Enrichment activities for fifth-grade mathematics are presented. They are intended to be a continuation of the program started in the fourth grade. Some of the activities reinforce principles taught in the regular program; others introduce new concepts to challenge students. The activities are divided into the following categories: number pictures; tic-tac-toe word problems; logic puzzles; crossword puzzles; mathematical word search; metric measurements; coded computations; suppressed digits; magic squares; graphing; geometric shapes; spatial perception; word problems (advertising puzzles); finding the square root of a number; patterns; base ten; base five; base two; and number sense (multiplication and division). Answers to the puzzles and other activities are appended. (DC)
MATHEMATICS ENRICHMENT

Grade 5

Curriculum Bulletin Number 237
Fort Worth Independent School District
Fort Worth, Texas

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These activities for the fifth grade were written as a continuation to the enrichment program started in the fourth grade. They are to be used throughout the year to create interest and enjoyment in mathematics. Some of the materials are for reinforcement of principles taught in the regular program; others introduce new concepts to challenge the pupils.

Patsy Johnston, curriculum writer/editor, wrote and edited these activities. The project was developed under the direction of Crawford Johnson, Program Director for Mathematics, and J. D. Shipp, Director of Elementary Schools. Appreciation is expressed for the creative work of these three as well as others who contributed to this project.

This publication was planned, written, edited, and published in the Department of Curriculum Development.

Dewey W. Mayr, Jr.
Director of Curriculum Development

August, 1981
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NUMBER PICTURES

Pictures can be made from numbers. See if you can make one.
Choose any problem. Solve it. Put an X or O on the tic-tac-toe board. If two people play, three Xs or Os in a line wins. If one person plays, two tic-tac-toes are needed to win.

1. Bob Stroud travels from Baltimore to Philadelphia, a distance of 156 km, once a month to purchase clothing for his boutique. A round trip plane fare costs $87.20. He could drive for about .20 a kilometer. How much could Henry save in a year by driving?

2. Silvia is going to make a pine bookcase. She needs 4 shelves and 2 sides. Each shelf will be 80 cm long and each side 130 cm long. If pine boards cost $4.95 a meter, how much will Silvia be spending?

3. It takes a lumber company 42 hours to clear a tract of land plus another 68 hours to replant. How long will it take to clear and replant a tract of land 40 times as large?

4. Logan can type 50 words a minute accurately. How many hours will it take Logan to type a 12,000 word research paper?

5. Elizabeth bought a ski parka for $78.44 and a tennis racket for $21.78. She charged both items on her new charge account. She made two monthly payments of $16 each. The interest charges for the two months were $2.78. If Elizabeth makes no further charges during the two months, how much does she owe?
6. A museum receives about 3450 visitors each month in June, July, and August. In September, October, April, and May, the museum receives about 1400 visitors monthly. In each of the other months, the museum receives about 250 visitors monthly. About how many hundreds of visitors are welcomed yearly?

7. In the month of March the Sweater Shop sold 33 sweaters at the regular price of $24 each and another 48 sweaters at a clearance price of $18 each. What were the total sales of sweaters in the month of March?

8. Ken is repairing the Agee’s color television. His repair rates are shown below. It took Ken 3 hours to find the trouble and will take another hour to repair the set. How much will Ken charge Mr. Agee for labor?

<table>
<thead>
<tr>
<th>Type of Work</th>
<th>Charge for 1st Hour</th>
<th>Each Hour More</th>
</tr>
</thead>
<tbody>
<tr>
<td>Black and White TV</td>
<td>$20</td>
<td>$15</td>
</tr>
<tr>
<td>Color TV</td>
<td>$26</td>
<td>$18</td>
</tr>
</tbody>
</table>

9. Twila’s basketball team won the city championship. She was high scorer for the game, scoring 40 points. She scored twice as many field goals (2 points) as free throws (1 point). How many field goals did Twila score in the championship game?
CHOOSE ANY PROBLEM. SOLVE IT.
PUT AN X OR A  0 ON THE TIC-TAC-TOE BOARD. IF TWO PEOPLE
PLAY, THREE XS OR O'S IN A LINE
WINS. IF ONE PERSON PLAYS, TWO TIC-TAC-TOES ARE NEEDED TO WIN.

1. Mr. and Mrs. Goldstein plan to see a play at the Cow Palace.
   How much will two balcony seats cost for a Friday night performance?

2. The sum of two numbers is 115. One number is 3 less than the
   other number. What is the smaller number?

3. On July 1, West Side Car Sales had 234 cars on the lot. By
   July 15, \( \frac{2}{3} \) of the cars were left. How many cars had been sold?

4. The Bottling Company has 2 bottling machines that bottle 240
   bottles of soft drinks a minute. How many sodas can be bottled
   by one machine in one second?

5. Virginia works 8 hours a day on the assembly line at Downtown
   Motors. For five days of work, the total of the deductions from
   her weekly paycheck is \$78.42. How many hours a week does
   Virginia work?

6. Walter began the day with \$100.52 in his cash drawer. During the
day, he sold 48 albums at \$5.99 each, 14 albums at \$8.79 each,
and 2 collector's editions at \$24.95 each. How much money should
Walter have in his cash drawer at the end of the day?
7. The three planets closest to the sun are Mercury, Venus, and Earth. The diameters of these three planets are 4,988 km, 12,289 km, and 12,755 km, respectively. How much larger is the diameter of Earth than the diameter of Mercury?

8. Jay drives a total of 84 km a day to and from work. If his car travels about 6 km on a liter of gasoline, in how many days will Randy use 70 liters of gasoline?

9. Leon bought a 1.59 kg chicken costing $3.15, 0.95 kg of hamburger costing $2.98, and a bag of apples costing $1.77. What was the total cost of Leon's purchases?
Choose any problem. Solve it. Put an X or a O on the tic-tac-toe board. If two people play, three Xs or Os in a line wins. If one person plays, two tic-tac-toes are needed to win.

1. The Millers buy a $487 stove and a $736 refrigerator on credit. The finance charge is $289 if the loan is paid back in 24 equal monthly payments. How much will each monthly payment be?

2. The County Arena has a seating capacity of 2000. Tickets normally sell for $4.00 a piece, but students receive a $2.00 discount. For the high school tournament being held there, the manager has made a special effort to allow as many students as possible to attend. For the manager to meet expenses, $5000 worth of tickets must be sold. How many student tickets can he afford to sell and still meet expenses?

3. Mr. Wilson buys a $899 sofa on credit. In addition to the sales tax of $44.96, he must pay a finance charge of $113. If he repays the loan in 12 equal monthly payments, what is the amount of each payment?

4. Alex is buying fencing to enclose a rectangular play area for his dog. The vet recommends that the dog have a yard at least 36 m². What is the least amount of fencing he will need?
5. The population of Quebec in 1901 was 69,000. By 1931, the population had increased by 62,000. By 1971, the population had increased another 55,000. How many thousands was the population of Quebec in 1971?

6. In 1947, a town began taking a population count of its citizens. The population that year totaled 25,476. Ten years later, the population increased about 8,500. The last count was taken in 1975 and the population was 42,789. How many years passed between the time the first and the last count was made?

7. As an electrician, Sherry charges $14.95 an hour for her services on weekdays and $22.95 an hour on weekends. If Sherry estimates the job can be finished in 4 hours, how much will she earn working during the week?

8. Mr. Mays, an appliance salesperson, earns $55 for each dryer, $90 for each refrigerator, and $65 for each stove. In a recent month he sold 12 stoves, 12 refrigerators, and 5 dryers. What were his earnings from these sales?

9. The sum of four consecutive numbers is 174. What is the largest of these numbers?
TIC-TAC-TOE

Choose any problem below. Solve it. Put an X or a 0 on the answer on the tic-tac-toe board. If two people play, three Xs or Os in a line are needed to win.

1. If you multiply my number by 6 and then divide by 12, you get 4. What's my number? 13
2. If you divide my number by 4 and then multiply by 10, you get 40. What's my number? 14
3. If you divide my number by 7 and then add 18, you get 20. What's my number? 7
4. If you add 53 to my number and then divide by 7, you get 10. What's my number? 4
5. Seven times my number is 20 more than 5 times my number. What's my number? 9
6. If you multiply my number by 9 and subtract 49, you get 50. What's my number? 7
7. My number is between 10 and 20. If you divide it by 11, you get a remainder of 4. What's my number? 8
8. If you multiply my number times itself and add 19, you get 100. What's my number? 9
9. If you multiply my number by 5 and add 10, you get 75. What's my number? 10
TIC-TAC-TOE

5  8  19

2  7  4

9  18  6

Choose any problem. Solve it. Put an X or a 0 on the tic-tac-toe board. If two people play, three Xs or Os in a line wins. If one person plays, two tic-tac-toes are needed to win.

1. The postage for 5 small boxes is 95¢. How much postage for each box?

3. One box holds $\frac{1}{4}$ pound of lemon drops. How many boxes can be filled with $1\frac{3}{4}$ pounds of lemon drops?

3. What is the perimeter of an equilateral triangle with a side 6 ft. long?

4. A box of assorted chocolates contains $\frac{1}{4}$ pounds of each kind. How many kinds of chocolates in a 1$\frac{1}{2}$ pound box?

5. Robert earns $3.25 per hour working at a garage. One week he worked 15 hours and the next week he worked 23 hours. How many more hours did he work the second week?

6. The cement truck has 8 cubic yards of concrete in it. The driver dumps $\frac{3}{4}$ of it for a driveway. How many cubic yards are left in the truck?

7. Write this Roman numeral IX as an Arabic numeral.

8. A bulldozer driver grades $\frac{2}{3}$ of a 15-acre lot. How many acres does he still have to grade?

9. How many sides does a rhombus have?
Choose any problem below. Solve it. Put an X or a 0 on the answer on the tic-tac-toe board. If two people play, three Xs or Os in a line wins. If one person plays, two tic-tac-toes are needed to win.

1. A rug 9 ft. x 12 ft. costs $34.56. This is how much per square foot?

2. If 30 oranges cost $9.30, what do 2 oranges cost?

3. At 3 for 15¢, how much do 9 apples cost?

4. Peggy has 42 jacks, Beth has \( \frac{1}{2} \) as many as Peggy, and Sue has \( \frac{1}{3} \) as many as Peggy. How many more jacks does Beth have than Sue?

5. A boy walks 42 ft. in 7 seconds. This is how many ft./sec?

6. James ran 40 miles in 10 hours. This is how many miles per hour?

7. Write this Roman numeral LXI as an Arabic numeral.

8. How many inches are there in \( \frac{1}{3} \) of 12 feet?

9. There are 64 beef and dairy cattle in the pasture. If \( \frac{3}{4} \) of them are beef cattle, how many dairy cattle are there?
TIC-TAC-TOE

Choose any problem below. Solve it.

Put an X or a 0 on the answer on the tic-tac-toe board. If two people play, three Xs or Os in a line win. If one person plays, two tic-tac-toes are needed to win.

1. How many legs on 12 cows and 9 chickens?

2. I had $20. I spent $2.50, $4.25, and $9.25. How much is left?

3. How much money is 44 dimes and 12 nickels?

4. Roller coaster rides cost 75¢ each. Ferris wheel rides cost 50¢ each. What's the cost for 4 roller coaster rides and 6 Ferris wheel rides?

5. How many hours in 2 days and 12 hours?

6. How many legs on 12 dogs and 4 cats?

7. I bought 4 records at $4.25 each. I gave the clerk $20. How much change did I receive?

8. There are 6 teams of fourth graders and 3 teams of fifth graders. Each team has 6 players. How many players are there?

9. I had 47 comic books. I gave my friend 15 of them and he gave me 23 of his. How many do I have now?
LOGIC PUZZLES

IT IS INTERESTING TO OBSERVE THAT PUZZLES OF THE PURELY LOGICAL TYPE ARE REPRESENTATIVE OF THE ENTIRE SCIENTIFIC PROCESS. AT THE BEGINNING ONE IS CONFRONTED WITH A MASS OF MORE OR LESS UNRELATED DATA. FROM THESE FACTS A FEW POSITIVE INFERENCES CAN BE SEEN IMMEDIATELY, BUT USUALLY IT IS NECESSARY TO MAKE ASSUMPTIONS TO GUIDE THE SEARCH FOR A SOLUTION. THE VALIDITY OF THESE ASSUMPTIONS MUST BE CHECKED BY TESTING FOR CONSISTENCY WITH THE ORIGINAL DATA. IF INCONSISTENCIES APPEAR, THE ASSUMPTIONS MUST BE REJECTED AND OTHERS TRIED UNTIL FINALLY A CONSISTENT SET OF CONCLUSIONS EMergES.

THE SOLUTION OF LOGIC PUZZLES CANNOT BE REDUCED TO A FIXED PATTERN. NEVERTHELESS, THERE ARE SOME GENERAL SUGGESTIONS ON HOW TO ATTACK PUZZLES OF THIS SORT.

Example:

Jones, Smith, Johnson, and Mays are four talented artists, one a dancer, one a painter, one a singer, and one a writer (not necessarily in that order).

1) Jones and Johnson were in the audience the night the singer made his debut on the concert stage.
2) Both Smith and the writer have sat for portraits by the painter.
3) The writer, whose biography of Mays was a best-seller, is planning to write a biography of Jones.
4) Jones has never heard of Johnson. What is each man's artistic field?

TO KEEP TRACK MENTALLY OF THESE MANY FACTS AND THE HYPOTHESES
AND CONCLUSIONS BASED UPON THEM IS CONFUSING AND DIFFICULT. THEREFORE, MAKING A CHART WHICH SHOWS ALL POSSIBILITIES IS A GOOD WAY TO START YOUR SOLUTION.

<table>
<thead>
<tr>
<th></th>
<th>DANCER</th>
<th>PAINTER</th>
<th>SINGER</th>
<th>WRITER</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jones</td>
<td></td>
<td></td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Smith</td>
<td></td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Johnson</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mays</td>
<td></td>
<td></td>
<td></td>
<td>X</td>
</tr>
</tbody>
</table>

Now from 1), it is known that neither Jones nor Johnson is the singer. Place an X opposite their names in the column headed by the singer.

From 2), it is known that Smith is neither the painter nor the writer. Place an X opposite Smith's name in the two columns headed painter and writer.

From 3), the writer is neither Mays nor Jones. Place an X opposite the names Mays and Jones in the column headed writer.

The chart now looks like the following:

<table>
<thead>
<tr>
<th></th>
<th>DANCER</th>
<th>PAINTER</th>
<th>SINGER</th>
<th>WRITER</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jones</td>
<td></td>
<td></td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Smith</td>
<td></td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Johnson</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mays</td>
<td></td>
<td></td>
<td></td>
<td>X</td>
</tr>
</tbody>
</table>

If a clue had stated that Smith was the singer, a check would have been placed opposite Smith's name in the column headed singer.
However, in this problem, there were no direct facts given. But at this stage when the chart is examined, it is clear that Johnson is the writer because all other possibilities are marked off. So place a check opposite Johnson's name in the column headed writer and fill the remaining squares in his row with X's.

According to 2), Johnson has sat for the painter and in 4), Jones does not know Johnson. Therefore, Jones is not the painter. Place an X by Jones' name in the column headed painter. By elimination on the chart, Jones must be the dancer. Place X's in the other squares in the dancer column.

Observe that the singer must be Smith. Add a check and X's in the correct squares.

Finally, Mays must be the painter, and the solution is complete.

PROBLEMS

1) Shepard, Johnston, and Nichols make their living as carpenter, painter, and plumber, though not necessarily respectively. The painter recently tried to get the carpenter to do some work for him, but was told that the carpenter was out doing some remodeling for the plumber.

The plumber makes more money than the painter. Johnston makes more money than Shepard. Nichols has never heard of Johnston. What is each man's occupation?
2) The following is a part of a report submitted by an investigator for a market analysis agency with standards of accuracy so high that it boasts that an employee's first mistake is his last.

- Number interviewed: 100
- Number who drink coffee: 79
- Number who drink tea: 69
- Number who drink both: 47

Why was the interviewer discharged?

3) Martha just got four new stamps for her collection. She is confused about which stamp comes from which country. Can you sort the stamps?

1. The stamp with the train on it is pink.
2. The German stamp has a picture of a runner.
3. The flower is not on the French stamp.
4. The Swedish stamp is not pink.
5. The plane is not on a yellow stamp.
6. The United States stamp is blue.
7. The flower is on a violet stamp.

<table>
<thead>
<tr>
<th>Country</th>
<th>Color</th>
<th>Picture</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
4) This is the first basketball game Fred has ever seen. He's really mixed up about the players' names and positions. He doesn’t even know who is captain of the team. Can you help him understand what’s happening? Who is the captain of the team?

1. The right-guard was the low scorer.
2. Sam scored more than Ben but less than Ed.
3. The captain played left-forward.
4. Ed scored more than the captain.
5. Jack and Dave together scored less than the right-forward.
6. The center scored more than the guards but less than the forwards.
7. Dave played left-guard.
8. The right-forward scored more than the left-forward.

Points scored: 7 16 25 10 15

Position played: ______________________

Player's name: ________________________

5) There are only three houses on Main Street. They are all on the same side of the street. The Wilsons, the Watsons, and the Jones live in the houses. Can you tell where each family lives and what kind of car each drives?

1. The Watsons live next to the Jones;
2. Mr. Wilson’s son, Billy Bob, is friendly with Mr. Watson's son.
3. The family on the right does not drive a Ford.
4. The people who drive the Cadillac do not live next to the people who drive the Ford.
5. The family in the middle have no children.
6. Billy Bob thinks his dad’s Cadillac is great.
7. One family owns a Plymouth.
6) A cat, a dog, a goat, and a horse are named Sandy, Blacky, Duke, and Rusty. Read the clues below to find each animal's name.

1. Duke is smaller than either the dog or Rusty.
2. The horse is younger than Sandy.
3. Blacky is the oldest and is a good friend of the dog.

7) Sally, Betty, Patsy, Bill, Sam, and Joe are six young persons who have been close friends from their childhood. They went through high school and college together, and when they finally paired off and became engaged nothing would do but a triple announcement party. Naturally they wanted to break the news to their friends in an unusual way.

At just the right moment during the party everyone was given a card bearing the information.

Who now are six will soon be three,
and gaily we confess it,
but how we've chosen you may know
no sooner than you guess it.

Joe, who is older than Sam is Sally's brother. Patsy is the oldest girl. The total age of each couple-to-be is the same although no two of us are the same age. Sam and Betty are together as old as Bill and Sally. What three engagements were announced at the party.
Jack Donovan was killed on a lonely road two miles from Trenton at 3:30 a.m. on March 17, 1933. Shorty Malone, Tony Verelli, Hank Rodgers, Joey Freiberg, and Red Johnson were arrested a week later and questioned. Each of these men made four simple statements of which three were absolutely true and only one of them false. One of these five men killed Donovan. To solve this puzzle, place "T" or "F" over each statement as you decide whether it is true or false. Start with Tony's statement, "One of us is guilty!" This statement is given to be true. Red's two statements, "I did not kill Donovan" and "Shorty lied when he said, 'I'm guilty,'" are either both true or both false. Since only one statement can be false, both of these must be true.

Shorty: "I was in Chicago when Donovan was murdered. I never killed anyone. Red is the guilty man. Joey and I were pals."

Hank: "I did not kill Donovan. I never owned a revolver in my life. Red knows me. I was in Philadelphia the night of March 17..."

Tony: "Hank lied when he said he never owned a revolver. The murder was committed on St. Patrick's Day. Shorty was in Chicago at that time when the murder was committed. One of us five is guilty."

Joey: "I did not kill Donovan. Red has never been in Trenton. I never saw Shorty before. Hank was in Philadelphia with me the night of March 17."

Red: "I did not kill Donovan. I have never been in Trenton. I never saw Hank before now. Shorty lied when he said I'm guilty."

Which of the five men killed Donovan?
9) In a certain bank the positions of cashier, manager, and teller are held by Brown, Green, and White, though not necessarily respectively. The teller, who was an only child, earns the least. White, who married Brown's sister, earns more than the manager. What position does each man fill?

10) In this murder case you are to find the victim, the witness, the policeman, the judge, and the hangman, as well as the killer.

The victim died instantly of a gunshot wound inflicted at close range. The witness did not see the crime, but swore that he heard an argument followed by a shot. After a lengthy trial, the murderer was convicted, sentenced to death, and hanged.

The six men involved in the case are Craig, Dunn, Grover, Hill, Mays, Wilson. You also have the following facts.

1. Mays had not known the victim or the murderer.
2. In court the judge asked Craig to give his account of the shooting.
3. Wilson was the last person to see Dunn alive.
5. The policeman testified that he picked up Grover near the place where the body was found.

Identify the victim, witness, policeman, judge, hangman, and murderer by name. Make a chart with names down one side and occupations across the top. Mays cannot be the victim or the murderer.
11) The organization of the business office of a certain company consists of the following: president, vice-president, manager, auditor, clerk, and secretary. The names of the office personnel in alphabetical order are: Mr. Brown, Mr. Crawford, Miss Green, Mrs. Johnston, Miss Jones, and Mr. Smith.

1. The vice-president is the president's grandson.
2. The manager is the secretary's son-in-law.
3. The auditor is Miss Green's step-sister.
4. Mr. Brown is a bachelor.
5. Mr. Crawford is 25 years old.
6. Mr. Smith is the president's neighbor.

What is the manager's name?

12) One afternoon David, Joe, and Sam with their wives, whose names in one order or another are Sue, Jane, and Betty, went out and played eighteen holes of golf together.

1. Betty, Jane, Sue, and Joe shot 106, 102, 200, and 94 respectively.
2. David and Sam shot a 98 and a 96, but for some time they couldn't tell who had made which since they hadn't put their names on their scorecards.
3. When the men finally identified their cards, it turned out that two of the couples had the same score.
4. Joe's wife beat David's wife.

What is the name of each man's wife, and what scores did David and Joe make?
13) Jimenez, Rodriguez, Martinez, and Contreras are four men whose occupations are Baker, Carpenter, Postman, and Police Officer, though not necessarily respectively.

1. Jimenez and Rodriguez are neighbors and take turns driving each other to work.
2. Rodriguez makes more money than Martinez.
3. Jimenez beats Contreras regularly at backgammon.
4. The Baker always walks to work.
5. The Police Officer does not live near the Carpenter.
6. The only time the Postman and the Police Officer ever met was when the Police Officer arrested the Postman for speeding.
7. The Police Officer makes more money than the Carpenter or the Postman.

What is each man's occupation?

14) On an airplane flying from New York to San Francisco there are passengers named Bob, David, and Joe. The pilot, co-pilot, and navigator have the same first names, but not respectively.

1. The navigator lives halfway between New York and San Francisco.
3. The passenger who lives nearest to the navigator earns exactly three times as much a month as the navigator.
4. The passenger with the same name as the navigator lives in San Francisco.
5. Joe, a member of the crew, recently beat the co-pilot at handball.
6. Passenger David earns $200 a week.

What is the pilot's first name?
15) Five kids, Ruth, Jane, Phil, Mark, and Joe went for lunch. The kids bought:

2 doubleburgers... 90¢ each 1 cheeseburger... 75¢ each
2 hamburgers... 60¢ each 2 colas... 20¢ each
2 shakes... 35¢ each 1 milk... 20¢ each
3 french fries... 25¢ each

Unfortunately everything got mixed up on the tray. Can you figure out what each kid spent and just what he had for lunch?

1. Each person had one drink and a burger of some kind.
2. The people who had doubleburgers didn't have french fries.
3. The cheeseburger person drank a cola.
4. The girls had the same thing to drink.
5. Phil didn't have french fries but his lunch cost the most.
6. No, two people had the same lunch.
7. Phil and Joe drank the same thing.
8. Jane's lunch cost the least.

<table>
<thead>
<tr>
<th>Lunch</th>
<th>Cost</th>
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</table>

Name          
Cost  $      $   $   $   $   $
Six horses ran in the big race. The jockeys wore shirts of different colors. Can you figure out the order in which the horses finished the race and the color of the shirt each jockey was wearing?

1. Johnston's Darlin' won the race.
2. The red shirt came in just behind Martin's Folly.
3. The blue shirt was last.
4. The yellow shirt came in third.
5. Martin's Folly was slower than the yellow shirt.
6. The jockey on Johnston's Darlin' and the jockey in the violet shirt are brothers.
7. The orange shirt came in between Winner's Circle and Lucky Lady.
8. Fire Ball came in before the yellow shirt.
9. The violet shirt beat Winner's Circle.
10. Lucky Lady has never been beaten by Dancer.

<table>
<thead>
<tr>
<th>Color</th>
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<tbody>
<tr>
<td>Red</td>
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<tr>
<td>Orange</td>
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<tr>
<td>Violet</td>
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</table>
AFTER THE REGULAR SEASON HAD FINISHED, THE SCHOOL'S BASKETBALL COACH DECIDED TO HOLD A FREE THROW CONTEST. THE MANAGER HAD A LOT OF TIME, WHILE THE PLAYERS WERE TAKING THEIR SHOTS, SO HE MADE OUT THE FOLLOWING LIST OF STATEMENTS WHICH HE TURNED IN TO THE COACH AFTER THE PLAYERS HAD FINISHED SHOOTING:

1. THE FIVE WITH THE HIGHEST SCORES WERE LLOYD, CARLTON, RONALD, JOE, AND ROY, BUT NOT RESPECTIVELY.
2. THE PLAYER WITH THE HIGHEST SCORE AND THE PLAYER WHO FINISHED FIFTH HAD NEVER BEEN IN A SEASON GAME AT THE SAME TIME.
3. LLOYD AND ROY WERE BOTH STARTERS IN THE FINAL GAME OF THE SEASON.
4. THE PLAYER WITH THE HIGHEST SCORE AND THE PLAYER WHO FINISHED SECOND HAD NEVER PLAYED TOGETHER IN A GAME.
5. JOE FINISHED HIGHER IN THIS CONTEST THAN ROY DID EVEN THOUGH ROY HAD BEEN THE TEAM'S LEADING SCORER DURING THE SEASON.
6. CARLTON AND THE WINNER HAD BEEN STARTERS IN THE FIRST GAME OF THE SEASON.
7. THE PLAYER WHO FINISHED FOURTH HAD NOT PLAYED IN THE LAST FOUR GAMES OF THE SEASON DUE TO AN INJURY.
8. JOE HAD PLAYED AT SEPARATE TIMES DURING THE LAST GAME OF THE SEASON WITH BOTH THE HIGHEST SCORER AND THE RUNNER-UP.

USING THE FACTS PROVIDED, THE COACH WAS ABLE TO DETERMINE THE ORDER IN WHICH THESE FIVE FINISHED THE CONTEST.

IN WHAT ORDER DID THE PLAYERS FINISH?

1. _____ 2. _____ 3. _____ 4. _____ 5. _____
ANN, BETH, CLEO, DONNA, AND EDITH WERE CLOSE FRIENDS IN HIGH SCHOOL, AS WERE THE FELLOWS THEY DATED, NOT IN ORDER—LESTER, CALVIN, DICK, RALPH, AND WARREN. ALL TEN DECIDED TO APPLY FOR ADMISSION AT THE UNIVERSITY, AND ALL WERE ACCEPTED. THE GIRLS, WHO WERE ALL GOING TO MAJOR IN EDUCATION, DECIDED TO JOIN SOCIAL GROUPS, SINCE THEY HAD HEARD THAT THIS WAS A GOOD WAY TO MAKE NEW FRIENDS. EACH GIRL JOINED A DIFFERENT ORGANIZATION—ALPHA, BETA, DELTA, CHI, AND IOTA. THE BOYS EACH DECIDED ON A DIFFERENT COLLEGE MAJOR—PRE-MED, PRE-LAW, ENGLISH, ECONOMICS, AND HISTORY. FROM THE CLUES BELOW, DETERMINE WHICH GIRL EACH DATED AND WHICH GROUP SHE BELONGED TO AS WELL AS THE MAJOR SUBJECT EACH BOY DECIDED ON.

1. DICK DATED THE GIRL IN ALPHA, WHILE THE IOTA GIRL DATED THE PRE-MED STUDENT.
2. BETH’S USUAL ESCORT WAS THE PRE-MED STUDENT. LESTER DATED DONNA. ANN DATED THE ECONOMICS STUDENT.
3. THE FIRST WEEKS OF SCHOOL WERE HARD, FINANCIALLY, FOR CALVIN. HE COULDN’T AFFORD TO TAKE THE BETA GIRL OUT. SO THE TWO OF THEM STUDIED TOGETHER, AND SHE HELPED HIM WITH HIS HISTORY COURSES.
4. WARREN’S GIRL WAS A DELTA.
5. THE DELTA GIRL AND THE ECONOMICS MAJOR OFTEN DOUBLE-DATED WITH CLEO AND THE PRE-LAW STUDENT.

<table>
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<tr>
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<td>MAJOR SUBJECT</td>
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<td>SOCIAL GROUP</td>
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</table>
1. The batting order will be Alton, Bill, Carl, Dennis, Erik, Frank, George, Henry, and Justin.

2. The center fielder is taller than the right fielder.

3. Alton is not the catcher this year.

4. Erik's sister goes steady with the second baseman.

5. Erik, Frank, Justin, the right fielder, and the center fielder are all seniors; the rest are underclassmen.

6. Erik and the outfielders played ball last summer.

7. Justin, Bill, and the pitcher were on the all-conference team last year.

8. Harry and the third baseman ride to school in the same car.

9. Alton, Justin, and the shortstop are math club members.

10. All the battery and the infield, except Bill, Alton, and Harry, are shorter than Carl.

11. The pitcher is the junior class president.

12. The catcher and the third baseman are co-chairmen of the junior prom decorations committee this year.

13. Carl is the only junior in Spanish class.

14. Dennis, Harry, Justin, and the catcher all ride home each evening with the second baseman.

15. Frank is taller than Dennis, while George is shorter than Dennis, each of them is heavier than the third baseman.

16. Either Alton or George will play in the outfield, but not both.

17. The shortstop, the third baseman, and Dennis all have part-time jobs after practice.
Recently a group of students from a nearby high school were discussing their school's offensive football team. They knew the names of the players, who were Bibb, Carr, Day, Finks, Harte, Jiminez, Kahler, Leggett, Mosley, Norwood, and Tarvin. No one in the group could tell which position each man played. They finally came up with a list of statements which enabled them to tell the position of each man on the team.

1. Carr, Bibb, Leggett, and all the backs were seniors.
2. Jiminez, Leggett, Tarvin, and the guards were in the same Spanish class.
3. Norwood and the halfbacks were taking calculus.
4. Bibb and Tarvin challenged the ends to a race.
5. Day, Mosley, and the tackles were juniors.
6. Kahler and the right halfback lived on the same street.
7. Tarvin, the backs, and the center were in the same English class.
8. Bibb and the center were senior class officers.
9. Tarvin and the right tackle were the only redheads on the team.
10. Jiminez, Mosley, and the center were the fastest linemen.
11. Leggett and the center were math club members.
12. Day, Leggett, and Tarvin, all linemen, played on the same side of the line.
13. The quarterback threw a pass to Mosley for a touchdown.
14. Finks, Norwood, and the fullback rode to school together.
15. Harte, Mosley, and the fullback were the team captains.
16. Finks usually lined up behind Jiminez.
17. The team usually lined up in a T formation.

What are the positions played by each man on the offensive team?
ACROSS

1) DCCXXV
4) XIV
6) CCXV
8) DIII
9) I
10) XC
11) XXI
12) LV
14) CDX

DOWN

1) VII
2) XXII
3) DIX
4) CI
5) XLIII
7) DIV
8) DXX
9) MDLI
13) DXLIII
15) MDCXLII
16) CCLII
17) DXLI
18) IX
19) XI I I
20) 6CCXLI
21) DCCLXI
22) V
23) CDLXII
24) DXXXV

31 36
ACROSS
1) AREA OF SQUARE: SIDE 16
4) L.C.M. OF 2, 8, 7
6) 46 x 17
8) 1239 FT. = ? YD.
9) NO. OF SIDES OF A RHOMBUS
10) NO. OF EGGS IN A DOZEN
11) 1032 IN. = ? FT.
12) \( \frac{36}{35} \times \frac{175}{12} \)
14) 39156 ÷ 52
16) NO. OF SIDES OF A QUADRILATERAL
17) L.C.M. OF 15, 16, 18
18) G.C.F. OF 45, 60
19) \( \frac{1}{2} + \frac{1}{10} - \frac{5}{6} + \frac{2}{3} \)
20) 1674 - 986
21) 17280 ÷ 36
23) 982.25 - 493.25
24) 324.58 + 196.42

DOWN
2) \( \frac{1}{2} \times 114 \)
MATHEMATICAL WORD SEARCH

When using word searches, you might want to give the students only the puzzles and let them make lists of the mathematical words that they find. Any word that is in the mathematics dictionary counts. You might give a prize or an extra A to the one who finds the most words. Instead of giving awards, you might divide the room into teams and let each team work as a group on one puzzle. Set a time limit of one or two weeks for a contest.

If your library does not have a mathematics dictionary, contact Crawford Johnson, mathematics program director, and request one for your school.
GEOMETRYGRAM

There are at least 40 words pertaining to geometry in the puzzle below. They be be written up, down, across, diagonally, backward, or forward. Some words may overlap.

OSRADIUSNESUTBOO
RECTANGLEWEKSNIPA
EGORAFAAAPATHGIRP
TMNIEIPRISMEENEIOC
EEGAMNRETEMAIDLTP
MNRNUIITCIYIEIRL
ITUGLTBASEUUAGUAM
RXELOYPLAAALARCE
EENEVSEINTERSECTR
PTTIEERILPITEEOO
EROXOTADLOOAOSRRE
LEAAUDUAUPLANEEWH
GVECIAURAADDHFT
NRADUEASQUAREEKPA
AISTPIIQCRICLESC
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35

40
WORD SEARCH

There are at least 85 words pertaining to mathematics in the puzzle below. They may be written up, down, across, diagonally, backward, or forward. Some words may overlap.

MATHEMATICAL WORD SEARCH

There are at least 90 words pertaining to mathematics in the puzzle below. They may be written up, down, across, diagonally, backward, or forward. Some words may overlap.

ESUBTRACTAPERIMENT
SRINIEVBCDHUNDREDERIE
TAREAUFPGFHDRATERA
ICIOSJRFANEGATIVEEIS
MCPERKARRLIRMCGNGNAU
AULDIAGONALTCOTEIPNR
TRAIQRESATCUHETORTGE
EACSPOWERCFTEHNNVRLOL
XCEETADIVIDEFIXYTCAXA
PYVARBCTTDDEOGOWPRIME
OFANAGAHHIJUHBNIKLDM
NNCLRNUMBERTACDDPQ
EDUELRSTEMPTYSETOTAL
NIELEMENTOUPLEQUALHE
TUNCLVWXILSYRVUZZDEN
WWWIXYZACTIHAACAMDG
BCOFEMATHITNYNTXURDT
ENUMERALPIGEHILIBEH
SPHERESLJLVLKLSOMNOEP
GREATEREQQYERSVNPPOINT

43

38
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There are at least 105 words pertaining to mathematics in the puzzle below. They may be written up, down, across, diagonally, backward, or forward. Some words may overlap.

M T C A R T B U S E I R E S D I G I T C
C A B A S H C D L P A R A B O L A D D O
I X T R I G O N O M E T R Y P S E V E N
G I E H N I N E P A R A L L E L R I N G
O O H M E E T A E X A I N T E G E R F R
L M R P I M R P E I D N U A H P A R G U
A L G E B R A A L M I V M N P F I R E E
L M M R Z E P T L U U E B G U O R O O N
O O I P E T O R I M S R E E C O I W M C
G D N E O V S U P C S S S R N O R T N E E
A U F N Y S I O S I S E H T O P Y H T C
R L I D N T T F E T Y P Y M R C R Y R A
I U N I O N I U L R M L A E D I A X Y L
T S I C D O V N L I M A T R I X E B U C
H B T U E I E C U A E N H O N P N U E U
M A E L C T R T P N T E R E A T I M I L
E S E A I A E I U G R E E H T O L P U U
T E P R M U H O O L I M E T E R A U Q S
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MATHEMATICANS' HIDDEN NAMES

The names of 41 mathematicians are hidden in the array of letters. They are arranged horizontally, vertically, diagonally--forward and backward.

HALF SI OLAG SODIRICHLET OF
TS LARE AIXIADIOPHANTUSBE
IPOASTD NENDLLINESAIDSEW
RABACUSESEHECONIERLUEE
EROMASTRMKGCUSTONADRACI
LEIBNIZOEICNERYRETIRGRE
LOCITIIRRAHAALACSAPELUR
OBOEEOVERTEXCARSOELILAGS
HLBERTENFESAREIPANNRUST
EYEEDPOSTELKNAREIRUOFER
TRACETTRDIRKOHERMITTINA
AEMOWERONPAMUMINIMOSETS
LTHEABOOLEMOAKCICLKUELES
USNILAT THEIGHTICPHIVDNA
TESTLBE NIRMIESUAGALUCD
SVECIBUCRCAYLEYSCASYCEI
OLSOSAEIAHCOMPOSITESYOR
PYTHAGORASCAKOSLOPETAAA
RSRIBEGHTALINPNALVIXXRM
INASOITSUNATNOMOIGERIEY
MPURBOMCLAVIUSBEINNEDH
ERSBOTHE SIMPLETINNDAIR
VESFIYEQUATELESSFSUMEOS
## MATHEMATICIAN'S HIDDEN NAMES

<table>
<thead>
<tr>
<th>Mathematician</th>
<th>Term</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aiken</td>
<td>Gauss</td>
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<tr>
<td>Archimedes</td>
<td>Gibbs</td>
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<td>Harriot</td>
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<tr>
<td>Boole</td>
<td>Hilbert</td>
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<td>Pascal</td>
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<td>Venn</td>
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<td>Viete</td>
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<tr>
<td>Fourier</td>
<td>Wallis</td>
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<td>Galileo</td>
<td>Weierstrass</td>
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<tr>
<td>Galois</td>
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Some students may also discover mathematical terms hidden in the puzzle.
METRIC MEASUREMENTS

RULER MASTER AND METRIC PREFIXES

**Metric System Prefixes:**

<table>
<thead>
<tr>
<th>Prefix</th>
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<tr>
<td>MILLI</td>
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</tr>
<tr>
<td>CENTI</td>
<td>0.01</td>
</tr>
<tr>
<td>DECI</td>
<td>0.1</td>
</tr>
<tr>
<td>DEKA</td>
<td>10</td>
</tr>
<tr>
<td>HECTO</td>
<td>100</td>
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<td>KILO</td>
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**Metric Units of Length:**

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<th>Equivalent</th>
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<tbody>
<tr>
<td>1000 MILLIMETERS</td>
<td>1 METER</td>
</tr>
<tr>
<td>100 CENTIMETERS</td>
<td>1 METER</td>
</tr>
<tr>
<td>10 DECIMETERS</td>
<td>1 METER</td>
</tr>
<tr>
<td>10 METERS</td>
<td>1 DEKAMETER</td>
</tr>
<tr>
<td>100 METERS</td>
<td>1 HECTOMETER</td>
</tr>
<tr>
<td>1000 METERS</td>
<td>1 KILOMETER</td>
</tr>
</tbody>
</table>

**Useful Metric Measures**

**Length**

<table>
<thead>
<tr>
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<th>Equivalent</th>
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<tr>
<td>100 CENTIMETERS</td>
<td>1 M</td>
</tr>
<tr>
<td>10 DECIMETERS</td>
<td>1 M</td>
</tr>
<tr>
<td>10 METERS</td>
<td>1 DEKAMETER</td>
</tr>
<tr>
<td>1000 M</td>
<td>1 KILOMETER</td>
</tr>
</tbody>
</table>

**Mass**

<table>
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<tbody>
<tr>
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</tr>
<tr>
<td>1000 CENTIMETERS</td>
<td>1 M</td>
</tr>
<tr>
<td>1000 DECIMETERS</td>
<td>1 M</td>
</tr>
<tr>
<td>1000 METERS</td>
<td>1 DEKAMETER</td>
</tr>
<tr>
<td>1000 M</td>
<td>1 KILOMETER</td>
</tr>
</tbody>
</table>

**Land Area**

<table>
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<th>Equivalent</th>
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<tr>
<td>100 SQUARE METERS</td>
<td>1 ARE</td>
</tr>
<tr>
<td>100 A</td>
<td>1 HECTARE</td>
</tr>
<tr>
<td>100 HA</td>
<td>1 SQUARE KILOMETER</td>
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</tbody>
</table>

**Volume**

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<tbody>
<tr>
<td>1000 MILLILITERS</td>
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</tr>
<tr>
<td>1000 CUBIC CENTIMETERS</td>
<td>1 L</td>
</tr>
<tr>
<td>1000 CM</td>
<td>1 CUBIC DECIMETER</td>
</tr>
<tr>
<td>1000 L</td>
<td>1 CUBIC METER</td>
</tr>
<tr>
<td>1000 DM</td>
<td>1 M</td>
</tr>
</tbody>
</table>
MIXED MEASUREMENTS

There are at least sixty measurement terms listed below. They may be written up, down, across, diagonally, backward, or forward. Some words may overlap.

DEFATHOMCORDFMDMONTH
RODOORMILCELAANROEOH
AMZYGTLZEIATCRUOHMAR
MILEEALLRMYARGOTCEHS
ILGEMSISRINUIEAPINTEE
LOFTAILLAOLDECADEECP
LTEAUMIIBLLATEUHRDRTO
IAASHRTELILICEDIMEXOW
MCALORIEMAEELLOTGRME
ETRAUQERAGRNYYEAREER
THRENOPNURETEMICEDETM
EMMICRONHPTIONTHEYEOW
RIENEWTONEYLINKAKNRT
ENODRAAMICIINCHIOTT
DUPNCTMNCKOTVELMNILA
OTSATTPAKOOSECONDPILW
LEOHMAEIELARGILLMAO
LBALERRDLTRHRERTZRAL
AMJOULEASPANCENTURYI
ROGROSSRCMYRIAMETERK
| Mixed Measurements | ACRE | AMPERES | ARE | BALE | BARREL | BUSHEL | CALORIE | CARAT | CELSIUS | CENT | CENTILITER | CENTURY | CHAIN | CORD | DECADE | DECAGRAM | DECILITER | DECIMETER | DEGREE | DIME | DOLLAR | DOZEN | DRAM | ERG | FAHRENHEIT | FATHOM | FEET | KILOGRAM | KILOMETER | KILOWATT | DEGREE | FOOT | GALLON | GANGLON | GROSS | HAND | HECTOGRAM | HECTOMETER | HERTZ | HERTZ | HORSEPOWER | HOUR | HUNDREDTHS | INCH | LINK | LITER | METER | MICRON | MIL | MILE | MILL | MILLIER | MILLILITERS | MILLIGRAM | MILLILITER | MILLIMETER | MINUTE | MONTH | MYRIAMETER | SECOND | SPAN | TON | VOLT | WATT | YEAR |
MIXED MEASUREMENTS

There are at least thirty-five measurement terms listed below either horizontally or vertically. Circle as many of these terms as you can find.

<p>| M | U | D | B | C | E | N | T | V | B | K | I | C | S | E | C | O | N | D |
| G | C | E | L | H | D | M | O | N | J | F | D | E | G | K | H | J | L | M |
| S | E | C | N | B | M | I | P | D | E | C | A | L | W | A | T | T | Q | R | S |
| E | D | I | M | E | R | L | U | R | K | E | Y | S | S | R | T | T | A | V | D |
| C | W | M | V | X | Y | L | G | M | I | I | L | M | I | E | T | E | R | Y | E |
| B | W | E | E | K | Z | C | R | A | L | L | T | U | I | M | B | G | A | R | G |
| R | E | T | D | O | G | E | A | B | O | R | H | S | L | I | T | E | R | H | R |
| T | L | E | S | D | T | N | M | R | G | R | R | R | L | L | C | E | N | E | E |
| O | L | R | C | E | N | T | I | G | R | A | M | S | I | L | X | C | A | C | E |
| N | T | C | h | K | R | U | L | H | A | A | H | A | H | G | I | D | E | B | T | T |
| N | W | E | L | A | L | R | L | O | M | Z | O | A | R | E | O | N | S | O | E |
| E | H | N | O | M | M | Y | I | H | B | O | Z | O | A | H | L | T | C | M | R |
| B | I | H | R | E | K | L | O | W | A | T | T | M | A | L | I | H | E | X |
| K | M | O | N | T | H | H | I | X | D | E | C | I | G | R | A | M | O | T | H |
| I | I | U | Y | E | H | I | T | E | R | E | Y | O | U | R | E | O | E | I |
| L | N | R | Z | R | B | H | E | T | A | R | E | C | D | B | T | L | R | A |
| O | U | X | M | E | T | E | R | H | Y | K | I | L | O | H | Y | E | A | R | G |
| S | T | A | N | D | Y | C | E | N | T | I | L | I | T | E | R | R | H | B | A |
| S | E | R | D | E | C | A | D | E | C | A | T | M | I | C | R | O | N | A | I |
| O | K | K | K | I | L | O | M | E | T | E | R | N | O | P | E | D | E | C | I | N |</p>
<table>
<thead>
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<th>MIXED MEASUREMENTS</th>
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<tbody>
<tr>
<td>ARE KILOGRAM</td>
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<tr>
<td>AREA KILOMETER</td>
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<tr>
<td>KILOWATT</td>
</tr>
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<tr>
<td>CENT LITER</td>
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<td>DECADE MILLIGRAM</td>
</tr>
<tr>
<td>DECIGRAM MILLILITER</td>
</tr>
<tr>
<td>DECIMETER MIN</td>
</tr>
<tr>
<td>DEGREE MONTH</td>
</tr>
<tr>
<td>DEKAMETER MIN</td>
</tr>
<tr>
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</tr>
<tr>
<td>EON TON</td>
</tr>
<tr>
<td>GRAM TONNE</td>
</tr>
<tr>
<td>HECTARE WATT</td>
</tr>
<tr>
<td>HECTOMETER WEEK</td>
</tr>
<tr>
<td>HOUR YEAR</td>
</tr>
</tbody>
</table>
LINEAR MEASUREMENT

Using the code below, shade in the squares with the proper designs.

Examples:

1000 m = 1 km =

.001001 m = .001 m + .000001 m =

1 mm + 1 micron =

1 km 1 hm 1 dkm 1 m 1 dm 1 cm 1 mm 1 micron

<table>
<thead>
<tr>
<th>1000m</th>
<th>100m</th>
<th>1000m</th>
<th>.11m</th>
<th>100m</th>
<th>1000m</th>
<th>.11m</th>
<th>100m</th>
<th>1000m</th>
<th>100m</th>
</tr>
</thead>
<tbody>
<tr>
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<td>1m</td>
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<td>10m</td>
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<td>.011m</td>
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<tr>
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<td>100m</td>
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<tr>
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<td>0</td>
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<td>1m</td>
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<td>.001001m</td>
<td>10m</td>
<td>10m</td>
<td>1m</td>
</tr>
</tbody>
</table>
OPERATIONS WITH MASS MEASUREMENTS

CONNECT THE NUMBER OF EACH PROBLEM WITH THE ANSWER FOR THAT PROBLEM.

1. 9 g + 28 g = ___ g
2. 73 t - 46 t = ___ t
3. 15 kg + 19 kg = ___ kg
4. 8 g + 900 mg = ___ g
5. 8 g + 900 mg = ___ mg
6. 3 g - 2.9 g = ___ mg
7. 256 mg + 344 mg = ___ g
8. 454 g + 45 g = ___ kg
9. 1 kg - 800 g = ___ kg
10. 1 g - 730 mg = ___ g
11. .1 g + 100 mg = ___ mg
12. 2345 mg + 1055 mg = ___ g
13. 1 g - 911 mg = ___ mg
14. 59 g + 1000 mg = ___ g
15. 3 kg + 700 g = ___ kg
16. .1 t - 660 kg = ___ kg
17. .042 g + 8 mg = ___ mg
18. 13 mg + 7 mg = ___ g

--
VOLUME

Cut out the squares. Fit them together so that the touching edges name the same volumes.

<table>
<thead>
<tr>
<th>.56 m³</th>
<th>.42 ml</th>
<th>2 m³</th>
<th>560 ml</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.4 m³</td>
<td>340 ml</td>
<td>1 cm³</td>
<td>60 l</td>
</tr>
<tr>
<td>0.034 l</td>
<td>7 l</td>
<td>1 l</td>
<td>1000 l</td>
</tr>
<tr>
<td>1 ml</td>
<td>56 cm³</td>
<td>42 cm³</td>
<td>2 dm³</td>
</tr>
<tr>
<td>56 ml</td>
<td>370 ml</td>
<td>1 cm³</td>
<td>56 ml</td>
</tr>
<tr>
<td>2 ml</td>
<td>18 ml</td>
<td>6 l</td>
<td>1 m³</td>
</tr>
<tr>
<td>60 ml</td>
<td>2000 l</td>
<td>34 l</td>
<td>1000 ml</td>
</tr>
<tr>
<td>0.002 l</td>
<td>1 m³</td>
<td>2000 l</td>
<td>37 l</td>
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</table>

<table>
<thead>
<tr>
<th>1000 dm³</th>
<th>5.6 ml</th>
<th>37 ml</th>
<th>2 cm³</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.7 ml</td>
<td>3400 l</td>
<td>3.4 ml</td>
<td>2000 ml</td>
</tr>
<tr>
<td>34000 l</td>
<td>.6 ml</td>
<td>.34 l</td>
<td>3400 cm³</td>
</tr>
<tr>
<td>.56 l</td>
<td>42 l</td>
<td>1 l</td>
<td>.0056 l</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>4.2 l</th>
<th>2000 cm³</th>
<th>5600 ml</th>
<th>1 dm³</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 m³</td>
<td>2 l</td>
<td>70 ml</td>
<td>34000 ml</td>
</tr>
<tr>
<td>34 cm³</td>
<td>2 cm³</td>
<td>3.4 l</td>
<td></td>
</tr>
<tr>
<td>560 l</td>
<td>3.7 l</td>
<td>.056 l</td>
<td>5.6 l</td>
</tr>
</tbody>
</table>
CODED COMPUTATIONS

Puzzles constructed by the coding or suppression of digits in an arithmetical calculation require no more than attention to obvious numerical facts. Keep track of clues and conclusions in an orderly way.

In a certain multiplication problem each digit from 0 to 9 was replaced by a different letter, yielding the following coded calculation.

\[
\begin{align*}
\text{A} & \text{L} \text{E} \\
\text{R} & \text{U} \text{M} \\
\text{W} & \text{I} \text{N} \\
\text{E} & \text{W} \text{W} \text{E} \\
\text{E} & \text{R} \text{M} \text{P} \text{N} \text{E}
\end{align*}
\]

Find the numbers that correspond to each letter. Write in a row the different letters appearing in the problem:

\[
\text{A} \text{L} \text{E} \text{R} \text{U} \text{M} \text{W} \text{I} \text{N} \text{P}
\]

Over each letter write its numerical equivalent when you discover it. Under the letters record clues and guesses. In problems of this sort, the digits 0 and 1 can often be found by simple inspection. For instance, 0 can never occur as the left-most digit of an integer, and when any number is multiplied by zero the result consists only of zeroes. When any number is multiplied by 1, the result is that number itself. In the present problem, you can identify 0 by observing that \(N + L = N\) with nothing carried over from the column on the right. Therefore, \(L\) must be zero.
In the search for \(1\) eliminate \(R, U,\) and \(M\) since none of these, as multipliers, reproduce \(ALE.\) Also \(E\) cannot be \(1\) since \(U\) times \(E\) does not have a product ending in \(U.\) No further clues to the identity of \(1.\)

Now the partial product \(WUL\) ends in \(L,\) which is \(0.\) Therefore, one of the two letters \(U\) and \(E\) must be \(5.\) Since \(M \times E\) and \(R \times E\) are numbers ending in \(E,\) \(E\) must be \(5.\)

\[
\begin{array}{c}
ALE \\
RUM \\
\hline
WINE \\
\hline
WUWL \\
\hline
EWW \ \\
\hline
ERMPNE \\
\hline
A05 \\
\hline
RUM \\
\hline
WINS \\
\hline
WUWO \\
\hline
5WW5 \\
\hline
5RMNP5
\end{array}
\]

Notice: \(R \times A05 = 5WW5.\) Therefore, \(R \times 5 = W5\) and \(R \times A = 5W.\) Check multiplication facts: \(9 \times 5 = 45\) and \(9 \times 6 = 54.\) Hence \(R = 9, A = 6,\) and \(W = 4.\)

\[
\begin{array}{c}
605 \\
\hline
9UM \\
\hline
4IN5 \\
\hline
4U40 \\
\hline
5445 \\
\hline
59MPN5
\end{array}
\]

Notice that \(U\) must be even and \(M\) must be odd. Also \(M \times 6\) must be \(42,\) so \(M\) is either \(7\) or \(8.\) Since \(M\) is odd, then \(M = 7.\) Replace \(M\) with \(7\) and multiply to find \(I\) and \(N.\) Thus, \(N = 3\) and \(I = 2.\)
In conclusion, \( U \) and \( P \) must be 1 and 8; \( U \) is an even number so \( U = 8 \). Therefore, \( P = 1 \). Substitute the numerical equivalents into the problem and check the answers by multiplication.

\[
\begin{align*}
6059 & \quad 7423 \\
\text{ALERU} & \quad \text{MWINP}
\end{align*}
\]

\[
\begin{align*}
605 & \\
987 & \\
4235 & \\
4840 & \\
5445 & \\
597135 &
\end{align*}
\]

1) Addition

\[
\begin{array}{c}
X X X X X \\
Y Y Y Y Y \\
Z Z Z Z Z \\
Y X X X Z \\
\end{array}
\]

2) Multiplication

\[
\begin{array}{c}
P N X \\
N X \\
R N X \\
N X S \\
Z P N X \\
\end{array}
\]

3) Division

\[
\begin{array}{c}
H I L \\
I L P H I L \\
I L \\
T I \\
L S \\
H I L \\
H I L \\
\end{array}
\]

4) Division

\[
\begin{array}{c}
Y F Y \\
A Y N E L L Y \\
N L Y \\
P P L \\
P N H \\
N L Y \\
N L Y \\
\end{array}
\]

5) Addition

\[
\begin{array}{c}
H I T L E R \\
G O E R I N G \\
H T T L L H H H \\
T E H A W \\
\end{array}
\]

6) Addition

\[
\begin{array}{c}
A H A H A \\
T E H A W \\
\end{array}
\]
7) Addition
   TEN
   TEN
   FORTY
   SIXTY

8) Addition
   ALGE
   BRA
   IS
   GREAT

9) Subtraction
   SEVEN
   NINE
   EIGHT
   (Two possible solutions)

10) Division
    A B J
    E C A | F D B H J
    C G G.
    A G A H
    A A E A
    K D D J
    K D B H
    A J

Puzzles numbered 11 -- 18 can be deciphered with no further clues. However, when the letters are arranged in the order of the numbers they represent, they spell out a phrase.

11) USP
    RAP
    OHEP
    SSTS
    OUNR
    OEEEAP

12) ESP
    NRA
    TALP
    NPYI
    ESP
    PYILP

Go
SUPPRESSED DIGITS

Can you restore the unknown digits represented by stars?

1) In a certain problem in long division; every digit except 7 was suppressed. Restore the missing digits. Start by thinking of all the multiplication facts when products end in 7. They are 1 x 7 and 3 x 9, since all 7's are known, use 3 x 9. Hence the first digit in the quotient must be 3 or 9 and the last digit in the divisor must be the other.

   * 7 *
  * * 7 7
  * 7 *
  * 7 *
  * * *
  * * *

2) In the following example of multiplication, most of the digits have been suppressed. Those that remain are not necessarily all of the 4's, 5's, and 6's in the example. Notice that the last digit in the multiplier has to be 0 or 1 because the product is a three digit number.

   6 * *
   * * *
   * * *
   * * *
   * * *
   * 5 * 5
   * * 5 * 4

3) * 7 *
   * 6 *
   * * 3 *
   * * 6 *
   3 * * *
   * 1 * * 1 *

4) * 4 *
   * 9 8 *
   * 7 *
   * * 4 *
   * 7 *
   * 3 *
   * * *
5) \[ \frac{9}{3} \]
   \[ \begin{array}{c}
   \times 4 \times 9 \\
   \times 1 \\
   \times \times \\
   \times 1 \\
   \times 2 \times 5 \\
   \times 2 \times 5
   \end{array} \]

6) \[ \frac{4}{3} \]
   \[ \begin{array}{c}
   \times \times 7 \\
   \times \times 3 \times
   \end{array} \]

7) \[ \frac{\times \times \times \times}{\times \times \times \times \times} \]
   \[ \begin{array}{c}
   \times \times \times \times \times \\
   \times \times \times \\
   \times \times \\
   \times \times \\
   \times \times \\
   \times \times \\
   \times \times
   \end{array} \]

8) Addition
   \[ \begin{array}{c}
   \times 2 \times 2 \\
   \times 1 \times 5 \times 1_\times \\
   \times 3 \times 4 \times 9
   \end{array} \]

9) Addition
   \[ \begin{array}{c}
   1 \times 1 \times 3 \\
   6 \times 4 \\
   1 \times 4 \times \\
   \times 2 \times 6 \\
   \times 4 \times 1 \times \times \times \times \times \\
   \times 9 \times 1
   \end{array} \]

10) Multiplication
    \[ \times 7 \]

11) Subtraction
    \[ \begin{array}{c}
    6 \times 3 \times 5 \\
    8 \times 2 \times \\
    4 \times 7
    \end{array} \]

12) Subtraction
    \[ \begin{array}{c}
    4 \times 2 \times 2 \\
    \times 3 \times 5 \times \times \times \\
    \times 1 \times 2 \times 1
    \end{array} \]

13) Multiplication
    \[ \times 7 \]

14) ** ** **
    \[ \begin{array}{c}
    \times 6 \times 6 \times \times \times \times \times \\
    \times 6 \\
    \times 9 \times \\
    \times \times \times
    \end{array} \]

58

63
MAGIC SQUARES: SHEET 1

This array of numbers is a 3 x 3 Magic Square. Show that the sum of the numbers in each row, column, and diagonal is 15.

<table>
<thead>
<tr>
<th>8</th>
<th>1</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>9</td>
<td>2</td>
</tr>
</tbody>
</table>

8 1 6 8 3 4 8 6
3 5 7 1 5 9 5 5
4 9 2 6 7 2 2 4

Complete these magic squares using the same numbers 1 through 9. Be sure each row, column, and diagonal adds to 15.

<table>
<thead>
<tr>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>9</td>
</tr>
<tr>
<td>9</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>8</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
</tbody>
</table>

Can you find another magic square using the same numbers 1 through 9?
MAGIC SQUARES: SHEET 2

This array of numbers is a 3 x 3 Magic Square. Show that the sum of the numbers in each row, column, and diagonal is 12.

<table>
<thead>
<tr>
<th>7</th>
<th>0</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td>1</td>
</tr>
</tbody>
</table>

Perform the indicated operations on each number in the magic square above. Enter the results below. Then see if each new array is also a magic square.

<table>
<thead>
<tr>
<th>Add 9</th>
<th>Multiply by 12</th>
<th>Add 1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Multiply by 10</th>
<th>Add 4</th>
<th>Multiply by 7</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

If the same number is added to each entry in a magic square, is the result another magic square? ____________

If each entry in a magic square is multiplied by the same number, is the result another magic square? ____________
MAGIC SQUARES: SHEET 3

This array of numbers is a 3 x 3 Magic Square. Show that the sum of numbers in each row, column, and diagonal is 21.

\[
\begin{array}{ccc}
10 & 3 & 8 \\
5 & 7 & 9 \\
6 & 11 & 4 \\
\end{array}
\]

Perform the indicated operations on each number in the Magic Square above. Enter the results below. Then see if each new array is also a Magic Square.

Add 3

Subtract 3

Add \( \frac{3}{4} \)

Divide by 2

Subtract \( \frac{2}{3} \)

Subtract \( \frac{3}{4} \)
MAGIC SQUARES: SHEET 4

This array of numbers is a 3 x 3 Magic Square. Show that the sum of numbers in each row, column, and diagonal is 45.

<table>
<thead>
<tr>
<th>26</th>
<th>1</th>
<th>18</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>15</td>
<td>23</td>
</tr>
<tr>
<td>12</td>
<td>29</td>
<td>4</td>
</tr>
</tbody>
</table>

26 1 18 26 7 12 26 18
7 15 23 1 15 29 15 15
12 29 4 18 23 4 4 12

Perform the indicated operations on each number in the Magic Square above. Enter the results below. Then see if each new array is also a Magic Square.

Add 9

Divide by 2

Add $\frac{1}{2}$

Divide by 3

Subtract $\frac{3}{4}$

Subtract $\frac{2}{3}$
MAGIC SQUARES: SHEET 5

Fill in the blank spaces in the magic square using each of the following numbers: 4, 6, 7, 8, 10, 11, 12, 13, 14, 15, so that the sum of the integers in each column, row, and diagonal is 34.

\[
\begin{array}{ccc}
1 & & \\
& 5 & \\
2 & 16 & 9 \\
& 3 & \\
\end{array}
\]

Fill in the blank spaces in the magic squares so that the sum of the integers in each column, row, and diagonal is 34.

\[
\begin{array}{ccc}
1 & & \\
8 & 4 & 9 \\
11 & & \\
\end{array}
\]

\[
\begin{array}{ccc}
2 & 15 & 14 \\
 & 5 & \\
 & 4 & \\
\end{array}
\]
MAGIC SQUARES: SHEET 6

FILL IN THE BLANK SPACES IN THE MAGIC SQUARES SO THAT THE SUM OF THE INTEGERS IN EACH COLUMN, ROW, AND DIAGONAL IS 34.

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>11</td>
<td>10</td>
<td>8</td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

FILL IN THE BLANK SPACES IN THE MAGIC SQUARES SO THAT THE SUM OF THE INTEGERS IN EACH COLUMN, ROW, AND DIAGONAL IS 55.

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>7</td>
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<td></td>
</tr>
<tr>
<td>25</td>
<td></td>
<td></td>
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<tr>
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<td>13</td>
<td>21</td>
<td>9</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
GRAPHING

Ordered number pairs are used to describe points in a plane. Start with two signed (+, -) number lines, called coordinate axes, drawn at right angles to each other. The horizontal (east-west) line is called the x-axis. The vertical (north-south) line is called the y-axis. In a coordinate plane, the point 0 at which the two axes intersect (cross) is called the origin.

The x-axis and the y-axis divide the plane into four regions called quadrants. These quadrants are numbered I, II, III, and IV in a counterclockwise order.

It is understood in all number pairs that the first number always represents a distance along the x-axis; the second number always represents a distance along the y-axis. For this reason, it is necessary not to interchange the numbers in an ordered pair.

Distances measured to the right of y-axis, along the x-axis or along a line parallel to the x-axis, are considered to be positive (+); distances measured to the left of the y-axis are considered to be negative (-). Distances measured upward from the x-axis, along the y-axis or along a line parallel to the y-axis, are considered to be positive (+); distances measured downward from the x-axis are considered to be negative (-). All numbers are signed numbers. If there is no sign in front of the number, then it is understood to be positive.

The distance of a point from the y-axis, measured either along the x-axis or along a line parallel to it, is called the x-coordinate or abscissa. The distance of a point from the
X-axis, measured either along the y-axis or along a line parallel to it, is called the y-coordinate or ordinate. The two numbers which are associated with any particular point, the abscissa and ordinate of the point, are called the coordinates of the point.

To graph the point P, represented by the ordered pair (2,3), start at the origin and move 2 units (squares) to the right along the x-axis, then move 3 units (squares) upward in a direction parallel to the y-axis.

(-,+) Quadrant II \(\uparrow\) Quadrant I (+,+)

\[R(5,3)\] \[P(2,3)\]

(-,-) Quadrant III \(-\) Quadrant IV (+,-)

To graph R, represented by the ordered pair (-5,3), start at the origin and move 5 units to the left along the x-axis, then move 3 units upward in a direction parallel to the y-axis.
To graph point S, represented by the ordered pair (-8, -5), start at the origin and move 8 units to the left along the x-axis, then move 5 units downward in a direction parallel to the y-axis.

To graph point T, represented by the ordered pair (6, -5), start at the origin and move 6 units to the right along the x-axis, then move 5 units downward in a direction parallel to the y-axis.

To graph the exercises, plot the first point, plot the second point, and then connect the two as you would in a dot to dot drawing. In the exercise start at the top of the first column of the first problem and read down. Then go to the second column of the first problem and read down. Continue in the same manner for all problems. Continue plotting the points but be careful to connect the points as you go. If you wait to connect them until you finish graphing all of the points, you may connect the wrong points. If you plot the points and connect them correctly, each graph will form a picture.
<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
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<td>(3, 2)</td>
<td>(-1, 8)</td>
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<tr>
<td></td>
<td>(3, 4)</td>
<td>(-2, 7)</td>
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<tr>
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<tr>
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<tr>
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<tr>
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</tr>
<tr>
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</tr>
</tbody>
</table>

2) Start new line

3) Start new line

4) Start new line

73
<p>| | | |</p>
<table>
<thead>
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<th></th>
<th></th>
<th></th>
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<tbody>
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<td>4) (2, -1)</td>
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<tr>
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</tr>
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<td>(0, 15)</td>
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<tr>
<td>(-2, 12)</td>
<td>(3, -6)</td>
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<td>(9, -3)</td>
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<td>(-12, -11)</td>
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<td>(-12, 3)</td>
<td>START NEW LINE</td>
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</tr>
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<td>(0, -3)</td>
</tr>
</tbody>
</table>

| 7) | (6, 14) | (2, 8) | (1, 4) |
|    | (4, 13) | (2, 4) | (1, 2) |
|    | (2, 9) | (8, 1) | (1, 5) |
|    | (15, 10) | (9, -5) | (-5, 2) |
|    | (1, 9) | (2, 2) | (-7, 14) |
|    | (2, 9) | (2, 4) | (1, 8) |
|    | (2, 8) | (2, -1) | (1, 5) |
|    | (10, 14) | (1, -1) | (1, 9) |
|    | (8, 2) | (1, 2) | (-1, 13) |
|    | (2, 5) | (-6, -5) | (-3, 14) |
|    |   | (-5, 1) |   |
GEOMETRIC SHAPES

Definitions

CUBE is a solid bounded by six planes, with its twelve edges all equal and its face angles all right angles (dice).

PARALLELOGRAM is a quadrilateral with its opposite sides parallel.

POLYGON is a simple closed curve consisting of three or more segments, each pair of adjacent segments having a common end point.

PRISM is a solid with two congruent, parallel faces which are polygons and with the remaining faces parallelograms.

PRISM WITH TRAPEZOIDAL BASES is a prism whose two congruent, parallel faces are trapezoids.

PRISM WITH TRIANGULAR BASES is a prism whose two congruent, parallel faces are triangles.

QUADRILATERAL is a polygon with four sides.

RECTANGLE is a quadrilateral whose angles are all right angles. All squares are rectangles. All rectangles are not squares.

RHOMBUS is a quadrilateral with four equal sides.

SQUARE is a quadrilateral with equal sides and equal angles.

TRAPEZOID is a quadrilateral with exactly two parallel sides.

TRIANGLE is a polygon with three sides.
GEOMETRIC SHAPES

How many squares?

How many rectangles?

How many parallelograms?

How many quadrilaterals?

How many rhombuses?

How many triangles?

How many trapezoids?
GEOMETRIC SHAPES

How many rectangles?

How many triangles?

How many parallelograms?

How many trapezoids?

How many quadrilaterals?
GEOMETRIC SHAPES

---

How many squares?

How many rectangles?

How many parallelograms?

How many triangles?

How many cubes?

How many prisms with trapezoidal bases?

How many prisms?

How many trapezoids?

How many triangular prisms?
SPATIAL PERCEPTION

These are square sheets of paper. They are to be folded flat along the dotted lines. How will each sheet look after it is folded?
SPATIAL PERCEPTION

Here is a block on which someone has painted numbers. 1 is on the back face, 2 is on the left face, 3 is on the front face, 4 is on the right face, 5 is on the top face, 6 is on the bottom face.

Someone has moved the block. 2 is on the bottom face, 3 is on the left face. The front face is covered. What number is on

A. the top face?
B. the right face?
C. the front face?
P. the back face?
SPATIAL PERCEPTION

Someone stacked sugar cubes to make this large cube. Then this person painted top and bottom of the large cube red and all of the sides of the large cube blue.

1. How many sugar cubes are there?
2. How many have a red side?
3. How many have one blue side?
4. How many have two blue sides?
5. How many are all white?
6. How many are red, white, and blue?
How many triangles can you find in the diagram?

Draw this figure with one single stroke of the pencil. You may not retrace lines previously drawn.
WORD PROBLEMS

Problem Solving

Remember! Take time to read the problem with care to decide:

1. What is given.
2. What is to be found.
3. What is needed to solve the problem.
4. How you plan to solve the problem.

Examples:

1. Exact Information (Just Right Amount)

<table>
<thead>
<tr>
<th>Color Print &amp; Devel.</th>
<th>12 Exposure Roll</th>
<th>$1.79</th>
</tr>
</thead>
<tbody>
<tr>
<td>Color Print &amp; Devel.</td>
<td>20 Exposure Roll</td>
<td>$2.79</td>
</tr>
<tr>
<td>Color Print &amp; Devel.</td>
<td>24 Exposure Roll</td>
<td>$3.29</td>
</tr>
</tbody>
</table>

If David needs one 12 exposure roll, two 24 exposure rolls, and three 20 exposure rolls, how much will it cost him? How many pictures can he take from the rolls that he buys?

One roll of 12 exposure $1.79
Two rolls of 24 exposures
2 x $3.29 = $6.58
Three rolls of 20 exposures
3 x $2.79 = $8.37
It will cost David $16.74.
He will be able to take 120 pictures from the six rolls of film.

One roll of 12 exposure
12
Two rolls of 24 exposure 2 x 24
48
Three rolls of 20 exposure 3 x 20
60
120
2. **Extraneous Information (too much)**

Sue has $2.25. She found $5.00 on the way to the store. She spent $1.50 for a notebook, 25¢ for an eraser, 50¢ for a pen. How much money did she spend?

She spent:

- **Notebook**: $1.50
- **Eraser**: 25¢
- **Pen**: 50¢

Knowing that Sue had $2.25 and that she found $5.00 is unneeded information for solving the problem. Be sure to shift all the known facts to find only what is needed.

3. **Insufficient Information (not enough)**

   A. Bill has 17 marbles in 3 bags. He found 2 more bags of marbles. How many marbles does he have now?

   This problem cannot be solved until it is determined how many marbles are in each of the 2 bags that Bill found. If you find a problem with insufficient information, state what piece of information is needed to solve the problem.

   B. Mary is 5 years older than her brother. How old is Mary?

   It is necessary to know how old Mary's brother is before you can determine Mary's age.
4. **No Numerical Solution**

William is going to buy a new car. William's father said that he must know all the expenses connected with owning a car before William has permission to buy the car. Besides the car payment, list some additional expenses for owning a car.

This type of problem does not have a numerical solution. However, it is a practical problem which needs solving.
ADVERTISING PUZZLES
(DISREGARD SALES TAX)

1. David went to the store for his dad. He bought a sink faucet, 3 ice breakers, and a hammer. How much did David spend?

2. Mr. Woods had purchased 2 vise-grips, 4 gluematic pens, and a propane blow torch. How much more than $10.00 does he need?

3. Ann bought 4 tubes of liquid steel, a vinyl carpet runner, and a florescent light fixture. She returned 2 tubes of liquid steel because they had holes in them. If the store refunded the full cost, how much money did she receive?

4. If Mrs. Mays bought a powerlock rule and a vise-grip wrench for her husband and paid the clerk with a twenty dollar bill, how much change should she receive?

5. Sarah wants to buy some ice breakers. How much will they cost?
ADVERTISING PUZZLES

(DISREGARD SALES TAX)

MARY NEEDED FIVE POUNDS OF CARROTS, 12 POUNDS OF RUSSET POTATOES, AND 2 BUNDLES OF ONIONS TO MAKE POTATO SALAD FOR THE SCHOOL PICNIC. THE BAGS OF GROCERIES CANNOT BE SPLIT BUT THE BUNDLES OF ONIONS ARE SEPARATE. WHAT WILL THE COST OF THESE ITEMS BE?

JANE AND JOE ARE PLANNING A HALLOWEEN PARTY. THEY WANT 7 POUNDS OF APPLES AND 7 POUNDS OF ORANGES. REMEMBER THAT BAGS OF GROCERIES CANNOT BE SPLIT. WHAT WILL IT COST TO GET THE FRUIT THAT THEY WANT?

MRS. WATSON IS SERVING SNACKS FOR HER WEIGHT-WATCHERS CLUB MEETING. SHE WANTS ONE POUND OF CARROTS, 2 BROCCOLI, 5 POUNDS OF ORANGES, 2 POUNDS OF APPLES, AND 2 HEADS OF LETTUCE. HOW MUCH CHANGE WILL SHE GET FROM A TEN DOLLAR BILL?

MRS. MARTIN HAS 25 CHILDREN IN HER KINDERGARTEN CLASS. SHE IS GOING TO BUY ONE BAG OF RUSSET POTATOES FOR THEM TO GROW POTATO PLANTS IN CLASS. HOW MANY POUNDS ARE THERE IN A BAG OF POTATOES AT THIS STORE?
1) Bill and Sue are planning to get married and have $500 to spend on a washer, dryer, and refrigerator. Since they do not need them now, which should they buy to save the most money?

2) Sue's mother decides that she will give Sue $275 to spend now. Bill's father says that he will give them the rest of the money that they need to buy all three now. How much will he give them?

3) What is the total cost of all three?

4) When they go to buy them, the salesman says that there will be a $35 charge for storage until they are married and need the appliances. Sue and Bill discuss whether they should buy the appliances now, or wait. The sale will be over by the time that they are married. How much will it cost to buy now and store until needed?

5) What will it cost them to wait and buy after the sale is off?

6) What should they do?
1) Jack and Jill are on the track team in high school. The coach tells them that they should practice as much as possible. Therefore, they decide to purchase their own jogging suits to be used on weekends. Jill has $29 and wants to buy the most expensive suit for the least amount of money. How much will her suit cost?

2) Jack wants an acrylic suit and a pair of ski gloves for the cheapest price. What will it cost him?

3) Louise is going skiing and needs goggles, a jacket, and a pair of gloves. She wants the best apparel. Since she does not know about ski clothes, she is going to assume that the ones which cost the most are the best. How much will they cost?

4) Bill decides to go with Louise and also needs goggles, a jacket, and a pair of gloves. He wants the cheapest apparel that he can buy. How much money will he need?
FINDING A SQUARE ROOT OF A NUMBER

To square a number is to use it as a factor twice. For example, the square of 3 x 3 = 9.

Finding a square root of a number is to find one of its two equal factors. For example, a square root of 9, written \( \sqrt{9} \), is 3 because 3 x 3 = 9. Finding a square root of a number is the inverse (opposite) operation of squaring.

To indicate a square root of a number, a radical sign, \( \sqrt{ } \), is used. The symbol \( \sqrt{9} \) is called a radical; 9, the number under the radical sign, is called the radicand.

<table>
<thead>
<tr>
<th>Numbers</th>
<th>Perfect Squares</th>
<th>Square Roots</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>25</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>36</td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td>49</td>
<td>7</td>
</tr>
<tr>
<td>8</td>
<td>64</td>
<td>8</td>
</tr>
<tr>
<td>9</td>
<td>81</td>
<td>9</td>
</tr>
</tbody>
</table>
COMPUTING THE SQUARE ROOT OF A PERFECT SQUARE

Model Problems

A. Compute the positive square root of 3136.

How To Proceed

1. Starting at the decimal point and moving to the left, group the digits of the number in pairs of two digits. Place a decimal point directly above the decimal point in the number.

2. Below the first group at the left, write the largest perfect square which is not more than that group. Write the square root of the perfect square above the first group.

3. Subtract the perfect square from the first group and bring down and annex the next group to the remainder.

4. Form a trial divisor by doubling (multiplying by 2) the part of the root already found in step 3 and annexing a 0.

Solution

\[ \sqrt{31.36} = 5.6 \]

\[ \begin{array}{c}
5 \\
\hline
31 \\
6 \\
\hline
25 \\
\hline
100 \ 6 \ 36
\end{array} \]
5. Divide the remainder found in step 3 by the trial divisor found in step 4. Annex the quotient to the part of the root already found; also, add it to the trial divisor to form the complete divisor.

6. Multiply the complete divisor by the last digit which was placed in the root, and subtract the product from the remainder found in step 3. The remainder is 0. The required root is 56.

NOTE: When necessary, the procedure given in steps 4, 5, 6 is repeated until the remainder is zero.

B. Compute: \( \sqrt{817.96} \)

1. Starting at the decimal point, moving first to the left and then to the right, group the digits in pairs. The first group on the left may have one digit. If the last group on the right has one digit, annex a 0 to form a two-digit group. Place a decimal point directly above the decimal point in the number.

2. The largest perfect square not more than 8 is 4. Write 4 below 8. Write 2 above 8.
3. Subtract 4 from 8, obtaining 4. Bring down the next group, 17, and annex it to 4, forming 417.

4. Find the first divisor by doubling 2 and annexing a 0: 2 x 2 = 4; the trial divisor is 40.

5. Divide the remainder, 417, by the trial divisor, 40. The quotient is 8. Therefore, the complete first divisor is 40 + 8, or 48.

6. 8 x 48 = 384. Subtract: 417 - 384 = 33. Bring down the next group, 96, and annex it to 33, forming 3396.

7. Find the second trial divisor by doubling 28 and annexing a 0; 2 x 28 = 56; the trial divisor is 560.

8. Divide the remainder, 3396, by the trial divisor, 560. The quotient is 6. Therefore, the complete divisor is 560 + 6 = 566.

9. 6 x 566 = 3396. Subtract: 3396 - 3396 = 0. The required root is 28.6.
COMPUTING THE APPROXIMATE SQUARE ROOT OF A NUMBER

Find \( \sqrt{48} \) correct to the nearest tenth.

1. In order to approximate \( \sqrt{48} \) correct to the nearest tenth, carry the work to two decimal places and then round off the result to the nearest tenth.

2. In order to carry the result to two decimal places, annex to 48 (at the right of the decimal point) two groups, each containing two zeroes.

3. Perform the computation and round off the answer to the nearest tenth. Since \( 6.92 \approx 6.9 \), the required root is 6.9.

Find \( \sqrt{62} \) to the nearest hundredth.

1. In order to carry the work in the result to three decimal places, annex three groups, each containing two zeroes, at the right of 62.

2. The first trial divisor is 140.

Since \( 1300 \div 140 = 9 \), the complete divisor is 140 \( \div 8 \), or 148.

3. Complete the computation and round off the answer to the nearest hundredth. Since \( 7.874 \approx 7.87 \), the required root is 7.87.
### SQUARE ROOTS

#### Perfect Squares:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>(\sqrt{289})</td>
</tr>
<tr>
<td>2.</td>
<td>(\sqrt{324})</td>
</tr>
<tr>
<td>3.</td>
<td>(\sqrt{784})</td>
</tr>
<tr>
<td>4.</td>
<td>(\sqrt{1296})</td>
</tr>
<tr>
<td>5.</td>
<td>(\sqrt{1.8496})</td>
</tr>
<tr>
<td>6.</td>
<td>(\sqrt{11025})</td>
</tr>
<tr>
<td>7.</td>
<td>(\sqrt{484})</td>
</tr>
<tr>
<td>8.</td>
<td>(\sqrt{9.61})</td>
</tr>
<tr>
<td>9.</td>
<td>(\sqrt{56.25})</td>
</tr>
<tr>
<td>10.</td>
<td>(\sqrt{129.96})</td>
</tr>
<tr>
<td>11.</td>
<td>(\sqrt{90.25})</td>
</tr>
<tr>
<td>12.</td>
<td>(\sqrt{57.76})</td>
</tr>
<tr>
<td>13.</td>
<td>(\sqrt{190.44})</td>
</tr>
<tr>
<td>14.</td>
<td>(\sqrt{9409})</td>
</tr>
<tr>
<td>15.</td>
<td>(\sqrt{46.24})</td>
</tr>
<tr>
<td>16.</td>
<td>(\sqrt{1.4884})</td>
</tr>
</tbody>
</table>

#### To the Nearest Tenth:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>17.</td>
<td>(\sqrt{29})</td>
</tr>
<tr>
<td>18.</td>
<td>(\sqrt{26})</td>
</tr>
<tr>
<td>19.</td>
<td>(\sqrt{1.19})</td>
</tr>
<tr>
<td>20.</td>
<td>(\sqrt{1.38})</td>
</tr>
<tr>
<td>21.</td>
<td>(\sqrt{147})</td>
</tr>
<tr>
<td>22.</td>
<td>(\sqrt{5.2634})</td>
</tr>
<tr>
<td>23.</td>
<td>(\sqrt{1.07})</td>
</tr>
<tr>
<td>24.</td>
<td>(\sqrt{1.39})</td>
</tr>
</tbody>
</table>

#### To the Nearest Hundredth:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>25.</td>
<td>(\sqrt{59})</td>
</tr>
<tr>
<td>26.</td>
<td>(\sqrt{129})</td>
</tr>
<tr>
<td>27.</td>
<td>(\sqrt{43})</td>
</tr>
<tr>
<td>28.</td>
<td>(\sqrt{149})</td>
</tr>
<tr>
<td>29.</td>
<td>(\sqrt{23.16})</td>
</tr>
<tr>
<td>30.</td>
<td>(\sqrt{131})</td>
</tr>
<tr>
<td>31.</td>
<td>(\sqrt{83})</td>
</tr>
<tr>
<td>32.</td>
<td>(\sqrt{1.08})</td>
</tr>
</tbody>
</table>
PATTERNS

In each case how are the successive numbers being formed? Write the next 4 in each sequence.

1) 2, 4, 6, 8, 10, _, _, _, _,

2) 1, 2, 4, 8, 16, _, _, _, _,

3) 1, 4, 9, 16, 25, _, _, _, _,

4) 1, 8, 27, 64, 125, _, _, _, _,

5) 2, 5, 8, 11, 14, _, _, _, _,

6) 2, 3, 5, 7, 11, 13, _, _, _, _,

7) 11, 121, 1331, 14641, _, _, _, _,

8) 1, 3, 6, 10, 15, _, _, _, _,

9) 1, 1, 2, 3, 5, 8, 13, 21, _, _, _, _,

10) 1, 3, 5, 7, _, _, _, _,

11) 1, 3, 9, 27, _, _, _, _,

12) 1, 4, 7, 10, _, _, _, _,

13) 9, 18, 36, 72, _, _, _, _,

14) 3, 9, 27, 81, _, _, _, _,

15) 2, 6, 18, 54, _, _, _, _,
PATTERNS

STUDY THESE SUMS OF CONSECUTIVE ODD NUMBERS:

1 = 1 = 1^2
1 + 3 = 4 = 2^2
1 + 3 + 5 = 9 = 3^2
1 + 3 + 5 + 7 = 16 = 4^2
1 + 3 + 5 + 7 + 9 = 25 = 5^2
1 + 3 + 5 + 7 + 9 + 11 = 36 = 6^2

1) What did you discover?

2) What is the sum of the first 7 odd numbers? (You should be able to do this in your head if you have discovered the pattern.)

3) What is the sum of the first 12 odd numbers?

4) What is the sum of the first 20 odd numbers?

Find six ways to name 100 using all the digits 1 through 9 inclusive.

Example: 123 - 45 - 67 + 89 = 100
(12 : 6) + 58 + 3 + (4 X A) + 9 = 100

5) 

6) 

7) 

8) 

9) 

10) 

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PATTERNS

Study these sums of consecutive natural numbers.

1 = 1
1 + 2 = 3
1 + 2 + 3 = 6
1 + 2 + 3 + 4 = 10
1 + 2 + 3 + 4 + 5 = 15
1 + 2 + 3 + 4 + 5 + 6 = 21

Complete the following:

1) 1 + 2 + 3 + 4 + 5 + 6 + 7 = ________
2) 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 = ________
3) 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = ________
4) 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = ________
5) 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 = ________
6) Can you discover a pattern? ________
7) Can you write a rule for the sum of the first N consecutive natural numbers?

Find three ways to name 100 using all the digits 1 through 9 inclusive.

Examples: 1 + 2 + 3 + 4 + 5 + 6 + 7 + (8 X 9) = 100
(12 : 6) + 58 + 3 + (4 X 7) + 9 = 100

8) ________________________________
9) ________________________________
10) ________________________________
BASE TEN

The base ten (decimal) system of numeration is based upon grouping by tens. If man had only four fingers on each hand instead of five, people might have learned to group by eights instead of tens.

It is interesting and fun to represent numbers using bases other than ten; but remember that regardless of the symbols used or the way objects are grouped, the numbers are the same. Only the representations of the numbers are different.

Studying other bases (numeration systems) will help one become more familiar with exponents and will give a better understanding of place value in base ten.

A review of base ten will be helpful in working with other bases.

Base ten has ten symbols: (0, 1, 2, 3, 4, 5, 6, 7, 8, 9).

The place value in base ten is based on tens and powers of ten.

- **Ones place**: \(10^0\) or 1
- **Tens place**: \(10^1\) or 10
- **Hundreds place**: \(10^2\) or 100 (10 x 10)
- **Thousands place**: \(10^3\) or 1000 (10 x 10 x 10)
- **Ten-thousands place**: \(10^4\) or 10,000 (10 x 10 x 10 x 10)
- **Hundred-thousands place**: \(10^5\) or 100,000 (10 x 10 x 10 x 10 x 10)
- **Millions place**: \(10^6\) or 1,000,000 (10 x 10 x 10 x 10 x 10 x 10)
Remember: In the power of 10, 10 is the base and the raised numeral is the exponent.

\[
\begin{align*}
\text{Base} & \rightarrow 10^n \\
\text{Exponent} & \rightarrow 10
\end{align*}
\]

In the numeral 2,563,407 the 7 is in the ones or \(10^0\) place,
the 0 is in the tens or \(10^1\) place,
the 4 is in the hundreds or \(10^2\) place,
the 3 is in the thousands or \(10^3\) place,
the 5 is in the ten-thousands or \(10^4\) place,
the 6 is in the hundred-thousands or \(10^5\) place,
the 2 is in the millions or \(10^6\) place.

The place value of each digit in the numeral 2,563,407 is shown by the chart below:

<table>
<thead>
<tr>
<th>Place Value</th>
<th>Millions</th>
<th>Hundred-Thousands</th>
<th>Ten-Thousands</th>
<th>Thousands</th>
<th>Hundreds</th>
<th>Tens</th>
<th>Ones</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(10^6)</td>
<td>(10^5)</td>
<td>(10^4)</td>
<td>(10^3)</td>
<td>(10^2)</td>
<td>(10^1)</td>
<td>(10^0)</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>5</td>
<td>6</td>
<td>3</td>
<td>4</td>
<td>0</td>
<td>7</td>
</tr>
</tbody>
</table>

The place value of each digit in a base ten numeral can also be shown by using expanded notation.

**Expanded Notation** is expressing a place value numeral as a sum of the products formed when each digit is multiplied by its place value.
EXAMPLES OF EXPANDED NOTATION:

\[ 653 = (6 \times 10^2) + (5 \times 10^1) + (3 \times 10^0) \]
\[ 4702 = (4 \times 10^3) + (7 \times 10^2) + (0 \times 10^1) + (2 \times 10^0) \]

A DECIMAL NUMERAL WRITTEN IN EXPANDED NOTATION CAN EASILY BE CHANGED BACK TO THE NUMERAL IT REPRESENTS.

EXAMPLE:
\[ (5 \times 10^2) + (3 \times 10^1) + (7 \times 10^0) = \]
\[ (5 \times 100) + (3 \times 10) + (7 \times 1) = \]
\[ 500 + 30 + 7 = 537 \]
1. \[ 5960 = (5 \times 1000) + (\_ \times 100) + (\_ \times 10) + (0 \times 1) \]
   \[= 5000 + \_ + \_ + \_ \]
   \[= 5960 \]

2. \[ 23,579 = (2 \times 10^4) + (3 \times 10^3) + (5 \times \_) + (\_ \times 10^1) + (\_ \times 10^0) \]
   \[= (2 \times \_) + (3 \times 1000) + (5 \times \_) + (\_ \times 10) + (\_ \times 1) \]
   \[= \_ + 3000 + \_ + \_ + \_ \]
   \[= 23,579 \]

Write each base ten numeral in expanded notation. Show all steps as in Problem #1 above.

3. \[ 5294 = \]

4. \[ 36,058 = \]

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CHANGING FROM BASE FIVE TO BASE TEN

Perhaps if man had only one hand, people might be using the base five numeration system instead of the base ten (decimal) numeration.

Just as base ten uses ten basic symbols (0, 1, 2, 3, 4, 5, 6, 7, 8, 9) and its place values are powers of ten, base five uses five symbols (0, 1, 2, 3, 4) and its place values are powers of five.

Grouping by tens

\[
\begin{array}{c}
\text{\textbullet\textbullet\textbullet\textbullet\textbullet\textbullet\textbullet\textbullet\textbullet\textbullet\textbullet} \\
2 \text{ tens } + 4 \text{ ones } = 24_{\text{ten}}
\end{array}
\]

Grouping by fives

\[
\begin{array}{c}
\text{\textbullet\textbullet\textbullet\textbullet\textbullet\textbullet} \\
4 \text{ fives } + 4 \text{ ones } = 44_{\text{five}}
\end{array}
\]

The ten and five are called subscripts and denote the base of the numeration system. From this point on, the subscript for base ten will not be written; however, subscripts for the other bases will be written.

Numbers written in bases other than base ten are read differently since reading a base ten numeral involves place value. 3,462 is read "three thousand, four hundred sixty-two." Therefore, when reading numbers in other bases, simply read each digit starting at the left and then say the base.
The numeral 42 in base five is read as "four two, base five." It means 4 fives plus 2 ones.

In the numeral 2314 in base five:
- The 4 is in the ones or $5^0$ place;
- The 1 is in the fives or $5^1$ place;
- The 3 is in the twenty-fives or $5^2$ place ($5^2 = 5 \times 5 = 25$);
- The 2 is in the one hundred twenty-fives or $5^3$ place ($5^3 = 5 \times 5 \times 5 = 125$).

The place value of each digit in the numeral 2314 in base five is shown by the chart below:

<table>
<thead>
<tr>
<th>One Hundred Twenty-Fives</th>
<th>Twenty-Fives</th>
<th>Fives</th>
<th>Ones</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5^3$</td>
<td>$5^2$</td>
<td>$5^1$</td>
<td>$5^0$</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>1</td>
<td>4</td>
</tr>
</tbody>
</table>
Notice: Base five is grouped by ones, fives, five x fives, and five x five x fives, etc. Also, notice that no commas are used in writing other base numerals. The commas are used in base ten to make the numerals easier to read.

When the numeral 2314 is written in expanded notation and five simplified, the resulting numeral is in base ten.

$$2314_{\text{five}} = (2 \times 5^3) + (3 \times 5^2) + (1 \times 5^1) + (4 \times 5^0)$$

$$= (2 \times 125) + (3 \times 25) + (1 \times 5) + (4 \times 1)$$

$$= 250 + 75 + 5 + 4$$

$$= 334$$

Therefore, $2314_{\text{five}} = 334$ (base ten).

To change numbers written in other bases to base ten, write in expanded notation and simplify.
Changing a number written in base five to a base ten numeral can be shown by flow charts.

 Multiply the $5^1$ digit by five

 Multiply the $5^0$ digit by one

 Add the products

 Base ten numeral

 Multiply the $5^2$ digit by twenty-five

 Multiply the $5^1$ digit by five

 Multiply the $5^0$ digit by one

 Add the products

 Base ten numeral

 Multiply the $5^0$ digit by one

 Multiply the $5^1$ digit by five

 Multiply the $5^2$ digit by twenty-five

 Add the products

 Base ten numeral

 Multiply the $5^1$ digit by one

 Multiply the $5^2$ digit by twenty-five

 Multiply the $5^0$ digit by one

 Add the products

 Base ten numeral

 Multiply the $5^1$ digit by five

 Multiply the $5^0$ digit by one

 Add the products

 Base ten numeral

 Multiply the $5^2$ digit by twenty-five

 Multiply the $5^1$ digit by five

 Multiply the $5^0$ digit by one

 Add the products

 Base ten numeral

 Multiply the $5^1$ digit by five

 Multiply the $5^0$ digit by one

 Add the products

 Base ten numeral

 Multiply the $5^2$ digit by twenty-five

 Multiply the $5^1$ digit by five

 Multiply the $5^0$ digit by one

 Add the products

 Base ten numeral

 Multiply the $5^1$ digit by five

 Multiply the $5^0$ digit by one

 Add the products

 Base ten numeral

 Multiply the $5^2$ digit by twenty-five

 Multiply the $5^1$ digit by five

 Multiply the $5^0$ digit by one

 Add the products

 Base ten numeral

 Multiply the $5^1$ digit by five

 Multiply the $5^0$ digit by one

 Add the products

 Base ten numeral

 Multiply the $5^2$ digit by twenty-five

 Multiply the $5^1$ digit by five

 Multiply the $5^0$ digit by one

 Add the products

 Base ten numeral
CHANGING FROM BASE FIVE TO BASE TEN

WRITE A BASE FIVE NUMERAL FOR EACH SET.

1. + + + + +
   + + + + +
   + +

2. # # # # #
   # # # # #
   # # # # #
   # # # # #
   # # # # #
   # # # # #

3. * * * * * *
   * * * * * *
   * * * * * *
   * * * * * *

HINT FOR NUMBER 3: FIRST REGROUP IN GROUPS OF FIVE.

4. PLACE VALUE

<table>
<thead>
<tr>
<th>Base Ten</th>
<th>$10^4$</th>
<th>$10^3$</th>
<th>$10^2$</th>
<th>$10^1$</th>
<th>$10^0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10,000</td>
<td>10x10x10x10</td>
<td>10x10x10</td>
<td>___</td>
<td>OR</td>
<td>OR</td>
</tr>
<tr>
<td>Base Five</td>
<td>$5^4$</td>
<td>$5^3$</td>
<td>$5^2$</td>
<td>$5^1$</td>
<td>$5^0$</td>
</tr>
<tr>
<td>625</td>
<td>5x5x5x5</td>
<td>5x5x5</td>
<td>___</td>
<td>OR</td>
<td>OR</td>
</tr>
</tbody>
</table>

FILL IN THE BLANKS.

5. $132_{\text{FIVE}} = (1 \times _) + (\underline{3} \times 5^1) + (2 \times _)$
   = $(1 \times _) + (_, x 5) + (2 \times _)$
   = ___ + ___ + ___

6. $2034_{\text{FIVE}} = (\underline{2} \times 5^3) + (0 \times _) + (3 \times _) + (\underline{4} \times 5^0)$
   = $(\underline{2} \times 125) + (0 \times _) + (3 \times _) + (4 \times 1)$
   = ___ + ___ + ___ + ___
Change each base five numeral to base ten by using expanded notation.

7. $43_{\text{FIVE}}$

8. $1213_{\text{FIVE}}$

9. $2132_{\text{FIVE}}$

10. $444_{\text{FIVE}}$

11. $4030_{\text{FIVE}}$

12. $11432_{\text{FIVE}}$

Complete the flow charts to change each base five numeral to base ten.

13. $32_{\text{FIVE}}$

14. $324_{\text{FIVE}}$
CHANGING FROM BASE TEN TO BASE FIVE

To change a base ten numeral to base five (regrouping in groups of five) divide by powers of base five starting with the largest power of five just smaller than the base ten numeral. Then divide the remainder by each successive power of five less than the first divisor.

To change 85 to a base five numeral:

First, list the powers of five until a number larger than 85 is obtained.

\[
\begin{align*}
5^0 &= 1 \\
5^1 &= 5 \\
5^2 &= 25 \\
5^3 &= 125
\end{align*}
\]

\[85\]

STOP, because 125 is greater than 85. Therefore, the first divisor is 25.

Second, regroup 85 into powers of five by using successive divisions. Remember, the first divisor is 25.

\[
\begin{array}{c}
25)85 \\
75 \\
10
\end{array}
\]

(There are 3 groups of 25, or 5². Therefore, 3 will be in the 5² place.)

\[
\begin{array}{c}
5)10 \\
10 \\
0
\end{array}
\]

(There are 2 groups of 5, or 5¹. Therefore, 2 will be in the 5¹ place.)

\[1110\]

1105
(There are 0 groups of 1 or 5\(^0\). Therefore, 0 will be in the 5\(^0\) place.)

*Successive Division*—Dividing by numbers which are powers of the base beginning with largest possible power of the base, then dividing the successive remainders by the next largest power, and so on.

Study the chart showing the place value of each digit in the base five numeral.

<table>
<thead>
<tr>
<th>Twenty-fives place</th>
<th>Fives place</th>
<th>Ones place</th>
</tr>
</thead>
<tbody>
<tr>
<td>5(^2)</td>
<td>5(^1)</td>
<td>5(^0)</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>

The answer is: 85 = 320 \text{ five}

A shorter way to write this computation is as follows:

\[
\begin{array}{c|c|c}
\text{Divisor} & 25 & 85 \downarrow 3 \quad \text{Quotient} (3 \times 25 = 75) \\
\hline
-75 & 5 & 10 \downarrow 2 \\
\hline
-10 & 1 & 0 \\
\hline
\end{array}
\]

(2 \times 5 = 10)

(0 \times 1 = 0)

Therefore, 85 = 320 \text{ five}
**Example:** $348 = \_ \_ \text{FIVE}

1. **List the powers of five**

\[
\begin{align*}
5^0 &= 1 \\
5^1 &= 5 \\
5^2 &= 25 \\
5^3 &= 125 \\
5^4 &= 625
\end{align*}
\]

**Notice:** The first divisor is 125. Therefore, the base five numeral will have **four** digits, requiring **four divisions**.

2. **Regroup 348 into powers of five by successive division.**

\[
\begin{align*}
125 &\overline{)348}^2 \quad 2 \\
&\underline{-250} \\
25 &\overline{)98}^3 \quad 3 \\
&\underline{-75} \\
5 &\overline{)23}^4 \quad 4 \\
&\underline{-20} \\
1 &\overline{)3}^3 \\
&\underline{-3} \\
\hline
0 \\
\end{align*}
\]

Therefore, $348 = 2343 \text{FIVE}$
CHANGING FROM BASE TEN TO BASE FIVE

1. Complete the division to change 67 to a base five numeral.

\[ \begin{array}{c}
5^0 = 1 \\
5^1 = 5 \\
5^2 = 25 \\
5^3 = 125
\end{array} \]

\[ \begin{array}{c}
25 \longdiv{67} \quad \underline{50} \\
\underline{17} \quad 0
\end{array} \]

67 = \_\_\_\_\_\_\_ FIVE

Change the following base ten numerals to base five. Show all steps.

2. 58

3. 25

4. 124

5. 96

6. 365

7. 500
ADDITION, SUBTRACTION, AND MULTIPLICATION IN BASE FIVE

The following base five addition table can be used to compute sums.

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>10</td>
<td>11</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>4</td>
<td>10</td>
<td>11</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>10</td>
<td>11</td>
<td>12</td>
<td>13</td>
</tr>
</tbody>
</table>

**ADDITION EXAMPLES:**

A. \[ \begin{align*} 
1_{\text{FIVE}} + 4_{\text{FIVE}} &= 4 \\
10_{\text{FIVE}} + 2_{\text{FIVE}} &= 10 \\
11_{\text{FIVE}} + 2_{\text{FIVE}} &= 11 \\
12_{\text{FIVE}} + 2_{\text{FIVE}} &= 12 \\
13_{\text{FIVE}} + 2_{\text{FIVE}} &= 13 \\
\end{align*} \]

**Regroup the 1 FIVE in the FIVES place**

**Write the 2 in the ONES place**

**FIVES place:** \[ 1 + 1 + 2 = 4 \]

**ONES place:** \[ 3 + 4 = 12; 1 \text{ FIVE, 2 ONES} \]

B. \[ \begin{align*} 
413_{\text{FIVE}} + 222_{\text{FIVE}} &= 1140_{\text{FIVE}} \\
4_{\text{FIVE}} + 2 = 11; 1 \text{ FIVE, 1 ONE} \\
\text{FIVES place: } 1 + 1 + 2 &= 4 \\
\text{FIVE}^2 \text{ place: } 4 + 2 &= 11; 1 \text{ FIVE, 1 ONE} \\
\text{FIVE}^3 \text{ place: } 1 \\
\end{align*} \]
Refer to the base five addition table to subtract in base five.

To subtract using the table look at the example to the right of the table. Four at the top of the last column is subtracted from each of the circled base five numerals to obtain the base five numerals in the first column.

<table>
<thead>
<tr>
<th>+</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>10</td>
<td>11</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>4</td>
<td>10</td>
<td>11</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>10</td>
<td>11</td>
<td>12</td>
<td>13</td>
</tr>
</tbody>
</table>

Subtraction Examples:

\[ \begin{array}{c}
3_{\text{FIVE}} - 2 \quad 4_{\text{FIVE}} \\
\hline
14_{\text{FIVE}}
\end{array} \]

Remember!

\[ \begin{array}{c}
\text{Ones place: } 13_{\text{FIVE}} = 1 \text{ five} + 3 \text{ ones} \\
8 - 4 = 4 = 5 + 3 = 8 \text{ ones}
\end{array} \]

\[ \begin{array}{c}
\text{Fives place: } 3 - 2 = 1
\end{array} \]

\[ \begin{array}{c}
0_{\text{FIVE}} - 4 \quad 4_{\text{FIVE}} \\
\hline
3 \quad 3_{\text{FIVE}}
\end{array} \]

\[ \begin{array}{c}
\text{Ones place: } 12_{\text{FIVE}} = 1 \text{ five} + 2 \text{ ones} \\
7 - 4 = 3 = 5 + 2 = 7 \text{ ones}
\end{array} \]

\[ \begin{array}{c}
\text{Fives place: } 1 \text{ five}^2 + 2 \text{ fives} = 5 + 2 = 7 \text{ fives}
\end{array} \]
Use the following base five multiplication table to compute products.

<table>
<thead>
<tr>
<th>X</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>2</td>
<td>4</td>
<td>11</td>
<td>13</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>3</td>
<td>11</td>
<td>14</td>
<td>22</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>4</td>
<td>13</td>
<td>22</td>
<td>31</td>
</tr>
</tbody>
</table>

\[ (2 \times 4 = 8; 8 + 5 = 1 \text{ R} 3 = 13_{\text{five}}) \]
\[ (3 \times 4 = 12; 12 + 5 = 2 \text{ R} 2 = 22_{\text{five}}) \]
\[ (4 \times 4 = 16; 16 + 5 = 3 \text{ R} 1 = 31_{\text{five}}) \]

**Multiplication Examples**

**A.**

\[ 1 \text{\hspace{1cm}} (\text{REGROUP}) \]
\[ \begin{array}{c}
  \underline{1} \\
  \underline{2_{\text{five}}} \\
\end{array} \]
\[ \begin{array}{c}
  \underline{3} \\
  \underline{2_{\text{five}}} \\
\end{array} \]
\[ \times \begin{array}{c}
  \underline{3_{\text{five}}} \\
  \underline{4} \\
\end{array} \]
\[ \begin{array}{c}
  \underline{1_{\text{five}}} \\
  \underline{3_{\text{five}}} \\
  \underline{3_{\text{five}}} \\
\end{array} \]

- **Ones place:** \( 4 \times 2 = 8; 8 + 5 = 1 \text{ R} 3 = 3_{\text{ones}} \)
- **Fives place:** \( 4 \times 3 = 12; 12 + 1 = 13; 13 + 5 = 2 \text{ R} 3 \)
- **Five^2 place:** \( 2 \)

**B.**

\[ 1 \text{\hspace{1cm}} (\text{REGROUP}) \]
\[ \begin{array}{c}
  \underline{1} \\
  \underline{2_{\text{five}}} \\
\end{array} \]
\[ \begin{array}{c}
  \underline{3} \\
  \underline{4_{\text{five}}} \\
\end{array} \]
\[ \times \begin{array}{c}
  \underline{2_{\text{five}}} \\
  \underline{3_{\text{five}}} \\
\end{array} \]
\[ \begin{array}{c}
  \underline{2_{\text{five}}} \\
  \underline{1_{\text{five}}} \\
  \underline{1_{\text{five}}} \\
\end{array} \]

- **Ones x ones place:** \( 3 \times 4 = 12; 12 + 5 = 2 \text{ fives, } 2_{\text{ones}} \)
- **Ones x fives place:** \( 3 \times 3 = 9; 9 + 2 = 11; 11 + 5 = 2 \text{ R} 1 \)
- **Fives x one place:** \( 2 \times 4 = 8; 8 + 5 = 1 \text{ R} 3 \)
- **Fives x fives place:** \( 2 \times 3 = 6; 6 + 1 = 7; 7 + 5 = 1 \text{ R} 2 \)
OR,

\[
\begin{array}{c}
3 \\
\times 23
\end{array}
\]

\[
\begin{array}{c}
3 \times 4 = 22, \text{ write 2, "carry" 2.} \\
1442_{\text{FIVE}}
\end{array}
\]

\[
\begin{array}{c}
3 \times 3 = 14 \text{ plus } 2 \text{ (carried)} = 21. \\
123_{\text{FIVE}}
\end{array}
\]

1st partial product is 212.

\[
\begin{array}{c}
2 \times 4 = 13, \text{ write 3 under 1, "carry" 1.} \\
2 = 212
\end{array}
\]

\[
\begin{array}{c}
2 \times 3 = 11 \text{ plus } 1 \text{ (carried)} = 12, \text{ write to the left of 3 (in second row). 2nd partial product is 1230.} \\
1442_{\text{FIVE}}
\end{array}
\]

Sum of partial products is 1442.
ADDITION, SUBTRACTION, AND MULTIPLICATION

Compute each sum in base five.

1. \[33_{\text{FIVE}} + 12_{\text{FIVE}} = 44_{\text{FIVE}}\]
2. \[241_{\text{FIVE}} + 334_{\text{FIVE}} = 575_{\text{FIVE}}\]
3. \[223_{\text{FIVE}} + 314_{\text{FIVE}} = 537_{\text{FIVE}}\]

Compute each difference in base five.

4. \[32_{\text{FIVE}} - 23_{\text{FIVE}} = 9_{\text{FIVE}}\]
5. \[141_{\text{FIVE}} - 43_{\text{FIVE}} = 98_{\text{FIVE}}\]
6. \[423_{\text{FIVE}} - 124_{\text{FIVE}} = 299_{\text{FIVE}}\]

Compute each product in base five.

7. \[43_{\text{FIVE}} \times 2_{\text{FIVE}} = 13_{\text{FIVE}}\]
8. \[32_{\text{FIVE}} \times 42_{\text{FIVE}} = 134_{\text{FIVE}}\]
9. \[242_{\text{FIVE}} \times 34_{\text{FIVE}} = 820_{\text{FIVE}}\]
BASE TWO (BINARY NUMERATION SYSTEM)

As a review, base ten uses ten symbols (0, 1, 2, 3, 4, 5, 6, 7, 8, 9), and its place values are powers of ten. Base five uses five symbols (0, 1, 2, 3, 4), and its place values are powers of five.

Base two, also called BINARY NUMERATION SYSTEM, uses two symbols (0, 1), and its place values are powers of two.

Study the following table of place values in base two.

<table>
<thead>
<tr>
<th>$2^{10}$</th>
<th>$2^9$</th>
<th>$2^8$</th>
<th>$2^7$</th>
<th>$2^6$</th>
<th>$2^5$</th>
<th>$2^4$</th>
<th>$2^3$</th>
<th>$2^2$</th>
<th>$2^1$</th>
<th>$2^0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1024</td>
<td>512</td>
<td>256</td>
<td>128</td>
<td>64</td>
<td>32</td>
<td>16</td>
<td>8</td>
<td>4</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

Base two numerals can also be written in expanded notation to change them to base ten.

$1110_{\text{TWO}} = (1 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + (0 \times 2^0)$

$= (1 \times 8) + (1 \times 4) + (1 \times 2) + (0 \times 1)$

$= 8 + 4 + 2 + 0$

$= 14$

Therefore, $1110_{\text{TWO}} = 14$. 

113

114
Since the base two system of numeration uses only the digits 0 and 1, the numerals can easily be shown using the idea of electrical circuits. If a light is on in a row of lights, it represents 1 in that position. If a light is off, the digit in that place is 0.

<table>
<thead>
<tr>
<th>2⁶</th>
<th>2⁵</th>
<th>2⁴</th>
<th>2³</th>
<th>2²</th>
<th>2¹</th>
<th>2⁰</th>
</tr>
</thead>
<tbody>
<tr>
<td>ON</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

ON represents 1
OFF represents 0

1100110₂ =
= (1 x 2⁶) + (1 x 2⁵) + (0 x 2⁴) + (0 x 2³) + (1 x 2²) + (1 x 2¹) + (0 x 2⁰)
= (1 x 64) + (1 x 32) + (0 x 16) + (0 x 8) + (1 x 4) + (1 x 2) + (0 x 1)
= 64 + 32 + 0 + 0 + 4 + 2 + 0
= 102

Therefore, 1100110₂ = 102.

Notice: Many more digits are required to write a number in base two. For this reason alone it would not generally be as practical to use as base ten.
EXAMPLE A:

\[
\begin{array}{c|cccc}
2^4 & 2^3 & 2^2 & 2^1 & 2^0 \\
\hline
& & & & \\
1 & 1 & 0 & 1 & 1 \_\text{two} \\
\end{array}
\]

\[
= (1 \times 2^4) + (1 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (1 \times 2^0)
= (1 \times 16) + (1 \times 8) + (0 \times 4) + (1 \times 2) + (1 \times 1)
= 16 + 8 + 0 + 2 + 1
= 27
\]

Therefore, \(110011_{\text{two}} = 27\).

Base ten numerals can be changed to base two numerals by successive divisions using as the first divisor the largest power of two just smaller than the base ten numeral. \(\text{(Remember base five.)}\)

\[
83 = \_\text{two}
\]

First, list the powers of two until you obtain a number larger than 83.

<table>
<thead>
<tr>
<th>(2^0)</th>
<th>(2^1)</th>
<th>(2^2)</th>
<th>(2^3)</th>
<th>(2^4)</th>
<th>(2^5)</th>
<th>(2^6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>4</td>
<td>8</td>
<td>16</td>
<td>32</td>
<td>64</td>
</tr>
</tbody>
</table>

See if you can answer these questions by studying the list of powers of two.

How many digits will the base two numeral contain?

How many times will you have to divide?

Answer—Seven

\[
\begin{array}{l}
.83 \\
116 \quad 12_{\text{i}}
\end{array}
\]
\[
\begin{array}{c|c|c|c}
\text{Divisor} & 64 & \overline{83} & 1 \\
\hline
-64 & & & \\
\hline
32 & 19 & 0 & \\
-0 & & & \\
\hline
16 & 19 & 1 & \\
-16 & & & \\
\hline
8 & 3 & 0 & \\
-0 & & & \\
\hline
4 & 3 & 0 & \\
-0 & & & \\
\hline
2 & 3 & 1 & \\
-2 & & & \\
\hline
1 & 1 & 1 & \\
-1 & & & \\
\hline
\text{Quotient} & & & 1 \\
\hline
0 & & & \\
\end{array}
\]

(0 is necessary as a place holder in the \(2^2, 2^3, 2^5\) places. Therefore, it is necessary to divide by 4, 8, and 32.)

Therefore, \(83 = 1010011_{\text{two}}\)
ADDITION, SUBTRACTION, AND MULTIPLICATION EXAMPLES:

EXAMPLE B:

\[
\begin{array}{c}
11_{\text{TWO}} \\
+ 10_{\text{TWO}} \\
\hline
101_{\text{TWO}}
\end{array}
\]

Ones place: \(1 + 0 = 1\) (Regroup)

twos place: \(1 + 1 = 2\); \(2 + 2 = 1 \text{ R } 0\)
	\(\text{Two}^2\) place: \(1\)

EXAMPLE C:

\[
\begin{array}{c}
111_{\text{TWO}} \\
+ 101_{\text{TWO}} \\
\hline
1100_{\text{TWO}}
\end{array}
\]

Ones place: \(1 + 1 = 2\); \(2 + 2 = 1 \text{ R } 0\)

twos place: \(1 + 1 = 2\); \(2 + 2 = 2 \text{ R } 0\)
	\(\text{Two}^2\) place: \(1 + 1 + 1 = 3\); \(3 + 2 = 1 \text{ R } 1\)
	\(\text{Two}^3\) place: \(1\)

EXAMPLE D:

\[
\begin{array}{c}
10_{\text{TWO}} \\
- 1_{\text{TWO}} \\
\hline
1_{\text{TWO}}
\end{array}
\]

Ones place: \(10 = 1 \text{ TWO} + 0 \text{ ONES} = 2 \text{ ONES}\)

twos place: \(1 - 1 = 1\)

EXAMPLE E:

\[
\begin{array}{c}
010_{\text{TWO}} \\
- 11_{\text{TWO}} \\
\hline
11_{\text{TWO}}
\end{array}
\]

Ones place: \(1 \text{ TWO} + 0 \text{ ONES} = 2 \text{ ONES};\) \(2 - 1 = 1\)

twos place: \(1 \text{ TWO}^2 + 0 \text{ TWOS} = 2 \text{ TWOS};\) \(2 - 1 = 1\)
	\(\text{One}^2 = 2 \text{ TWOS}\)
### Base Two Addition Table

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<tr>
<td>1</td>
<td>1</td>
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</tbody>
</table>

\[ 1 + 1 = 2; \quad 2 = (2 \times 1) + 0 = 10_{\text{TWO}} \]

### Base Two Multiplication Table

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</table>

**Example F:**

\[ 101_{\text{TWO}} \times 11_{\text{TWO}} = 1111_{\text{TWO}} \]

**Example G:**

\[ 111_{\text{TWO}} \times 101_{\text{TWO}} = 100011_{\text{TWO}} \]
BASE TWO (BINARY NUMERATION SYSTEM)

1. What is another name for base two?

2. Why could the numeral 101121₂ not be a base two numeral?

3. Without dividing, how many digits are required to write 62 as a base two numeral?

Write each base two numeral in expanded notation and change to base ten.

4. \(1101_\text{Two} = (1 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0)\)
   \[= (\quad) + (\quad) + (\quad) + (\quad)\]
   \[= \quad + \quad + \quad + \quad\]
   \[= \quad\]

5. \(1010_\text{Two}\)

6. \(10101_\text{Two}\)

7. \(1001001_\text{Two}\)

\[125\]
Write the base two numeral represented by each row of lights.

8.

9.

10.

Change each base ten numeral to base two. (Show all work.)

11. \( 38 = \) \( \_
\)

12. \( 23 = \) \( \_
\)

\[
\begin{align*}
2^0 &= 1 \\
2^1 &= 2 \\
2^2 &= 4 \\
2^3 &= 8 \\
2^4 &= 16 \\
2^5 &= 32 \\
2^6 &= 64
\end{align*}
\]

13. \( 79 = \) \( \_
\)

14. \( 157 = \) \( \_
\)
PERFORM THE INDICATED OPERATIONS IN BASE TWO.

15. \(101_{\text{TWO}} + 110_{\text{TWO}}\)
16. \(1111_{\text{TWO}} + 101_{\text{TWO}}\)

17. \(1010_{\text{TWO}} - 101_{\text{TWO}}\)
18. \(1011_{\text{TWO}} - 111_{\text{TWO}}\)

19. \(101_{\text{TWO}} \times 10_{\text{TWO}}\)
20. \(1010_{\text{TWO}} \times 101_{\text{TWO}}\)

21. \(1010_{\text{TWO}} + 110_{\text{TWO}}\)
22. \(1100_{\text{TWO}} - 101_{\text{TWO}}\)
NUMBER SENSE

MULTIPLY BY 5
Take half of the number and multiply by 10.

- 36  THE NUMBER  36
- 18  HALF OF THE NUMBER  \times 5
- 180  MULTIPLY BY 10  180

- 47  THE NUMBER  47
- 23.5  HALF OF THE NUMBER  \times 5
- 235  MULTIPLY BY 10  235

MULTIPLY BY 15
Multiply the number by 10 and add half the product to it.

- 49  THE NUMBER  49
- 490  MULTIPLY BY 10  15
- 245  HALF THE PRODUCT  245
- 735  SUM OF PRODUCT AND  49
  HALF THE PRODUCT  735

MULTIPLY BY 9
Multiply the number by 10 and subtract the number from it.

- 93  THE NUMBER  93
- 930  MULTIPLY BY 10  \times 9
- 837  SUBTRACT THE NUMBER  837
MULTIPLY BY 99

MULTIPLY THE NUMBER BY 100 AND SUBTRACT FROM THIS PRODUCT THE ORIGINAL NUMBER.

236  THE NUMBER  236
23600  MULTIPLY BY 100  \times 99
23364  SUBTRACT THE NUMBER  2124

2124
23364

MULTIPLY BY 11

2. ADD TWO DIGITS AT A TIME STARTING AT THE RIGHT TO PRODUCE THE SUCCEEDING DIGITS OF THE ANSWER.

32415 \times 11  32415
356565  \times 11  
32415
32415
356565

START ON THE RIGHT AND WORK LEFT.
WRITE THE RIGHT-HAND DIGIT (5).
ADD 5 + 1 = 6 FOR THE NEXT DIGIT.
ADD 1 + 4 = 5 FOR THE NEXT DIGIT.
ADD 4 + 2 = 6 FOR THE NEXT DIGIT.
ADD 2 + 3 = 5 FOR THE NEXT DIGIT.
WRITE THE LEFT-HAND DIGIT (3).
4. Sometimes it is necessary to carry.

\[ 257861 \times 11 = 2836471 \]

Start on the right and work left.

Write the right-hand digit (1).

Add \(1 + 6 = 7\) for the next digit.

Add \(6 + 8 = 14\); write 4 and carry 1.

Add \(8 + 7 = 15\); \(15 + 1 = 16\); write 6 and carry 1.

Add \(7 + 5 = 12\); \(12 + 1 = 13\); write 3 and carry 1.

Add \(5 + 2 = 7\); \(7 + 1 = 8\); write 8.

Write 2 for the last digit on the left.

**Multiply by 12**

1. Double the right-hand digit of the number to be the right-hand digit of the answer.

2. Double each succeeding digit, and add to the digit on the right plus anything that is carried.

3. The left-hand digit of the number plus anything that is carried becomes the left-hand digit of the answer.

\[ 32761 \times 12 = 393132 \]

Double 1, write 2.

Double 6, add 1, write 3, carry 1.

Double 7, add 6, add 1, write 1, carry 2.

Double 2, add 7, add 2, write 3, carry 1.

Double 3, add 2, add 1, write 9.

Write 3.
**Multiply two-digit number by two-digit number**

1. Start multiplying as usual with the ones places.

   39
   x47

   3

2. Stop using the regular method.

   6
   39
   x47

3. Next, cross multiply.

   3 x 7 = 21
   9 x 4 = 36

4. Add the products.

   57
   6

5. Add the ones carried.

   63

6. Write the 3 in the tens place in the answer, and carry 6.

   6
   39
   x47

   33

7. Multiply the tens places, add the ones carried, and write this to complete the answer.

   3 x 4 = 12
   6
   39
   x47

   1833

13
Multiply a three-digit number by a two-digit number

\[
\begin{array}{c}
456 \\
x 28 \\
8 \\
456 \\
x 28 \\
8 \\
5 \times 8 = 40 \\
2 \times 6 + 12 \\
52 \\
4 \\
56 \\
54 \\
456 \\
x 28 \\
68 \\
5 \times 5 = 10 \\
4 \times 8 = 32 \\
42 \\
5 \\
47 \\
127 \\
132
\end{array}
\]

1. Start multiplying as usual with the ones place.
2. Stop using the regular method.
3. Next, cross multiply as shown.
4. Add the products.
5. Add the ones carried.
6. Write the 6 in the tens place in the answer, and carry 5.
7. Next, cross multiply as shown.
8. Add the products.
9. Add the ones carried.
10. Write the 7 in the hundreds place and carry 4.

11. Multiply the hundreds place by the tens place.

12. Add the ones carried.

13. Write this to complete the answer.
MULTIPLY NUMBERS BY 25

Divide the number by 4 and multiply the quotient by 100.

\[
25 \times 48 \quad \text{Divide } 48 \text{ by } 4 \rightarrow 12.
\]

\[
\frac{100}{4} \times 48 \quad \text{Multiply } 12 \text{ by } 100 \rightarrow 1200.
\]

\[
100 \times 12
\]

\[
1200
\]

MULTIPLY NUMBERS BY 50

Divide the number by 2 and multiply the quotient by 100.

\[
50 \times 74 \quad \text{Divide } 74 \text{ by } 2 \rightarrow 37.
\]

\[
\frac{100}{2} \times 74 \quad \text{Multiply } 37 \text{ by } 100 \rightarrow 3700.
\]

\[
100 \times 37
\]

\[
3700
\]

MULTIPLY NUMBERS BY 75

Divide the number by 4, multiply the quotient by 3, and multiply the product by 100.

\[
75 \times 28
\]

\[
100 \times \frac{3}{4} \times 28 \quad \text{Divide } 28 \text{ by } 4 \rightarrow 7.
\]

\[
100 \times 3 \times 7 \quad \text{Multiply } 7 \text{ by } 3 \rightarrow 21.
\]

\[
100 \times 21 \quad \text{Multiply } 21 \text{ by } 100 \rightarrow 2100.
\]

\[
2100
\]
Multiply number close to 100

93 x 97

1. Take the complements to 100 of both numbers and multiply them.
93-7 97-3

7 x 3 = 21

2. Write 21 in the ten's and one's places of the answer.

93-3 = 90

3. Subtract the complement of one number from the other number and place the result next to 21.

97-7 = 90

2021

Squaring a number

(47)²

1. Square 7 to get 49.

9

2. Put the 9 in the one's column of the answer and carry the 4.

3. Take 4 x 7 x 2 = 56 and add the 4 from step 2 to get 60.

09

4. Put the 0 in the ten's column of the answer and carry the 6.

5. Square 4 to get 16 and add the 6 from step 4 to get 22.

2209

6. Place this result next to 09.
SHORT CUT FOR SQUAREING NUMBERS 51-59

(53)^2
1. Write 3 as a two-digit number, 09
2. Square 5, 25
3. Add 3, 28
4. Answer 2809

SHORT CUT FOR SQUAREING A NUMBER ENDING IN 5

(75)^2
1. Write 5 in one's and ten's place, 25
2. Multiply 7 by its successor 8, 56
3. Answer 5625

MULTIPLY BY DIFFERENCE OF 2 SQUARES METHOD

47 x 53 is
1) Square 50, 2500
(50-3) x (50+3) 2) Square 3, 9
which is
50^2 - 3^2
3) Subtract.

Numbers must "center" around a number like 40, 50, 100, etc.

74 x 86 = (80 - 6) (80 + 6) = 80^2 - 6^2 = 6400 - 36 = 6364
**Using Difference of 2 Squares for Other Problems**

\[ 64^2 - 36^2 = (64+36)(64-36) = (100)(28) = 2800 \]

**Divisibility by 2**

A number is divisible by 2 if the last digit is even (0, 2, 4, 6, or 8).

**Divisibility by 3**

A number is divisible by 3 if the sum of its digits is divisible by 3.

267 Sum of its digits: 2 + 6 + 7 = 15.
Since 15 is divisible by 3, then 267 is divisible by 3.

**Divisibility by 4**

A number is divisible by 4 if its last two digits form a number that is divisible by 4.

624 Since 24 is divisible by 4, then 624 is divisible by 4.

737 Since 37 is not divisible by 4, then 737 is not divisible by 4.

**Divisibility by 5**

A number is divisible by 5 if the last number is 0 or a 5.

**Divisibility by 6**

A number is divisible by 6 if it is divisible by 2 and by 3.

366 is divisible by 6 because it is divisible by 2 (the last digit 6 is even) and it is divisible by 3 (the sum of the digits is 15).
**Divisibility by 7**

Double the last digit of the number and subtract it from the remaining digits. If the remaining number is divisible by 7, the original number is divisible by 7. If the second number is still too large to divide by inspection, continue doubling the last digit and subtracting from the remaining digits until the answer is a number that is small enough to divide by inspection.

- 238: Double 8 and subtract the result from the remaining digits. Since 7 is divisible by 7, then 238 is divisible by 7.

- 4327: Double 7 and subtract the result from the remaining digits.
  - 14
  - 418
  - 16
  - 25

Since 25 is not divisible by 7, then 4327 is not divisible by 7.

**Divisibility by 8**

A number is divisible by 8 if the last three digits are divisible by 8.

- 4234: Since 234 is divisible by 8, then 4234 is divisible by 8.

- 62311: Since 311 is not divisible by 8, then 62311 is not divisible by 8.
**Divisibility by 9**

A number is divisible by 9 if the sum of its digits is divisible by 9.

261  
Sum of the digits: 2 + 6 + 1 = 9 which is divisible by 9. Therefore, 261 is divisible by 9.

**Divisibility by 10**

A number is divisible by 10 if the last digit is 0.

**Divisibility by 11**

1. Starting with the one's place, place an “x” over every other digit in the number.
2. Add the digits with an “x” above them.
3. Add the digits without an “x” above them.
4. Find the difference of the two sums.
5. If the difference is divisible by 11, then the number is divisible by 11.

\[
\begin{align*}
4 & \quad 3 \quad \underline{5} \quad 6 \\
\underline{4} & \quad 5 & \quad 6 = 15 & \text{This number is not divisible by 11.} \\
3 & \quad 5 = \underline{8} & \text{divisible by 11.} \\
7 & \\
9 & \quad 8 \quad \underline{6} \quad 1 \quad 5 \\
\underline{9} & \quad 6 & \quad 5 = 20 & \text{This number is divisible by 11.} \\
8 & \quad 1 = \underline{9} & \text{by 11.} \\
11 &
\end{align*}
\]

**Divisibility by 25**

A number is divisible by 25 if the last two digits are 00, 25, 50, or 75.

**Divisibility by 50**

A number is divisible by 50 if the last two digits are 00 or 50.
<p>| | |</p>
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<th></th>
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<tr>
<td>2)</td>
<td>$25 \times 24 =$</td>
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<tr>
<td>3)</td>
<td>$25 \times 80 =$</td>
</tr>
<tr>
<td>4)</td>
<td>$25 \times 284 =$</td>
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<td>5)</td>
<td>$25 \times 328 =$</td>
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<tr>
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<td>$25 \times 456 =$</td>
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<td>$10 \times 85 =$</td>
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<td>$100 \times 63 =$</td>
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<td>$1000 \times 74 =$</td>
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NUMBER SENSE

MULTIPLICATION

1) $49 \times 15 = $
2) $73 \times 5 = $
3) $734 \times 11 = $
4) $296 \times 9 = $
5) $956 \times 99 = $
6) $624 \times 12 = $
7) $90 \times 25 = $
8) $432 \times 50 = $
9) $592 \times 75 = $
10) $92 \times 93 = $
11) $525 \times 11 = $
12) $92 \times 25 = $
13) $36 \times 15 = $
14) $54 \times 5 = $
15) $95 \times 96 = $
16) $93 \times 94 = $
17) $1932 \times 11 = $
18) $500 \times 25 = $
19) $526 \times 50 = $
20) $48 \times 75 = $
21) $424 \times 15 = $
22) $196 \times 5 = $
23) $1085 \times 11 = $
24) $92 \times 75 = $
25) $96 \times 97 = $
26) $556 \times 11 = $
27) $2536 \times 50 = $
28) $44 \times 15 = $
29) $324 \times 75 = $
30) $250 \times 50 = $
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</table>
SQUARING A NUMBER ENDING IN 5

1) \(15^2 = 15 \times 15 = 225\)
2) \(25^2 = 25 \times 25 = 625\)
3) \(35^2 = 35 \times 35 = 1225\)
4) \(45^2 = 45 \times 45 = 2025\)
5) \(55^2 = 55 \times 55 = 3025\)
6) \(65^2 = 65 \times 65 = 4225\)
7) \(75^2 = 75 \times 75 = 5625\)
8) \(85^2 = 85 \times 85 = 7225\)
9) \(95^2 = 95 \times 95 = 9025\)
10) \(105^2 = 105 \times 105 = 11025\)
11) \(115^2 = 115 \times 115 = 13225\)
12) \(125^2 = 125 \times 125 = 15625\)
13) \(135^2 = 135 \times 135 = 18225\)
14) \(145^2 = 145 \times 145 = 21025\)
15) \(155^2 = 155 \times 155 = 24025\)
MULTIPLYING BY DIFFERENCE OF 2 SQUARES METHOD

1) \(42 \times 48 = (45 - 3)(45 + 3) = 45^2 - 3^2 = \)

2) \(56 \times 54 = \)

3) \(92 \times 98 = \)

4) \(74 \times 76 = \)

5) \(81 \times 89 = \)

6) \(64 \times 66 = \)

7) \(71 \times 79 = \)

8) \(21 \times 29 = \)

9) \(32 \times 38 = \)

10) \(12 \times 18 = \)

11) \(13 \times 17 = \)

12) \(22 \times 28 = \)

13) \(34 \times 36 = \)

14) \(31 \times 39 = \)

15) \(33 \times 37 = \)

\[144\]
### DIVISIBILITY

Place an "X" in the box if the number in the left column is divisible by the number in the top row.

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Page 17

1) Shepard---Carpenter
Johnston---Painter
Nichols---Plumber

Page 18

2) Of the 100, 79 (coffee drinkers) + 69 (tea drinkers) - 47 (number drinking both) = 101 (number of coffee only + number of tea only drinkers). However, the investigator gave the total number interviewed as 100.

3) County
   Color
   Picture

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Page 19

4) Points scored 7 16 25 10 15
Position played R. G. L. F. R. F. L. G. Center
Player's name  Jack Sam Ed Dave Ben

Sam is the captain of the team.

5) left house center house right house
   Ford Plymouth Cadillac
   Watson Plymouth Jones

Page 20

6) cat--Duke
dog--Sandy
goat--Blacky
horse--Rusty

7) Sam--Sally
Bill--Patsy
Joe--Betty

Page 21

8) Hank
9) Brown--manager
   Green--teller
   White--cashier

10) Hill--victim
    Grover--witness
    Craig--policeman
    Mays--judge
    Wilson--hangman
    Dunn--murderer

11) President--Mrs. Johnston
    Manager--Mr. Crawford
    Vice-President--Mr. Brown
    Auditor--Miss Jones
    Clerk--Miss Green
    Secretary--Mr. Smith

12) David scored 98
    Sam scored 96
    David--Betty
    Sam--Sue
    Joe--Jane

13) Jimenez--carpenter
    Rodriguez--policeman
    Martinez--banker
    Contreras--police officer

14) Pilot--Joe

15) Jane
    Ruth
    Phil
    Mark
    Joe
    Ham 60¢
    D. B. 90¢
    D. B. 90¢
    Ham 60¢
    Cola 20¢
    Cola 20¢
    Cheese 75¢
    Shake 35¢
    Milk .20¢
    Shake 35¢
    Fries 25¢
    Fries 25¢
    105¢
    110¢
    125¢
    120¢
    120¢

16) Color
    Red
    Yellow
    Orange
    Green
    Blue
    Violet
    Name
    Lucky Lady
    Winner's Circle
    Martin's Folly
    Johnston's Darlin'
    Dancer
    Fire Ball
    Place
    5th
    3rd
    4th
    1st
    6th
    2nd

17) Ronald--1st
    Joe--3rd
    Roy--5th
    Lloyd--2nd
    Carlton--4th
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Page 30

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Page 31

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17  18  6  9  5
1  19  3  20  22  2
21  3  3  4  2
7  6  1  24  5
4  6  2  5  3  5
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149
LINEAR MEASUREMENT

Using the code below, shade in the squares with the proper designs.

Examples:

1000 m = 1 km =

.001001 m = .001 m + .000001 m =

1 mm + 1 micron =

1 km 1 dm 1 cm 1 mm 1 micron
CONNECT THE NUMBER OF EACH PROBLEM WITH THE ANSWER FOR THAT PROBLEM.

1. $8 \text{ g} + 28 \text{ g} = 37 \text{ g}$
2. $73 \text{ t} - 46 \text{ t} = 27 \text{ t}$
3. $15 \text{ kg} + 19 \text{ kg} = 34 \text{ kg}$
4. $8 \text{ g} + 900 \text{ mg} = 8.9 \text{ g}$
5. $8 \text{ g} + 900 \text{ mg} = 8900 \text{ mg}$
6. $3 \text{ g} - 2.9 \text{ g} = 100 \text{ mg}$
7. $256 \text{ mg} + 344 \text{ mg} = 6 \text{ g}$
8. $454 \text{ g} + 45 \text{ g} = 5 \text{ kg}$
9. $1 \text{ kg} - 800 \text{ g} = 2 \text{ kg}$
10. $1 \text{ g} - 730 \text{ mg} = .27 \text{ g}$
11. $.1 \text{ g} + 100 \text{ mg} = 200 \text{ mg}$
12. $2345 \text{ mg} + 1055 \text{ mg} = 3.4 \text{ g}$
13. $1 \text{ g} - 911 \text{ mg} = 89 \text{ mg}$
14. $59 \text{ g} + 1000 \text{ mg} = 60 \text{ g}$
15. $3 \text{ kg} + 700 \text{ g} = 3.7 \text{ kg}$
16. $1 \text{ t} - 660 \text{ kg} = 340 \text{ kg}$
17. $.042 \text{ g} + 8 \text{ mg} = 50 \text{ mg}$
18. $13 \text{ mg} + 7 \text{ mg} = .02 \text{ g}$
Cut out the square... fit them together so that the touching edges name the same volumes.

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<tr>
<td>339</td>
<td>1371</td>
<td>97</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>315</td>
<td>17366</td>
<td>96</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>245</td>
<td>245</td>
<td>108</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>108</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>9.</td>
<td>113</td>
<td>10.</td>
<td>47</td>
<td>11.</td>
<td>4472</td>
</tr>
<tr>
<td>624</td>
<td>53</td>
<td>4351</td>
<td>898</td>
<td></td>
<td></td>
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<tr>
<td>147</td>
<td>141</td>
<td>121</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>526</td>
<td>235</td>
<td>43736</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1410</td>
<td>2491</td>
<td>49203</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>43736</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>4909366</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>13.</td>
<td>6235</td>
<td>14.</td>
<td>181</td>
<td>36</td>
<td>1616</td>
</tr>
<tr>
<td>2828</td>
<td>36</td>
<td>291</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>407</td>
<td>288</td>
<td>36</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>36</td>
<td>36</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| 149 | 159 |
**MAGIC SQUARES: SHEET 1**

This array of numbers is a 3 x 3 Magic Square. Show that the sum of the numbers in each row, column, and diagonal is 15.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>9</td>
<td>2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>8</th>
<th>1</th>
<th>6</th>
<th>8</th>
<th>3</th>
<th>4</th>
<th>8</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>5</td>
<td>7</td>
<td>1</td>
<td>5</td>
<td>9</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>9</td>
<td>2</td>
<td>6</td>
<td>7</td>
<td>2</td>
<td>2</td>
<td>4</td>
</tr>
</tbody>
</table>

15 15 15 15 15 15 15 15 15

Complete these magic squares using the same numbers 1 through 9. Be sure each row, column, and diagonal adds to 15.

<table>
<thead>
<tr>
<th>6</th>
<th>1</th>
<th>8&lt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
<td>4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>4</th>
<th>3</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>6</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>2</th>
<th>9</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>8</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>9</td>
</tr>
<tr>
<td>6</td>
<td>7</td>
<td>2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>6</th>
<th>7</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>9</td>
</tr>
<tr>
<td>8</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>4</th>
<th>9</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
<td>6</td>
</tr>
</tbody>
</table>

Can you find another magic square using the same numbers 1 through 9?

<table>
<thead>
<tr>
<th>4</th>
<th>3</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>6</td>
</tr>
</tbody>
</table>
MAGIC SQUARES: SHEET 2

This array of numbers is a 3 x 3 Magic Square. Show that the sum of the numbers in each row, column, and diagonal is 12.

```
  7  0  5
  2  4  6
  3  8  1
```

7  0  5  7  2 -3  7  5
2  4  6  0  4  8  4  4
3  8  1  5  6  1  1  3
12 12 12 12 12 12 12 12

Perform the indicated operations on each number in the Magic Square above. Enter the results below. Then see if each new array is also a Magic Square.

Add 9

```
16  9  14
11 13  15
12 17  10
```
Total 39

Multiply by 12

```
84  0  60
24 48  72
36 96  12
```
Total 144

Add \( \frac{1}{2} \)

```
7.5  1  5.5
2.5 4.5  6.5
3.5 8.5  1.5
```
Total 13.5

Multiply by 10

```
70  0  50
20 40  60
```
Total 120

Multiply by 7

```
49  0  35
14 28  42
```
Total 84

If the same number is added to each entry in a Magic Square, is the result another Magic Square?

If each entry in a Magic Square is multiplied by the same number, is the result another Magic Square?
MAGIC SQUARES: SHEET 3

This array of numbers is a $3 \times 3$ Magic Square. Show that the sum of numbers in each row, column, and diagonal is 21.

```
10  3  8
5  7  9
6 11  4
```

Perform the indicated operations on each number in the Magic Square above, enter the results below. Then see if each new array is also a Magic Square.

```
<table>
<thead>
<tr>
<th>13</th>
<th>6</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>10</td>
<td>12</td>
</tr>
<tr>
<td>9</td>
<td>14</td>
<td>7</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
</tbody>
</table>
```

```
<table>
<thead>
<tr>
<th>7</th>
<th>0</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td>1</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
</tbody>
</table>
```

```
<table>
<thead>
<tr>
<th>10</th>
<th>3</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
</tbody>
</table>
```

Add 3

```
<table>
<thead>
<tr>
<th>5</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
</tbody>
</table>
```

```
<table>
<thead>
<tr>
<th>9</th>
<th>2</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>8</td>
<td>3</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>3</td>
</tr>
<tr>
<td>----</td>
<td>----</td>
<td>----</td>
</tr>
</tbody>
</table>
```

Subtract 3

```
<p>| 9  | 4  |
|----|
| 4  |</p>
<table>
<thead>
<tr>
<th>5</th>
</tr>
</thead>
</table>
```

```
<p>| 5  | 1  |
|----|
| 6  |</p>
<table>
<thead>
<tr>
<th>10</th>
</tr>
</thead>
</table>
```

Divide by 2

```
<table>
<thead>
<tr>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>---</td>
</tr>
</tbody>
</table>
```

```
<table>
<thead>
<tr>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
</tr>
<tr>
<td>---</td>
</tr>
</tbody>
</table>
```

Add $\frac{1}{3}$

```
<table>
<thead>
<tr>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>---</td>
</tr>
</tbody>
</table>
```

```
<table>
<thead>
<tr>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>---</td>
</tr>
</tbody>
</table>
```

```
<table>
<thead>
<tr>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>---</td>
</tr>
</tbody>
</table>
```

```
<table>
<thead>
<tr>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>---</td>
</tr>
</tbody>
</table>
```

```
<table>
<thead>
<tr>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>---</td>
</tr>
</tbody>
</table>
```

Add $\frac{1}{2}$

```
<table>
<thead>
<tr>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>---</td>
</tr>
</tbody>
</table>
```

```
<table>
<thead>
<tr>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>---</td>
</tr>
</tbody>
</table>
```

```
<table>
<thead>
<tr>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>---</td>
</tr>
</tbody>
</table>
```

```
<table>
<thead>
<tr>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>---</td>
</tr>
</tbody>
</table>
```

Subtract 2

```
<table>
<thead>
<tr>
<th>19</th>
</tr>
</thead>
<tbody>
<tr>
<td>19</td>
</tr>
<tr>
<td>---</td>
</tr>
</tbody>
</table>
```

```
<table>
<thead>
<tr>
<th>19</th>
</tr>
</thead>
<tbody>
<tr>
<td>19</td>
</tr>
<tr>
<td>---</td>
</tr>
</tbody>
</table>
```

```
<table>
<thead>
<tr>
<th>19</th>
</tr>
</thead>
<tbody>
<tr>
<td>19</td>
</tr>
<tr>
<td>---</td>
</tr>
</tbody>
</table>
```

Subtract $\frac{3}{4}$

```
<table>
<thead>
<tr>
<th>19</th>
</tr>
</thead>
<tbody>
<tr>
<td>19</td>
</tr>
<tr>
<td>---</td>
</tr>
</tbody>
</table>
```

```
<table>
<thead>
<tr>
<th>19</th>
</tr>
</thead>
<tbody>
<tr>
<td>19</td>
</tr>
<tr>
<td>---</td>
</tr>
</tbody>
</table>
```

```
<table>
<thead>
<tr>
<th>19</th>
</tr>
</thead>
<tbody>
<tr>
<td>19</td>
</tr>
<tr>
<td>---</td>
</tr>
</tbody>
</table>
```

153
MAGIC SQUARES: SHEET 4

This array of numbers is a 3 x 3 Magic Square. Show that the sum of numbers in each row, column, and diagonal is 45.

<table>
<thead>
<tr>
<th>26</th>
<th>1</th>
<th>18</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>15</td>
<td>23</td>
</tr>
<tr>
<td>12</td>
<td>29</td>
<td>4</td>
</tr>
</tbody>
</table>

Perform the indicated operations on each number in the magic square above. Enter the results below, then see if each new array is also a magic square.

Add 9

<table>
<thead>
<tr>
<th>35</th>
<th>10</th>
<th>27</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>24</td>
<td>32</td>
</tr>
<tr>
<td>21</td>
<td>38</td>
<td>13</td>
</tr>
</tbody>
</table>

Total 72

Divide by 2

<table>
<thead>
<tr>
<th>13</th>
<th>$\frac{1}{2}$</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3\frac{1}{2}$</td>
<td>$7\frac{1}{2}$</td>
<td>$11\frac{1}{2}$</td>
</tr>
<tr>
<td>6</td>
<td>$14\frac{1}{2}$</td>
<td>2</td>
</tr>
</tbody>
</table>

Total $22\frac{1}{2}$

Add $\frac{1}{3}$

<table>
<thead>
<tr>
<th>26$\frac{1}{3}$</th>
<th>$1\frac{1}{3}$</th>
<th>$18\frac{1}{3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$7\frac{1}{3}$</td>
<td>$15\frac{1}{3}$</td>
<td>$23\frac{1}{3}$</td>
</tr>
<tr>
<td>$12\frac{1}{3}$</td>
<td>$29\frac{1}{3}$</td>
<td>$4\frac{1}{3}$</td>
</tr>
</tbody>
</table>

Total 46

Divide by 3

<table>
<thead>
<tr>
<th>$8\frac{2}{3}$</th>
<th>$\frac{1}{3}$</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2\frac{1}{3}$</td>
<td>5</td>
<td>$7\frac{2}{3}$</td>
</tr>
<tr>
<td>4</td>
<td>$9\frac{2}{3}$</td>
<td>$1\frac{1}{3}$</td>
</tr>
</tbody>
</table>

Total 15

Subtract $\frac{3}{4}$

<table>
<thead>
<tr>
<th>25$\frac{1}{4}$</th>
<th>$\frac{1}{4}$</th>
<th>$17\frac{1}{4}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$6\frac{1}{4}$</td>
<td>$14\frac{1}{4}$</td>
<td>$22\frac{1}{4}$</td>
</tr>
<tr>
<td>$11\frac{1}{4}$</td>
<td>$28\frac{1}{4}$</td>
<td>$3\frac{1}{4}$</td>
</tr>
</tbody>
</table>

Total $42\frac{3}{4}$

Subtract $\frac{2}{3}$

<table>
<thead>
<tr>
<th>25$\frac{1}{3}$</th>
<th>$\frac{1}{3}$</th>
<th>$17\frac{1}{3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$6\frac{1}{3}$</td>
<td>$14\frac{1}{3}$</td>
<td>$22\frac{1}{3}$</td>
</tr>
<tr>
<td>$11\frac{1}{3}$</td>
<td>$28\frac{1}{3}$</td>
<td>$3\frac{1}{3}$</td>
</tr>
</tbody>
</table>

Total 43
**MAGIC SQUARES: SHEET 5**

Fill in the blank spaces in the magic square using each of the following numbers: 4, 6, 7, 8, 10, 11, 12, 13, 14, 15, so that the sum of the integers in each column, row, and diagonal is 34.

```
<table>
<thead>
<tr>
<th>14</th>
<th>1</th>
<th>8</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>5</td>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>16</td>
<td>9</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
<td>13</td>
<td>6</td>
</tr>
</tbody>
</table>
```

Fill in the blank spaces in the magic squares so that the sum of the integers in each column, row, and diagonal is 34.

```
<table>
<thead>
<tr>
<th>14</th>
<th>15</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>16</td>
<td>12</td>
</tr>
<tr>
<td>8</td>
<td>4</td>
<td>9</td>
<td>13</td>
</tr>
<tr>
<td>11</td>
<td>10</td>
<td>7</td>
<td>6</td>
</tr>
</tbody>
</table>
```

```
<table>
<thead>
<tr>
<th>3</th>
<th>2</th>
<th>15</th>
<th>14</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>16</td>
<td>5</td>
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**MAGIC SQUARES: SHEET 6**

**Fill in the blank spaces in the magic squares so that the sum of the integers in each column, row, and diagonal is 34.**

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**Fill in the blank spaces in the magic squares so that the sum of the integers in each column, row, and diagonal is 65.**

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**155 161**
Page 68.

2) Lighthouse

3) Lantern
4) Castle

5) Factory
6) Rocket

7) Butterfly
### Page 72

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### Page 73

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### Page 74

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### Page 75

- ![Diagram](image)
- ![Diagram](image)

### Page 76

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<td>B. 1</td>
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<tr>
<td>C. 6</td>
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<tr>
<td>D. 5</td>
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</tbody>
</table>

### Page 77

1. 36
2. 24
3. 18
4. 12
5. 2
6. 8

### Page 78

### Page 79

- Triangles 100

### Page 80

1. $35.83
2. $16.65
3. $1.98
4. $5.86
5. You need to know how many ice breakers Sarah wants.

### Page 81

1. $4.84
2. $5.05
3. $3.68
4. 5 pounds

### Page 82

1. $224.90
2. $999.90
3. $1034.90
4. $1189.90
5. Buy now

### Page 83

1. $24.95
2. $41.90
3. $159.85
4. $59.85

### Page 84

1. refrigerator
2. $224.90
3. $999.90
4. $1034.90
5. $1189.90
6. Buy now

### Page 85

1. $24.95
2. $41.90
3. $159.85
4. $59.85
The sum of a specific number of consecutive odd numbers is that specific number squared.

There are a number of different answers. Here are some possible answers for 5-10.

1. $1 + 2 + 3 + 4 + 5 + 6 + 7 + (8 \times 9) = 100$
2. $(4 \times 5) + (7 \times 8) + (3 \times 6) + 9 - 2 - 1 = 100$
3. $(5 \times 7) + 8 - 4 + (3 \times 2) + (9 \times 6) + 1 = 100$
4. $(7 \times 8) + 45 + 1 - (36 \div 9) + 2 = 100$
5. $72 + 38 - (45 \div 9) + 1 - 6 = 100$
6. $92 + (56 \div 7) - 3 - 4 + 8 - 1 = 100$

There are a number of different answers. Here are some possible ones for 8-10.

8. $123 - 45 - 67 + 89 = 100$
9. $79 + 56 - 32 + 1 - 6 + 4 = 100$
10. $89 - 62 + 71 + 4 - 5 + 3 = 100$
1. \[5960 = (5 \times 10^3) + (9 \times 10^2) + (6 \times 10^1) + (0 \times 10^0)\]
   \[= (5 \times 100) + (9 \times 100) + (6 \times 10) + (0 \times 1)\]
   \[= 5000 + 900 + 60 + 0\]
   \[= 5960\]

2. \[23,579 = (2 \times 10^4) + (3 \times 10^3) + (5 \times 10^2) + (7 \times 10^1) + (9 \times 10^0)\]
   \[= (2 \times 10000) + (3 \times 1000) + (5 \times 100) + (7 \times 10) + (9 \times 1)\]
   \[= 20000 + 3000 + 500 + 70 + 9\]
   \[= 23,579\]

3. \[5294 = (5 \times 10^3) + (2 \times 10^2) + (9 \times 10^1) + (4 \times 10^0)\]
   \[= (5 \times 1000) + (2 \times 100) + (9 \times 10) + (4 \times 1)\]
   \[= 5000 + 200 + 90 + 4\]
   \[= 5294\]

4. \[36,058 = (3 \times 10^4) + (6 \times 10^3) + (0 \times 10^2) + (5 \times 10^1) + (8 \times 10^0)\]
   \[= (3 \times 10000) + (6 \times 1000) + (0 \times 100) + (5 \times 10) + (8 \times 1)\]
   \[= 30000 + 6000 + 0 + 50 + 8\]
   \[= 36058\]

Page 103

1. \[\text{Place Value}\]

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5. \[132_{\text{five}} = (1 \times 5^2) + (3 \times 5^1) + (2 \times 5^0)\]
   \[= (1 \times 25) + (3 \times 5) + (2 \times 1)\]
   \[= 25 + 15 + 2\]
   \[= 42\]

\[162_{16\circ} \]
6. \[2034_{\text{five}} = (2 \times 5^3) + (0 \times 5^2) + (3 \times 5^1) + (4 \times 5^0)\]
\[= (2 \times 125) + (0 \times 25) + (3 \times 5) + (4 \times 1)\]
\[= 250 + 0 + 15 + 4\]
\[= 269\]
1. \(5^0 = 1\)  
   \(5^1 = 5\)  
   \(5^2 = 25\)  

\[
\begin{array}{c|c|c|c}
& 25 & 67 & 2 \\
\hline
\ \  & 5 & 17 & 3 \\
\end{array}
\]

\(5^3 = 125\)  
67 = 232\text{five}

2. 213\text{five}

3. 100\text{five}

4. 444\text{five}

5. 341\text{five}

\[
\begin{array}{c|c|c|c}
& 1 & 2 & 2 \\
\hline
\ \  & 15 & 1 & 1 \\
\end{array}
\]

2.

3.

4.

5.

Page 108

Page 113

Page 120

1. Binary Numeration System

2. There are no 2's in base two.

3. Six

4. 1101\text{two} = (1 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0)
   = (1 \times 8) + (1 \times 4) + (0 \times 2) + (1 \times 1)
   = 8 + 4 + 0 + 1
   = 13

5. 1010\text{two} = (1 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (0 \times 2^0)
   = (1 \times 8) + (0 \times 4) + (1 \times 2) + (0 \times 1)
   = 8 + 0 + 2 + 0
   = 10

Page 113

1. 100\text{five}

2. 1130\text{five}

3. 2003\text{five}

4. 4\text{five}

5. 43\text{five}

6. 244\text{five}

7. 1001001\text{two} = (1 \times 2^6) + (1 \times 2^3) + (1 \times 2^0)
   = (1 \times 64) + (1 \times 8) + (1 \times 1)
   = 64 + 8 + 1
   = 73

8. 2444\text{five}

9. 20433\text{five}
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