtained by measuring latencies in responding for each of various definable steps in the process of solving a problem. Such measurement has been accomplished in research settings for tasks involved in reading and inductive reasoning, for example, where it has been shown that latency measures are highly predictive of success in the larger task. Latency measures might be useful diagnostically to reveal precisely what retards performance, and thus

make possible appropriate remediation. Such diagnostic information might also be useful in adapting instruction to the capabilities of a particular student.

The value of feedback is related to the promptness with which it is available to the learner. Therefore scoring procedures are needed that yield information expeditiously. A system that would automatically reveal the diagnostic information while the problem is being solved, so that a student could evaluate his performance while it is in pr gress, would be ideal.

The use of computers would obviously facilitate the scoring of performance on the instructional materials. Programs have already been developed that measure latencies in responding during practice sessions and match the scores with normative information. Other programs make inferences about a student's problem-solving procedures and offer suggestions if he or she is on the wrong track. Others provide access to stored information for problems requiring formulation and evaluation of hypotheses. In view of the increasing availability of computers for instruction, such resources obviously should not be neglected. However, one should consider the possibility that any constraints imposed by the machines with regard to the kinds and the settings of problems that can be presented may limit the generality of the skills acquired.

Some Issues

The review of the literature has revealed a number of issues concerned not only with how problem-solving skills can best be taught, but also with whether they can and should be taught explicitly. The lack of consensus on such issues would no doubt be reduced, however, if they were viewed in terms of specific kinds of skills and problems and, perhaps, specific kinds of learners.

Do we know enough to teach problem-solving skills? Some skepticism has been expressed as to whether enough is known about problem solving to justify any general recommendations about how to teach problem-solving processes. Most writers, however, are convinced that we do know enough at least to teach certain problem-solving skills. For well-structured problems in relatively narrow domains of knowledge, such as mathematics, there is no doubt that we do know how to teach such specific skills as the use of al rithms. Presumably we also know how to teach pattern recognition and automatic processing for problems of a sort that are encountered repeatedly. We know how to teach the development of problem representations at least for verbally stated problems that can be translated into equations or diagrams.

But as we go into domains where problems are increasingly ill-structured, we can be much less certain about the adequacy of our knowledge. We know little about how to teach students to develop representations of ill-structured problems, to develop plans for solving such problems, or to employ appropriate strategies or heuristic approaches. Still less can we advise students about efficient methods for accessing relevant information in LTM. Much needs to be learned about how ill-structured problems are solved.

Should processes be taught explicitly or by discovery? There is some disagreement as to whether problem-solving processes should be taught explicitly or by allowing the learner to discover them for himself. The former method would surely be more efficient in bringing students to the point where they can cope successfully with a specific kind of problem, while the discovery method would be more likely to lead to ability to generalize the acquired procedures to problems that do not closely fit the problem type being taught. There is

some evidence that learning by discovery rather than by rule results in a better problem representation. More research is clearly in order.

A related question has to do with the wisdom of making explicit the tacit knowledge that apparently is available to some skilled problem solvers. There may be some risk that such an attempt interferes with automatic processing of information, but it seems wise at least to try to find out what the tacit procedural knowledge is, so that it could be taught to others. Again, more research is needed.

<u>How general should instruction in problem solving be?</u> There is a difference of opinion on the issue of how general instruction in problem solving should be, with some holding that instruction would best be taught in the context of defined areas of expertise, while others hold that very general instruction is needed in order to prepare students to deal with the unknown problems of the future. For problem solving in such areas as electronics troubleshooting or auto repair, it would be wise to teach specific rules and procedures because of the many instances where they can be applied. But if we consider instruction concerned with the unknown ill-structured problems of the future, generality would be essential; basic skills and aptitudes with wide applicability should be taught as well as such very general processes as use of heuristics and strategies.

While knowledge of the cognitive processes involved in solving problems is far from complete, and information on instructional methods and their value is even more spotty, it seems that the arguments for the reforms in education suggested by cognitive psychologists are compelling, particularly in those areas involving development of problem representations, procedural knowledge, pattern recognition, and automatic processing. Such instructional reforms

require close collaboration of cognitive psychologists with teachers and with educators concerned with curriculum. There is need for a great deal of research on instruction in which ideas such as those here described are tried out and evaluated in educational settings. Continuation of research on cognitive processes is also essential. It would be highly desirable that, as Glaser (1976b) has suggested, a linking science be developed that would be based on knowledge derived both from scientific investigations and educational practice and that would provide a conceptual framework for instructional procedures and educational evaluations.

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