An editorial and abstracts for 12 research reports are contained in this issue. The editorial by Robert E. Reys focuses on the dangers of publishing poor research. The abstracts, each with a critique, concern research on a game for logical reasoning, wait-time and sex differences, vocabulary instruction on ratio and proportion, male-female enrollment across mathematics tracts, multiplication, partitioning, cognitive style, teacher use of materials, counting strategies in addition, attitudes of precalculus college students, question placement in word problems, and preservice teachers' concept of zero. Research studies reported in RIE and CIJE for October through December 1983 are also listed. (HNS)
INVESTIGATIONS IN MATHEMATICS EDUCATION

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An editorial comment...ROBERT E. REYS.......

Bright, George W.; Harvey, John G. and Wheeler, Margariete Montague. USE OF A GAME TO INSTRUCT ON LOGICAL REASONING. School Science and Mathematics 83: 396-405; May-June 1983. Abstraced by THEODORE EISENBERG...


Scott, Patrick B. A SURVEY OF PERCEIVED USE OF MATHEMATICS MATERIALS BY ELEMENTARY TEACHERS IN A LARGE URBAN SCHOOL DISTRICT. School Science and Mathematics 83: 61-68; January 1983. Abstracted by JAMES H. VANCE. 37


Stones, Ivan; Beckmann, Milton and Stephens, Larry. FACTORS INFLUENCING ATTITUDES TOWARD MATHEMATICS IN PRE-CALCULUS COLLEGE STUDENTS. School Science and Mathematics 83: 430-435; May-June 1983. Abstracted by SANDRA PRYOR CLARKSON. 49


When Poor Research is Published—The Bell Tolls For Us

Robert E. Reys
University of Missouri

Have you ever started reading an article and then gotten the feeling of deja vu? In some cases a feeling of deja vu provides security or stimulates a warm nostalgia. There are other times when this warm feeling is the type associated with getting sick.

Have you ever read a research study reported in a journal that made you cringe? I am not talking about quality research studies which report findings that are philosophically or theoretically in conflict with your own, but simply a poor piece of research. Such "research" in mathematics education is a painful thorn for all of us. Whenever such articles appear under the guise of research there is the possibility that someone somewhere sometime will read them, perhaps even believe them, and, worse yet, cite this "research evidence" to others.

Although we can and should write letters to editors expressing our dissatisfaction with such practice, once something of poor quality is published much has already been lost for several reasons. It may be the only "research" article in mathematics education the person has read. It may be that the reader is unable to discriminate between good and poor research articles. The reader may be unable to place the findings from this research in any kind of overall context and is therefore unable to judge its contribution.
As you may know, part of the rationale for creating INVESTIGATIONS IN MATHEMATICS EDUCATION was to establish a dialogue among interested parties on research in mathematics education. It is to the credit of our mathematics education community that this concept has not only proven viable but is now well established. The journal not only provides different perspectives of published research but has been instrumental in improving the overall quality of research in mathematics education that is reported.

The only redeeming value of publishing low-quality research articles is that they provide cannon fodder for graduate students in mathematics education. Research seminars have a field day dissecting and critiquing such research. Unfortunately most readers do not take such a critical eye toward research. Consequently when they read a research article they may give it far more creditability than it deserves. Whenever a poor or weak research manuscript related to mathematics education is published anywhere—it hurts all of us.

Mathematics education is a young discipline and much of the theoretical framework is still in the formative stage. Even among carefully conducted, high-quality studies, the findings are often mixed. Counterexamples abound and they must be carefully examined in this theory-building process.

Journals vary greatly in quality, as do the articles published in them. This is a fact of life and one that often determines which journals we subscribe to and read regularly. This editorial is simply a reminder to all of us that only high-quality research articles should be published. Although the criteria for accepting research articles in journals does vary, it is my hope that high among these criteria is a theoretical base for the research. A wide variety of pragmatic research must be encouraged, but if significant progress is to be made in mathematics education, the research must in some way contribute toward theory-building. This includes not only research which addresses current theories, but provides for new theory-building as well.
Time is too short to deal with trash which is published under the guise of research. We should continue to do everything possible to produce high-quality research manuscripts. This role is assumed by editors, editorial boards, referees, and readers. However, it is the individual authors who have the ultimate responsibilities for not only conducting significant, high-quality research but also reporting it in a clear, honest, and readable form.
Abstract and comments prepared for I.M.E. by THEODORE EISENBERG, Ben Gurion University of Negev.

1. Purpose

The purpose of this study was to determine the extent to which playing the game Mastermind enhances one's logical reasoning ability.

2. Rationale

Deductive logic can be considered as an independent mathematical structure, and most mathematical structures can be learned through a motivating game format. The principles of basic logic are needed in order to successfully play a popular decoding game called Mastermind. This study, therefore, investigated the extent to which this game enhances formal logical reasoning skills.

3. Research Design and Procedures

A pretest/posttest/control-group design was used. Eleven classes (five sixth grades and six eighth grades) were in the experimental group; four classes (two at each grade level) were in the control group. These 15 classes came from four different schools, with one class in each school serving in the control group.

Two pretests and two posttests were administered, hereafter noted as PT(1), PT(2), P0T(1), and P0T(2). PT(1) was the well-known Wason and Johnson-Laird four-card problem. This test was given to the intact classes and the students were allowed 20 minutes to complete it. PT(1) was used to determine the students' formal operational level. PT(2) was a 40-item author-constructed logical reasoning test.
which the students were allowed 15 minutes to complete. PoT(1) was a randomization of items on PT(2). PoT(2) consisted of 12 pictorial representations of two different versions of the Mastermind game board. The students were given a score on PoT(2) depending on their ability to develop a winning end-game strategy. Students were allowed 20 minutes to complete PoT(2).

The study was conducted over 12 consecutive weeks. The first two weeks and the last two weeks were used for testing. The students played one of two versions of Mastermind during weeks 3 through 10. The students played the game twice weekly in 20-minute sessions and only during this allotted time. (It is not known if the students played Mastermind outside of class.)

4. Findings

For each class the mean and standard deviation for those playing the two versions of Mastermind were presented for PT(2), PoT(1) and PoT(2). (The control group did not take PoT(2).)

Using ANOCOVA it was shown that there were no differences in the way the classes (and grades) handled the different versions of Mastermind, PoT(2).

Experimental and control classes were compared on PoT(1) by ANOCOVA using PT(2) as the covariate. Again, no differences were observed.

The PoT(1) scores were further analyzed for two subgroups of students, those who had high and low PT(1) scores, signifying their formal operational level. No additional comprehensive information was determined by this procedure.
5. **Interpretations**

Playing Mastermind alone did not enhance the students' reasoning ability. "Hence, teachers should be cautious of claims that Mastermind teaches logical reasoning."

**Abstractor's Comments**

The authors are well known for their studies in this area. From a design point of view, this study is really very nice. I particularly like that it was carried out over a 12-week period; the write-up is also very clear and to the point. But by now the authors have quite a bit of information in this area and it would have been nice if they would have embedded their findings from this study into a more general context. There are also several procedural points to question.

1. The purpose of PT(1) is not clear. It was used to determine the student's formal operational level, but it should have also been used as a posttest. Indeed, it would certainly have given more credibility to the findings.

2. Reliability coefficients (or a correlation coefficient) should have been computed for PT(2) and PoT(1). It is incredible that for the 12 groups of students in the sixth grade taking this test, eight of these groups had lower mean scores on the second testing than they did on the first testing! Looking at only those sixth graders in the experimental group, we see that 60% of the groups had lower mean scores on the second testing than they did on the first testing. For the eighth graders, 33% of the classes in the experimental groups had lower mean scores on the second testing. Indeed, Mastermind may well decrease logical reasoning ability, not enhance it.
3. It is unclear why the authors placed so much emphasis on PoT(2). A simple statement that the two versions were equivalent would certainly have been sufficient.

Overall this study is carefully done. Although more posttests (which also served as pretests) could have been used, the results are quite convincing. As the authors admonish, we should be wary of claims that logical reasoning can be taught without explicit instruction.
1. Purpose

The research questions explored in this study were:

1) Will teacher wait-time [i.e., the period of time the teacher waits for a child to begin answering his/her question] be significantly greater for boys than for girls during fourth grade mathematics instruction?

2) For the group as a whole, will teacher wait-time be significantly greater for high, medium, or low achievers? (p. 273)

2. Rationale

Teacher expectations (Cooper, 1979) is one aspect of classroom environmental factors that is examined in this study. The wait-time concept as a reflection of teacher expectation is based on the work of Rowe (1974) and Tobin (1979). In addition to considering wait-time as a function of perceived student achievement, the researchers, citing the work of Burton (1978), also considered wait-time as possibly being affected by teachers' expectations of boys outperforming girls in mathematics.

3. Research Design and Procedures

Seventy-six fourth-grade girls and 79 fourth-grade boys in five classes (in the same rural school in a small Arkansas town), with their five female teachers, participated in the study. The teachers,
who voluntarily participated, represented diverse backgrounds. The teachers used the results of the California Achievement Test to rank the children as low, medium, or high in general academic achievement.

Teacher wait-time was recorded by an observer for boys and girls individually (i.e., questions directed to more than one child were not considered) as they responded during mathematics questioning. For each of the five classes, ten mathematics teaching sessions which included "routine mathematics instructional activities" were observed and audiotaped. The observations were approximately 20 minutes each. Data were recorded during the last 15 minutes of instruction.

The audiotapes were transcribed "to aid in timing the responses" (p. 274). Wait-time was measured by using a stopwatch. Two raters (one of whom was the observer) analyzed the data independently. Interrater reliability of wait-time of the five teachers ranged from .86 to .93, using the Pearson product-moment coefficient of correlation.

To determine whether the difference between the mean wait-time for boys and girls was significant (p < .05) for each teacher as well as for the whole group, t-tests were calculated. To determine whether the differences among mean wait-time for low, medium, and high achievers were significant (p < .05) for each teacher as well as for the whole group, analysis of variance was used.

4. Findings

Based upon the results of the t-tests, "teachers gave significantly more wait-time to boys than to girls" (p. 273). Based upon the results of the analysis of variance, there were no significant differences among achievement levels.
5. Interpretations

With respect to teachers' expectations that boys outperform girls in mathematics, the results reflected these expectations because significantly more wait-time was given to boys than to girls.

With respect to teachers giving more wait-time to high-ability students than low-ability students, the results did not support the findings of other research (e.g., Rowe, 1974).

Since teachers who participated in this study represented diverse backgrounds, it is possible "that the phenomenon of longer wait-time for boys in mathematics instruction may be something that exists outside this small school" (p. 275). More extensive research should be undertaken to examine whether this phenomenon "is a generalized occurrence" (p. 275).

If research results suggest wait-time as a factor that might account for sex-related differences in mathematics performance, teachers should give all students adequate time to answer mathematics questions. Systematically giving boys more time and girls less time over the course of children's school years may possibly have a cumulative effect, gradually helping to discourage girls in their mathematics and possibly causing them to achieve less and less relative to boys. (p. 275)

Abstractor's Comments

This research is a contribution to the literature on sex-related differences. Implications of the results provide insight into the possible cumulative effects of an environmental factor (i.e., teacher expectation as reflected by wait-time) which might contribute to sex-related differences in mathematics performance in later years. Further research is needed to verify the effects of wait-time differences.
The statistical analysis of the data would have been more complete (and perhaps more revealing) if sex-by-achievement level interactions were examined and reported (e.g., comparing teachers' wait-time of high-achieving boys with high-achieving girls, low-achieving boys with low-achieving girls, and low-achieving boys with high-achieving girls). Even though this was not a concern reflected in either of the two research questions, the possible interaction cannot be overlooked. Perhaps future research will examine this.

In comparing the results of this study with Rowe's (1974) results, it is important to note that a measure of achievement (operationally defined as a score on the California Achievement Test in this study) cannot be equated with a measure of verbal ability (of which there is no formal operational definition in Rowe's [1974] work). One cannot expect to replicate or support findings when two conceptually different independent variables (i.e., achievement and ability) are being used.

Teachers should be aware of their behavior towards male and female students in mathematics classes. The results of this study can encourage teachers to examine and monitor their own behavior and consider how their behavior might affect their students psychologically. It is hoped that future research will explore this phenomenon and provide classroom teachers with guidance so that children are given an equal chance to enjoy and excel in the study of mathematics— as proposed by the National Council of Teachers of Mathematics in a position statement (NCTM, 1980).

References


Abstract and comments prepared for I.M.E. by DOUGLAS T. OWENS, University of British Columbia.

1. Purpose

The purpose of the study was to determine whether the inclusion of vocabulary-oriented activities using terms and symbols related to ratio and proportion in the instructional program would result in a higher level of achievement on the topic.

2. Rationale

Previous research was cited which indicates "...a positive correlation between the ability to comprehend written mathematical material and achievement in mathematics" (p. 337). Also, a lack of knowledge of technical terms has been cited as a source of difficulty for students.

Little research has been done on how vocabulary instruction might best be integrated into the mathematics curriculum. Current practice offers no consensus.

3. Research Design and Procedures

A list of terms and symbols used in seventh- and eighth-grade mathematics was compiled and reviewed by a panel of educators. Of the 117 terms and 36 symbols deemed necessary, six terms and five symbols were rated as essential to ratio and proportion. Instructional
activities incorporating these terms were designed to be consistent with previous practice and research on facilitating the formation of mathematical concepts.

Subjects were chosen from three suburban schools having at least two appropriate seventh-grade mathematics classes meeting concurrently. One school was located in each of the high (N = 46), moderate (N = 106), and low (N = 39) SES areas. Two seventh-grade mathematics classes were pooled and reassigned randomly to the two treatment groups. The two experienced mathematics teachers became the teachers of the experimental and control groups meeting at that time. The teachers cooperated to plan lessons and use the same materials and procedures. The experimental groups used the vocabulary-oriented activities for 5 to 10 minutes each day; "...whereas the control classes spent the time working computational problems" (p. 340). Each class period was 50 minutes long and the experiment took place within four weeks.

Data from three existing measures were collected from school records: Metropolitan Achievement Test (MAT) (1) mathematics and (2) reading comprehension, and (3) previous mathematics mark. The MAT had been administered at the beginning of the school term and the seventh grade mathematics mark was the letter grade earned (presumably first semester) the same year. The posttest developed was similar to the chapter test in the textbook except that vocabulary-oriented items were included. The posttest yielded two measures: (1) verbal (11 items) and (2) computational (15 items). Internal consistency coefficients of .51 and .75, respectively, are reported, and judged adequate.

A posttest-only control-group design was appropriate. Data were analyzed using a general linear model analysis of variance under which each of the five effects was considered an additional contribution to the variance already explained by the other effects.
4. Findings

Means are reported by school and treatment group for all five measures. The statistical model accounted for about 40% of the variance in computational scores and 52% of the variance in the verbal scores. Treatment made a significant contribution to the model’s performance when entered last in both cases of computation and verbal measures. Additionally, for verbal score, MAT-mathematics and school made a significant contribution at the point at which they were entered.

5. Interpretations

The results support findings of previous research relating mathematical achievement to vocabulary knowledge. Also, the students in the experimental treatment outperformed the control group on computational items even though they had less computational practice. Thus, increased achievement can be the result of concentrating on a few essential terms and symbols for only a few minutes each day.

This study was carried out in essentially normal classrooms by regular teachers and without tightly controlled clinical or laboratory conditions. While this may be seen as a limitation, the authors prefer the view that the conditions enhance the potential for applying the results to other classrooms. The teachers found the vocabulary-oriented activities easy to integrate into their lessons, which would indicate that there is good potential for use by others. It is likely that activities of this type can be integrated into a mathematics curriculum with minimum disruption and high potential for payoff.

Further research is needed to verify the results and give further evidence of generalizability. In particular, follow-up studies should include a sample large enough that the class rather than the individual may be used as the experimental unit.
Hilgard (1964) describes six steps in research on learning ranging from basic pure research which does not apply directly to a classroom setting to step 6, "advocacy and adoption" of the practice for the classroom. Hilgard's step 5 is "a tryout of the results of prior research in a 'normal' classroom with a typical teacher [p. 409]." The present authors need not apologize and in fact should be commended for undertaking applied research.

The authors repeatedly referred to the treatment as characterized by "vocabulary-oriented activities." It would seem that an alternative might be to rationalize as activities which teach terms and symbols as an integral element of the concepts related to ratio and proportion. Is it preferable, on the other hand, to establish a rationale in terms of the reading comprehension of mathematical material? Perhaps a commitment from the start to applied or laboratory research can clarify which rationale is preferable among the several choices. In any event the reader is assisted by knowing a theoretical basis for the study.

The authors give examples of the vocabulary activities, but not of the test. It is easy to imagine what the computational items look like, but we are at a loss without examples of the "verbal" items. The verbal test is described as "...10 vocabulary-oriented items and 1 word problem" (p. 339). Perhaps this can explain the low internal consistency coefficient for this test. Future studies of this type might use "problem solving" or "applications" as an additional measure.

The authors call for further research to verify and determine generalizability of the findings. It would appear that the most crucial would be to determine the generalizability of this teaching
strategy to other topics and other age levels. Perhaps the present curriculum as implemented could be substantially strengthened by more emphasis on concepts expressed as terminology and symbols.

References

The purpose of the study was to compare male and female enrollment in mathematics courses of students in predominately black high schools.

2. Rationale

Recent research has found that males and females have similar course-taking patterns in mathematics. However, little is known of male-female enrollment differences in minority settings. The authors sought to expand existing knowledge on sex differences by examining differences in enrollment of students in predominately black high schools.

3. Research Design and Procedures

Six senior high schools (grades 10 to 12) in an East Coast city participated in the study. The percentage of black students in these schools ranged from 64% to 99%, with three of the schools reporting that 99% of their students were black.

The students were divided by the school district into two broad groups on the basis of their scores on a standardized mathematics achievement test. Students scoring below the 70th percentile formed one group, while students scoring above this point formed the other group. Students scoring below the 70th percentile were further subdivided into three groups, depending upon their scores, and
assigned specific mathematics courses. In addition to the required
courses, these students could enroll in electives especially designed
for them. The authors grouped all the required and elective courses
for students scoring below the 70th percentile on the test into a
single category and called them the lower track courses.

Students scoring above the 70th percentile on the test could
enroll in courses ranging from Prealgebra to Calculus. All schools
did not offer the same courses; only one school offered calculus and
one school offered no course beyond Algebra II. All of these courses
were categorized by the authors as the higher track courses.

The proportion of males and females enrolled in each of the two
tracks and in each course in the higher track were computed for each
school. Data on the specific courses twelfth-grade students were
taking also were presented. The authors note that classroom counts
were used in the study, and so it is possible that some students were
included more than once in computing the proportions.

4. Findings

When all six schools were combined, no male-female differences by
track appeared. It also was found that the same proportion of females
as males (about 80%) who were taking mathematics were in the lower
track. There was variation among the schools, however. In three of
the schools, the proportion of males in the higher track exceeded that
of females by at least 10 percentage points. In two of the remaining
schools, the percent of females exceeded that of males by between
and 7 percent. Three of the schools had a difference of 5 percent
between the percent of males and females enrolled in lower track
courses with males exceeding females in two of the schools. In one
school, males were dominant in both tracks and in each course in the
upper track, whereas in another school, more females than males were
enrolled in almost all of the courses.
No consistent sex differences in enrollments in Algebra I and Geometry were found when data from all six schools were combined. However, more males than females were enrolled in all of the Algebra II sections. In the most advanced courses, males outnumbered females in three of the schools, whereas females outnumbered males in one of the two remaining schools that offered courses beyond Algebra II. The authors note that because the number of students enrolled in each of these advanced courses was small, the differences may not be reliable.

The authors were interested in male-female differences for black students. No racial information was available for the students and so the authors limited their consideration to the three schools that were 99% black. In two of these schools, more females than males were in the higher track courses. There was no tendency for males to be more heavily represented in the higher level courses.

The data did point to some between-school differences in tracking patterns. Enrollment of students in the lower track courses was more likely in some schools than in others; the percent of students enrolled in lower track courses ranged from 65% to 86%. In an effort to determine if these differences were associated with differences between the schools in socio-economic background of the students they served, the authors examined various characteristics of the census tracks that surround the schools. They found the income and educational levels were not substantially different across schools, and the differences that did emerge were not consistent with the mathematics enrollment trends. For example, the school located in the least affluent of the areas had the greatest proportion of students enrolled in advanced courses.

5. Interpretations

The authors conclude that although the schools were similar in terms of being predominately black and having similar socio-economic make-up of the student body, there was wide variation between the schools in male-female enrollment patterns in mathematics courses.
The authors suggest that if we wish to increase female students' involvement in mathematics, then we will need to consider possible reasons for the variations across schools in mathematics participation. Schools, rather than students, should become the unit of study.

**Abstractor's Comments**

The authors investigated a topic that is the intersection of two important concerns in mathematics education, the participation of minorities and the participation of women in mathematics. Results of the most recent National Assessment of Educational Progress (1983) indicate that about the same proportion of females as males are enrolling in high school mathematics courses. The authors reach the same conclusion in their study of schools with predominately black enrollment. However, they note that there are mathematics enrollment differences among the schools. The authors view the differences as a significant result. In fact, they interpret the results to mean that we should shift our research attention to school characteristics rather than student characteristics. Because of the importance ascribed to the between-school differences, they deserve another look.

Two types of between-school differences are identified. One is the difference between schools in the male/female ratio of students enrolled in mathematics courses. A second difference between schools is the proportion of mathematics students enrolled in higher (or lower) track courses. However, the word "differences" must be used cautiously. Only percentages are reported in the article; no statistical analyses are presented. Although some studies, in which the number of subjects is very large, sometimes err in attributing educational significance to differences that are small but statistically significant, this study erred by attributing educational
significance to differences that were not statistically significant. While differences in percentages may be suggestive of a pattern that should be more closely examined, one has to be careful in attributing significance to differences that may be the result of chance. With regard to the first kind of differences, male/female differences, an analysis of the reported data shows that, in most cases, the proportion of males (and females) enrolled in a given track (or course) within a particular school is not significantly different from .50. Therefore, the differences between schools in terms of the ratio of males and females participating in mathematics do not appear to be substantial.

The second kind of between-school differences is more pronounced. An analysis of the reported data shows statistically significant differences between several of the schools in the proportion of mathematics students enrolled in higher (or lower) track courses. But these differences are extremely difficult to interpret because several factors contribute to the number of students enrolled in a particular track course. Apparently, the primary factor is the score on the standardized screening test that places students into one track or the other. A second factor is the required versus elective status of students enrolled in a particular course. The article provides no information about the second factor, nor about the relative importance of the two factors in determining student placement.

In spite of these problems, the authors conclude that "...if we want to increase female involvement in mathematics, then we will need to consider possible reasons for variations across schools in mathematics involvement. In other words, schools should become the targets of our interest" (p. 118). It is not clear whether the authors are referring to variations of kind one (male/female differences) or kind two (tracking differences), but in either case there are problems. As just noted, the first kind of alleged variations between schools does not exist, except in a few cases.
The second kind of variation, which does exist, is difficult to interpret. Apparently it is mostly a consequence of the standardized screening test that tracks students and is not a function of the high schools at all.

Moving beyond the interpretation problems in this study, there remains the issue of whether we can ignore student characteristics (e.g., gender, affective and cognitive characteristics) and shift our attention to school characteristics (e.g., course offerings, counseling services). Will females participate equally in mathematics opportunities available to all students, and gain equally from them? Or do we need to continue the study of differences in student characteristics (e.g., perception of the usefulness of mathematics) in order to fully understand the persistent sex-related differences in achievement and ultimately develop optimal learning environments for both males and females?

Reference

1. Purpose

The stated purposes of the study were "...to test the generalizability of two previous researches [See References] and to extend the categorization begun by Goodall and Casey [See References]."

2. Rationale

The impetus for the study comes directly from research reported by McIntosh (See References) and replicated by Goodall and Casey. Both studies presented evidence that ability to compute using multiplication was not necessarily an indication that a student has an understanding of multiplication. Based on the two referenced studies, O'Brien and Casey wanted to provide empirical evidence that students could be strong at computation in multiplication but weak in understanding "logical multiplication."

It was felt, based on the cited studies, that extending the categories used by Goodall and Casey could give clearer evidence as to the differences with respect to computational proficiency and understanding multiplicative context.
3. **Research Design and Procedures**

While the study is presented in two articles and discussed as Part I and Part II, there was only one data-gathering episode. The two articles emphasize the two different analyses of the data.

Twenty-seven grade 4, 27 grade 5, and 33 grade 6 students solved five multiplicative computations, one of which was \( 6 \times 3 = \). The students were then asked to write a story problem for \( 6 \times 3 = \).

The story problems were placed into six categories based on the "multiplicativeness" of the content; this analysis is reported in the first article. The story problems were also placed into seven categories based on the logic used and the realism of the information; this analysis is reported in the second article.

4. **Findings**

The success rates for the five computations were 82% for grade 4; 75% for grade 5, and 97% for grade 6. All three groups of students did well on all five problems except \( 13 \times 16 = \); on which only 40% of the grade 4 and 25% of the grade 5 students were successful.

Of the six categories concerning the "multiplicativeness" of the story problem, Categories 1 - 3 were for story problems that were judged to involve multiplication or repeated addition; Categories 4 - 6 were for story problems that were judged to not have a multiplicative context, e.g., involved \( 6 + 3 \) instead of \( 6 \times 3 \).

Seventy-four percent of the grade 4 students' story problems were judged to be in Categories 4 - 6; the figure for grade 5 was 84%; and for grade 6 the figure was 30%. Thirty-seven percent of the grade 4 students' story problems and 44% of the grade 5 students' story problems "were clearly additive in context."
For the second article, seven categories of logic and realism were created and are presented below:

A: Didn't pose a question: made a statement or left question unasked.
B: Incomplete logical structure: left out essential information.
C: Added extraneous information or extraneous computation.
D: Nonsensical or impossible arithmetic operation.
E: Unrealistic data.
F: Nonsensical question.
G: Child's written language makes classification impossible.

Of the 34 story problems judged to be in Categories 1 - 3 in Part I of the study, there were 14 "errors"; i.e., 14 story problems judged to be in one of Categories A - G; 10 of the 14 were in Category E. Of the 53 story problems judged to be in Categories 4 - 6 in Part I of the study there were 54 "errors": 16 were in E, 15 in A, and 12 in C.

5. Interpretations

The conclusions presented in Part I of the study were that while the students were proficient in multiplication computation few students provided evidence of an ability to construct a multiplication context for even a combination as simple as 6 X 3; a large proportion of the stories were clearly additive; throughout all grades and all categories the stories seemed artificial; much of the difficulty in constructing a multiplicative context was resolved by grade 6.

In summary, the authors state that for those children who wrote stories judged to be in Categories 4 - 6, "It seems fair to say then that these children do not know what multiplication is. They have algorithmic skill but no mathematical knowledge of multiplication."
The conclusions presented in Part II of the study were that those children who did not present a multiplicative context showed difficulty with necessary and sufficient information, the relationship of information to question, and the "common sense" (in either the mathematical or everyday-life sense) of information and question. There were 42 logical errors made and 90% were made in stories from Categories 4 - 6; the unrealistic data errors were split 10 in stories from Categories 1 - 3 and 16 in stories from Categories 4 - 6.

In summary, in a situation where computing devices are widely available mathematics education "should generate logical mathematical knowledge."

Abstractor's Comments

First, there were the following three technical irritants:

1. The data for the multiplication computations don't seem to "jive." As one example, 27 grade 4 students gave responses to the item 60 X 1 = ____. If only one student missed the item, the success rate would be 96.3%; if two students missed the item, the success rate would be 92.3%. The given success rate is 95%. Did the authors actually round the results to the nearest 5%?

2. The authors (or the editors) couldn't decide whether to use the proportion or the percent of stories judged to be in each of Categories 1 - 6. They could have used, for example, .37 or 37%; in the table, however, they use .37!%

3. On page 250 of Part I an example is given to illustrate "logical multiplication." The example is purposively non-numerical. The problem is as follows:
Using red, yellow and blue for the roof, and white and black for the front, color these houses so that they are totally different.

After the statement of the problem there is a picture of some houses. After the picture there is discussion of mapping the members of one class to each of the members of the second class. When I work the problem I keep getting six as the answer, but there are eight houses given.

Another shortcoming that may be a criticism of the article as opposed to the study is that there is no discussion of how the story problems were categorized. The placement of a story in Categories 1-6 and Categories A-G is, of course, the heart of the entire study. Were reliability checks made? Were the definitions of the categories specified before the data analysis or did they evolve with the data analysis? Were the judges "calibrated" before beginning the analysis of the data reported in the article?

Categories B, D, F, and G are categories where the child actually has something wrong with their story. Category A, on the other hand, may contain stories that are wrong only because they contradict the instructions given. The article states simply that, "the children were asked to write a story problem for 6 X 3 = _____." Some of the stories in Category A may have been written by children who do not understand the difference between a "story" and a "story problem."

I see nothing inherently wrong with stories in Category C; in fact, some problem-solving researchers would encourage story problem writers to create more problems of the Category C variety. Category E certainly calls for a very subjective judgment. The judges have called something "unrealistic" based, I assume, on their own personal experiences. The example given is:
There was a store that had 6 oranges at $3 a piece. How much would all 6 oranges cost?

I don't see why such a story should be judged as having an error and placed in Category E.

In summary, I do feel the two articles are worth reading and heeding. As computing devices become even more available the emphasis in the curriculum must shift (shift, not abandon) from the "how" of computation to the "when, why, and what" of computation.

References


Goodall, Joyce and Casey, Shirley. "What are We Teaching When We Teach Multiplication?" Submitted for Publication.
1. Purpose

This study sought to trace the emergence and differentiation of the process of partitioning as revealed in children's attempts to subdivide a continuous whole into equal parts.

2. Rationale

The authors cite Kieren (1976, 1980) as having created a new theoretical context for inquiring into the child's acquisition of rational number concepts and claim that basic to Kieren's perspective is the process of dividing a whole into parts.

3. Research Design and Procedures

"The method can be characterized as a clinical interaction technique set within a discovery paradigm. ... The interaction was characterized by flexibility in questioning." "The initial question for each task was standard, but the subsequent questions, although following a general pattern, were varied, as were the numbers in the problem, depending upon the behavior of the child."

The sample consisted of 43 children in kindergarten and grades 1-3 in Alberta, Canada. The interviewer was known to all the children. Five partitioning tasks were used in the study, with the cake problem being presented as representative. The participants were given little sticks to demonstrate how they would cut a cake into 2, 4, 3, and 5 equal parts. There were one circular and four rectangular cakes and one large circular cookie.
The tasks were analyzed using a three-stage scheme: (1) the interview, characterized by flexibility in questioning based upon the child's behavior; (2) daily reflection on the day's interaction; and (3) a final systematic examination made of the data collected in stages 1 and 2.

4. Findings

The authors did not report their findings per se; instead they proposed a five-level theory that describes the development of the process as they saw it. They claim the first four levels are imbedded in their data; the fifth level following logically even though hypothetical.

"The first four levels are outlined below in terms of three distinctive characteristics: (a) the construct, or key concept, developing during the level; (b) the algorithm, or procedure, employed to produce the partitions; and (c) the domain, or extent, of the partitioning capabilities within the level.

Level I: Sharing
  Construct--breaking; sharing; halving
  Algorithm--allocating pieces ("a piece for you")
  Domain--social setting; counting numbers

Level II: Algorithmic halving
  Construct--systematic partitioning in two
    --no notion of equality
  Algorithm--repeated dichotomies
  Domain--one-half and other unit fractions whose denominators are powers of 2

Level III: Evenness
  Construct--equality; congruence
    --repeated dichotomies becoming meaningful
. Algorithm -- halving algorithm; geometric transformations
   -- extension of algorithmic halving to doubling the number of partitions and adding two parts
. Domain -- unit fractions with even denominators

Level IV: Oddness
. Construct -- even and odd
   -- search for a new first move
   -- use of the new first move
   -- geometric transformations
. Algorithm -- exploratory measures; trial and error
   -- counting; one-by-one procedure
. Domain -- all unit fractions

Level V is called composition, hypothesized as a natural extension of level IV.

5. Interpretations

The authors interpret their levels to mean that a child first learns to partition in two; then, with the acquisition and eventual mastery of the halving algorithm, in powers of 2; then, with the use of geometric motions, in even numbers. Partitioning in odd numbers follows the learning of a first move other than a median cut. With the discovery of the new first move, children are able to partition in thirds, fifths, and other odd numbers; thus, thirds and fifths are achieved together.

Abstractor's Comments

Pothier and Sawada present an interesting theory as to how young children develop understanding of dividing a whole into equal parts. Being able to characterize each level by (a) the construct or by
concept, (b) the algorithm or procedure of partitioning, and (c) the domain or extent of the partitioning capabilities lends creditability to their theory. It is interesting that there appears to be a correlation between the author's levels and the general way that addition of fractions is presented. Even though a different interviewer might reach slightly different conclusions, this reviewer can find little fault with the author's clinical study.
Abstract and comments prepared for I.M.E. by SAMUEL P. BUCHANAN, University of Central Arkansas.

1. **Purpose**

The stated purpose of this investigation was to study "the effects of field dependence – independence and the level of operativity on mathematics achievement" of upper elementary school students (p. 345).

2. **Rationale**

The authors report extensively on previous studies that investigated the relationship between cognitive styles (field dependence – independence) and the mathematics achievement of elementary school students. Also reported was a study of the relationship between students' level of operativity, as defined by Piaget, and their performance on standardized mathematics tests. This study was to take into consideration the IQ differences of the students, something that reportedly had not been a part of previous investigations.

3. **Research Design and Procedures**

The subjects were 450 sixth, seventh, and eighth graders who were separated into groups according to sex, grade level, level of operativity, and cognitive styles. The Lorge-Thorndike Intelligence Test was used to determine IQs. The Group Embedded Figure Test was utilized to determine field dependency – independency. The Formal Operational Reasoning Test was selected to indicate the level of reasoning for formal operational thought. Lastly, the Metropolitan
Achievement Test, which was part of the students' academic records, was selected to measure mathematics achievement.

The subjects were tested in groups of 15 to 25 during regularly scheduled 45-minute classes. The Group Embedded Figures Test was administered first, with the Formal Operational Reasoning Test administered two weeks later.

A 3x3x2 (Grade Level X Cognitive Style X Operativity) analysis of covariance with IQ as the covariate was performed on the students' standard scores on the mathematics test. Also, a 3x3x2 (Grade Level X Cognitive Style X Operativity) multivariate analysis of covariance with IQ as the covariate was performed on the mathematics test scores for computation, concepts, and problem solving.

4. Findings

While the ANCOVA results indicated significant main effects for grade level, cognitive style, and operativity, no significant interaction was indicated. Similar results were obtained from the MANCOVA.

5. Interpretations

This study extended to upper elementary students the findings of previous investigations of lower elementary students; that is, that both cognitive style and level of operativity have a significant effect on mathematics achievement.

Abstractor's Comments

This study was simply the application of statistical tools to a wealth of data obtained from the administration of two standardized tests to a group of elementary students to test an hypothesis. The
design of the investigation was well-conceived and the choice of statistical tools was appropriate. While the design was painfully simple, the questions being investigated were worthy of consideration based on the extensive review of literature.
Scott, Patrick B. A SURVEY OF PERCEIVED USE OF MATHEMATICS MATERIALS BY ELEMENTARY TEACHERS IN A LARGE URBAN SCHOOL DISTRICT. School Science and Mathematics 83: 61-68; January 1983

Abstract and comments prepared for L.M.E. by JAMES H. VANCE, University of Victoria, British Columbia

1. Purpose

The survey was conducted to gather information regarding the use of manipulative materials in Grades K to 5 in an urban school district. Relationships between teachers' use of instructional aids and variables such as grade level, years of experience, textbook use, and student achievement were also investigated.

2. Rationale

It is widely held that concrete manipulative materials should be an integral part of mathematics instruction in the elementary school. Fennema (1981) has noted that while primary programs encourage the use of concrete materials in mathematics instruction, symbolic representations are used almost exclusively with older children. Information about the current use of mathematics materials by teachers was sought by district staff to assist them in making decisions relating to materials adoption and in-service needs.

3. Research Design and Procedures

Copies of a survey form listing 25 teaching aids were sent to the mathematics representative in each of the district's 75 elementary schools. In addition to concrete materials and devices such as attribute blocks, geoboards, and balances, the list included such items as flash cards, calculators, and thermometers. Teachers were asked to indicate which of the aids they had in the classroom and the frequency with which they used each of them. Information regarding
years of experience, textbook use, and in-service desires was also sought. Responses to the survey were obtained from 88% of the schools and 60% of the teachers.

Percentages of teachers at each grade level (K = 5) using each of the materials were determined. Statistical analyses were conducted to study differences in concrete material use across grade levels and to examine the relationships between material use and textbook use, material use and years of experience, years of experience and requests for more materials or in-service on materials, requests for in-service or materials and material use, and material use and student achievement in Grade 5.

4. Findings

Of the 25 teaching aids, only flash cards and calculators were used by more than half of the teachers. Cuisenaire rods, geoboards, and popsicle sticks were used at least once a year by over 40% of the teachers.

There was a steady decline in the use of 17 of the materials considered concrete manipulative as grade level increased. "Average use by first grade teachers was significantly higher than each of grades two through five (at the 0.0001 level)" (p. 65). Most measurement materials were used fairly equally at all grade levels, with compasses and protractors showing an increased use in the upper grades. Calculator use also increased with grade level (from 9% in kindergarten to 27% in fifth grade).

Teachers who did not use textbooks used significantly more materials than the 86% of the teachers who reported using textbooks. The correlation between years of teaching experience and concrete material use (-0.13) was statistically significant. Over 80% of the teachers requested more materials and over 50% requested in-service on their use. Requests for materials and inservice were not
significantly related to either years of experience or use of materials. There was a non-significant correlation between use of manipulatives and achievement in grade 5.

5. Conclusions

1. In mathematics instruction teachers use few materials other than textbooks.
2. Use of most materials decreases as grade level increases.
3. Calculator use is low but increases with grade level.
4. Teachers who do not use textbooks use significantly more manipulative materials.
5. Teachers with more recent training tend to use more materials.
6. Most teachers requested more materials; but only a "slight majority" (p. 67) requested in-service on their use.
7. There was no significant correlation between material use and achievement at the fifth-grade level.

Abstractor's Comments

The major finding of the survey was that in general teachers use few manipulatives and that the use of most materials declines with grade level. While most educators would agree that this is an undesirable result, it is important to recognize that higher figures would not necessarily have reflected a better situation. The real issues are how and why materials are used, not simply how often something is used in some way (Reys, 1971). For example, just over two percent of the fifth-grade teachers reported using small toy figures and less than two percent of kindergarten teachers said they used protractors. Ideally, should these figures be higher? Presumably five-year-olds are not taught to measure angles with a protractor, so one could wonder just how this device was used. The same question could be asked about rulers in kindergarten (76% use).
Are children taught how to measure with a ruler or do they simply use rulers to draw "straight lines?"

The mathematics materials are listed on the survey form in what appears to be a random order. It is possible that more useful information might have been obtained and a clearer message delivered to teachers if the items had been grouped according to topic (place value, geometry, measurement, number, and operations) and if teachers had been invited to write in other aids not listed for each topic. That would have meant including some devices such as geoboards and Cuisenaire rods under more than one heading. The following materials would have been listed under place value: Cuisenaire rods, popsicle sticks, abacus, place value pocket charts, bean sticks, base ten blocks, unifix cubes, chip trading program, calculators, and (even) flash cards. Note that some of these materials are suitable for the early grades (popsicle sticks) while others are more appropriate in later grades (abacus). It would seem desirable that all classes use at least one appropriate manipulative while studying place value, but certainly not all of them. Such a listing would make teachers aware of the range of materials (beyond the textbook) available for each topic, and the information obtained from the survey would be more useful to the researcher and other readers.

Another variable that may have affected the survey results is the curriculum that was actually taught. Some teachers may, for example, make little use of geoboards and geoblocks because they devote little or no time to geometry in their mathematics program. Compasses and protractors were used by about 60% of fifth grade teachers, but we are not told whether work with these instruments is part of the fifth-grade curriculum in the district.

The appropriateness of some of the statistical procedures employed to relate material use to other variables and the conclusions stemming from these analyses might be questioned. For example, although the correlation between years of teaching experience and
material use was found to be statistically significant, should one conclude that teachers with more recent training tend to use more materials? A correlation of 0.13 accounts for less than two percent of the variance. Furthermore, can years of experience be equated to recency of training?

To investigate the relationship between student achievement and material use, the percent of fifth graders scoring in the lower or upper quartile on the Comprehensive Test of Basic Skills was correlated with reported use of manipulatives by teachers. This was clearly inappropriate as the survey was not designed to examine the very complex question of the effect of manipulatives on student achievement. The problem has been studied extensively by other researchers (Suydam, 1984).

In summary, while the survey did indicate the need for in-service with district elementary teachers on the use of manipulative materials in mathematics, questions on the relationships between material use and other curricular and instructional variables should have been left for another study specifically designed for that purpose.

References


1. Purpose

"The purpose of this study was to evaluate a component-skill analysis of the child's transition from using the solution procedure counting-all to using the solution procedure counting-on." (p. 47)

The analysis was designed to help identify specific subskills that a child must acquire to move from using the counting-all procedure to using the counting-on procedure.

2. Rationale

Two procedures have been identified for solving addition problems of the form m + n. In the counting-all procedure, entities must be present for each addend, and children count all the entities. In the counting-on procedure, children begin with "m" and count to "m + n."

There is evidence that American children spontaneously move from counting-all to counting-on. It was felt that a component-skills analysis of the counting-on procedure would help to clarify the conceptual advances made during this transition. Three subskills were proposed: (1) counting-up from an arbitrary point, (2) shifting from the cardinal to the counting meaning of the first addend, and (3) beginning the count of the second addend with the next counting word.

3. Research Design and Procedures

The subjects were 73 first-grade children who were being taught addition number facts for single digits.
Each child was given a counting-on test, a component subskills assessment, and a second counting-on test. The counting-on test consisted of six trials. In the first three trials, children were shown a long card containing \( m \) dots. Above this was an index card on which was written the numeral \( m \). The child was told by the experimenter that "There are \( m \) dots here" and that the index card "Tells you how many dots there are on the card." The first dot array card was then turned face down and a second dot array card was placed to the right of the first. A second index card was also provided, giving the numeral \( n \). The child was asked to tell how many dots there were on both cards all together.

In the next three trials, the procedure was repeated except that the first dot array card was left face up each time and the question for the child was preceded by the hint: "See, this card (indicating first addend numeral card) tells you how many dots there are here, so you don't have to count them over again, but you can if you need to."

![Figure 1. The counting-on task](image-url)
The first addend was always between 12 and 19 and the second addend was between 6 and 9. If a child counted-on on one or more trials, he/she was classified as capable of counting-on. Evidence of counting-on included verbal counting-on and a relatively rapid solution time during which the child looked at the second array. Further questions such as "How did you figure that out?" were also asked after the second and subsequent trials.

Four trials were used to assess Subskill 1. In each trial, the child was asked to "Start counting from m and keep going until I tell you to stop." The child had to start counting at m (12 < m < 19) and continue to at least m + 3.

For Subskill 2, the child was presented cards such as those used for the counting-on test. The child had to tell what count number the last dot on the first array card would have if he/she were to count all the dots on the two cards together. Subskill 3 was assessed similarly except the child had to tell what count number the first dot on the second array card would get. If a child exhibited a subskill on three consecutive trials, then he/she was classified as demonstrating the subskill.

Subjects who did not display counting-on, did count all, and displayed Subskill 1 but neither Subskill 2 nor 3 were assigned to a teaching procedure. In this teaching session, the experimenter helped the child focus on the appropriate concepts to develop Subskills 2 and 3. The teaching stopped when a child exhibited these subskills on four consecutive trials. There were 16 children in this group. Eight of these children were assigned to the teaching session and the other eight served as a control, receiving no treatment before the posttest.
4. Findings

On the addition pretest, 28 children counted-on and 45 counted-all but did not count-on. Of the 28 children who counted-on, all demonstrated Subskill 1, 24 displayed Subskill 2, and 28 displayed Subskill 3.

The 45 children who did not count-on fell into the following categories of the subskills assessment:

- 6 children displayed none of the three subskills
- 16 displayed only Subskill 1
- 14 displayed Subskills 1 and 2, but not Subskill 3
- 9 displayed all three subskills

It was proposed that this distribution suggests the subskills are different and follow a consistent sequence of acquisition: Subskill 1, followed by Subskill 2, followed by Subskill 3.

Examples of specific responses made by the count-all subjects were also reported and these were suggested as further support for the component-skills analysis. There were only 2 correct responses out of 88 trials for those children who did not demonstrate Subskill 2 and only 2 correct out of 144 trials for those children not demonstrating Subskill 3. Of the 9 children who did not count-on but who did display all three subskills, seven counted-on in the posttest. It was suggested that the skills assessment may have induced counting-on for these children.

All of the children in the teaching condition reached criterion on Subskills 2 and 3, with an average of 6.8 trials for each subskill. Seven of the eight children receiving instruction counted-on on the posttest while only one of the eight control children counted-on on the posttest. The difference between the two groups was significant, with $\chi^2 = 8.63, p < .01$. The instructed children counted-on more often when the dots for the first addend were visible. Having the first addend visible did not seem to make a difference for the 28 children who counted-on on the pretest.
5. Interpretations

The investigators suggested that a strong case had been made for the proposed component-analysis. Evidence for the conclusion included the initial match between counting-on and possession of the three subskills and the fact that teaching children missing subskills induced counting-on for almost all of them.

It was proposed that Subskill 2 does not require children to consider the first addend simultaneously as the addend and as a part of the sum. Subskill 3 requires this focus for both addends. The investigators concluded that "The key to counting-on therefore seems to be the ability to consider both addends simultaneously as parts and as composing the whole while counting the second addend" (p. 56).

Subskills 2 and 3 seemed to be accessible to these children as evidenced by the success of the teaching procedure. The materials used, tasks required, and assessment procedures appeared to help the children organize and focus their thinking. The procedure of interrupting the child's usual solution procedure and pointing out relevant connections seems to be very useful. It was suggested that further exploration be given to the use of these materials, tasks, and methods in the classroom.

Abstractor's Comments.

This study has several very important, positive features. The questions investigated have a solid theoretical rationale and are relevant to mathematics curriculum and instruction. The investigators have taken a complex task and broken it into three identifiable subskills. Their careful procedures and analyses have provided a rather thorough and precise study of these subskills as well as the transition from counting-all to counting-on. The teaching procedures and results are of particular interest since these may be applicable in classroom settings.
There are several questions and concerns that should be discussed.

1. At the time of the study, the children were being taught addition number facts for single digits and initial subtraction concepts. It was reported that teachers did not encourage or discourage the use of counting. It would be useful to have more information concerning the background of these children. What counting experiences had they had prior to the study? Children often count-all even when working with two one-digit addends. Had they been encouraged to simply recognize and memorize single digit facts?

2. It was stated that behavioral evidence of counting-all included taking extended time in staring at the first addend array. This, in itself, might be a bit misleading. A child might appear to be focusing on the first array, while he/she is actually thinking in a count-on manner (deciding what to do). The length of time may not be very significant. The investigators indicated that probe questions were asked of the children to further clarify their solution processes. This would seem to be much stronger evidence for a particular child's solution process.

3. On the pretest, a child was classified as capable of counting-on if he/she counted-on on one or more trials out of six. Why was this used as the criterion? How many of the 28 children who counted-on on the pretest demonstrated this on only one trial? Is count-on behavior on one trial out of six sufficient evidence to conclude that a child is capable of counting-on? A more detailed description of the rationale for this would be helpful.

4. The results of the teaching procedure are encouraging. The small number of students in this group (8) make any generalizations somewhat tentative. It would be very interesting to see if the results of this teaching procedure can be replicated with a larger group of children. As pointed out by the investigators, the use of the materials, tasks, and methods used in this study should be more fully explored.
This could provide information that would be quite useful in a classroom setting and might even provide some techniques that could be used to help children develop counting-on solution procedures.

The instructed children counted-on more when all of the dots were visible and they were given a hint not to count-all. It could be important to investigate more fully this result. Can we identify which children will need this conceptual support? Which of these factors is most influential? How do these factors interact to affect a child's development of counting-on? The results of this study should be used to generate further important research questions.
1. Purpose

To investigate how a student's sex, high school mathematics background, size of graduating class, and college grade level relate to the student's attitudes toward mathematics.

2. Procedures

Information was gathered from 1054 students enrolled in "pre-calculus" (college algebra, mathematics for elementary teachers, and applied mathematics) courses in four state and six community colleges. A mathematics attitude scale was administered and the background information indicated above was collected. An analysis of variance was performed.

3. Findings

The results were as follows:

- "The mean score for all 1054 students was 45.39 with a standard deviation of 16.225." (Eighty was the highest possible score.)
- "...no significant difference existed between males and females."
- "...differences in attitudes toward mathematics are clearly related to the high school mathematics background of the student."
- "...the college grade level at which a student enrolls in a pre-calculus college mathematics course is related to the attitude of the student."
4. **Interpretations**

After indicating that the students' attitudes toward mathematics were rather neutral, the investigators state that "This is not surprising since many of the students with a very positive attitude are likely to start in a calculus course rather than a pre-calculus course." They also indicate that results show that "Students with an above average or strong college preparatory background showed a significantly higher attitude than those with an average college preparatory background or below." In general, "students with good attitudes toward mathematics take their pre-calculus mathematics early in the college career. Those with poor attitudes tend to put their mathematics off until later."

**Abstractor's Comments**

That there is considerable interest in how student attitudes relate to student performance and what conditions influence attitudes is not in question; however, there are additional questions that these researchers could have asked that are pertinent to studies of this sort.

1. How does a student's attitude affect his or her performance in a pre-calculus course?
2. Did the students' high school mathematics background affect their attitudes, or did their attitudes affect how many mathematics courses they took in high school?
3. Are the students being investigated taking required or elective pre-calculus courses?
4. At what point in the course are these students being surveyed— at the beginning, midpoint, or end?

This reviewer feels that data on attitudes might be more useful if "attitude profiles" could be identified and student interviews used to see how such profiles influence performance, self-concept, and persistence in mathematics.
Threadgill-Sowder, Judith A. QUESTION PLACEMENT IN MATHEMATICAL WORD PROBLEMS. *School Science and Mathematics* 83: 107-111; February 1983.

Abstract and comments prepared for I.M.E. by TRUDY B. CUNNINGHAM, Bucknell University.

1. **Purpose**

To test the hypothesis that word problems with the question stated first prompt the student to find information necessary for the correct solution and to disregard extraneous information.

2. **Rationale**

In the context of Rothkopf's (1965) mathemagetic behaviors, this investigation extends the work of Williams and McCreight (1965) and Arter and Clinton (1974) to older students and longer problems. Threadgill-Sowder assumed that length of problem and age of student affect search behavior.

3. **Research Design and Procedures**

The Necessary Arithmetic Operations Test R-4 was administered to 52 students enrolled in two community college algebra classes in which the content was "similar to that of second year algebra" (p. 108). One week later the same students were asked to solve 14 word problems during a 50-minute class. Four problems contained extraneous data and required one- or two-step solutions. The other problems required at least two steps. Two versions of the test, with the questions stated first and with all questions stated last, were randomly assigned. Each problem was given one point for partially correct procedure, two points for correct procedure but incorrect answer, or three points for correct procedure and answer. An analysis of covariance, with the Necessary Arithmetic Operations Test score as covariate, was performed.
on the scores obtained for the four problems with extraneous information and again on the scores obtained for the ten remaining problems. Summaries of these analyses and of the descriptive statistics for each set of problems were presented in tables.

4. Findings

Questions placed before or after other information has no effect on the in-class performance of community college algebra students solving one- and two-step word problems, some of which include extraneous information.

5. Interpretations

Threadgill-Sowder concludes: "Question placement apparently has no effect on the ability of students to solve word problems, regardless of length and complexity of problems or age of students" (p. 110). She argues that additional research on question placement in word problems is unnecessary and that research involving the arousal and motivational potential of word problems may only reflect the nature of word problems. Threadgill-Sowder suggests further that experience teaches the student to expect a question and therefore "could negate placement effects which might occur were not this expectation present" (p. 111). In her judgment, the complexity of word problems both arouses the student and serves as a motivating factor.

Abstractor's Comments

The description of the investigation and the statistical analysis of data were both clear and concise. A more careful researcher might have elected incorrectly a less powerful test than analysis of covariance.
The findings, however, are stated more objectively in the abstract than in the original report of the research; the original generalizes beyond the sample. After claiming agreement with two similar studies involving elementary school subjects and shorter, if not simpler problems, Threadgill-Sowder states:

Question placement apparently has no effect on the ability of students to solve word problems, regardless of length and complexity of problems or age of students. The mathemagenic model does not seem to have any carry-over to this area of mathematical study. (p. 110)

If the community college students varied substantially in age, that variance was not noted. Nor is there any indication of variable length and complexity of the problems used in the study (except that the problems without extraneous information required at least two steps). The fact that no significant treatment effects were found in this sample of college-age students does not justify the researcher's conclusion. Taken with the results of earlier studies, these data only suggest that question placement has no effect across age. The complexity hypothesis requires testing one of more groups of subjects on word problems of varied complexity and a reliable method of rating problems for complexity.

The significance of this investigation lies not in its conclusion but in the implied, yet unanswered question: Is it possible that patterns of question placement can be used to teach students to understand and solve word problems more effectively? Longitudinal studies which compare the problem-solving success of students who are first given problems with questions preceding other information with the success of a control group solving problems with random question placement may or may not indicate a significant difference.
Threadgill-Sowder notes that experience with word problems breeds expectation. The challenge is to develop a teaching strategy in which this expectation, a mathemagenic behavior, is converted into the skills and confidence needed for success in solving word problems.

References


Williams, M. H. and McClelland, R. W. "Shall We Move the Question?" Arithmetic Teacher 12: 418-421; October 1965.

Abstract and comments prepared for I.M.E. by DAVID L. STOUT, Pensacola Junior College.

1. Purpose

The study examined the question, "Are elementary school teachers knowledgeable about zero?" (p. 147)

2. Rationale

The authors state that research concerning elementary school teachers' understanding of zero and the effects of this on their students' difficulty with zero is sparse. A small-scale study by Reys (1974) produced no researchable questions regarding elementary teachers' understanding of zero; however, it was speculated that a teacher's difficulty with zero could contribute to the students' difficulty. The present study "focused on preservice teachers' understanding of zero" (p. 147).

3. Research Design and Procedures

The subjects were preservice elementary teachers enrolled in two sections of an elementary school mathematics methods course. Of the 62 subjects, the authors had complete data for 52—47 females and 5 males. The study was completed prior to subjects receiving specific instruction regarding zero. Group tasks were given prior to interviews in one section and in reverse order in the other section.

Two sets of tasks were used, with each set consisting of four tasks. The first task was a group-administered written test consisting of 18 (randomly ordered) division problems and three...
elaboration tasks. Of the 18 problems, six had a zero dividend and a nonzero divisor, six had a nonzero dividend and a zero divisor, and six had a nonzero dividend and a nonzero divisor. Six of the problems were in form a:b, six in the form b:a, and six in the form a/b. Three open-ended questions made up the three elaboration tasks. Each question was presented on a separate sheet of paper. The three questions were "What is zero?", "Is zero a number? Why? Why not?", and "What is zero divided by zero?".

The second task consisted of four interview tasks:

1. Nominal uses of zero: Each subject read aloud nine seven-digit telephone numbers, six three- or four-digit street addresses, and five license plates having three letters followed by three digits. The order of presentation of the telephone numbers, street addresses, and license-plate numbers was random.

2. Mathematical uses of zero: Each subject was presented, in random order, with four subtasks: counting back from a single-digit number, responding with the cardinality of partitions of a nine-element set, responding with the cardinality of a nine-element set when diminished by two, then two, then three, and then two elements, and calculating and reading aloud items on basic arithmetic facts involving zero.

3. Classification: The authors used two subtasks, Classification I and II. In Classification I, 15 attribute cards were to be sorted into two mutually exclusive sets. If a subject failed to sort the cards dichotomously as blanks and non-blanks, the sort was provided by the interviewer. In any case, each subject was asked to describe this sort. In the Classification II subtask, 12 more cards were added to the 15 of the Classification I subtask. The desired trichotomous sort was of two-object cards, one-object cards, and blank cards. If a subject
4. Partitioning: Each subject was asked to generate possible combinations of how five fish were caught by two fishermen. Subjects who did not generate a 5-0 combination were encouraged to do so.

The order of presentation of the four "Elaboration Tasks" was: "nominal uses of zero" first, "mathematical uses of zero" second, and the remaining two in random order.

Subjects were interviewed individually using a common protocol by one of the two investigators who also kept a verbatim record of each interview.

4. Findings

1. Division test: The nonzero dividend and nonzero divisor problems proved easiest, with 47 of the 52 respondents getting all six correct; 39 of the 52 subjects correctly worked the six zero dividend, nonzero divisor problems; but only 12 of the 52 subjects correctly answered the six nonzero dividend, zero divisor problems. In fact, 33 of the 52 subjects missed all six.

2. Elaboration tasks:
   a. "What is zero?": The most frequent responses were "symbol" and "number." Almost 15% of the responses were considered ambiguous and therefore unclassifiable.
   b. "Is zero a number?": Eight (15%) of the subjects said zero was not a number.
   c. "What is zero divided by zero?": 77% of the responses were incorrect. The most frequent response was zero divided by zero was zero.

3. Interview tasks:
a. Nominal uses of zero: When reading telephone numbers, only one of the subjects said "zero" where appropriate. Almost all subjects (98%) incorrectly read three- and four-digit street addresses by saying "oh" for zero. When the subjects read the alpha-numeric content of license plates, only two subjects read them appropriately by saying "oh" for 0 and "zero" for 0.

b. Mathematical uses of zero: Every subject, when counting back from a given single-digit number, appropriately used "zero." When presented with a "nine-zero" partition of a nine-element set, 32 subjects described the cardinality of the empty set as "zero." However, 41 subjects used "zero" appropriately when describing the cardinality of the nine-element set when two, then two more, then three more, and finally two more elements were removed and not replaced. Furthermore, 44 subjects appropriately read and answered basic fact items involving zero.

c. Classification Tasks: Most subjects correctly generated and described the dichotomous sort in Classification I and the trichotomous sort in Classification II. Twelve subjects had to be shown the dichotomous sort and 17 had to be shown the trichotomous sort.

d. Partitioning: All but one of the subjects generated a "5 - 0" combination. "None" and "zero" were used by 24 and 22 of the subjects, respectively, while the rest used words such as "nothing" or "not any" to describe the "5 - 0" combination.

5. Interpretations

The authors claim the results of their study imply the subjects (preservice elementary school teachers) "did not possess an adequate understanding of the number zero" (p. 154). Furthermore, the range
of difficulties of subjects "suggests that explicit attention should be given to concepts of zero in mathematics education courses" (p. 154).

The authors also state "it is not clear that successful performance in Classification I suggests an awareness of zero" and that "the relationship of an understanding of the empty set to an understanding of zero needs investigation" (p. 154).

Abstractor's Comments

1. The authors' "exploratory study" seems to have provided some disturbing evidence which should cause mathematics educators to double efforts to enhance preservice elementary teachers' understanding of zero.

2. I also agree with the authors when they state "for better communication, however, it would seem desirable that mathematics teachers and teacher educators vocalize 0 as 'zero' when appropriate" (p. 155).

3. The authors' suggestions for further research and action follow nicely from their study.

4. Cultural biases were noted as factors which could contribute to the vocalization of 0 as "oh" in informal social settings.

5. The authors' study was well-conceived and carried out. It provides evidence which, I feel, cannot be ignored, especially in this fast-moving technological age.

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A correction:

In the Fall 1983 issue of IME (volume 16, number 4, page 41), a reference by Brophy is incorrectly cited. It should read:

Brophy, J. E. Teacher behavior and its effect. Journal of Educational Psychology, 1979, 71(6), 733-750.