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ABSTRACT

This collection of brief essays covers a range of topics pertaining to mathematics education thought by the author to be of relevance. These include thoughts on: philosophy of education, noting diverse schools of thought; goals, noting ways to organize mathematics instruction; process objectives, with a brief list of objectives and comments on evaluation; trends; learning activities; psychology of learning, noting different ways to guide learners; and issues in mathematics education. (MNS)

PHILOSOPHY OF EDUCATION AND THE MATHEMATICS CURRICULUM

Diverse schools of thought in the philosophy of education have much to offer in terms of objectives, learning activities, and appraisal techniques for pupils in ongoing units of study in the curriculum area of mathematics. Teachers and supervisors need to study, analyze, and implement selected strands from diverse philosophical schools of thought.

Experimentalism and the Curriculum

A teacher emphasizing experimentalism as a philosophy of teaching and learning may well emphasize generalizations such as the following:

1. pupils are to solve realistic problems. These problems identified by learners need to be life-like and emphasize what is relevant in society.

2. interesting experiences are important for pupils which then sustains effort in learning.

3. subject matter is learned to solve problems and is not an end in itself.

4. solutions related to problems are tentative and subject to change.

Experimentalism, as a philosophy of education, would definitely not recommend the following:

1. pupils being required to work content on each page sequentially in a reputable series or multiple series of mathematics textbooks.

2. Learners working on basic operations and story problems within the framework of a reputable mathematics workbook. These situations do not provide opportunities for pupils to identify and attempt to solve relevant problems.

3. the teacher determining objectives, learning activities, and appraisal techniques for pupils.

Experimentalism then emphasizes that problem solving be the heart of the curriculum. Real problematic situations existing in society are important. Learners should have ample opportunities to work within the committee framework on interesting activities and experiences. Active involvement in learning is to be wholeheartedly stressed in ongoing units of study. The consequences of each deed or act is important to consider.

Existentialism and the Curriculum

Existentialists emphasize the following generalizations in the curriculum:

1. pupils individually actively making choices in terms of what to learn and the means of learning.

2. individual choices are to be stressed as being important rather than group or committee decisions.

3. individuals lose their humanness if choices are not made on a personal basis.

4. Alienation is possible, of course, when choices are made.

5. situations in life may appear to be ridiculous or absurd.

7. creative, products, acts and deeds of individuals are important! Conformity behavior is definitely not an end goal.

Existentialists would not emphasize the following:

1. the teacher selecting what pupils are to learn (ends), the activities to achieve these ends, and means of appraising learner achievement.

2. pupils completing sequentially pages of content from a mathematics textbook or workbook.

3. pupils exhibiting conformity behavior in terms of learnings gained.

Realism and the Curriculum

Realists emphasize precise, exact learnings which learners may acquire. Many realists stress the curriculum areas of science and mathematics as being more important generally for pupils as compared to other curriculum areas. Thus, the real environment can be known as it truly exists. Mathematical concepts and generalizations may be utilized to express precise content pertaining to reality.

A second set of realists may emphasize the importance of using precise, measurable objectives in the curriculum. These educators believe that what exists can be identified in terms of behaviorally stated objectives. Learning activities may then be provided for pupils to attain these desired ends. Ultimately, it can be measured if these chosen objectives have been achieved by pupils. Thus, all curriculum areas may be emphasized adequately in terms of balance in the school-class setting. Cognitive objectives (use of intellect) and psychomotor objectives (use of neuro-muscular skills) may receive primary emphasis in teaching-learning situations. Affective or attitudinal ends are more difficult, of course, to state in measurable terms as compared to either cognitive or psychomotor goals.

Realists, then, may emphasize the following generalizations:

1. pupils can know the real enivronment as it truly exists.

2. learnings gained by pupils individually can be measured in terms of gains made.

3. selected realists believe that science and mathematics should be emphasized more than other curriculum areas in the school-class setting.

4. values in life would reflect reality in the environment. Thus, pollution, for example, it its many forms would hinder the beauty and goodness within nature. This would suggest that the natural environment be protected and nurtured to truly reflect positive affective ends in society.

Realists deemphasize the following:

1. ideas, concepts, and generalizations being separated from what the human can truly know in terms of nature and the natural environment.

2. cognitive, psychomotor, and affective goals that cannot be identified with precision and in measurable terms.

3. pupils being heavily involved in determining what to learn (the objectives), the means of learning (activities to attain desired ends), and appraisal techniques. The teacher is in a much better position to determine these three parts of the curriculum.

4. many abstract learnings to the detriment of concrete and semi-concrete experiences for the pupils.

Idealism and the Curriculum

Idealists in teaching-learning situations would tend to stress abstract concepts and generalizations as being highly significant in terms of pupil attainment in ongoing units of study. The learner, as well as all individuals, cannot know the real world as it truly exists. However, pupils can acquire ideas about that which is real, actual and life-like.

Thus, pupils would be guided in achieving the following, as advocated by idealists:

1. relevant ab tract content, meaningfully presented.

2. universals in terms of broad encompassing ideas, presented as challenging ideas for learner attainment.

3. deductive learnings being emphasized for pupils in terms of methods of teaching utilized by the instructor. Clear, meaningful presentations given by the teacher then are important within the framework of ongoing learning activities.

In Closing

Teachers and supervisors need to study, evaluate, and ultimately implement relevant strands from diverse schools of thought in the philosophy of education. The philosophy or philosophies of education adopted may well make for an improved mathematics

GOALS IN THE MATHEMATICS CURRICULUM

There is considerable debate pertaining to which objectives learners are to attain. The mathematics curriculum is no exception. One hears much about a return to the basics. The basics generally are perceived as emphasizing the three R's (reading, writing, and arithmetic). Thus, the third R--arithmetic--has essential content for all learners to master. Within the framework of essentialism, which objectives, methods of teaching, and appraisal procedures need to be in evidence?

Instructional Management Systems

Instructional Management Systems (IMS) advocate the utilization of precise, measurable ends. Vagueness and ambiguity need to be eliminated from goals of instruction according to IMS tenants. With clarity of intent in objectives, the teacher knows precisely which sequential ends students are to attain. Thus, learning activities may be selected by the teacher to guide pupils to achieve each objective on an individual basis. An objective needs to be attained by the student before progressing to the next sequential end. The teacher can then measure if a learner has/has not achieved a specific goal. Uncertainty on the teacher's part is not in evidence to determine if a student has mastered content necessary in goal attainment.

The Missouri Department of Elementary and Secondary Education[1] listed the following characteristics of IMS:

1. High expectations for learning. Teachers and administrators expect a high level of achievement by all students and communicate their expectations to students and parents. No students are expected to fail, and the school assumes responsibility for seeing that they don't.

2. Strong leadership by building principals. The building principal is an instructional leader who participates in all phases of instruction. The principal is a visible leader of instruction, not just an office-bound administrator.

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[1]Department of Elementary and Secondary Education. Jefferson City, Missouri, 1982.

3. Emphasis on instruction in the basic skills. Since mastery of the basic skills is essential to learning in all other subjects, the effective schools make sure students at least master the basic skills.

4. Clear-cut instructional objectives. Each teacher has specific instructional objectives within the overall curriculum which are communicated to students, parents and the general public. In effective schools, teachers and administrators--not textbooks--are clearly in charge of the curriculum and teaching activities.

5. Mastery learning and testing for mastery. Students are taught, tested, retaught and retested to the extent necessary to assure mastery of important objectives.

6. School Discipline and climate. The effective schools may not be shiny and modern, but they are at least safe, orderly and free of distractions. All teachers and students, as well as parents, know the school's expectations about behavior and discipline.

The following are definitely not emphasized by IMS:

1. open-ended general objectives in the mathematics curriculum.

2. leaway in interpretation as to which subject matter should be taught so that students may choose sequential goals to achieve in a flexible mathematics curriculum.

3. pupil -teacher planning in selecting objectives.

4. learners in a classroom achieving at a similar/same level of progress. Each student progresses as rapidly as possible in achieving objectives.

Learning Centers and Mathematics

Educators, advocating humanism as a psychology of learing, believe that students should be involved in decision-making. Thus, the mathematics teacher, alone,does not select objectives, learning activities, and evaluation procedures for students. Rather, within a flexible framework developed by the teacher, the learner may select from among alternatives which sequential activi' es to pursue. A learning centers approach might then be in evidence. An adequate number of centers and tasks needs to be available so that the involved student may truly choose which activities to pursue and which to omit. Continuous progress must be made by the learner in completing personal suitable tasks. Each student may then achieve at a unique optimal rate of progress. Diverse objectives in mathematics may be achieved when comparing one student with another.

Choices made by learners in tasks pursued depend upon personal interests, abilities, capacity, and motivation. The kinds of tasks chosen may emphasize individual or committee endeavors, an activity centered or subject matter emphasis, inductive or deductive methods, as well as concrete or abstract experiences.

Morris and Pai[2] wrote the following pertaining to the thinking of Carl Rogers:

> But what are the conditions for such learning, and what must the teacher do to facilitate them? Like other humanistic educators, Rogers assumes that human beings have a natural potentiality for learning and curiosity. John Holt argues that this potentiality and desire for knowledge develops spontaneously unless smothered by a repressive and punitive climate. Consequently, humanistic educators seek to remove restrictions from our schools so that the child's capacity for learning can be cultivated. They attempt to provide the child with a more supportive, understanding, and nonthreatening environment for self-discovered learning. For example, if Jimmy is having serious difficulty in reading, he should not be forced to recite or read aloud in front of his peers, whose reactions may strengthen his own perception of himself as a failure. Rogers believes that significant learning can be promoted by allowing children to confront various problematic situations directly. If students choose their own direction, discover their own resources, formulate their own problems, decide their own course of action, and accept the consequences of their choice, significant learning can be maximized. This suggests that significant learning is not possible unless the learner's feelings and the intellect are both involved in the learning process.

Advocates of learning centers do not emphasize:

1. precise, measurable objectives for student attainment. What is specific to measure in pupil progress may not be relevant. Interests and purposes of learners are significant, but can not by any means be precisely measured.

2. teachers selecting objectives, learning activities, and evaluation techniques for students.

3. a rigid, formal curriculum. Rather, input for students in curriculum development is important.

4. each pupil being assigned the same/similar tasks as compared to other learners in the classroom.

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[2] Van Cleve Morris and Young Pai, Philosophy and the American School. Boston: Houghton Mifflin Company, 1976, page 367.

Structure of Knowledge and the Mathematics Curriculum

Mathematics may be perceived as having considerable structure. There are selected concepts and generalizations which hold true consistently. Thus, concepts, such as the following may be stressed in teaching and learning:

1. The commutative property of addition and multiplication.
2. The associative property of addition and multiplication.
3. The distributive property of multiplication over addition.
4. The identity elements for addition and multiplication.
5. The property of closure for addition and multiplication.

Key concepts and generalizations, as advocated by mathematicians on the higher education level, then become objectives for students to attain on the elementary, junior high school or middle school, and senior high school years.

To achieve these structural ideas, the teacher of mathematics needs to have students utilize inductive methods of learning. Lecture and heavy use of explanations is not recommended. Rather, the teacher identifies problems and questions. To secure content in answer to the questions and problems, a variety of reference sources need to be utilized. Answers to problematic situations come from students. Methods of learning used by students should be similar to those emphasized by professional mathematicians.

Woolfolk and Nicolich[3] wrote:

> Jerome Bruner is a well-known modern cognitive theorist. . . . Bruner has been especially interested in instruction based upon a cognitive learning perspective. He believes that teachers should provide problem situations that stimulate students to discover for themselves the structure of the subject matter. Structure is made up of the fundamental ideas, relationships, or patterns of the subject matter, that is, the essential information. Specific facts and details are not part of the basic structure. However, if students really understand the basic structure they should be able to figure out many of these details on their own. Thus Bruner believes that classroom learning should take place inductively, moving from specific

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[3]Anita E. Woolfolk and Lorraine McCune Nicolich, Educational Psychology for Teachers. Englewood Cliffs, New Jersey: Prentice Hall, Inc., 1980, page 209.

examples presented by the teacher to generalizations, about the structure of the subject, that are discovered by the students.

Structure of knowledge advocates in mathematics do not believe in:

1. student-teacher planning as to objective the former is to attain. Rather, structural ideas need to be achieved as identified by subject matter specialists.

2. teachers presenting subject matter deductively for learners to acquire.

3. content for student attainment being chosen by others than professionals in the mathematics curriculum.

4. emphasizing abstract experiences for students as compared to the concrete and semi-concrete. Sequence in learning activities must progress from manipulative (real objects and items), to the iconic (pictures, films, filmstrips, slides, and transparencies), to the symbolic (abstract words, letters, and numerals).

The Mathematics Laboratory

Mathematics laboratories philosophy in teaching and learning believe that students are active, not passive beings. Learners need to choose and select, rather than to listen to lectures and lengthy explanations of subject matter. Concrete experiences need to be at the heart of the mathematics curriculum. An adequate number of real objects need to be in the offing to stimulate student achievement. Thus, for example, objects and materials need to be in evidence from which learners may select to weigh, measure lengths and widths, determine the volume, as well as find areas, perimeters, and circumferences.

Within the framework of concrete experiences, students use abstract learnings to record weights, measurements, areas, and circumferences.

Involving the mathematics laboratory concept, Ediger[4] wrote:

> Pupils should have ample opportunities to experience the mathematics laboratory concept of working. The mathematics laboratory emphasizes tenets of teaching and learning such as the following:
>
> (a) Pupils are actively involved in ongoing learning activities.
> (b) A variety of experiences is in evidence so that pupils may select materials and aids necessary for problem solving.

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[4] Marlow Ediger, The Elementary Curriculum, A Handbook. Kirksville, Missouri: Simpson Publishing Company, 1977, page 170.

(c) Practical experiences are emphasized for learners in that they actually measure the length, width, and/or height of selected people and things; weigh real objects and record their findings; find the volume of important containers; as well as determine areas of selected geometric figures.

(d) Pupils become interested in mathematics due to reality being involved in ongoing learning activities.

(e) Provision is made for individual differences since there is a variety of learning opportunities for pupils from which to select on an individual basis.

(f) Meaning is attached to what is being learned since pupils individually and in committees work on tasks adjusted to their present achievement levels.

A mathematics laboratory philosophy does not advocate:

1. a textbook methodology in teaching and learning situations.

2. students being recipients of facts, concepts, and generalizations from teachers.

3. lecture and extensive explanation approaches in teaching mathematics.

4. abstract, symbolic learnings to the exclusion of using realia in the mathematics curriculum,

A Miniature Society Concept in the Mathematics Curriculum

There are selected mathematics educators who believe strongly in guiding students to acquire and apply facts, concepts, and generalizations useful in society. The community becomes an ideal place then in having learners attain understandings, skills, and attitudinal goals. Thus, for example, students with appropriate readiness experiences and with teacher stimulation might engage in finding unit prices for soap, cereal, flour, and cake mixes. How much then does each brand name and generic brand cost per ounce or gram? Other factors also need to be evaluated, in addition to unit pricing, and that is quality within each item.

Students in a miniature society context, might determine the cost of:

1. a given number of items from a supermarket.

2. selected items purchased from a hardware store.

3. items of clothing from a clothing store.

4. cost of gasoline, after buying a certain number of liters or gallons.

A miniature supermarket may be developed in the classroom. Empty cereal, fruit and vegetable, as well as other containers may be placed on shelves in the classroom setting. Appropriate clearly labeled prices need to be attached to each food item. Play money may be used by learners in shopping for needed items. Paper and pencil, as well as the hand held calculator may be used to determine cost of a given set of items purchased, as well as change to be received from money given in payment.

John Dewey[5] wrote:

> The development within the young of the attitudes and dispositions necessary to the continuous and progressive life of a society cannot take place by direct conveyance of beliefs, emotions, and knowledge. It takes place through the intermediary of the environment. The environment consists of the sum total of conditions which are concerned in the execution of the activity characteristic of a living being. The social environment consists of all the activities of fellow beings that are bound up in the carrying on of the activities of any one of its members. It is truly educative in its effect in the degree in which an individual shares or participates in some conjoint activity. By doing his share in the associated activity, the individual appropriates the purpose which actuates it, becomes familiar with its methods and subject matters, acquires needed skill, and is saturated with its emotional spirit.
>
> The deeper and more intimate educative formation of disposition comes, without conscious intent, as the young gradually partake of the activities of the various groups to which they belong. As a society becomes more complex, however, it is found necessary to provide a special social environment which shall especially look after nuturing the capacities of the immature. Three of the more important functions of this special environment are: simplifying and ordering the factors of the disposition it is wished to develop; purifying and idealizing the existing social customs; creating a wider and better balanced environment than that by which the young would be likely, if left to themselves, to be influenced.

A miniature society mathematics curriculum does not emphasize:

1. a textbook centered method of teaching mathematics.

2. a teacher initiated curriculum whereby the instructor selects objectives, learning activities, and appraisal procedures for pupils.

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[5]John Dewey, Democracy and Education. New York: The Macmillan Company, 1961, page 22.

3. minimizing concrete, life-like experiences for students.

4. students being recipients of content in a highly structured mathematics curriculum.

In Closing

Numerous philosophies are in evidence pertaining to goals in mathematics for learners to attain. These include:

1. IMS with its emphasis upon precise, measurable ends for learner attainment.

2. Learning centers with its stress placed up students becoming quality decision makers in ongoing experiences.

3. Structure of knowledge with its advocacy of students acquiring major concepts and generalizations as identified by professional mathematicians.

4. A mathematics laboratory with emphasis placed up students using concrete materials in mathematics achievement.

5. A miniature society philosophy in which learners use mathematics in the functional real world.

Teachers and supervisors need to study and evaluate each philosophy. Ultimately, those philosophies which guide each pupil to achieve optimally should be emphasized in the mathematics curriculum.

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PROCESS OBJECTIVES AND THE MATHEMATICS CURRICULUM

There are relevant processes that learners need to develop proficiency in. Units of study in the mathematics curriculum may contribute much in having pupils achieve vital process objectives. Utilizing desireable processes in teaching-learning situations assists pupils to acquire relevant products, such as accurate facts, concepts, and generalizations. Which process objectives might be emphasized in ongoing units of study in the mathematics curriculum?

PROCESSES AND THE LEARNER

Process ends that pupils achieve should have wide application in mathematics units of study in the school-class setting as well as in the curriculum of life. Teachers, principals, and supervisors may well consider the worth of emphasizing the following processes in the mathematics curriculum:

1. thinking critically and creatively.
2. engaging in realistic problem solving activities.
3. working together harmoniously with others in committee endeavors.
4. developing proficiency in achieving well on an individual basis.
5. utilizing structural ideas and content in ongoing units of study.
6. achieving a positive attitude toward the curriculum area of mathematics.
7. developing an adequate self concept.
8. participating adequately in decision-making procedures.
9. utilizing learnings obtained in the mathematics curriculum within school and societal settings.

10. devising relevant hypotheses.

Each of these process objectives may be stated either as a general or a specific, measurable end. Learning activities may then be chosen to guide pupils in achieving the desired objectives. Ultimately, evaluation must take place to determine if learners have achieved the chosen ends.

Evaluating Pupil Achievement

Each pupil's progress may be appraised using a five point scale for each process objective. Thus, a value of five would indicate high proficiency in achieving a process objective. A rating of one, of course, would indicate low achievement in any one process objective. The classroom teacher or teaching team may evaluate each pupil in terms of having acquired diverse identified process objectives. The evaluator may be a competent consultant hired from outside the domain of the local school system. Mathematics supervisors in the local school district or school system as a team or together with involved teachers of pupils may also serve as appraisers of pupil achievement in the process objectives domain.

At selected intervals, additional evaluations may be made of pupil achievement in the process ends arena. Educators may then notice pupil achievement comparing present with past intervals in learning. These processes are vital to develop in the school curriculum, as well as in society.

Recommended Trends in the Mathematics Curriculum

There are selected relevant trends recommended in the mathematics curriculum.

1. More emphasis is placed upon using microcomputers in the class setting. Thus, learners may receive computer assisted instruction (CAI) as learning activities. Drill and practice, programed learning, simulations, games, and problem solving experiences are inherent in CAI.

2. Learners need to experience realistic word problems from textbooks and workbooks, as well as solve lifelike problems existing in society. Identifying and solving problematic situations are vital for the learner in school, as well as in society. Higher levels of cognition (use of the intellect) is involved in the problem solving area.

3. Continued emphasis must be placed upon the use of the hand held calculator. The calculator is easy to carry, and inexpensive. Experiences using the caluulator may well make mathematics more enjoyable for learners. For example, to check addition, subtraction, multiplication, and division problems, the calculator may be utilized. This eliminates lengthy paper-pencil procedures to determine if a final product to an operation was performed correctly.

4. Learners need to understand and attach meaning to diverse relevant concepts in mathematics. These concepts, among others, may include the commutative property of addition and multiplication, the associative property of multiplication over addition, identity elements of addition and multiplication, as well as the inverse properties of subtraction and division.

5. Recommended principles of learning need emphasis in ongoing lessons and units. Students need to experience interesting, meaningful, and purposeful activities. Each learner might then be guided to achieve optimally.

6. A variety of activities need to be provided. Textbooks, workbooks, worksheets, slides, films, filmstrips, transparencies, illustrations, models, and realia may be utilized as materials to provide for individual differences.

7. Numerous methods need to be utilized to appraise student progress. Among others, the techniques of evaluation should include teacher written tests, teacher observation, anecdotal records, checklists, rating scales, learner self evaluation, standardized tests, and criterion referenced tests.

Each student needs guidance to attain in an optimal manner in the nathematics curriculum.

LEARNING ACTIVITIES IN THE MATHEMATICS CURRICULUM

Each learner needs to experience a quality mathematics curriculum to achieve optimally. Which learning activities might then be provided in ongoing lessons and units?

1. To guide learners to perceive meaning, use a flannel board and felt cutouts. Thus, for example, to understand $\frac{1}{2} \div \frac{1}{4} =$ __, the teacher may take $\frac{1}{2}$ a circle and place the number of $\frac{1}{4}$'s needed to cover the $\frac{1}{2}$. Learners may then see that $\frac{1}{2} \div \frac{1}{4} = 2$. It takes two $\frac{1}{4}$'s to equal $\frac{1}{2}$. In each experience, learners need to comprehend what is being learned. Meaningful, not rote learning, must be in evidence in teaching - learning situations.
2. To stimulate interest, utilize a variety of materials as activities. To guide pupils to understand place value, use sticks in bundles of tens and ones. Additional methods include
   (a) placing felt cutouts on a flannel board. Put a piece of yarn around each set of ten to show tens and ones.
   (b) manipulating an abacus to reveal tens and ones.
   (c) make a place value chart, containing pockets for a ten's column and a one's column. Congruent slips of construction paper may be used to show place value in any two digit number.
3. To establish purpose for learning, guide learners to perceive value in ongoing lessons and units. If pupils are to perceive value in studying the meaning of 1/3, have cookies available in the classroom to guide learners in understanding the necessity of learning to divide a cookie into 3 congruent parts. Each set of three learners may then receive 1/3 of a cookie. A need then exists for learning what 1/3

means. Practical use can be made of content learned.

4. To provide for individual differences, guide students to achieve optimally on a personal basis. Diagnose specific learnings that a student does not understand. Provide activities which assist the involved learner to overcome deficiencies. Sequential experiences are highly significant for each learner.

PSYCHOLOGY OF LEARNING IN THE MATHEMATICS CURRICULUM

There are numerous psychologies which teachers and supervisors need to study, evaluate, and ultimately implement selected tenets aiding individuals to achieve in an optimal manner. Each psychology of learning needs to be appraised in terms of guiding students to

1. perceive meaning and understanding in learning.
2. reflect purpose or accept reasons for participating in ongoing lessons and units.
3. achieve increased levels of motivation for achieving and accomplishing.
4. develop interest in the mathematics curriculum.

Humanism and the Psychology of Learning

Humanists believe that students should be involved in determining what to learn (the objectives), as well as the means (activities and experiences) of learning. A psychological curriculum is then in evidence in the classroom. Each learner might then select which station and task to pursue sequentially. Each center or station can have a list of activities typed on a task card. The involved student must choose which tasks to pursue and which to omit. A humane mathematics curriculum may then be in evidence.

Humanism, emphasizes strongly that individuals attempt to achieve self-actualization. Self-actualization stresses that each person desires to attain what he/she believes to be optimal achievement intellectually, socially, emotionally, and physically. To attain self-actualization, intermediary goals need achievement. A. H. Maslow[1] (late humanist psychologist) emphasized the following levels that individuals must achieve to ultimately realize self-actualization:

[1]Robert Zais, Curriculum, Principles, and Foundations. New York: Thomas Y. Crowell Company, 1976, page 221.

On the basis of data gathered over many years of study and research, Maslow identifies man's basic psychological needs, in order of prepotency, as: (1) safety, (2) love and belongingness, and (3) respect and self-esteem. It should be emphasized that these basic needs are essentially unconscious, and when they are not filled, man's behavior is more or less dominated by the drive to fill them. Maslow calls this behavior deficiency-motivated (or coping) behavior. To the extent that the environment does not permit basic needs to be filled, psychopathology occurs: The individual becomes "starved" for safety, love, or esteem; he perceives himself and the world around him from an extremely limited, narrow, and distorted perspective; and he behaves (to a greater or lesser degree) neurotically. At the risk of great oversimplification. we might say that according to this theory, paranoia, for example, results from extreme deprivation of the safety need.

In contrast to the neurotics described above, psychologically healthy people have more or less gratified their basic needs for safety, love, and esteem. When these needs are satisfied, Maslow hypothesizes that individuals tend to be motivated . . . . This behavior of self-actualizing individuals is called growth-motivated (or expressive) behavior. Because growth-motivated individuals are less encapsulated (by unconscious basic needs), they interpret environment situations in more objective terms.

Humanism does not emphasize that:

1. teachers alone select objectives, learning activities, and evaluation procedures.

2. what is measurable in terms of student learning represents the most relevant goals to attain. Rather, the interests, purposes, and personal meanings that a learner brings to any situation is vital. These basically are intrinsic and not subject to measuring.

3. a logical mathematics curriculum for students which stresses a teacher ordered sequence of objectives for student attainment. In contrast, the involved student chooses sequential activities, from among alternatives. Thus, a psychological, not logical, set of learnings is emphasized.

Instructional Management Systems (IMS)

Advocates of IMS believe in students achieving precise, behaviorally stated objectives. After instruction, the mathematics teacher may measure if a student has or has not achieved a measurable objective. The objectives are arranged sequentially by the teacher or a committee of teachers. A logical, not a psychological curriculum in mathematics is then in evidence. The instructor has arranged the objectives from those which are easier to attain to an increasing level of complexity. Arrangement of these goals is made so that ideally students will be successful in achievement. For example, objective number one is a prerequisite to achieving goal number two. Goal number two needs to be attained prior to objective three, and so on. The mathematics teacher has then ascertained which the proper or appropriate sequence of objectives should be. The involved learner does not decide individually or through student-teacher planning which objective needs to be achieved first, second, third, fourth, and so on. The teacher is the key person in determining and ordering objectives for learners to achieve.

A comprehensive system of recording of achievement for each learner may be implemented. Since each objective is measurable if a student has/has not been successful in goal attainment, records may be kept to determine precisely which ends have been attained by any one learner. Also, the student and the teacher can notice which objectives still need to be achieved. The objectives are arranged in an ordered sequential arrangement to optimize learner achievement.

Pertaining to the utilization of measurable objectives in teaching, Morris and Pai[2] wrote:

> In establishing behaviors that will be beneficial to the learner and to society, behavior engineers are most concerned with reaching target behaviors (aims of education) by gradually and systematically modifying "old" behaviors and/or shaping new responses. In other words, overall aims of education are first translated into specific objectives involving competencies in various disciplines and other areas of the learner's life, such as learning the three R's, responsible citizenship, and so on. These objectives in turn are formulated into specific programs, courses, and learning activities, the purposes of which are also defined in terms of specific behavioral changes. Each of these competencies are then analyzed into still smaller and simpler behaviors, so that by learning them the child can evantually read the target behavior. Hence, educational goals must be stated in terms of specific and directly observable behaviors.
>
> It is for this reason that the use of behavioral objectives in instructional and curricular planning is indispensable to education as behavior engineering, because these objectives serve not only as clear guides to learning activities but also as standards by which the teaching-learning processes can be evaluated.

IMS advocates do not stress:

1. the use of open-ended general objectives in the mathematics curriculum. Precise objectives provide more direction and guidance in terms of what students are to learn. In fact, clarity of intent as to what pupils are to learn is

[2] Van Cleve Morris and Young Pai, Philosophy and the American School. Boston: Houghton Mifflin Company, 1976, page 324.

inherent in measurbaly stated objectives.

2. teacher-pupil planning in developing the mathematics curriculum. Rather, the math teacher determines what students are to learn (the objectives), as well as the means of learning (activities and experiences).

3. a psychological mathematics curriculum in which pupils with teacher guidance sequence their own learning. The teacher selects and logically arranges objectives for students to achieve, according to IMS advocates.

Structure of Knowledge

Structure of knowledge advocates believe that students should achieve structural ideas (major generalizations) identified by professional mathematicians. The structure of mathematics emphasizes learners acquire key ideas that held true again and again when progressing through the formal years of schooling. These might well include the

1. commutative properties of addition and multiplication.
2. associative properties of addition and multiplication.
3. distributive property of multiplication over addition.
4. identify elements of addition and multiplication.
5. property of closure for addition and multiplication.
6. inverse properties of subtraction and division.

Those emphasizing a structural ideas psychology believe that students should acquire content inductively. To learn inductively, the student with teacher guidance needs to discover these main ideas as determined by professors of mathematics on the higher education level. Reputable, quality textbooks in mathematics generally emphasize the six above named key generalizations, among other structural ideas. Main ideas, such as the structure of knowledge can be emphasized again again as learners progress through the diverse sequential years of schooling. For example, a first grade learner discovering that 3 + 2 = 2 + 3 (commutative property of addition) or the fourth grade student realiz ing that 4623 + 3641 = 3641 + 4623 reflects a spiral curriculum. Thus, structural content provides a framework to the learner in continually perceiving relationships, as well as methods of learning--inductive procedures. The mathematics teacher must be a good asker of questions, rather than a lecturer or explainer in utilizing induction as an approach in guiding pupil learning.

Sequence in learning activities is important. Manipulative (enactive) materials should be used first when appropriate. Thus, an abacus, markers and objects are used as concrete materials to aid student discovery of structural ideas. Secondly, iconic materials, in sequence, need adequate emphasis. These include pictures, slides, filmstrips, films, and drawings to illustrate objectives reflecting structural ideas. Ultimately, symbolic learnings need to be stressed. The symbolic represents abstract phases in learning, such as the use of numerals and letters in the mathematics curriculum.

Woolfolk and Nicolich[3] wrote the following comparing Jerome Bruner's structure of knowledge psychology with that of Jean Piaget.

> At the enactive stage, which corresponds roughly to Piaget's sensorimotor stage, children learn to represent objects by acting on them. These early years are filled with solving problems of how to crawl, walk, play with toys, and generally use the body to be effective in the world. Children at this stage learn by doing and by seeing what others do. Telling is helpful only if the child can also act it out or see someone else act it out. It is the action that will be represented internally.
>
> At the iconic stage, which corresponds to the early years of Piaget's preoperational stage, children begin to form pictures or images to represent what is going on in their world. At this point, they can remember events from the past and imagine the future in terms of visions of what might happen again. These images are much like photographs in that they are highly realistic and closely tied to actual physical experience.
>
> At the symbolic stage, which corresponds roughly to the later years in Piaget's preoperationaly stage and to the other years as well, children are able to represent their world through symbols, the most important of which is language. These symbols need not copy physical reality but can be abstractions. With such abstract symbols, people can ultimately hypothesize about possibilities, people, places, and things they have never experienced.

Programmed Learning

Programmed learning, whether in textbook or micro-computer software form, emphasizes precise, measurable ends in the mathematics curriculum. Programmers

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[3]Anita E. Woolfolk and Lorraine McCune Nicolich, Educational Pyschology for Teachers. Englewood Cliffs, New Jersey: Prentice-Hall, Incorporated, 1980, pg. 66.

determine the sequential objectives for students to attain. Pilot studies are run to ascertain the best order of presenting a set of programmed items. Basically, if a student is ready to pursue a given program, he/she should be highly successful in its completion. Perhaps, a ninety percent rate of success should be in evidence for a student in completing any specific program.

In the utilization of programmed materials, the involved student reads a few sentences or a very short selection, responds to a multiple choice or completion item, and receives immediate feedback as to which the correct response is. If a response is correct, in a computerized item, the learner may be given a second chance to respond. Should be student have still responded incorrectly, the correct answer is shown on the screen. In using programmed textbooks, the correct answer is uncovered by the student as given by the programmer, the learner also knows immedicately if he/she is correct or incorrect in responding. In linear programming, the student sees the correct answer and is ready for the next sequential item. Read, respond, and check are concepts that are used continuously, in using programmed textbooks or software and the microcomputer.

The programmer selects the subject matter, order of learning content, and the correct response for each item in programmed learning.

Preston and Hermann[4] wrote the following pertaining to programmed instruction:

> The pupils read one frame at a time and respond to it. It may call on them to answer a multiple-choice question by pushing a key on a machine (a minority of available programs is presented by teaching machines) or by making their choice in a programmed textbook. Or they may be directed to construct an answer.
>
> After the child responds, he or she is informed of the correctness or incorrectness of the response by a light or some other signal in a machine or by an answer appearing at a designated place in the programmed book. This information is known as "feedback." If the response is false, he or she is given either the correct information or information that can be advantageously utilized in making another choice. This feedback feature is a central element in programmed instruction: It informs the child of his

[4]Ralph C. Preston and Wayne L. Hermann, Jr. Teaching Social Studies in the Elementary School, fifth eduction. New York: Holt, Rinehart and Winston, 1981, pages 313 and 314.

or her progress and indicates the next step--whether to repeat an exercise, respond to a similar exercise, or go on to the next step. In any case, whatever, he or she is directed to do next is designed to reinforce learning --to increase his or her chances of responding correctly to the question or task the next time it is encountered. It is evident that the program performs two functions that a busy teacher handling an entire class cannot accomplish: (1) It presents the subject matter to each child at a rate appropriate to his or her needs, and (2) each child responds overtly to each presentation and receives, without delay, reinforcement for every response he or she makes.

Programmed learning does not advocate:

1. student-teacher planning in determining objectives, learning activities, and appraisal procedures.

2. learners acquiring much content before feedback is given on the correctness/incorrectness of each step of learning.

3. pupils practicing what is incorrect in terms of subject matter learning. Rather, immediate knowledge of results is provided to students after having acquired a small amount of subject matter in each sequential step of learning.

4. internal processes being important in instruction. Instead, that which is observable and measurable is important.

The Basics

Much has been written pertaining to students learning the basics--reading, writing, and arithmetic. Proponents of the basics believe there is an essential body of knowledge that all students should master in mathematics, as well as in other organized bodies of knowledge. Frills and fads are to be eliminated in the curriculum. Basic, essential learnings can then be identified. The chosen subject matter can be ordered properly for students to achieve on each grade level. Mastery in acquiring subject matter in mathematics is important.

General or behaviorally stated objectives may be utilized in teaching. To achieve objectives on the part of students, the teacher may assign work to be completed by learners using reputable basal mathematics textbooks, workbooks, and worksheets. A deductive method of teaching involving lecture, explanations, and demonstrations by the teacher might well be emphasized heavily. A comprehensive program of evaluation using teacher written tests, standardized tests, instructor observation of student progress, anecdotal statements, checklists, and rating scales can be used in the mathematics curriculum. Remedial work biased on diagnosis of specific problems in learning must also be stressed in teaching situations.

Shepherd and Ragan[5] wrote:

> Each disequilibrium and its companion, accommodation by society, has had its impact upon the school as an institution of that society. Therefore, the schools and their curriculum vehicles have also experienced a testing of mission and purpose, of program, and of product, as well as a critical appraisal of service and spirit. The tensions of the school and its curriculum vehicles seem to orignate from three sources: the potential displacement of the family through the school as an agency of a nation-state; the movement from a industrial culture to a yet, unrealized future culture; and the accommodations to the constitutional conditions of independence and dependence, individuality and conformity, and inclusion and exclusion. In general, the functioning of the curriculum in the socialization of individuals is the prime source of the powerful tensions being experienced by the schools.
>
> During this period of tension and disequilibrium, the number and variety of curriculum alternatives available have rapidly multipled. The alternatives range from "back to the basics," to "forward to the future." In unsettled times, alternatives are apt to be presented as dichotomies. Although it is most unlikely that a movement can go backward and forward in the same instant, it is possible that the continuity involved in the development of an individual and an institution can be maintained and furthered.

A subject matter curriculum in mathematics does not emphasize:

1. learning by discovery by students. Rather the teacher through lecture and explanations provides essential content to learners.

2. variety in learning activities. Basically, the use of the textbook, workbook, and worksheets, contain what is necessary for pupils to acquire.

3. planning the objectives, activities to achieve goals, and appraisal procedures with students. The mathematics teacher is in the best position, due to training and experinece, in determining what pupils are to attain.

In Summary

There are numerous psychologies available to provide guidance in assisting learners to achieve in an optimal manner.

1. Humanism emphasizes that students become proficient decision makers to achieve sequence in learning.

2. IMS advocates that teachers of mathematics arrange order of goals for pupils to achieve.

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[5]Gene Shepherd and William Ragan, Modern Elementary Curriculum, Sixth Edition New York: Holt, Rinehart and WInston, 1982, page 5.

3. Structure of knowledge psychology stresses an inductive procedure to assist students to progress sequentially in acquiring major generalizations.

4. Programmed learning proponents believe that a programmer is in the best position, through field testing, to order precise objectives for pupils to achieve.

5. Advocates of the basics believe there is a core of essential knowledge in mathematics for each and every student ot acquire.

Hopefully, teachers will select those psychologies in an integrated whole to assist each student to learn as much as possible in ongoing lessons and units in mathematics.

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ISSUES IN THE MATHEMATICS CURRICULUM

There are salient issues in the mathematics curriculum which need discussing and resolving. Each teacher and supervisor needs to take a position on vital issues. Which issues might be relevant to synthesize?

Inductive Versus Deductive Teaching

Pupils may attain significant concepts and generalizations through induction. Thus, learners with teacher guidance achieve learnings in mathematics using discovery techniques. Problems and questions then are identified by pupils. Pupils individually and in committees, using a variety of reference sources, secure necessary facts, concepts, and generalizations to solve problematic situations. From specific understandings attained, pupils develop broad ideas or generalizations. The generalizations supported by facts are usable to answer questions and solve problems.

Opposite of inductive procedures in teaching is deductive methodology. In deductive means (1) the teacher in a meaningful manner explains a new process to pupils. Learners might then apply that subject matter provided by the teacher(s). (2) A one way street of communication exists, i.e. from the teacher to the learner. However, the pupil must attach meaning to what has been acquired in order that the cognitive level of application may be utilized by pupils on an individualized basis.

Active Involvement Versus Passive Recipient

Educators emphasizing active learner involvement within lessons and units believe that individuals learn by doing. Thus, pupils with teacher guidance identify and solve life-like problems in mathematics. To become proficient in problem solving, a learner needs to practice specific skills therein. The subskills in problem solving include problem selection, gather data or information in answer to the problem, develop a hypothesis or answer to the problem, test the hypothisis and revise the hypothesis if necessary. The sequential steps in problem solving are flexible and not rigid.

A passive receiver in learning may secure information (facts, concepts, and generalizations) from a sender. The sender usually is the classroom teacher. Thus, content moves from the teacher to the learner in explanation/ lecture form. Individual differences need adequate provision. Hopefully, learners can apply that which was received from the sender. In contrast, active pupil involvement indicates the whole person (intellectual, emotional, social, and physical) is involved in projects and activities to solve problems relevant in society. Thus, school and society become integrated entities in the mathematics curriculum.

Advocates of active pupil involvement in learning believe that (1) learners are capable and interested in making curricular decisions. (2) Learners are in a better position to sequence their own content, as compared to a logical curriculum offered by adults. (3) Learners need to be involved in self appraisal for evaluation techniques to be effective. Otherwise, adult means of appraising learning performance may lack impact upon the learner.

Measurably-stated Versus General Objectives

How precise should objectives for pupil attainment be stated? With measurable ends, the teacher may select learning activities which guide learners to attain the chosen objectives. After which, the teacher may measure if a pupil has/ has not achieved the stated goal. Successful learners may then attempt to attain the next sequential objective. Unsuccessful learners may need a new teaching strategy to achieve the previously unattained objective.

Related procedures to measurable objectives in teaching mathematics are Instructional Management Systems (IMS), mastery learning, criterion referenced testing (CRT), and exit objectives. In each of the above named plans of instruction, precise measurable objectives are utilized in teaching and learning situations. Advocates of measurable ends believe that teachers need to possess clarity of intent in teaching. Thus, the teacher and pupils are clear on what the latter specifically will be learning. Vagueness and ambiguity as to what pupils will be learning is not in evidence.

The teacher can more effectively select learning activities if measurable rather than general objectives are used. Each experience selected is chosen on the basis of one criterion-Do the activities guide learners to attain specific objectives? If the activity is too complex or lacks challenge, it should be omitted. The teacher may measure personal success in teaching by obtaining objective data in determining if pupils have/have not achieved desired objectives. Furthermore, results of learner progress may be communicated clearly and precisely to parents. Evidence needs to exist to show to responsible individuals that pupils are/ are not achieving measurable objectives.

If pupils are not attaining measurable goals, feedback is received by the teacher. The teacher may then need to select a different teaching strategy to aid one or more learners to attain a previously unachieved objective.

Opposite of measurably stated objectives are (1) broad general goals to provide some kind of direction in determining the kinds of learners a teacher wishes to develop. (2) Evaluation procedures which lack precision in determining if pupils have/have not attained desired ends.

Learner-centered Versus Society-centered Curriculum

Should most of the objectives in teaching and learning come from pupils themselves? Or, should attainable goals for pupils be selected on the basis of what society needs and deems to be highly significant.

How might ends be chosen which reflect personal interests and purposes of pupil? First of all, pupils can decide which tasks to pursue and which to omit when interacting with learning centers in the school/class setting. An adequate number of tasks needs to be in evidence at diverse learning centers in order that pupils may select, as well as omit, sequential experiences. Thus, learners individually might truly select interesting tasks to pursue. Hopefully, learners may also perceive purpose or reasons for participating in ongoing activities.

Additional teaching strategies emphasizing personal interests and purposes of learners include

1. individualized reading. Each pupil selects and reads a library book pertaining to mathematics which has interesting content and is on the reading level of the involved reader. The pupil may also select how to be evaluated in terms of using appropriate word recognition techniques and comprehension skills. Thus, the learner may read a self-chosen selection orally to the teacher. The teacher might then assist the learner in appraising quality utilization of word recognition techniques. To reveal comprehension, a pupil may develop a mural, diorama, model, or creative dramatics presentation to reveal what has been comprehended from

the reading of the library book.

2. the contract system. The pupil with teacher guidance may specify which activities to complete involving mathematics within a particular period of time. The contractual agreement needs to be reasonable in terms of activities to be completed by the learner within the specified time in the contract. Mathematics activities in the contract need to reflect learner enthusiasm and reasons for its contents.

To emphasize societal needs in the mathematics curriculum, teachers and supervisors need to ascertain what society emphasizes that needs to be learned by pupils. Among other learnings, these might include

1. being able to compute the total cost of goods/services purchased in any given situation.

2. possessing skill to ascertain the amount of change to be returned from a larger cash denomination given after purchasing needed items.

3. being skillful in writing checks and keeping a responsible checkbook balance.

4. knowing how to obtain loans to make satisfactory investments.

5. possessing applicable concepts involving interest rates.

6. realizing specific abilities involved in ordering materials from mail order companies.

7. shopping intelligently for necessary goods and services used in the home setting.

8. buying insurance for property and health in an effective manner.

9. learning to live within budget requirements.

10. completing job application forms as well as being knowledgable about required diverse forms used in taxation--local, state, and national levels.

In Conclusion

Selected issues in the mathematics curriculum need studying, analyzing, and synthesizing. Teachers and supervisors need to become students of philosophical issues in curriculum development. Each issue needs to be resolved in terms of guiding learners to achieve optimally in intersts, purposes, and meanings in the mathematics curriculum.