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-- Multilingual/Bilingual Materials (171)

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This supplementary math curriculum guide for use with Spanish-speaking, fourth through sixth grade students in Chicago public schools employs a contrastive analysis approach. Lessons are presented for objectives for which the instructional strategies used in the United States differ from those used in Spanish-speaking countries. (Objectives for which the methodology is the same are taught from the standard math curriculum.) Every lesson has four parts: (1) an explanation of the differences in the instructional strategies used in the Spanish-speaking countries and in the U.S.; (2) a student activity, in Spanish, to reinforce students' skills in using their native language and methodologies; (3) suggestions for facilitating the students' transition from their native methodology to the U.S. methodology; and (4) a transitional activity which provides students with practice in solving problems using both methodologies and languages. The major topics covered are place value, operations with whole numbers, rational numbers, and measurement. An English-Spanish vocabulary list is provided. (CMG)
SUPPLEMENT FOR CURRICULUM GUIDE FOR MATHEMATICS:
SPANISH-SPEAKING STUDENTS

GRADES 4-6

SUPLEMENTO DE LA GUIA DIDACTICA DE MATEMATICAS
PARA LOS ESTUDIANTES DE HABLA HISPANA

CUATRO-SEXTO GRADOS

FIELD TEST

RUTH B. LOVE
General Superintendent of Schools

BOARD OF EDUCATION
CITY OF CHICAGO
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A contrastive analysis approach to curriculum development is used in the Supplement for Curriculum Guide for Mathematics: Spanish-Speaking Students to enlighten both the bilingual teacher and the English-speaking classroom teacher regarding differences in the teaching methodology of the United States and Spanish-speaking countries.

Lessons have been developed for the objectives for which the instructional strategies used in the United States differ from those used in Spanish-speaking countries. Teachers are requested to use the Curriculum Guide for Mathematics to teach those objectives for which the same methodology is used in the United States and Spanish-speaking countries. It is important to note that instruction in every objective taught in the school system is provided for the Spanish-speaking student.

Every lesson has four parts:

- an explanation for the teachers to acquaint them with the differences in the instructional strategies used in Spanish-speaking countries and in the United States
- an activity for the students to reinforce their skills in using the language and methodology of their native countries
- suggestions for the teachers to facilitate the students' transition from the methodology used in Spanish-speaking countries to the methodology used in the United States
- a transitional activity designed to prepare students to use the standard algorithm of the Chicago public schools curriculum by providing practice in solving problems using the methodology and language of Spanish-speaking countries and the United States.

The sequence of objectives in the mathematics program of the Spanish-speaking countries may vary from that of the Chicago curriculum. Where there is variation in the order of the objectives, the sequence of the Chicago public schools curriculum is used.
ACKNOWLEDGMENT

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INTRODUCTION

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The major topics included for grades 4-6 are place value, operations with whole numbers, rational numbers, and measurement.

The standard page format provides basic information for each objective. Information includes the following: STRAND, OBJECTIVE, OBJECTIVE CODE, and SUGGESTED ACTIVITIES.

*Differentiated curriculum activities were not developed for Levels J2, L2, and L3.
GRADE 4

Place Value

2-J1-2 Identify and name the place value of each digit in a seven-place number.

2-J1-3 Write numbers, including seven-digit numbers, in expanded notation.

Operations with Whole Numbers

3-J1-16 Divide a two-digit dividend by a one-digit divisor, with remainder.

3-J1-17 Divide a three-digit dividend by a one-digit divisor, with remainder.

3-J1-19 Divide a three-digit dividend by a two-digit divisor.

Measurement

5-J1-3 Perform operations with numbers representing money values.

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OBJECTIVE Code: 2-J1-2

OBJECTIVE Identify and name the place value of each digit in a seven-place number.

SUGGESTED ACTIVITIES

Word names for numbers used in the Spanish-speaking countries differ from those used in the United States.

PART I

In some Spanish-speaking countries, such as Spain, the place value for numbers greater than 999 is shown by using a point instead of a comma to separate each of the periods. The periods of three places are called classes.

The place value for millions is shown by a small index number placed above and between the period of the thousands and the millions. Successively, billions and trillions have a 2 and a 3 as indexes.

<table>
<thead>
<tr>
<th>Spanish-Speaking Countries Method</th>
<th>United States Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>6,596,860</td>
<td>6,596,860</td>
</tr>
<tr>
<td>6,596,860</td>
<td>6,596,860</td>
</tr>
<tr>
<td>5,286,370</td>
<td>5,286,370</td>
</tr>
</tbody>
</table>

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PART II

Usa el punto para separar los períodos y las clases de los números. Escribe los números dados en palabras:

Ejemplo:

75321,46 = 71,532,146
siete millones, quinientos treinta y dos mil, ciento cuarenta y seis

1) 8975236
2) 8194517
3) 7532137
4) 9254378
5) 5894238
6) 8937923
7) 7523787
8) 9276537

Usa la coma para separar los períodos y las clases de los números. Escribe los números dados en palabras:

Ejemplo:

7626387 = 71,626,387
siete millones, seiscientos veintiséis mil, trescientos ochenta y siete

1) 6875321
2) 9276751
3) 7824567
4) 2356896
5) 2357682
6) 8245783
7) 9234671
8) 9236752
PART III 2-J1-2

Review place value terminology and period indexes used in Spanish-speaking countries; then introduce the United States method.

Examples:

1) 2,795,431

3) 4,873,522

5) 2,804,006

Have the students read the numbers and give the place value of each digit in both English and Spanish.

The first number above is read as follows:

1) Dos millones, setecientos noventa y cinco mil, cuatrocientos treinta y uno.

2) Two million, seven hundred ninety-five thousand, four hundred thirty-one.

Make the students aware of the differences in using the plural and the singular forms for:

- millones
- setecientos
- noventa y cinco mil
- cuatrocientos
- treinta y uno
- million
- seven hundred
- ninety-five thousand
- four hundred
- thirty-one

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Rewrite the following exercises using both the United States method and one of the methods used in the Spanish-speaking countries.

Escribe los siguientes números en ambos métodos: el requerido en los Estados Unidos y uno de los métodos usados en los países de habla hispana.

<table>
<thead>
<tr>
<th>Spanish-Speaking Countries Method</th>
<th>United States Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ejemplo: 2 4 7 4 5 3 6</td>
<td>Example: 2 4 7 4 5 3 6</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>2 1 4 7 4, 5 3 6</td>
<td>2 4 7 4 5 3 6</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>2 4 7 4 5 3 6</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>1) 3476879</td>
<td></td>
</tr>
<tr>
<td>2) 3467921</td>
<td></td>
</tr>
<tr>
<td>3) 8564367</td>
<td></td>
</tr>
<tr>
<td>4) 2479652</td>
<td></td>
</tr>
<tr>
<td>5) 3576543</td>
<td></td>
</tr>
<tr>
<td>6) 2795723</td>
<td></td>
</tr>
<tr>
<td>7) 9275684</td>
<td></td>
</tr>
<tr>
<td>8) 5457956</td>
<td></td>
</tr>
<tr>
<td>9) 1247865</td>
<td></td>
</tr>
<tr>
<td>10) 5276953</td>
<td></td>
</tr>
</tbody>
</table>

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OBJECTIVE

Write numbers, including seven-digit numbers, in expanded notation.

SUGGESTED ACTIVITIES

Expanded notation in some Spanish-speaking countries is worked vertically rather than horizontally.

PART I

<table>
<thead>
<tr>
<th>Spanish-Speaking Countries Method</th>
<th>United States Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>4,069,503</td>
<td>4,069,503</td>
</tr>
<tr>
<td>4 x 1,000,000 + 0 +</td>
<td>4,000,000 + 0 +</td>
</tr>
<tr>
<td>6,000 + 9,000 +</td>
<td>60,000 + 9,000 +</td>
</tr>
<tr>
<td>500 + 0 + 3</td>
<td>500 + 0 + 3</td>
</tr>
</tbody>
</table>

Multiples of Ten

| 4,000,000                          | 4 x 1,000,000         |
| 0,000                               | 0 x 100,000          |
| 60,000                             | 6 x 10,000           |
| 9,000                              | 9 x 1,000            |
| 500                                 | 5 x 100              |
| 00                                  | 0 x 10               |
| + 3                                 | 3 x 1                |
| 4,069,503                           | 4,069,503            |
PART II

Escribe los siguientes números en notación desarrollada:

1) 9,142,326

2) 7,257,431

3) 4,375,249

4) 6,235,432

5) 3,294,386
PART III

Have the students convert the vertical expanded notation to the horizontal expanded notation form.

Example:

\[
1,349 = \begin{cases} 
1,000 \\
300 \\
40 \\
+ 9 \\
\end{cases} 
\begin{array}{c}
1,000 + 300 + 40 + 9 = 1,349
\end{array}
\]

\[
1,349,528 = 1,000,000 + 300,000 + 40,000 + 9,000 + 500 + 20 + 8
\]

\[
1,349,528 = \begin{cases} 
1,000,000 \\
300,000 \\
40,000 \\
9,000 \\
500 \\
+ 20 \\
+ 8 \\
\end{cases}
\begin{array}{c}
\end{array}
\]
PART IV

Compare the two forms of expanded notation. Write the missing numbers in the squares.

Compara las dos formas de notación desarrollada y escribe el número que falta en los cuadros en blanco.

Ejemplo: $3,458,232 = 3,000,000$

\[
\begin{align*}
&\phantom{0}400,000 \\
&\phantom{0}50,000 \\
&\phantom{0}30 \\
&\phantom{0}2 \\
\end{align*}
\]

$3,000,000 + 400,000 + 50,000 + 30 + 2$

\[
\begin{align*}
\phantom{0}0 + 0 + 0 + 0 \\
\end{align*}
\]

1) $2,541,325 = 2,000,000$

\[
\begin{align*}
&\phantom{0}000,000 \\
&\phantom{0}0,000 \\
&\phantom{0}00 \\
&\phantom{0}0 \\
\end{align*}
\]

2) $4,358,241 = \square,000,000$

\[
\begin{align*}
&\phantom{0}00,000 \\
&\phantom{0}0,000 \\
&\phantom{0}0,000 \\
&\phantom{0}00 \\
&\phantom{0}0 \\
\end{align*}
\]

3) $8,575,268$

\[
\begin{align*}
&\phantom{0}400,000 \\
&\phantom{0}50,000 \\
&\phantom{0}30 \\
&\phantom{0}2 \\
\end{align*}
\]

$3,000,000 + 400,000 + 50,000 + 30 + 2$

\[
\begin{align*}
\phantom{0}0 + 0 + 0 + 0 \\
\end{align*}
\]

4) $3,506,939$

\[
\begin{align*}
&\phantom{0}00,000 \\
&\phantom{0}0,000 \\
&\phantom{0}0,000 \\
&\phantom{0}00 \\
&\phantom{0}0 \\
\end{align*}
\]

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OBJECTIVE: Divide a two-digit dividend by a one-digit divisor, with remainder.

SUGGESTED ACTIVITIES

In the Spanish-speaking countries the methodology for dividing a two-digit number requires that after the student multiplies each of the digits of the quotient by the digit of the divisor, the partial products obtained are subtracted mentally from the respective digits of the dividend. Products are not written under the dividend. The remainder is placed under the dividend. The division symbol used is a right angle facing up from left to right: (\(\frac{5}{5}\)).

PART I

<table>
<thead>
<tr>
<th>Dividendo</th>
<th>Cociente</th>
<th>Residuo</th>
</tr>
</thead>
<tbody>
<tr>
<td>57 15</td>
<td>7</td>
<td>2</td>
</tr>
</tbody>
</table>

Note: The process is explained on page 11.
PART I (continued)

Spanish-Speaking Countries Method

Ejemplo:

The steps in the process are as follows:

\[
\begin{array}{r|c}
57 & 5 \\
\hline
1 & 5 \\
\end{array}
\]

Use the 5 tens as the first step. Five goes into 5 one time. Write the one under the line.

\[
\begin{array}{r|c}
57 & 5 \\
\hline
7 & 1 \\
\end{array}
\]

Think: \( 1 \times 5 = 5 \).
Think: \( 5 - 5 = 0 \). Write 0 under the 5.

Bring down the next digit, 7.

\[
\begin{array}{r|c}
57 & 5 \\
\hline
07 & 1 \\
\end{array}
\]

5 goes into 7 one time.
Think: \( 1 \times 5 = 5 \).
Think: \( 7 - 5 = 2 \). Write 2 under 7 as the remainder.

Ejemplo:

Since 4 is less than 5, divide 5 into 49. 5 goes into 49 nine times. Write 9 under the line.

\[
\begin{array}{r|c}
49 & 5 \\
\hline
9 & \\
\end{array}
\]

Think: \( 9 \times 5 = 45 \).
Think: \( 49 - 45 = 4 \). Write 4 under 9 as the remainder.
Divide cada uno de los siguientes números usando el símbolo con el cual estás familiarizado y restando mentalmente:

1) 84 dividido entre 2
2) 72 dividido entre 3
3) 95 dividido entre 3
4) 65 dividido entre 5
5) 30 dividido entre 5
6) 72 dividido entre 6
7) 83 dividido entre 8
8) 91 dividido entre 7
9) 54 dividido entre 6
PART III

Write an example on the chalkboard and have students explain both methods, step by step, at the same time that another student is solving it. Compare the two division algorithms.

\[
\begin{array}{c}
\text{74} \\
\text{2} \\
\text{28} \\
\end{array}
\]

\[
\begin{array}{c}
\text{8} \\
\text{9} \\
\text{72} \\
\end{array}
\]

\[
\begin{array}{c}
\text{9} \\
\text{74} \\
\end{array}
\]

\[
\begin{array}{c}
\text{8} \\
\text{72} \\
\text{2} \\
\end{array}
\]

Note: The number in the box does not appear in the Spanish form. The number is subtracted mentally immediately after the product is obtained.
PART IV

Divide the following numbers by using both methods:

Divide los siguientes números usando los dos métodos:

<table>
<thead>
<tr>
<th>Spanish-Speaking Countries Method</th>
<th>United States Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ejemplo:</td>
<td>Example:</td>
</tr>
<tr>
<td>98 ( \div 4 ) = 24 ( \div 6 )</td>
<td>9 ( \div 6 = 1 )</td>
</tr>
<tr>
<td>( \frac{98}{4} ) = ( \frac{98}{6} )</td>
<td>( \frac{9}{6} = \frac{24}{4} )</td>
</tr>
<tr>
<td>98 ( \div 4 ) = 24 ( \div 6 )</td>
<td>9 ( \div 6 = 1 )</td>
</tr>
<tr>
<td>( \frac{9}{5} ) = ( \frac{18}{5} )</td>
<td>( \frac{9}{5} = \frac{18}{5} )</td>
</tr>
</tbody>
</table>

1. 68 \( \div 5 \)
2. 83 \( \div 7 \)
3. 97 \( \div 4 \)
4. 69 \( \div 5 \)

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STRAND OPERATIONS WITH WHOLE NUMBERS OBJECTIVE CODE 3-J1-17

OBJECTIVE Divide a three-digit dividend by a one-digit divisor, with remainder.

SUGGESTED ACTIVITIES

The division algorithm in the Spanish-speaking countries is different from the one used in the United States.

PART I

<table>
<thead>
<tr>
<th>Spanish-Speaking Countries Method</th>
<th>United States Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dividendo</td>
<td>Divisor</td>
</tr>
<tr>
<td>7 6 7</td>
<td>3</td>
</tr>
<tr>
<td>1 6</td>
<td>2 5 5</td>
</tr>
<tr>
<td>1 7</td>
<td>-6</td>
</tr>
<tr>
<td>2</td>
<td>-1 5</td>
</tr>
<tr>
<td>Residuo</td>
<td>Remainder</td>
</tr>
</tbody>
</table>

Some of the differences are as follows:

The symbol is different. In South American countries the division sign ( ) is used; in Mexico and Cuba, the symbol is an angle opened up ( ). In some regions within a country, the symbol opens down as in the United States ( ). The dividend is written to the left of the symbol. The divisor is written to the right of the symbol.

The subtraction is done mentally. The partial products do not appear in the algorithm; only the remainders are shown below the digits that were divided.
These are the steps in the division process that students in Spanish-speaking countries follow.

\[
\begin{array}{c}
\longdiv{767}{3} \quad \text{3 goes into 7, two times.} \\
\underline{3} \\ \\
\frac{1}{2} \\
\leftarrow 6 \quad \text{Think: } 3 \times 2 = 6. \\
\text{Think: } 7 - 6 = 1. \\
\text{Write 1 under the 7.} \\
\end{array}
\]

\[
\begin{array}{c}
\longdiv{767}{3} \quad \text{Bring down the 6 next to the 1.} \\
\underline{\frac{6}{6}} \\
\frac{16}{25} \\
\leftarrow 15 \quad \text{3 goes into 16, five times.} \\
\text{Think: } 3 \times 5 = 15. \\
\text{Think: } 16 - 15 = 1. \\
\text{Write 1 under the 16, below the 6.} \\
\end{array}
\]

\[
\begin{array}{c}
\longdiv{767}{3} \quad \text{Bring down the 7 next to the 1.} \\
\underline{7} \\
\frac{16}{25} \\
\leftarrow 15 \quad \text{3 goes into 17, five times.} \\
\text{Think: } 3 \times 5 = 15. \\
\text{Think: } 17 - 15 = 2. \\
\text{Write 2 under the 17, below the 7.} \\
\end{array}
\]

In some of the Latin American countries the division symbol is the same as the one used in the United States (\(\overline{\text{——}}\)). However, the abbreviated form is used to subtract mentally, as follows:

\[
\begin{array}{c}
\longdiv{53}{6} \quad \text{6 goes into 53, eight times.} \\
\underline{6} \\
\frac{53}{6} \\
\leftarrow 5 \quad \text{Think: } 6 \times 8 = 48. \\
\text{Think: } 53 - 48 = 5. \\
\text{Write 5 under the 3.} \\
\end{array}
\]

\[
\begin{array}{c}
\quad + \leftarrow 15 \\
\end{array}
\]

\[
\begin{array}{c}
\quad \downarrow 48 \quad \text{Bring down the 6 and repeat the step above.} \\
\end{array}
\]

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6 goes into 56 nine times.

Think: \( 6 \times 9 = 54 \).
Think: \( 56 - 54 = 2 \).
Write 2 under the 56, below the 6.
PART II

Divide y comprueba los resultados usando la prueba del nueve. Sigue el ejemplo.

Ejemplo:

Mentalmente dí:

\[ \begin{array}{c}
3 \ 3 \ 8 \ \underline{7} \\
5 \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \\
\ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \\
\ \ \ \ \ \ \ \ \ \ \8 \\
\end{array} \]

7 \times 4 = 28.
33 - 28 = 5.

Baja el 8 para formar 58.

\[ \begin{array}{c}
3 \ 3 \ 8 \ \underline{7} \\
5 \ 8 \ 4 \ 8 \\
\ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \\
\ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \\
\ \ \ \ \ \ \ \ \ \ \56 \\
\end{array} \]

58 - 56 = 2.

El 7 es el divisor.

El 3 es la suma de los números del cociente.

El 5 es el producto de 7 \times 3, más el residuo.

7 \times 3 + 2 = 23
2 + 3 = 5.

El otro 5 es la suma de las cifras del dividendo.

1. 963

2. 737

3. 596

4. 489

5. 781

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Part III

3-J1-17

After the students have mastered the division algorithm in the short form, teach them to make the transition to the method used in the United States. Use two steps.

Step One. Move from the algorithm form used in Spanish-speaking countries to the form used in the United States by changing the symbol and the order of the numbers.

<table>
<thead>
<tr>
<th>Spanish-Speaking Countries Method</th>
<th>United States Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples:</td>
<td></td>
</tr>
<tr>
<td>a) Divide 365 por 7</td>
<td>365 by 7</td>
</tr>
<tr>
<td>365</td>
<td>7</td>
</tr>
<tr>
<td>365</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
<tr>
<td>b) Divide 365 entre 7</td>
<td>How many 7's are there in 365?</td>
</tr>
<tr>
<td>365</td>
<td>7</td>
</tr>
<tr>
<td>365</td>
<td>15</td>
</tr>
<tr>
<td>15</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For both of these forms, write the remainders only under the dividend immediately after multiplying the divisor by the quotient. Do not write the products.

**STEP ONE**

<table>
<thead>
<tr>
<th>365</th>
<th>7</th>
<th>52</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td></td>
<td>15</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>1</td>
</tr>
</tbody>
</table>

**Step Two.** Show the products of each digit of the quotient times the digit of the divisor by writing them under the divisor.

**STEP TWO**

<table>
<thead>
<tr>
<th>365</th>
<th>52</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>35</td>
</tr>
<tr>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

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PART IV

Compare both algorithm forms in the examples given and fill in the blanks in the following problems:

Compara ambas formas algorítmicas en los ejemplos dados y llena los blancos en los problemas siguientes:

Ejemplo:

\[
\begin{array}{c}
831 \\
2192 \\
3
\end{array}
\quad
\begin{array}{c}
92 \\
91831 \\
21 \\
3
\end{array}
\quad
\begin{array}{c}
92 \\
831 \\
-81 \\
-183
\end{array}
\]

1)

\[
\begin{array}{c}
403 \\
4 \\
6
\end{array}
\quad
\begin{array}{c}
6403 \\
403 \\
-363 \\
-421
\end{array}
\]

2)

\[
\begin{array}{c}
518 \\
1112 \\
38 \\
2
\end{array}
\quad
\begin{array}{c}
4518 \\
518 \\
111 \\
38
\end{array}
\quad
\begin{array}{c}
4518 \\
518 \\
-411 \\
-838 \\
-2
\end{array}
\]

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OBJECTIVE
Divide a three-digit dividend by a two-digit divisor.

SUGGESTED ACTIVITIES
In Spanish-speaking countries the division algorithm follows a different process from the one used in the United States. The division by a two- or more digit number is accomplished by multiplying the digit of the quotient by the digits of the divisor, one at a time. The product obtained is subtracted mentally from the respective digit of the dividend immediately after the multiplication is done. Only the remainder is written down.

PART I

<table>
<thead>
<tr>
<th>Spanish-Speaking Countries Method</th>
<th>United States Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>8 6 0</td>
<td>1 5</td>
</tr>
<tr>
<td>1 1 0</td>
<td>5 7</td>
</tr>
<tr>
<td>0 5</td>
<td>1 1 0</td>
</tr>
<tr>
<td></td>
<td>- 1 0 5</td>
</tr>
<tr>
<td></td>
<td>5</td>
</tr>
</tbody>
</table>

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PART I  (continued)  3-J1-19

Cast-Out-Nines Method

To check if the division algorithm is done correctly, the cast-out-nines method is used as follows:

Sum all digits of the dividend, divisor, quotient, and remainder. If a two-digit number is obtained as the sum, add again until a one-digit number results,

\[ Q \times D + R = D \]
Quotient \times Divisor + Remainder = Dividend.

\[
\begin{align*}
\text{Quotient} & \quad 5 + 7 = 12 \\
& \quad 1 + 2 = 3 \\
\end{align*}
\]

\[
\begin{align*}
\text{Dividend} & \quad 5 \times 6 + 5 = 23 \\
& \quad 2 + 3 = 5 \\
\end{align*}
\]

\[
\begin{align*}
\text{Divisor} & \quad 1 + 5 = 6 \\
\end{align*}
\]
Divide cada uno de los siguientes números entre 23.
Usa el ángulo de dividir que mejor te parezca:

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1)</td>
<td>8 7 9</td>
<td>2 3</td>
</tr>
<tr>
<td>2)</td>
<td>8 7</td>
<td></td>
</tr>
<tr>
<td>3)</td>
<td>9 8 7</td>
<td></td>
</tr>
<tr>
<td>4)</td>
<td>9 7 8</td>
<td></td>
</tr>
<tr>
<td>5)</td>
<td>7 8 9</td>
<td></td>
</tr>
<tr>
<td>6)</td>
<td>6 7 9</td>
<td>2 3</td>
</tr>
<tr>
<td>7)</td>
<td>9 6 7</td>
<td></td>
</tr>
<tr>
<td>8)</td>
<td>7 0 6</td>
<td></td>
</tr>
<tr>
<td>9)</td>
<td>9 8 6</td>
<td></td>
</tr>
<tr>
<td>10)</td>
<td>9 9 8</td>
<td></td>
</tr>
</tbody>
</table>

Ejemplo:

\[
\begin{array}{c@{\!}c@{\!}c@{\!}c}
7 & 9 & 8 & | \underline{2} \underline{3} \\
1 & 0 & 8 & \underline{3} \underline{4} \\
\hline
1 & 6 & & \\
\end{array}
\]

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PART III

Compare the two division algorithms below. Explain that the numbers in the boxes do not appear in the Spanish form and that the subtraction is done mentally, immediately after the products are obtained.

\[ \begin{array}{c}
2 & 8 \\
\downarrow \\
16 & \rightarrow + \rightarrow 8
\end{array} \]

\[ \begin{array}{c}
2 & 8 \\
\downarrow \\
18 \\
\downarrow \\
4 + 2 & \rightarrow +
\end{array} \]

* This 2 is the 2 from the 24.

\[ \begin{array}{c}
2 \\
\downarrow \\
188
\end{array} \]

\[ \begin{array}{c}
2 & 8 \\
\downarrow \\
188
\end{array} \]

The 8 is brought down to make 188.

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*This 4 is the 4 from the 48.
Divide the following numbers by using both algorithm forms:

Divide los siguientes números usando los tres métodos:

**Ejemplo:**

\[
\begin{align*}
\begin{array}{c|c}
814 & 18 \\
94 & 45 \\
04 & 45 \\
\end{array} & \begin{array}{c|c}
18 & 814 \\
094 & 94 \\
04 & 4 \\
\end{array} & \begin{array}{c|c}
18 & 814 \\
72 & 94 \\
90 & 4 \\
\end{array}
\end{align*}
\]

\[
\begin{align*}
\begin{array}{c|c}
983 & 21 \\
21 & 983 \\
21 & 983 \\
\end{array}
\end{align*}
\]

\[
\begin{align*}
\begin{array}{c|c}
891 & 48 \\
48 & 891 \\
48 & 891 \\
\end{array}
\end{align*}
\]

Divide using three different forms. Write the quotient and remainder on the lines provided.

Divide en tres formas distintas los números dados. Escribe el cociente y el residuo en los espacios dados.

<table>
<thead>
<tr>
<th>DIVIDENDO</th>
<th>DIVISOR</th>
<th>COCIENTE</th>
<th>RESIDUO</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) 755</td>
<td>34</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2) 849</td>
<td>27</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3) 316</td>
<td>51</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4) 279</td>
<td>11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5) 900</td>
<td>46</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
OBJECTIVE  Perform operations with numbers representing money values.

SUGGESTED ACTIVITIES

Among Spanish-speaking countries different symbols and/or abbreviations are used to denominate monetary systems.

PART I

In Spanish-speaking countries the following symbols are used to represent the UNIT of money: B, B/,, , G, P or P, L, SB, S/, $, ¥, Cs. In Spain the abbreviation Pta is used to indicate Pesetas, and a comma is used to separate pesetas from cêntimos. In other countries the abbreviation cent is used to indicate centavos, cêntimos, and/or centésimos. Note: The symbol $ is used in El Salvador and in Costa Rica to indicate the unit of money; the symbol does not refer to cents.

<table>
<thead>
<tr>
<th>Spanish-Speaking Countries Method</th>
<th>United States Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,35Pta</td>
<td>$1.25</td>
</tr>
<tr>
<td>Una Peseta y treinta y cinco cêntimos</td>
<td>One dollar and thirty-five cents.</td>
</tr>
<tr>
<td>C 3.50</td>
<td>tres Cordobas y cincuenta centavos</td>
</tr>
<tr>
<td>S/3.50</td>
<td>tres Soles y cincuenta centavos</td>
</tr>
<tr>
<td>3.50p</td>
<td>tres Pesos y cincuenta centavos</td>
</tr>
<tr>
<td>3.50</td>
<td>tres Quetzales y cincuenta centavos</td>
</tr>
</tbody>
</table>
The basic operations with units of money should be emphasized by giving special attention to the operations of subtraction and division.

Daily newspapers may be used by students to find the rate of exchange for the monetary unit of their native countries.
PART II

Encuentra en el periódico del día el equivalente en dólares de la unidad monetaria de cada país indicado. Luego ejecuta las siguientes conversiones:

Ejemplo:
México: Peso

<table>
<thead>
<tr>
<th>Peso</th>
<th>42.12</th>
</tr>
</thead>
<tbody>
<tr>
<td>$926.70</td>
<td>22</td>
</tr>
<tr>
<td>46</td>
<td>2 7</td>
</tr>
<tr>
<td></td>
<td>50</td>
</tr>
<tr>
<td></td>
<td>6</td>
</tr>
</tbody>
</table>

(1) España: Pesetas (Pta)

185, 55 PTAS = $_______ Dólares

(2) Perú: Sol (S/.)

Ecuador: Sucre (S/.)

S/. _______ = $453.45
S/. _______ = $453.45

(3) El Salvador: Colón ($)

189.49 $ = $______ Dólares

(4) Venezuela: Bolívar (B)

140,357.15B = $______

(5) Honduras: Lempira (L)

879.85 L = $______ Dólares

879.85 L = $879.85

(6) México: Peso ($)  

$23,450.75 = $______

$______ = $7.50

(7) Argentina: Peso ($)

$678.23 = $______ Dólares

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PART III

Help the students to compare the monetary system used in their respective countries with the system used in the United States. Compare symbols, the divisions of the monetary unit, and the way units are separated from cents.

Note: In some countries the divisions of the unit are smaller than the cent. In Chile, the smallest part is called a milesimo.

Comparison of Symbols

Explain that--

The symbol for the Peso has only one line written over the S. $ The symbol for the dollar has two lines written over the S. $ The symbol for the Sol has a slanted line and a period to the right of the S. $ The symbol for the cent varies among countries. cts, c, ¢, cmo

Comparison of Coins

The United States has a 25¢ coin but not a 20¢ coin.

Mexico has a 20¢ coin but not a 25¢ coin.

Comparison of Notation

Some Spanish-speaking countries use a comma to separate monetary units from decimal parts of the unit.

3,34Pesetas $17.05 16.30¢
3.45Ptas $17.05 16.30¢

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Use multiplication or division to convert the following amounts of money from one monetary system to another.

Usa el proceso algorítmico de multiplicación y de división según se usa en los Estados Unidos para convertir las siguientes cantidades monetarias de un sistema a otro:

1) 146.80 L = $ ________ (dólares)
2) $B 39,45 = $ ________ (dólares)
3) $46.75 = $ ________ (Pesos mexicanos)
4) $/. 49,580.00 = $ ________ (dólares)
5) £ 1,895.75 = $ ________ (dólares)
6) 3956.40 PTA = $ ________ (dólares)
7) 1,595.50 Ø = $ ________ (dólares)
8) $145.50 = $B ________ (Pesos bolivianos)
9) $568.50 = £ ________ (Colones, Salvador)
10) $375.50 = £ ________ (Colones, Costa Rica)
GRADE 5

Place Value

2-K1-1  Read and write eight-digit numerals.
2-K1-3  Write numerals in expanded notation, including eight places.

Operations with Whole Numbers

3-K1-2  Subtract whole numbers, five digits or more, with regrouping.
3-K1-6  Find the quotient of a five-digit dividend (or less) and a two-digit divisor.
3-K1-7  Estimate the quotient of a dividend of five digits or less and a two-digit divisor.

Rational Numbers

4-K1-7  Subtract rational numbers with like denominators, with regrouping.
4-K2-3  Subtract two rational numbers with unlike denominators.
4-K2-5  Subtract mixed numerals, including regrouping.
4-K3-5  Subtract rational numbers with unlike denominators, grouping required.
4-K3-6  Read and write a three-place decimal.
OBJECTIVE  Read and write eight-digit numerals.

**SUGGESTED ACTIVITIES**

In some Spanish-speaking countries, such as Spain, points are used in lieu of commas in dividing each group of three digits in a numeral (1,000 = 1,000). The names of the groupings vary. In the United States, a three-place grouping is called a period; in Spanish-speaking countries, a three-place grouping is called a class and two classes form a period.

**PART I**

Spanish-Speaking Countries Method

<table>
<thead>
<tr>
<th>Segundo Período</th>
<th>Primer Período</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cuarta Clase Millares de Millón</td>
<td>Tercera Clase Millones</td>
</tr>
<tr>
<td>12° 11° 10° orden</td>
<td>9° 8° 7° orden</td>
</tr>
<tr>
<td>Centenas de Millones</td>
<td>Decenas de Millón</td>
</tr>
<tr>
<td>8 7</td>
<td>5 1 4</td>
</tr>
<tr>
<td>Unidades de Millón</td>
<td>Centenas de Millar</td>
</tr>
<tr>
<td>2 6 3</td>
<td>Decenas de Simple</td>
</tr>
<tr>
<td></td>
<td>Unidades de Simple</td>
</tr>
</tbody>
</table>

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PART I (continued)

In some countries, such as Spain, the numeral is written with points in place of commas.

Example: $87,514,963$

In some countries, such as Peru, the numeral is written with exponents 1, 2, 3 ... to denote millions, billions, trillions... respectively.

Example: $87,514,963$

When writing numerals with words --

The thousands are written in the singular form (2,000 = dos mil) but hundreds and millions in the plural form (200 = doscientos; 3,000,000 = tres millones).

Words used to name numbers greater than or equal to 200 are formed by using the name of the place (ciento) after the name of the digit. For example, 200 is doscientos and 300 is trescientos.

The following are exceptions to the procedure described above:

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td></td>
<td>quinientos</td>
</tr>
<tr>
<td>700</td>
<td></td>
<td>setecientos</td>
</tr>
<tr>
<td>900</td>
<td></td>
<td>novecientos</td>
</tr>
</tbody>
</table>

The number $87,514,963$ is written as: ochenta y siete millones, quinientos catorce mil, novecientos veintitres.

United States Method

<table>
<thead>
<tr>
<th>Third Period</th>
<th>Second Period</th>
<th>First Period</th>
</tr>
</thead>
<tbody>
<tr>
<td>Millions</td>
<td>Thousands</td>
<td>Ones</td>
</tr>
<tr>
<td>Ten Millions</td>
<td>Hundred</td>
<td>Ten</td>
</tr>
<tr>
<td>Millions</td>
<td>Thousands</td>
<td>Hundreds</td>
</tr>
<tr>
<td>8</td>
<td>7,514,963</td>
<td>9,63</td>
</tr>
</tbody>
</table>

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**PART II**

Escribe los números en el lugar correcto para indicar el valor relativo de cada cifra. También escribe los números con letra.

<table>
<thead>
<tr>
<th>Número</th>
<th>Valor Relativo de los Números</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Decenas de Millón</td>
</tr>
<tr>
<td>42 648 912</td>
<td>4</td>
</tr>
<tr>
<td>64 169 462</td>
<td></td>
</tr>
<tr>
<td>91 523 193</td>
<td></td>
</tr>
<tr>
<td>8 410 277</td>
<td></td>
</tr>
<tr>
<td>61 079</td>
<td></td>
</tr>
<tr>
<td>18 010 498</td>
<td></td>
</tr>
<tr>
<td>79 832 100</td>
<td></td>
</tr>
</tbody>
</table>

Cuarenta y dos millones, seiscientos cuarenta y ocho mil novecientos doce
PART III

Review Spanish terms for place values and then teach the English terms such as:

unidades ----> units (ones)

Review the term for place value groupings and then teach the United States terminology --

1 clase (for 3 places) ----> units to hundreds...
1 período (for 6 places) ----> units to thousands...

If necessary, review the reason for place value division markings and teach the use of commas --

1,000 (mil) ----> 1,000 (thousand)

Review the rules for writing numerals with words and then teach the translation--

dos ----> two  catorce ----> fourteen

Point out that hundreds, thousands, and millions are written as singular in English--

doscientos ----> two hundred
dos mil ----> two thousand
dos millones ----> two million

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Use the correct punctuation and write the numbers in words:

<table>
<thead>
<tr>
<th>Spanish-Speaking Countries Method</th>
<th>United States Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) 64 162 942</td>
<td>1) 64 162 942</td>
</tr>
<tr>
<td>2) 19 123 456</td>
<td>2) 91 765 320</td>
</tr>
<tr>
<td>3) 10 000 020</td>
<td>3) 12 000 060</td>
</tr>
</tbody>
</table>

Write the digits for the following numerals:

(1) cuarenta y tres millones, ochocientos dieciocho mil, setecientos sesenta y ocho
(1) twenty-four million, three hundred seventy-six thousand, four hundred fifty-one

(2) treinta y tres millones, cuatrocientos noventa y dos mil, seiscientos veinticinco
(2) fifty-nine million, two hundred forty-three thousand, six hundred seventy-nine
OBJECTIVE Write numerals in expanded notation, including eight places.

SUGGESTED ACTIVITIES

In most Spanish-speaking countries a vertical arrangement is used to show a number in expanded notation. In the United States a horizontal arrangement is used. Also, in some Spanish-speaking countries an index is used to denote millions (5₄₀₀₀₀₀₀ = 5,000,000). In some Spanish-speaking countries a point is used in place of the comma (1.000 = 1,000).

PART I

Write 82,654,217 in expanded form.

<table>
<thead>
<tr>
<th>Spanish-Speaking Countries Method</th>
<th>United States Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>80₁,000,000</td>
<td>80,000,000 + 2,000,000 + 600,000 +</td>
</tr>
<tr>
<td>2₁,000,000</td>
<td>50,000 + 4,000 + 200 + 10 + 7 =</td>
</tr>
<tr>
<td>600,000</td>
<td>82,654,217</td>
</tr>
<tr>
<td>50,000</td>
<td></td>
</tr>
<tr>
<td>4,000</td>
<td></td>
</tr>
<tr>
<td>200</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
<tr>
<td></td>
<td>82₁,654,217</td>
</tr>
</tbody>
</table>

In some Spanish-speaking countries the students are instructed to write a small index ₁, ₂, ... to single out the millions in the seventh place, the billions in the thirteenth place... respectively.

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**PART II**

Marca las clases y los períodos y escríbelos en notación desarrollada.

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>42 619 411</td>
<td>5</td>
<td>16 700 412</td>
</tr>
<tr>
<td>2</td>
<td>17 416</td>
<td>6</td>
<td>272 419</td>
</tr>
<tr>
<td>3</td>
<td>6 425 698</td>
<td>7</td>
<td>72 454 321</td>
</tr>
<tr>
<td>4</td>
<td>6 942 333</td>
<td>8</td>
<td>91 543 298</td>
</tr>
</tbody>
</table>

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PART III

Relate the place value terms in Spanish to the English terms.

- unidades
- decenas
- centenas
- unidades de millar
- decenas de millar
- centenas de millar
- unidades de millón
- decenas de millón

unidades → unite (ones)
decenas → tens
centenas → hundreds
unidades de millar → thousands
decenas de millar → ten thousands
centenas de millar → hundred thousands
unidades de millón → millions
decenas de millón → ten millions

If necessary, review the use of the index in the millions-place. Point out that the index is not used in the United States.

\[ 2{,}000{,}000 = 2{,}000{,}000 \]

Review the reason for using marks to group the digits. Stress the use of commas instead of periods in the United States method.

\[ 1{,}000 = 1{,}000 \]

Review the format for number expansion that is used in the Spanish-speaking countries. Then teach the horizontal format used in the United States.

\[
\begin{align*}
40,000,000 + 2,000,000 + 500,000 + 30,000 + 7,000 + 900 + 60 + 1 &= 42,537,961 \\
\end{align*}
\]
PART IV

Write numbers in expanded notation. Use both vertical and horizontal forms.

Escribe los siguientes números en notación desarrollada en la forma vertical y horizontal.

<table>
<thead>
<tr>
<th>Spanish-Speaking Countries Method</th>
<th>United States Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) 9,142,029</td>
<td>5) 13,998,417</td>
</tr>
<tr>
<td>2) 4,828,949</td>
<td>6) 29,384</td>
</tr>
<tr>
<td>3) 27,179,144</td>
<td>7) 100,554</td>
</tr>
<tr>
<td>4) 75,397,208</td>
<td>8) 98,402,634</td>
</tr>
</tbody>
</table>

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OBJECTIVE

Subtract whole numbers, five digits or more, with regrouping.

SUGGESTED ACTIVITIES

In the subtraction process used in the Spanish-speaking countries, the subtrahend is increased mentally by one instead of renaming the minuend.

PART I

<table>
<thead>
<tr>
<th>Ones and Tens Places</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>43, 67 5</td>
<td>6</td>
</tr>
<tr>
<td>12, 56 6</td>
<td>12566</td>
</tr>
</tbody>
</table>

The steps in the process are as follows:

- Borrow 1 ten from 7 tens to make 10 ones. Add the 10 ones to the 5 ones to make 15 ones.
- Say: 15 minus 6 is 9.
- Add the 1 ten borrowed to the 6 tens in the subtrahend to form 7 tens.
- Rename the 7 tens as 6 tens and ten ones.
- Add the 10 ones to the 5 ones to form 15 ones.
- Say: 15 minus 6 is 9.
Spanish-Speaking Countries Method

United States Method

<table>
<thead>
<tr>
<th>Ones and Tens Places</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>4 2,3 3(\frac{1}{6})</td>
<td>4 2,3 3(\frac{1}{6})</td>
</tr>
<tr>
<td>3 5,4 6(\frac{1}{8})</td>
<td>- 3 5,4 6(\frac{1}{8})</td>
</tr>
<tr>
<td>8</td>
<td>8</td>
</tr>
</tbody>
</table>

Analyze this process by studying the example given on the preceding page.

<table>
<thead>
<tr>
<th>Tens and Hundreds Places</th>
<th>2(\frac{1}{2})</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 2,3 3(\frac{1}{6})</td>
<td>4 2,3 3(\frac{1}{6})</td>
</tr>
<tr>
<td>3 5,4 6(\frac{1}{8})</td>
<td>- 3 5,4 6(\frac{1}{8})</td>
</tr>
<tr>
<td>6 8</td>
<td>6 8</td>
</tr>
</tbody>
</table>

Find the number for the tens place by repeating the steps used in the ones place.

<table>
<thead>
<tr>
<th>Hundreds and Thousands Places</th>
<th>1(\frac{1}{2})</th>
</tr>
</thead>
<tbody>
<tr>
<td>4(\frac{1}{2}) 3(\frac{1}{6})</td>
<td>4(\frac{1}{2}) 3(\frac{1}{6})</td>
</tr>
<tr>
<td>3(\frac{1}{2}) 4(\frac{1}{8})</td>
<td>- 3 5,4 6(\frac{1}{8})</td>
</tr>
<tr>
<td>6,8 6 8</td>
<td>6,8 6 8</td>
</tr>
</tbody>
</table>

Repeat the steps above for each of the places, until all of the places have been subtracted.
Revisa la lectura y escritura de números de cinco cifras. Presente problemas escritos para que los alumnos averigüen la operación necesaria para resolverlos.

¿Cuál es la diferencia entre cada uno de los siguientes números?

<table>
<thead>
<tr>
<th></th>
<th>Diferencia entre</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. 35,806</td>
<td></td>
</tr>
<tr>
<td>b. 17,492</td>
<td></td>
</tr>
<tr>
<td>c. 29,957</td>
<td></td>
</tr>
<tr>
<td>d. 55,376</td>
<td></td>
</tr>
<tr>
<td>e. 23,094</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Diferencia entre</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>a y b</td>
</tr>
<tr>
<td></td>
<td>a y c</td>
</tr>
<tr>
<td></td>
<td>a y d</td>
</tr>
<tr>
<td></td>
<td>a y e</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Diferencia entre</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>c y d</td>
</tr>
<tr>
<td></td>
<td>c y e</td>
</tr>
<tr>
<td></td>
<td>d y e</td>
</tr>
<tr>
<td></td>
<td>c y a</td>
</tr>
</tbody>
</table>

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PART III

3-K1-2

Explain the subtraction method used in the United States. Compare it with the method used in Spanish-speaking countries.

Example:

<table>
<thead>
<tr>
<th>Spanish-Speaking Countries Method</th>
<th>United States Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>2\textsuperscript{1}4\textsuperscript{1}2\textsuperscript{1}6</td>
<td>1\textsuperscript{1}3\textsuperscript{1}1\textsuperscript{1}1</td>
</tr>
<tr>
<td>1\textsuperscript{1}9\textsuperscript{1}8\textsuperscript{1}7\textsuperscript{1}9</td>
<td>2\textsuperscript{4}\textsuperscript{8}\textsuperscript{8}\textsuperscript{16}</td>
</tr>
<tr>
<td>1\textsuperscript{1}1\textsuperscript{1}\textsuperscript{1}1</td>
<td>1\textsuperscript{9}\textsuperscript{8}\textsuperscript{7}\textsuperscript{9}</td>
</tr>
<tr>
<td>0\textsuperscript{4}\textsuperscript{3}\textsuperscript{4}\textsuperscript{7}</td>
<td>0\textsuperscript{4}\textsuperscript{3}\textsuperscript{4}\textsuperscript{7}</td>
</tr>
</tbody>
</table>

Borrow 1 ten from 2 tens. Add the 10 ones to the 6 ones to make 16 ones. Think: 16 - 9 = 7. Write 7 under the line, below the 9. Return the 1 borrowed to the tens place of the subtrahend to make 8 tens. Repeat the borrowing and returning from minuend to subtrahend as needed.

Renamed the 2 tens as 1 ten and 10 ones. Add the 10 ones to the 6 ones to make 16 ones. Say: 16 - 9 is 7. Write 7 under the line. Repeat the process by renaming the digit in the hundreds place...

Place the following problem on the chalkboard:

\[ 25327 - 18729 \]

Ask questions that will lead the students to an understanding of the differences in the two methods.
### PART IV

Subtract these numbers:

Rreste los siguientes números:

<table>
<thead>
<tr>
<th>Spanish-Speaking Countries Method</th>
<th>United States Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 2 1 6 4</td>
<td>3 2 1 6 4</td>
</tr>
<tr>
<td>-1 7 8 6 5</td>
<td>-1 7 8 6 5</td>
</tr>
<tr>
<td>1 3 9 7 6</td>
<td>1 3 9 7 6</td>
</tr>
<tr>
<td>-1 2 9 8 7</td>
<td>-1 2 9 8 7</td>
</tr>
<tr>
<td>2 5 6 0 4</td>
<td>2 5 6 0 4</td>
</tr>
<tr>
<td>-8 7 9 6</td>
<td>-8 7 9 6</td>
</tr>
<tr>
<td>1 6 3 4 1 1</td>
<td>1 6 3 4 1 1</td>
</tr>
<tr>
<td>-9 8 7 6 5</td>
<td>-9 8 7 6 5</td>
</tr>
</tbody>
</table>

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OBJECTIVE Find the quotient of a five-digit dividend (or less) and a two-digit divisor.

SUGGESTED ACTIVITIES

The division algorithm in the Spanish-speaking countries follows a process different from the one used in the United States.

PART I

<table>
<thead>
<tr>
<th></th>
<th>Spanish-Speaking Countries Method</th>
<th>United States Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>23451 ÷ 24</td>
<td>892 14 12500</td>
<td>14 12500</td>
</tr>
<tr>
<td></td>
<td>130 892</td>
<td>-112</td>
</tr>
<tr>
<td></td>
<td>40 125</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>40</td>
</tr>
</tbody>
</table>

These symbols are used:

The dividend is written to the left of the symbol.

23451 ÷ 24

The divisor is written to the right of the symbol.

The subtraction is done mentally. The partial products of divisor times quotient do not appear in the algorithm. Only the remainders are shown below the digits that were divided.

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Analyze the steps of the division algorithm as it is done in the Spanish-speaking countries.

Think: 32 goes into 135 as 3 goes into 13, -- four times.

Now multiply: 4 x 2 = 8,
Think: 15 - 8 = 7.
Write 7 below the 5.

Now multiply: 4 x 3 = 12.
Add the 1 from the 15; 12 + 1 = 13.
Subtract 13 from 13.
Write 0 as a remainder under the 13.

Follow the steps above for each of the digits of the quotient.

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PART I (continued)

The inverse operation of division or the cast-out-nines method is used to check a division problem.

```
1 5 3 4 8  | 4 5
1 8 4      | 3 4 1
0 4 8      |     X
0 3        |   4 5
```

Divisor 3 4 1
Quotient 4 5
Divisor 1 7 0 5
Product 1 3 6 4
Remainder 1 5 3 4 8
Dividend 3

The cast-out method uses the principle of addition of the digits of the divisor, dividend, quotient, and remainder, if any. The nines or digits whose sum is nine are deleted from each of the sums. If a two-digit number is obtained in any of the four sums, the digits are added again until a one-digit number is obtained. The sums are placed around a figure X.

Example:

```
1 7 5 1 1
\[ \begin{array}{c}
1 7 \\
8 6 9 7 \\
0 1 9 \\
2 7 \\
1 0 \\
\end{array} \]
```

\[ \begin{array}{c}
\text{1 + 7 = 8} \\
\text{5 + 1 + 1 = 7} \\
\text{7 x 8 = 56} \\
\text{56 + 10 = 66} \\
\text{6 + 6 = 12} \\
\text{1 + 2 = 3} \\
\end{array} \]

The sum of the digits of the divisor 17 is 8 \( (1 + 7 = 8) \). The 8 is written in the bottom part of the X.

The sum of the digits of the quotient 511 is \( 7 \) \( (5 + 1 + 1 = 7) \). The 7 is written in the top part of the X.

The sum of the digits of the dividend 8697 is \( 21 \) \( (8 + 6 + 7 = 21) \). The sum of the digits of 21 is \( 3 \) \( (2 + 1 = 3) \). The 3 is written in the left part of the X.

The product of 7 and 8 in the X is 56.

The sum of the product 56 and the remainder 10 is 66.

The sum of the digits of 66 is 12; the sum of the digits of 12 is 3. The 3 is written in the right part of the X. The digits in the left and right parts of the X are compared. If they are identical, the algorithm is correct.

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Sigue el ejemplo dado. Encuentra el cociente de los siguientes números y comprueba los resultados usando la prueba del nueve:

1) 7 6,5 9 2 | 4 2

Ejemplo:

\[
\begin{array}{c|c}
5 & 4 & 3 & 7 & 6 \\
\hline
4 & 7 & 7 & 8 & 7 \\
4 & 3 & 6 & 0 & 2 \\
\end{array}
\]

Cociente

\[
\begin{align*}
8 + 7 + 7 &= 22 \\
2 + 2 &= 4
\end{align*}
\]

Dividendo

\[
\begin{align*}
5 + 4 + 3 + 7 + 6 &= 25 \\
2 + 5 &= 7
\end{align*}
\]

Producto

\[
\begin{align*}
4 \times 8 &= 32 \\
32 + 2 &= 34 \\
3 + 4 &= 7
\end{align*}
\]
Facilitate the students' transition from one algorithm form to the other form, Figure A to Figure C, by using Figure B as an in-between step. See illustration below.

Note: Figure A shows the division algorithm used more frequently in the Spanish-speaking countries.

Figure B shows an in-between step. This form is also used in some Spanish-speaking countries. Students who have mastered form A must go to form B before making the transition to form C.

Figure C is the form used in the United States for the division algorithm.

Neither Figure A nor B shows the partial products. The subtraction is done by mentally subtracting the numbers as the products are obtained from the multiplication.

**FIGURE A**

```
1 2,500 | 14
130
0 4 0
12
```

**FIGURE B**

```
892
14 | 12,500
14 | 12,500
130
0 4 0
12
```

**FIGURE C**

```
892
14 | 12,500
-112
130
-12 6
0 4 0
-2 8
12
```

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PART IV

Divide the following numbers using both division algorithm forms:

Divide los siguientes números usando las dos formas:

<table>
<thead>
<tr>
<th>Spanish-Speaking Countries Method</th>
<th>United States Method</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>1) 41,596</td>
<td>18</td>
</tr>
<tr>
<td>2) 30,587</td>
<td>52</td>
</tr>
<tr>
<td>3) 1,457</td>
<td>12</td>
</tr>
<tr>
<td>4) 75,410</td>
<td>84</td>
</tr>
<tr>
<td>5) 10,087</td>
<td>25</td>
</tr>
</tbody>
</table>

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OBJECTIVE Estimate the quotient of a dividend of five digits or less and a two-digit divisor.

SUGGESTED ACTIVITIES

In Spanish-speaking countries the estimation is done by counting the number of digits in the dividend to determine the number of digits in the quotient instead of by doing the division algorithm.

PART I

Spanish-Speaking Countries Method

United States Method

Rounding 73 to 70 gives an estimate of 6 which makes 73 X 6 > 430.

\[ \text{6} \times 70 \leq 430 \]
\[ 6 \times 73 = 438 \]

The number of digits in the dividend and in the divisor are counted. An attempt is made to divide the divisor into the first digit of the dividend. If the division is not possible, the divisor is divided into the first two digits, the first three digits...of the dividend until the division process is possible. When the division is possible, the first digit of the quotient is obtained and the number of digits in the quotient is determined by counting the digits to the right as a digit 2, digit 3... In the example above, the quotient is between 6,000 and 6,999 because it is a four-digit number.

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PART II

Estima los cocientes. Indica la primera cifra.

1) \[ 10,535 \overline{15} \]  
5) \[ 19,219 \overline{33} \]

2) \[ 19,204 \overline{11} \]  
6) \[ 49,108 \overline{11} \]

3) \[ 13,933 \overline{13} \]  
7) \[ 11,538 \overline{25} \]

4) \[ 58,103 \overline{12} \]  
8) \[ 20,253 \overline{22} \]

El cociente del problema número 1) tiene ____ cifras.
El cociente del problema número 2) tiene ____ cifras.
El cociente del problema número 3) es mayor que ____.
El cociente del problema número 4) es mayor que ____.
El cociente del problema número 5) es mayor que ____.
El cociente del problema número 6) es mayor que ____.
El cociente del problema número 7) es mayor que ____.
El cociente del problema número 8) es mayor que ____.
PART III

Review the Spanish terms and guide the students in drawing the division symbols.

<table>
<thead>
<tr>
<th>Spanish-Speaking Countries Method</th>
<th>United States Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>dividendo</td>
<td>divisor</td>
</tr>
<tr>
<td>residuos</td>
<td>cociente</td>
</tr>
</tbody>
</table>

Have the students write the dividend outside and to the left side of the right angle and the divisor inside the angle.

Explain that in the United States the divisor is written outside and to the left of the right angle and the dividend inside the angle. Point out that the angle opens downward.

When solving problems with the students, be sure that they place each digit of the quotient in its proper place. Tell them to write 0 (zero) if the remainder is zero.
PART IV

Compare both ways. Write the number of digits needed in the quotient.

Compara ambas formas luego escribe el número de dígitos necesarios en el cociente.

\[
\begin{array}{c}
19,348 \div 12 \quad \text{(four digits)} \\
\hline
1,389 \\
\end{array}
\quad
\begin{array}{c}
12 \times XXX \quad \text{(four digits)} \\
\hline
19,834 \\
\end{array}
\]

1) 40,017 \div 16
2) 16 \div 34,701

3) 67,248 \div 23
4) 23 \div 16,824

5) 12,006 \div 14
6) 14 \div 11,600
**OBJECTIVE** Subtract rational numbers with like denominators, with regrouping.

---

**SUGGESTED ACTIVITIES**

In Spanish-speaking countries subtracting mixed numerals is done by converting them to improper fractions and then subtracting. This process is carried out horizontally.

**PART I**

<table>
<thead>
<tr>
<th>Spanish-Speaking Countries Method</th>
<th>United States Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2 \frac{1}{4} - 1 \frac{3}{4} =$</td>
<td>$2 \frac{1}{4} - 1 \frac{3}{4} =$</td>
</tr>
<tr>
<td>$\frac{9}{4} - \frac{7}{4} =$</td>
<td>$\frac{2}{4} = 1 \frac{5}{4}$</td>
</tr>
<tr>
<td>$\frac{9}{4} - \frac{7}{4} =$</td>
<td>$-1 \frac{3}{4} = 1 \frac{3}{4}$</td>
</tr>
<tr>
<td>$\frac{2}{4} = 1 \frac{1}{2}$</td>
<td>$0 \frac{2}{4} = \frac{1}{2}$</td>
</tr>
</tbody>
</table>

In order to subtract mixed numerals, it is necessary to convert them to improper fractions.

If renaming a whole number is required in order to subtract mixed numerals, the mixed numerals are regrouped as illustrated above.
Step 1

The mixed numeral is converted to an improper fraction by multiplying the whole number by the denominator and then adding the numerator to the product. The denominator is common to both fractions.

\[ 2 \frac{1}{4} = \frac{(2 \times 4) + 1}{4} = \frac{9}{4} \]

Step 2

The second mixed numeral is converted by proceeding in the same manner as explained above.

\[ 1 \frac{3}{4} = \frac{(1 \times 4) + 3}{4} = \frac{7}{4} \]

Step 3

The second fraction is subtracted from the first fraction.

\[ \frac{9}{4} - \frac{7}{4} = \frac{2}{4} \]

\[ \frac{9 - 7}{4} = \frac{2}{4} \]

Step 4

The fraction is renamed in simplest form.

\[ \frac{\frac{1}{2}}{2} = \frac{1}{4} \]

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Encuentra la diferencia y expresa la en la forma más simple.

1) \(8 \frac{4}{8} - 4 \frac{7}{8} = \)
2) \(2 \frac{3}{7} - 1 \frac{5}{7} = \)
3) \(1 \frac{7}{9} - \frac{8}{9} = \)
4) \(2 \frac{2}{3} - 2 \frac{1}{3} = \)
5) \(1 \frac{2}{9} - \frac{5}{9} = \)
6) \(4 \frac{1}{4} - 3 \frac{3}{4} = \)
7) \(3 \frac{2}{6} - 1 \frac{5}{6} = \)
8) \(5 \frac{3}{5} - 2 \frac{4}{5} = \)
9) \(7 \frac{1}{4} - 3 \frac{3}{4} = \)
10) \(6 \frac{5}{7} - 4 \frac{6}{7} = \)
PART III

4-KL-7

Write a subtraction problem on the chalkboard. Have the students explain both methods of subtracting fractions. Analyze the differences in position and process.

<table>
<thead>
<tr>
<th>Spanish-Speaking Countries Method</th>
<th>United States Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>$6 \frac{1}{8} - 4 \frac{7}{8} =$</td>
<td>$6 \frac{1}{8} = 5 \frac{9}{8}$</td>
</tr>
<tr>
<td>$\frac{49}{8} - \frac{39}{8} = \frac{10}{8} =$</td>
<td>$-4 \frac{7}{8} = 4 \frac{7}{8}$</td>
</tr>
<tr>
<td>$1 \frac{2}{8} = 1 \frac{1}{4}$</td>
<td>$1 \frac{2}{8} = 1 \frac{1}{4}$</td>
</tr>
</tbody>
</table>
Subtract the following numbers and express each difference in the simplest form:

Resta las siguientes fracciones y reduce a la mínima expresión:

1) \(2 \frac{2}{5} - 1 \frac{3}{5} = \)
2) \(3 \frac{2}{6} - 2 \frac{5}{6} = \)
3) \(4 \frac{1}{6} - 2 \frac{2}{6} = \)
4) \(4 \frac{3}{8} - \frac{7}{8} = \)
5) \(5 \frac{2}{6} - \frac{5}{6} = \)
6) \(3 \frac{3}{7} - 2 \frac{5}{7} = \)
7) \(2 \frac{3}{8} - 1 \frac{7}{8} = \)
8) \(3 \frac{3}{7} - 2 \frac{4}{7} = \)
9) \(1 \frac{4}{9} - \frac{8}{9} = \)
10) \(7 \frac{1}{3} - 5 \frac{2}{3} = \)
OBJECTIVE 
Subtract two rational numbers with unlike denominators.

SUGGESTED ACTIVITIES

To subtract fractions with unlike denominators, the minuend and the subtrahend are written horizontally instead of vertically. The rational numbers are converted to improper fractions in the Spanish-speaking countries.

PART I

<table>
<thead>
<tr>
<th>Spanish-Speaking Countries Method</th>
<th>United States Method</th>
</tr>
</thead>
</table>
| \[
\frac{5}{8} - \frac{3}{5} = \frac{25}{40} - \frac{24}{40}
\]
| \[
\frac{5}{8} = \frac{25}{40}
\]
| \[
- \frac{3}{5} = \frac{24}{40}
\]
| \[
\frac{1}{40}
\]

| \[
\frac{7}{8} - \frac{5}{12} = \frac{21}{24} - \frac{10}{24}
\]
| \[
\frac{7}{8} = \frac{21}{24}
\]
| \[
- \frac{5}{12} = \frac{10}{24}
\]
| \[
\frac{11}{24}
\]

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Students are required to simplify the fractions to the lowest terms when possible by dividing numerator and denominator by the largest factor.

Example:

\[ \frac{2}{4} - \frac{3}{9} = \frac{1}{2} - \frac{1}{3} \]

The factor two is the divisor for the first fraction and the factor three is the divisor for the second fraction.

\[ = \frac{3-2}{6} = \frac{1}{6} \]

When denominators do not have a common factor, the unlike denominators are multiplied to find the least common denominator.

\[ \frac{5}{6} - \frac{2}{5} = \frac{25}{30} - \frac{12}{30} = \frac{13}{30} \]

The unlike denominators are multiplied.

\[ = \frac{25-12}{30} = \frac{13}{30} \]

When unlike denominators have a factor in common, the following process is used:

\[ \frac{7}{8} - \frac{7}{12} = \frac{21}{24} - \frac{14}{24} = \frac{21-14}{24} = \frac{7}{24} \]

The denominators are placed side by side.

The lowest common factor is written at the right of the line.

The quotients are written below the numbers.

Another factor is used until all common factors are at the right of the line.

All the prime numbers are multiplied.

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Resta las siguientes fracciones. Reduce si es posible, las fracciones a su expresión mínima antes de restar.

Ejemplo:

\[
\frac{4}{6} - \frac{7}{14} = \frac{2}{3} - \frac{1}{2} = \frac{4 - 3}{6} = \frac{1}{6}
\]

1) \[\frac{1}{2} - \frac{9}{12} = \frac{3}{5}\]

2) \[\frac{2}{12} - \frac{10}{16} = \frac{10}{16}\]

3) \[\frac{7}{9} - \frac{3}{7} = \]

4) \[\frac{25}{30} - \frac{1}{3} = \]
After students have mastered the subtraction of rational numbers with unlike denominators, facilitate the transition to the method used in the United States as follows:

**Spanish-Speaking Countries Method**

**Example:**

\[
1 \frac{3}{4} - \frac{2}{3} = \]

**United States Method**

\[
1 \frac{3}{4} = 1 \frac{9}{12}
\]

Convert the mixed numeral to an improper fraction.

\[
\frac{7}{4} - \frac{2}{3} = \]

\[
\frac{21}{12} = \frac{8}{12}
\]

Find a common denominator.

\[
\frac{21}{12} - \frac{8}{12} = \frac{13}{12}
\]

\[
= 1 \frac{1}{12}
\]
**PART IV**

Subtract the fractions with unlike denominators. Use both methods.

Resta estas fracciones de diferente denominador. Usa ambos métodos.

<table>
<thead>
<tr>
<th>Spanish-Speaking Countries Method</th>
<th>United States Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) $\frac{14}{16} - \frac{2}{3} = \frac{14}{16}$</td>
<td>1) $\frac{14}{16}$</td>
</tr>
<tr>
<td></td>
<td>$- \frac{2}{3}$</td>
</tr>
<tr>
<td>2) $\frac{9}{12} - \frac{3}{5} = \frac{9}{12}$</td>
<td>2) $\frac{9}{12}$</td>
</tr>
<tr>
<td></td>
<td>$- \frac{3}{5}$</td>
</tr>
<tr>
<td>3) $\frac{8}{9} - \frac{2}{5} = \frac{8}{9}$</td>
<td>3) $\frac{8}{9}$</td>
</tr>
<tr>
<td></td>
<td>$- \frac{2}{5}$</td>
</tr>
</tbody>
</table>
OBJECTIVE Subtract mixed numerals, including regrouping.

SUGGESTED ACTIVITIES

Subtraction of mixed numerals is accomplished in Spanish-speaking countries in a manner different from the United States method.

PART I

The main differences are as follows:

The subtraction is done horizontally not vertically.

The mixed numerals are converted to improper fractions.

<table>
<thead>
<tr>
<th>Spanish-Speaking Countries Method</th>
<th>United States Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minuend</td>
<td>Subtrahend</td>
</tr>
<tr>
<td>$3 \frac{1}{8} - 2 \frac{3}{4}$</td>
<td>$3 \frac{1}{8} \rightarrow 2 \frac{9}{8}$</td>
</tr>
<tr>
<td>$\frac{25}{8} - \frac{11}{4}$</td>
<td>$- 2 \frac{3}{4} \rightarrow 2 \frac{6}{8}$</td>
</tr>
<tr>
<td>$\frac{25}{8} - \frac{22}{8} = \frac{3}{8}$</td>
<td>$\frac{3}{8}$</td>
</tr>
</tbody>
</table>
PART II

Haz los siguientes ejercicios:

Ejemplo:

\[3 \frac{1}{8} - 2 \frac{3}{4} = \frac{25}{8} - \frac{11}{4}\]

\[= \frac{25 - 22}{8} = \frac{3}{8}\]

1) \[8 \frac{1}{8} - 6 \frac{2}{16} = \]

2) \[19 \frac{2}{4} - 10 \frac{1}{12} = \]

3) \[18 \frac{2}{5} - 4 \frac{2}{10} = \]

4) \[6 \frac{2}{3} - 3 \frac{11}{18} = \]

5) \[8 \frac{1}{20} - 6 \frac{1}{4} = \]
PART III

After students have mastered the Spanish-speaking method of subtracting mixed numerals, emphasize that the main difference is in regrouping.

**Spanish-Speaking Countries Method**

\[ \frac{1}{3} \rightarrow \frac{25}{8} \]

Explain that \(3\frac{1}{8}\) is changed to \(\frac{25}{8}\) by multiplying \(\frac{1}{8} \times 3\) and adding \(\frac{1}{8}\) to the product.

\[ \frac{2}{4} \rightarrow \frac{11}{4} \]

Show that \(2\frac{3}{4}\) is changed to \(\frac{11}{4}\) by multiplying \(\frac{3}{4} \times 2\) and adding \(\frac{3}{4}\) to the product.

\[ \frac{25}{8} - \frac{11}{4} \]

In order to subtract, both fractions must have the same denominator.

\[ (11 \times 2) \]

\[ \frac{25}{8} - \frac{22}{8} = \frac{3}{8} \]

**United States Method**

\[ \frac{1}{3} \rightarrow \frac{9}{8} \]

Explain that \(3\frac{1}{8}\) is changed to \(\frac{9}{8}\) by renaming one whole as \(\frac{8}{8}\) and adding it to \(\frac{1}{8}\).

\[ \frac{3}{4} \rightarrow \frac{6}{8} \]

Show that \(2\frac{3}{4}\) is changed to \(\frac{6}{8}\) by finding an equivalent fraction of \(\frac{3}{4}\) with the same denominator as the other fraction.

\[ \frac{3}{8} = \frac{3}{8} = \frac{9}{8} \]

\[ \frac{3}{4} = \frac{6}{8} = \frac{2}{8} \]

\[ \frac{3}{8} - \frac{2}{8} = \frac{1}{8} \]

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### PART IV

Solve the following problems by using both methods:

Resuelve los problemas usando ambos métodos:

<table>
<thead>
<tr>
<th>Spanish-Speaking Countries Method</th>
<th>United States Method</th>
</tr>
</thead>
</table>

1. \[ \frac{5}{5} - \frac{3}{15} = \]

2. \[ 2 \frac{1}{4} - 1 \frac{1}{12} = \]

3. \[ 8 \frac{1}{6} - 6 \frac{1}{12} = \]

4. \[ 15 \frac{1}{2} - 8 \frac{1}{4} = \]

5. \[ 18 \frac{1}{2} - 2 \frac{1}{3} = \]
OBJECTIVE
Subtract rational numbers with unlike denominators, grouping required.

SUGGESTED ACTIVITIES

In Spanish-speaking countries the most common way to subtract fractions is to write them horizontally. The method used changes mixed numerals to improper fractions.

PART I

<table>
<thead>
<tr>
<th>Spanish-Speaking Countries Method</th>
<th>United States Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>$6 \frac{1}{4} - 4 \frac{7}{8}$</td>
<td>$6 \frac{1}{4} = 6 \frac{2}{8} = 5 \frac{10}{8}$</td>
</tr>
<tr>
<td>$\frac{25}{4} - \frac{39}{8}$</td>
<td>$- \frac{7}{8} = -4 \frac{7}{8} = 4 \frac{7}{8}$</td>
</tr>
<tr>
<td>$\frac{50}{8} - 39 \frac{11}{8} = \frac{1}{8} = 1 \frac{3}{8}$</td>
<td>$1 \frac{3}{8}$</td>
</tr>
<tr>
<td>$6 - 3 \frac{2}{3}$</td>
<td>$6 = 5 \frac{3}{3}$</td>
</tr>
<tr>
<td>$\frac{18}{3} - 11 \frac{2}{3}$</td>
<td>$-3 \frac{2}{3} = 3 \frac{2}{3}$</td>
</tr>
<tr>
<td>$\frac{18}{3} - 11 \frac{7}{3} = 2 \frac{1}{3}$</td>
<td>$2 \frac{1}{3}$</td>
</tr>
</tbody>
</table>
### PART I (continued)

<table>
<thead>
<tr>
<th>Spanish-Speaking Countries Method</th>
<th>United States Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{3}{4} - \frac{2}{5}$</td>
<td>$\frac{3}{4} = \frac{15}{20}$</td>
</tr>
<tr>
<td>$\frac{15}{20} - \frac{8}{20} = \frac{7}{20}$</td>
<td>$\frac{2}{5} = \frac{8}{20}$</td>
</tr>
<tr>
<td>$\frac{3}{4} - \frac{3}{10}$</td>
<td>$\frac{3}{4} = \frac{15}{20}$</td>
</tr>
<tr>
<td>$\frac{15}{20} - \frac{6}{20} = \frac{9}{20}$</td>
<td>$\frac{3}{10} = \frac{6}{20}$</td>
</tr>
</tbody>
</table>

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Resuelve estos problemas. Convierte los números mixtos a fracciones impropias.

1) \[
\frac{7}{4} - 2 \frac{9}{12} =
\]
2) \[
\frac{1}{4} - 24 \frac{7}{6} =
\]

Resuelve estos problemas restando los números enteros de los números enteros, y las fracciones de las fracciones.

3) \[
\frac{5}{3} - 2 \frac{1}{4} =
\]
4) \[
\frac{2}{3} - 16 \frac{2}{5} =
\]

Resuelve estos problemas cambiando el entero a número racional y el número mixto a fracción impropia. Encuentra el mínimo común denominador.

5) \[
5 - 3 \frac{1}{6} =
\]
6) \[
7 - 2 \frac{2}{4} =
\]

Resta.

7) \[
\frac{1}{3} - \frac{4}{8} =
\]
8) \[
\frac{9}{6} - \frac{3}{1} =
\]
9) \[
6 - 1 \frac{3}{5} =
\]
10) \[
7 \frac{3}{4} - 6 =
\]
PART III

Permit the students to learn their own method correctly. Make comparisons between both methods. Identify the minuend and the subtrahend in each problem.

Discuss the functions of each part of a problem. Focus students’ attention on the final answer.

Perform subtraction operations. Use both methods.

<table>
<thead>
<tr>
<th>Spanish-Speaking Countries Method</th>
<th>United States Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) ( \frac{1}{8} - \frac{2}{4} = )</td>
<td>( 3 \frac{1}{8} ) Minuend</td>
</tr>
<tr>
<td>Este es el sustraendo.</td>
<td></td>
</tr>
<tr>
<td>Este es el minuendo.</td>
<td>- 2 ( \frac{1}{4} ) Subtrahend</td>
</tr>
</tbody>
</table>

| 2) \( 7 - 3 \frac{2}{5} = \)     | \( 7 = 6 \frac{5}{5} \) This is the minuend after regrouping. |
| \( \frac{7}{5} - \frac{17}{5} = \) |                     |
| El sustraendo está indicado como fracción impropia. |                     |
| El minuendo está indicado como fracción impropia. |                     |
PART IV

Solve these problems by using the horizontal method. Convert each mixed numeral to an improper fraction.

Resuelve estos problemas usando el método horizontal en español. Convierte a fracción impropia.

1) \( \frac{3}{5} - 2 \frac{3}{4} = \)
2) \( 17 \frac{1}{3} - 8 \frac{4}{5} = \)

Subtract whole numbers from whole numbers and fractions from fractions. Rename the minuend.

Resta separadamente las partes enteras y las fracciones. Renomea el minuendo.

3) \( 12 \frac{1}{3} - 4 \frac{5}{12} = \)
4) \( 11 \frac{2}{3} - 6 \frac{3}{4} = \)

Solve these problems by using either method.

5) \( 6 \frac{1}{3} = \)
6) \( 8 \frac{3}{4} - 6 \frac{4}{5} = \)

\(-2 \frac{5}{6} = \)

Copyright © 1982 Board of Education of the City of Chicago. Chicago, Illinois
OBJECTIVE Read and write a three-place decimal.

SUGGESTED ACTIVITIES

In Spanish-speaking countries the students must recall decimal orders. The tenths are called primer orden; the hundredths, segundo orden; the thousandths, tercer orden; and the ten-thousandths, cuarto orden.

PART I

<table>
<thead>
<tr>
<th>Spanish-Speaking Countries Method</th>
<th>United States Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>When the decimal point is written in the Spanish-speaking countries, it is not necessary to read it. The whole unit is read first, and then the decimal. If the decimal point is read, however, it is read as con (with).</td>
<td>In English the whole number is read first; then the word and is said to indicate the decimal point; and finally the number and the name of the place with the last significant digit are given.</td>
</tr>
</tbody>
</table>

1st order decimal:

1.1 It is read: un entero, un decimo; un entero con un decimo.

2nd order decimal:

1.01 It is read: un entero, un centesimo; un entero con un centesimo.

3rd order decimal:

1.001 It is read: un entero, un milésimo; un entero con un milésimo.
PART II

Lee los siguientes números:

1) 25.102
   veinticinco enteros y ciento dos milésimos

2) 18.02
   dieciocho enteros con dos centésimos

3) 40.001
   cuarenta enteros y un milésimo

4) 1.45
   un entero con cuarenta y cinco centésimos

5) 1.060
   un entero con sesenta milésimos

6) 5.401
   cinco enteros y cuatrocientos un milésimo

Escribe los siguientes números:

1) trescientos treinta milésimos

2) quinientos cuatro milésimos

3) cuarenta y cinco milésimos

4) un entero, cuarenta y cuatro centésimos

5) cuarenta enteros con cinco milésimos
PART III

write several three-place decimal numbers in the appropriate columns of a table displayed on the chalkboard or a chart. Have the students write and read the decimal numbers using and to indicate the decimal point. Compare the English form with the Spanish form.

<table>
<thead>
<tr>
<th>Whole Unit Order</th>
<th>Decimal Order</th>
</tr>
</thead>
<tbody>
<tr>
<td>6th 5th 4th 3rd 2nd 1st</td>
<td>1st 2nd 3rd 4th 5th 6th</td>
</tr>
<tr>
<td>100,000 10,000 1,000 100 10 1</td>
<td>.1 .01 .001 .0001 .00001</td>
</tr>
</tbody>
</table>

Example:

1.709   un entero con setecientos nueve milésimos  
one and seven hundred nine thousandths

1) 7.04

2) 132.747

3) 326.085

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PART IV

Write the following numbers:

Escribe los números siguientes:

1) five and five thousandths
   cinco enteros y cinco milésimos

2) forty and seven thousandths
   cuarenta enteros y siete milésimos

3) one hundred ten and twelve thousandths
   ciento diez enteros y doce milésimos

4) seven hundred thousandths
   setecientos milésimos

5) one hundred five and one thousandth
   ciento cinco enteros y un milésimo

6) one hundred fifty thousandths
   ciento cincuenta milésimos

7) eighty-one hundredths
   ochenta y un centésimos

8) four hundred and ten thousandths
   cuatrocientos enteros con diez milésimos

9) fifty and thirteen hundredths
   cincuenta enteros y trece centésimos

10) one hundred five thousandths
    ciento cinco milésimos

Note: If help is needed in reading Spanish, ask the teacher to allow one of the fluent Spanish-speaking students to read the numbers.
GRADE 6

Place Value

2-L1-1 Write whole numbers, including seven-place, in expanded form using exponents.

Operations with Whole Numbers

3-L1-6 Divide a three- or four-digit dividend by a three-digit divisor. (Estimate the partial quotient by rounding to the nearest hundred.)

Rational Numbers

4-L1-13 Divide fractions.
OBJECTIVE
Write whole numbers, including seven-place, in expanded form using exponents.

SUGGESTED ACTIVITIES
The translation of terms for place values is necessary for some Spanish-speaking students. The students from some Spanish-speaking countries, such as Spain, use a point rather than a comma to separate each period of a number.

PART I

Spanish-Speaking Countries Method

3,692 = (3\times10^3) + (6\times10^2) + (9\times10^1) + (2\times10^0)

8,495,526

(8\times10^6) + (4\times10^5) + (9\times10^4) + (5\times10^3) + (5\times10^2) + (2\times10^1) + (6\times10^0)

United States Method

3,692 = (3\times10^3) + (6\times10^2) + (9\times10^1) + (2\times10^0)

8,495,526

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PART II

Marca correctamente los períodos de los números. Luego escribéllos en notación desarrollada.

1) 6 125 426 =
2) 3 969 =
3) 7 941 643 =
4) 123 456 =
5) 841 907 =
6) 6 413 216 =
7) 308 341 =
8) 4 491 613 =
9) 82 624 =
10) 213 001 =

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PART III

Review place values in the Spanish-speaking system and then teach the English terms. If the students use points in place of commas to separate digits for place value divisions, teach the use of commas. Use the exponential forms to help show the similarities of the two systems.

Review--

Then teach--

Place

<table>
<thead>
<tr>
<th>Place</th>
<th>Exponential Form</th>
<th>Equivalent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unidades = 1 x 10^0</td>
<td>= Units</td>
<td></td>
</tr>
<tr>
<td>Decenas = 1 x 10^1</td>
<td>= Tens</td>
<td></td>
</tr>
<tr>
<td>Centenas = 1 x 10^2</td>
<td>= Hundreds</td>
<td></td>
</tr>
<tr>
<td>Unidades de Millar = 1 x 10^3</td>
<td>= Thousands</td>
<td></td>
</tr>
<tr>
<td>Decenas de Millar = 1 x 10^4</td>
<td>= Ten Thousands</td>
<td></td>
</tr>
<tr>
<td>Centenas de Millar = 1 x 10^5</td>
<td>= Hundred Thousands</td>
<td></td>
</tr>
<tr>
<td>Unidades de Millón = 1 x 10^6</td>
<td>= Millions</td>
<td></td>
</tr>
</tbody>
</table>

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PART IV

Write numbers by using both systems of period division. Expand the numbers and use the correct exponent.

Escribe los números siguientes, usando ambos sistemas de separación de los periodos. Escríbelos en notación desarrollada usando el exponente correcto.

<table>
<thead>
<tr>
<th>Número / Number</th>
<th>Spanish-Speaking / Countries</th>
<th>United States / Methods</th>
<th>Expansion with Exponents</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) 648 912</td>
<td>648,912</td>
<td>648,912</td>
<td>((6 \times 10^5) + (4 \times 10^4) + (8 \times 10^3) + (9 \times 10^2) + (1 \times 10^1) + (2 \times 10^0))</td>
</tr>
<tr>
<td>2) 2 462 110</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3) 72 812</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4) 3 651 000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5) 79 864</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6) 9 016 242</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7) 1 214</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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OBJECTIVE
Divide a three or four-digit dividend by a three-digit divisor. (Estimate the partial quotient by rounding to the nearest hundred.)

SUGGESTED ACTIVITIES
The division algorithm in the Spanish-speaking countries follows a process different from the one used in the United States. (See Objective 3-K1-6 for the division algorithm process.)

PART I

<table>
<thead>
<tr>
<th>Spanish-Speaking Countries Method</th>
<th>United States Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>9918 ÷ 236 = 42 R6</td>
<td>9918 ÷ 236 = 42 R6</td>
</tr>
<tr>
<td>478 ÷ 42 = 11 R14</td>
<td>944 ÷ 42 = 22 R16</td>
</tr>
<tr>
<td>006</td>
<td>472 ÷ 42 = 11 R10</td>
</tr>
<tr>
<td></td>
<td>6</td>
</tr>
</tbody>
</table>
PART II

Indica el número de cifras de cada uno de los cocientes que se te piden. Luego divide para encontrar el cociente exacto.

Ejemplo:

\[
\begin{array}{c|cc}
4 & 9 & 3 \\
3 & 0 & 1 \\
\hline
1 & 2 & 0 \\
\end{array}
\]

El cociente tiene 2 cifras.

\[
\begin{array}{c|cc}
4 & 9 & 3 \\
0 & 1 & 3 \\
\hline
1 & 2 & 0 \\
0 & 1 & 3 \\
\hline
1 & 0 & 4 \\
\end{array}
\]

El cociente es mayor que 40.

\[
\begin{array}{c|cc}
4 & 9 & 3 \\
0 & 1 & 3 \\
\hline
1 & 2 & 0 \\
0 & 1 & 3 \\
\hline
1 & 0 & 4 \\
\end{array}
\]

El cociente es 41.

1) 3 9,4 7 3 \[ \underline{2 0 8} \]
El cociente tiene ___ cifras.
El cociente es mayor que ____.
El cociente es ____________.

2) 9 3,0 0 8 \[ \underline{5 1 6} \]
El cociente tiene ___ cifras.
El cociente es mayor que ____.
El cociente es ____________.

3) 1 0,8 4 3 \[ \underline{7 4 1} \]
El cociente tiene ___ cifras.
El cociente es mayor que ____.
El cociente es ____________.

4) 9,5 4 0 \[ \underline{4 2 5} \]
El cociente tiene ___ cifras.
El cociente es mayor que ____.
El cociente es ____________.

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Help the Spanish-speaking students to make the transition from their own form of division to the form used in the United States. Follow this procedure:

Ask the students to write the symbol with angle opening down. ( ) Place the divisor at the left-hand side of the dividend. Select the first division step that uses the same number of digits in the dividend that are in the divisor.

799 will be the first group of digits to be divided. The first three digits will give the first digit on the quotient line.

Each digit after this step will give a digit on the quotient line.

Divide the first group of digits. Fill the second and third places with zeros.

Bring the next digit down to make the second division step. Divide a second time.

Write this quotient above the zero in the tens place of the first quotient.
Write a zero in the ones place.

Bring the next digit down to make the third division step. Divide for the third time.
Place this digit above the ones place in the other quotients.

The quotient is the sum of the numbers on the quotient line.
PART IV

3-L1-6

Assist the students to make the transition from one algorithm form to the other by giving them individual help.

Divide los siguientes números usando las dos formas; primero calcula cuantos lugares (cifras) tendrá el cociente.

1) \[ \begin{array}{c|c}
69,545 & 225 \\
\end{array} \]

2) \[ \begin{array}{c|c}
98,761 & 506 \\
\end{array} \]

3) \[ \begin{array}{c|c}
9,779 & 512 \\
\end{array} \]

4) \[ \begin{array}{c|c}
10,911 & 801 \\
\end{array} \]

5) \[ \begin{array}{c|c}
98,007 & 111 \\
\end{array} \]

6) \[ \begin{array}{c|c}
19,681 & 820 \\
\end{array} \]

7) \[ \begin{array}{c|c}
716,909 & 716 \\
\end{array} \]

8) \[ \begin{array}{c|c}
65,832 & 410 \\
\end{array} \]

9) \[ \begin{array}{c|c}
10,780 & 324 \\
\end{array} \]

10) \[ \begin{array}{c|c}
66,887 & 999 \\
\end{array} \]

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STRAND  RATIONAL NUMBERS

OBJECTIVE  Divide fractions.

SUGGESTED ACTIVITIES

Division of fractions is performed in various ways in Spanish-speaking countries but the crisscross method is used most frequently. This method is used after mixed numerals have been converted to improper fractions. Some teachers advise the students to reduce the fractions to lowest terms before dividing.

In Spanish-speaking countries the division symbols (\(\div\) and \(\frac{\cdot}{\cdot}\)) are used.

PART I

Spanish-Speaking Countries Method

Crisscross Method with Common Fractions

The numerator of the first fraction is multiplied by the denominator of the second fraction to form the numerator of the quotient.

\[
\frac{2}{3} \cdot \frac{4}{6} = \frac{12}{18}
\]

The denominator of the first fraction is multiplied by the numerator of the second fraction to form the denominator of the quotient.

\[
\frac{2}{3} \cdot \frac{4}{6} = \frac{12}{18}
\]

The quotient is reduced to the lowest terms.

\[
\frac{2 \div 4}{3 \div 6} = \frac{1}{1} = 1
\]

United States Method

2 \(\div\) 4 = \(\frac{2}{4}\)

3 \(\div\) 6 = \(\frac{3}{6}\)

Divisor is inverted.

2 \(\div\) 6 = \(\frac{2}{6}\)

3 \(\div\) 4 = \(\frac{3}{4}\)

Fractions are reduced to lowest terms.

\[
\frac{2}{3} \times \frac{2}{1} = \frac{4}{3}
\]

\[
\frac{2}{3} \times \frac{2}{1} = \frac{4}{3}
\]

Numerators and denominators are multiplied.

\[
\frac{1}{2} \times \frac{2}{1} = \frac{1}{2}
\]

\[
\frac{1}{2} \times \frac{2}{1} = \frac{1}{2}
\]
PART I (continued)

Spanish-Speaking Countries Method

Crisscross Method with Mixed Numerals

Mixed numerals are converted to improper fractions and then reduced to lowest terms, when possible. Then the crisscross operation is performed.
PART II

Resuelve los siguientes ejercicios de dividir:

1) \( \frac{8}{9} : \frac{4}{5} = \)

2) \( \frac{2}{3} : \frac{2}{3} = \)

3) \( \frac{4}{6} : \frac{7}{8} = \)

4) \( \frac{6}{13} : \frac{2}{4} = \)

5) \( \frac{5}{8} : \frac{1}{7} = \)

6) \( \frac{8}{9} : \frac{13}{14} = \)

7) \( \frac{5}{9} : \frac{2}{3} = \)

8) \( \frac{8}{9} : \frac{4}{6} = \)

9) \( \frac{5}{6} : \frac{5}{8} = \)

10) \( \frac{8}{11} : \frac{5}{9} = \)
PART III

Explain that the division process is usually performed in the United States by using the reciprocal of the divisor and in Spanish-speaking countries by multiplying the numerator and denominator in a crisscross pattern or by using the reciprocal method.

<table>
<thead>
<tr>
<th>Spanish-Speaking Countries Method</th>
<th>United States Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Note: Division symbols are : and ÷</td>
<td></td>
</tr>
</tbody>
</table>

1) \[ \frac{2}{3} \div \frac{4}{3} = \frac{1}{2} \] \[ \frac{2}{3} \div \frac{4}{3} = \frac{2}{3} \times \frac{3}{4} \]

2) \[ \frac{2}{3} \div \frac{2}{5} = \frac{1}{3} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{2} \]

\[ \frac{14}{3} \div \frac{7}{5} = \]

\[ \frac{14}{3} \times \frac{5}{7} = \]

\[ \frac{2}{14} \times \frac{5}{1} = \frac{10}{3} \]

\[ \frac{10}{3} = \frac{3}{3} \]
PART III (continued)

Spanish-Speaking Countries Method

3) \[
\frac{2}{3} \div \frac{4}{3} =
\]

\[
\frac{1}{2} \times \frac{1}{2} = \frac{1}{2}
\]

or

\[
\frac{2 \times 3}{3 \times 4} = \frac{1}{2} = \frac{1}{2}
\]
PART IV

Solve the following problems:

Resuelve los siguientes problemas:

1) \( \frac{1}{3} \div \frac{5}{6} = \) 
6) \( 18\frac{2}{9} \div \frac{4}{6} = \)

2) \( \frac{8}{9} \div \frac{5}{7} = \) 
7) \( \frac{3}{4} \div \frac{2}{3} = \)

3) \( \frac{4}{8} \div \frac{6}{4} = \) 
8) \( \frac{3}{6} \div \frac{7}{8} = \)

4) \( \frac{5}{10} \div \frac{10}{11} = \) 
9) \( \frac{14}{4} \div \frac{2}{3} = \)

5) \( \frac{11}{12} \div \frac{3}{4} = \) 
10) \( \frac{2}{7} \div \frac{6}{10} = \)
ENGLISH/SPANISH VOCABULARY

The equivalent or most nearly accurate equivalent Spanish terms have been listed for the English terms used in the mathematical context.

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<thead>
<tr>
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<th>Spanish</th>
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<td>block, cube</td>
<td>cubo, hexaedro</td>
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<tr>
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<td>orilla, borde, margen</td>
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<tr>
<td>boundary</td>
<td>límite, linde, lindero</td>
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<td>centavo, céntrico, centésimo</td>
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<td>palillos de colores usados para contar o medir</td>
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<td>ocho, octavo</td>
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<td>encerrar en un círculo</td>
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<td>pie, pies</td>
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<td>tablas de multiplicar</td>
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narrow
nine, ninth
October 12, 1981, 10/12/1981
odd number
one; ones place
one hundredth
one tenth
one thousandth
ordered pairs
ordinal numbers
outcome

pair
percent, percentage
pint (nonexistent in Spanish)
place value
plane
power of a number
protractor
quart
quarter
quarterly
rate pair (nonexistent in Spanish)
regrouping
remainder
rename
right-hand side
right triangle
round off
same
sandpaper
Saturday
scale
score
second
sequence
sets
seven, seventh
short, shorter, shortest

angosto
nueve, noveno
12 de octubre de 1981, 12/10/1981
número no, -impar
uno, una; unidades
un centésimo
un décimo
un milésimo
números ordenados (1, -4)
números ordinales
resultado
par, emparejar, formar pares
porciento, porcentaje
equivalencia a dos tazas
valor relativo de los números
plano
potencia de un número
transportador
un cuarto de galón
un cuarto de dólar, cuarto, cuadrante
trimestralmente
razón de dos números
reagrupar
residuo
convertir números de un valor a otro (ej: unidades a decenios o centenios o viceversa)
a la derecha
triángulo rectángulo
redondear
el mismo, lo mismo
papel de lija
sábado
escala, balanza, romana, báscula
puntuación
segundo (orden)
orden, secuencia
conjunto
siete, séptimo
corto, más corto, el más corto

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<table>
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<tbody>
<tr>
<td>six, sixth</td>
<td>seis, sexto</td>
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<td>cuerpo geométrico</td>
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<td>boletín, tablero de etiquetas</td>
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<td>take away</td>
<td>restar, quitar, sacar</td>
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<td>mil, milésimo</td>
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<td>poniente, occidente</td>
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Use this form to evaluate the supplement. Put a number from 1 to 4 in each box according to the scale below:

1 poor  2 fair  3 good  4 excellent

Return the completed form to the Bureau of Mathematics, Room 838, Department of Curriculum, Mail Run #84.

School _____________ District _____ Date ________ Teacher _____________

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<th>Part of Lesson</th>
<th>Evaluation</th>
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<tbody>
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<td>Significance of Differentiation</td>
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<td>Presentation</td>
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