Theory-based prototype computer-video instructional modules were developed to serve as an instructional supplement for students experiencing difficulty in learning mathematics, with special consideration given to students underrepresented in mathematics (particularly women and minorities). Modules focused on concepts and operations for factors, prime numbers, and fractions. Task analysis and demand specification procedures were used to sequence instructional objectives within and across topics. Cognitive social theory provided a framework for the incorporation of attentional, retentional, and motivational variables into the video sequences. Results of field trials show that the modules were effective in teaching/reteaching mathematical skills/concepts to secondary school students who had not made normal progress in mathematical learning. In the first trial, students who used materials gained a significant advantage over control subjects in involving factors and prime numbers. The pattern of gains attributable to these modules was replicated in a second trial, but without a control group. Comparable effects were also found to be associated with use of fractions modules, but these results should be interpreted with caution because of lack of a control group. Additional results indicate that the interactive computer-video modules had a beneficial effect on affective as well as cognitive outcomes. (JN)
Theory-Based Interactive Mathematics

Summary

A vast literature on computer applications in education provides clear evidence of the effectiveness of computer assisted instruction, but the same body of literature also shows that, after twenty years of development, this powerful technology has had little impact on education. Similarly, research has demonstrated the capacity of instructional television to serve as an effective instructional medium, but systematic applications of it in educational settings are not widespread. The advent of new, low cost information technologies which can be understood and managed by teachers may now offer the means by which some previous barriers to the adoption of these potentially useful tools might be overcome. Systems that join microcomputers and video instruction into a single, interactive instructional system make it possible to overcome some of the inherent limitations of each medium when used singly, while enabling developers of instructional materials to take advantage of the unique characteristics of each approach.

The work reported here was undertaken to develop theory-based prototype instructional materials designed to serve as an instructional supplement for students experiencing difficulty in learning mathematics. The special needs of students who are underrepresented in mathematics, particularly women and minorities, were given special consideration.

Field trials were conducted to validate theory-based computer/video instructional modules designed to teach or review concepts and operations for factors, prime numbers, and fractions. Task analysis and demand specification procedures were used to sequence instructional objectives within and across topics. Cognitive social learning theory
Theory-Based Interactive Mathematics

provided a framework for the incorporation of attentional, retentional, and motivational variables into the video sequences. Field testing of the factors and prime numbers modules revealed statistically and practically significant pre- to posttest improvement of experimental over control 9th grade students. More limited testing of the fractions modules with high school students who had failed a basic skills competency test required for graduation also provided evidence supporting the efficacy of the approach.

Attributions of success and failure in mathematics and non-mathematical subjects were measured in a posttest only design. Subscale scores were analyzed with a principal components factor analysis with varimax rotation. This analysis yielded four factors, the first of which was a general attribution factor for nonmathematical subjects while the second was a parallel factor for attributions in mathematics. All of the attribution scores involving effort loaded on the third factor, and the final factor consisted of ability attributions in success situations. Subsequent regression analyses using the MAXR procedure revealed that the best one variable model for prediction of final score on the factors' prime numbers criterion test consisted of the mathematics attributions factor. The best two variable model added factor 3, effort attributions. Zero order correlations also revealed a significant relationship between effort attributions and group (experimental and control) membership, such that experimental subjects attributed their academic outcomes to effort to a greater degree than did control subjects. These results indicate that the interactive
computer-video modules had a beneficial effect on affective as well as cognitive outcomes.
The computer has become a ubiquitous force in American society within a relatively brief period of time. Its influence now pervades almost every aspect of science, commerce, and industry. There have also been optimistic predictions that computers will revolutionize education, but in spite of very visible and growing interest in educational applications of this powerful technology, the impact of the computer and associated technology is only beginning to be felt (Watson, 1983). There is widespread interest in developing the means whereby the powerful potential of this technology might be more fully realized. As a measure to promote such advances, the National Science Foundation and the National Institute of Education undertook, in 1980, a joint effort to stimulate the exploration of ways in which new information technologies could be used to facilitate effective instruction in mathematics. The effort was especially geared to bring together at least two independent developments; 1) the availability of low cost information recording and playing devices, and 2) recent advances in the understanding of cognitive processes.

The specific objectives of the joint NSF-NIE program included the development of prototypes of relevant courseware, and methods for the assessment of student progress. Additionally, there was a special interest in the exploration of approaches that would be responsive to the special problems and needs of underrepresented groups such as women and minorities. The project reported in this document was both a response to this set of priorities, and a union and natural extension of
research and development work previously conducted independently by the principal investigators. Specifically, the effort focused on the design and development of prototypes for the theory-based computer-video instructional modules designed to address the general need for effective instructional software in mathematics, with special attention to the incorporation of features chosen to make the materials appropriate for minority and women students.

The Problem

A Crisis in Mathematics and Science Education

Throughout the nation there is a growing sense of concern about the quality and status of mathematics and science education. The perception of crisis in these areas of education is based on a variety of different forms of evidence that mathematics and science achievement is declining among American secondary school and college students. For example, mathematics scores on college entrance examinations have dropped steadily for the past 20 years (National Academy of Sciences, and National Academy of Engineering 1982).

Underrepresented Students

Within the context of this general problem, particular concern has been voiced regarding the special plight of female and minority students, both of whom tend to be underrepresented in the high school mathematics tracks that lead to career opportunities in scientific and technical fields. The study of mathematics has been described as the "critical filter" for entrance into many occupations (Sells, 1973). The status of women students in mathematics has been reported more fully than that of minority students, perhaps because the situation for
minority students is often regarded as part of a more general pattern of poor education and consequent underpreparedness in a number of curricular areas.

Sociologist Lucy Sells (1975) found that of the freshmen admitted to the University of California, Berkeley in the fall of 1972, 57% of the men had taken four full years of mathematics, compared with only 8% of the women who had comparable preparation. The four year mathematics sequence is required for admission to the calculus sequence which, in turn, is required for majoring in every field at the university except for traditionally lower paying fields. Similar differences in mathematics preparation for men and women first year students have been reported at the University of California, Santa Barbara. Even though their participation in mathematics courses is relatively low, increasing numbers of women are shifting their career interests away from "traditional" female careers, expressing instead interest in science and in technical fields (Magarrell, 1980). Similarly, minority students are grossly underrepresented in advanced mathematics courses in high school (Sells, 1979, 1980), effectively eliminating a larger spectrum of educational and career choices.

Clearly there is a need to reverse this trend of declining achievement in mathematics, and to accomplish this in ways that can afford women and minority students the opportunity to participate more fully in scientific and technical careers. The problems involved in responding to this challenge are exacerbated by a severe shortage of teachers in mathematics and science, and by general declines in appropriations for education. Reductions in state and local funding for education in Cali-
Tornia, for example, have resulted in larger classes and corresponding restrictions on the amount of time teachers can spend with individual students who encounter difficulties.

Research along several lines is needed to address the educational and social problems presented by this situation. One approach involves basic research on the underlying causes of group differences in mathematics achievement and participation (Lantz & Smith, 1981; Reyes, Note 3). While this research on socialization and other antecedent conditions is of critical importance, it is also essential to develop immediate and effective means by which students now in school might be helped. The application of educational technology has been suggested as one approach to this problem. The feasibility research reported in the present study falls within this category. The general goal of this project was to develop and test the efficacy of an approach employing educational technology as an adjunct to classroom instruction.

Although the general impact of instructional applications of computers is barely perceptible at the present time, there is a large body of research and evaluation literature on the ways in which computers have been employed in an instructional capacity, and on the outcomes of the applications. The findings reported in that literature are summarized in the following section to provide a context for the approach taken in the present project.

Research on Computer Assisted Instruction

The literature on computer assisted instruction (CAI) has expanded rapidly ever since articles started appearing in the mid fifties. So
much has been written, in fact, that a recent computerized search of the CAI literature netted over 500 titles (Kulik, Kulik, and Cohen, 1986). Considering the enormity of this body of literature, the present summary is drawn from review and synthesis articles.

Reviews of the CAI literature fall into two categories. The most common type, the box-score reviews, generally tally the number of studies showing favorable or unfavorable results for CAI, elaborated by descriptions and discussions of the specific studies. Although these reviews have been quite useful in summarizing the literature, they have not proved to be definitive in solving the sometimes controversial issues surrounding the field. More recent reviews that combine the findings of separate investigations through meta-analysis (Glass, 1975) provide more definitive information. In these reviews, multivariate statistics are used to synthesize findings. This technique makes it possible to draw reliable, reproducible, and general conclusions from diverse studies on a given topic.

The early literature on CAI was very enthusiastic about the potential of the medium. One writer even likened CAI to having the tutorial services of Aristotle available to the student (McDougall, 1975). Other early visions of AI were equally enthusiastic. Classrooms were foreseen as filled with computers acting as infinitely patient tutors, scrupulous examiners, and tireless schedulers of instruction. This situation was expected to free teachers to work individually with their students while the students themselves would be free to follow their own paths and paces (Kulik, Kulik, and Cohen, 1980).
Advantages of CAI

Many writers agree that CAI has specific advantages as a teaching medium. CAI actively involves the individual in the learning process by interacting with the learner, and it can also be paced to the individual learner (Chambers & Sprech-1980), thus making it possible for teachers to spend more time with each student individually. Furthermore, it is claimed that these unique features of CAI make it especially well suited for remedial education. Commissioner of Education Terrell Bell (1974) claims that instruction in computer programming facilitates problem solving. He believes that computer programming reflects realistic problem solving because the problems could have a number of approximate solutions. Feurzeig (1981) stressed that both the capacity of CAI to present individually adapted instruction by giving the student control of both the selection and pacing of material, and its ability to provide immediate feedback could serve to increase student motivation. Hall (1980) believes that CAI presents a "no nonsense" learning environment in which information is presented compactly without any surprises during the presentation. He also notes that, in addition to the advantage of being used to the student's abilities and needs, the computer can act as an impartial judge of the student's progress.

Varieties of Computer Assisted Instruction

Over the last 20 years, computers have come to be used in the classroom in a variety of different ways. A number of authors agree on two main uses of CAI: 1) as a supplement to the regular classroom materials and activities, and 2) as substitutes for other modes of instruction (Chambers and Sprech-her, 1980; Edwards, Norton, Taylor, Van...
In addition to supplementary or substitute applications, several other kinds of CAI have been developed. Coburn, Kelman, Roberts et al (1982) presented a good overview of these varieties of CAI. By far, the most common and the most discussed application has been for drill and practice programs. In this form of CAI, the computer simply presents a problem (often a mathematical problem) for the student to solve. The student then attempts the solution and the computer responds accordingly with the appropriate feedback or reinforcement, followed by another problem. Another variant of CAI is the tutorial which provides instructional material in addition to the simple presentation of problems found in drill and practice. Computer simulations are a relatively new form of CAI in which the computer illustrates a real or imaginary system based on the modeler's theory of how that system operates. Instructional games, which constitute another type of CAI, are similar to video games in that they operate by a clear set of rules, usually have a winner at the end, and are designed to be fun to play. However, instructional games have some concept, knowledge or skill embedded in the game which the student must master in order to play successfully. Another approach, computer managed instruction (CMI), tests students' skills and then recommends additional instruction.

Besides these CAI applications, regular off-the-shelf software, such as word processing and spreadsheet programs, have been used to teach writing, typing, and accounting skills. Finally, programming the computer itself has been used as a teaching tool to solve a great variety of problems in different disciplines.
Effectiveness of Computer Assisted Instruction

Perhaps the most commonly asked questions in the CAI literature are concerned with its effectiveness. Several different facets of CAI effectiveness have been studied, and the major variables are reviewed below.

Student Achievement

Student achievement was the most commonly studied variable dealing with the effectiveness of CAI. Vinsonhaler and Bass (1972) ended their review with the conclusion that the effectiveness of CAI over traditional instruction was reasonably well established when performance was measured by SAT and MAT type scores, especially in the field of elementary education. Chambers and Sprecher (1980) found CAI either improved learning or showed no difference when compared to regular classroom instruction. Kearsley, Hunter, and Seidel (1983b) concluded their review with the observation that CAI made instruction both more effective and efficient. McDougall (1975) also concluded CAI was at least as effective as traditional instruction, while Hallworth and Brebner (1980) found that CAI created effective learning in a variety of learners in a wide variety of subjects.

A few researchers have explored the conditions under which CAI has proved most effective. For example Edwards et. al (1974) found differences in the effectiveness of CAI, depending on whether it was used as supplementary instruction, or to substitute for other instructional approaches. CAI as a classroom supplement increased student achievement scores, while results were mixed when CAI was used to substitute for regular instruction. Different results for different kinds of CAI have
also been reported. Drill and practice appears to be effective, while tutorials, problem solving, and simulations all show mixed results.

The results of meta-analyses help to delineate these previous findings further. Burns and Bozeman (1981) reported that learning was enhanced significantly by supplementary use of CAI in mathematics. In a meta-analysis of college level CAI, Kulik et al. (1980) found that CAI raised achievement by a quarter of a standard deviation in a typical class. A subsequent meta-analysis at the secondary school level revealed that classes using CAI performed at the 63rd percentile versus the traditional classes at the 50th percentile (Kulik et al., 1983). These scores translated into an increase of .32 standard deviation advantage for the CAI classes. These authors also found stronger effects in the more recent studies, suggesting that computer assisted instructional techniques might be used more appropriately as experience with the technology increases.

**Learning Time**

Several researchers have found CAI to reduce the amount of time students need to learn (Chambers and Sprecher, 1980; Fourzeig, 1981; Hallworth and Brebner, 1980). One meta-analysis (Kulik et al., 1980) reported substantial and statistically significant advantages for CAI over traditional instruction, in that CAI took less time than traditional instruction at the college level. In fact, CAI students used 2/3 of the amount of time required for traditional instruction. Similar results were obtained in Kulik et al.'s (1983) meta-analysis in which secondary school level CAI classes were found to have time savings of 39% and 88%, although statistical tests could not be conducted because
of an insufficient sample size.

**Student Attitudes**

Various reviews have explored the effect of CAI on student attitudes. Chambers and Sprecher (1980) report CAI improved student attitudes towards computers in the learning situation. Generally, student attitudes towards CAI have been reported as positive (McDougall, 1975). A meta-analysis at the college level showed a small positive effect on student attitudes towards instruction (Kulik et al., 1980), while another such analysis at the secondary level revealed positive changes in students' attitudes towards computers were more positive after CAI than before, and there was a small positive increase in attitudes towards instruction.

**Low Ability Students**

There is some evidence that CAI may be particularly useful for low ability individuals and for special educational students. For example, Chambers and Sprecher (1980) found that low ability students gained more from CAI than did higher ability students. Feurzeig (1981) also concluded CAI was more effective for low ability students. A meta-analysis at the secondary school level (Kulik et al., 1983) indicated that the effects of CAI were stronger for low aptitude than for talented students, but the number of studies was too small for a test of significance. A University of Calgary (Hallworth and Brebner, 1980) study suggested that the special qualities of CAI, such as patient repetitiveness, individual instruction, and immediate feedback, were particularly well suited for education of the handicapped. Finally, Jamison, Suppes,
and Wells (1974) reported evidence which favored CAI versus discussion groups for special education. These authors also maintained that CAI can be used successfully to improve achievement scores of disadvantaged students at both secondary and college levels.

Effects of Specific Instructional Variables.

Although CAI has been found to be effective for a number of different subjects and with different populations, relatively little is known about the specific characteristics of the medium that are responsible for effective computer assisted teaching and learning. Hallworth and Brebner (1980) observed that outcomes on the effectiveness of CAI depended a great deal on the instructional design of the curriculum materials, but, as Kearsley, Hunter, and Seidel (1983) point out, there has been little research on the effects of specific instructional design variables. Moreover, there is little actual empirical knowledge about how to individualize instruction. In particular, we are relatively uninformed about the specific effects of graphics, sound, motion, or humor. McDougall (1975) declares that we really know little about how to utilize the unique characteristics of computers. Jamison et al. (1974) have expressed a similar concern, indicating that all forms of alternative media (eg. instructional radio, instructional television, programmed instruction, and CAI) have closely emulated traditional instruction. They speculate that quite possibly different research results would be found if more imaginative uses of each media were explored. Finally, McDougall (1975) has noted that very little attention has been paid to the actual integration of computers into the schools.
Status of CAI Applications

Most reviewers of the CAI literature agree that it has not fulfilled its early promise. As early as 1970 it was apparent that the predicted rapid growth of CAI was not occurring, and that the impact on education has been minimal (Chambers & Sprecher, 1980; Feurzeig, 1981). In fact, a survey conducted in 1971 found only 7.7% of teachers reported that CAI was used in their schools (McDougall, 1975). The general consensus of those who have examined the impact and status of CAI is captured in a statement by Branson and Foster (1979-80), who observed that:

There can be little argument that after almost 20 years of effort and great financial investment, very few CAI systems have established a market despite the great expectations of computer specialists, vendors, and educators.

Why has CAI failed to meet the enthusiastic expectations that greeted its arrival on the educational scene? The opinions of a number of informed observers are summarized below.

Costs: The most frequently cited reason for the failure of CAI to meet expectations is cost. Chambers and Sprecher (1980) note that high costs have contributed to the lack of use at both the elementary and secondary levels. Although hardware costs have rapidly diminished, CAI development costs have still remained high. From a marketing viewpoint (Branson and Foster, 1979-80) CAI has failed because of high costs and an inability to develop a stable profit making product line. However, some writers have seen a bright spot on the horizon. Educational materials for microcomputers have lower costs than those for mainframe computers. Also, during a three year period analyzed by Chambers and Sprecher
(1980) costs for traditional instruction increased by 13% per year while CAI costs decreased by 5% per year with a 10% improvement in performance. Feurzeig (1980) basically agrees the costs issue has been the biggest hindrance to CAI applications, but he noted that microcomputers are helping to decrease costs. Edwards, et al. (1974) argued that CAI is about equal in costs to other alternative instructional media such as television. It should also be noted that the cost effectiveness of CAI looks more favorable when reductions in learning times are taken into consideration, but the bottom line seems to be, as Kearsley et al. (1983) observed, that educators are more influenced by what they can afford than by any cost effectiveness argument.

**Teacher Training:** The shortage of appropriate training for teachers is frequently cited as a prime reason for the limited utilization of CAI. Teachers are currently trained to work with groups of children and not to provide individual tutoring on material delivered by computers (Kearsley, et al., 1983). Without enthusiastic acceptance by teachers, and provisions for local control, widespread support for CAI seems unlikely (Hallworth and Brebner, 1980). It seems clear that the potential of CAI cannot be realized until it is integrated into the school curriculum more effectively, with more personal support for teachers and better coordination between educators and CAI experts (Chambers and Sprecher, 1980).

**Distribution:** Distribution problems are also cited as factors limiting the expansion of CAI. A major one of these problems is a severe shortage of good instructional programs (McDougall, 1975). To further complicate the distribution of what good software does exist, the incom-
patibility of different computer systems has been a major obstacle (Kearsley et al., 1983b).

**Comparisons With Other Media**

Comparing one instructional medium with another has certain pitfalls, as various authors have pointed out. Lipson (1980-81) stresses the point that the talent of materials developers may well constitute the critical difference in the effectiveness of one medium as compared with another. Molnar (1982) made a similar point, noting that "Whether a student learns more from one medium than another is at least as likely to depend on HOW the medium is used rather than that medium" (p. 106). With these warnings in mind, the results of studies in which different media have been compared are examined here.

In a comparative study by Edwards et al. (1974), CAI was found to be about equal to human tutoring as supplemental instructional support. No differences were found between programmed instruction, films, film strips, and video versus CAI. CAI did as well or better as language laboratories. In an extensive review by Jamison, Suppes and Wells (1974), several teaching media were compared with traditional instruction (TI). Instructional radio (IR) was found to teach effectively especially when appropriate visual material was supplied. IR taught as well as other media including TI and instructional television (ITV). No significant differences were found between ITV and TI in studies where all variables except the media were controlled, and no significant differences were found between TI and programmed instruction (PI). In addition, PI may have resulted in reducing learning times. CAI was found to be effective as a supplement to TI, and it took less time.
ITV, in itself, has also been the subject of a large number of studies. ITV has demonstrated its effectiveness in over 100 experiments, at every level and in a great variety of subject matters (Molnar, 1982). ITV or video has the advantage of being able to teach cognitive skills. Allen (1981) at North Carolina State University produced a video tape series to teach studying and learning strategies to entering students. Altogether six video tapes were completed with the following titles: 1) "How to Succeed at NCSU," 2) "Roadblocks to Academic Success," 3) "Effective Reading," 4) "How to Take Tests," 5) "Using the Library," 6) "Welcome to Freshman English." The preliminary results indicated that this video program was well received and had the potential for achieving substantial success.

Interactive Video: Recently a new form of CAI has emerged which combines computers with video tape or videodisc. This new teaching media has been referred to as interactive video (IVT for interactive video tape and IVD for interactive videodisc). With this medium the computer is able to control video sequences illustrating various concepts, and to respond in typical CAI fashion.

There are some distinctive differences between the two forms of interactive video. IVT uses video cassettes, which are cheaper than disks and can be much more easily modified. However, IVT video access is sequential and it therefore requires longer access times than video disk. IVD has tremendous storage capacity for video frames and these frames can be looked at singly or as a motion sequence. Also, IVD has two audio tracks as opposed to one for IVT. Furthermore, IVD has random access ability and therefore faster access times. However, on the nega-
tive side, IVD is much more expensive to produce than IVT, and it is much more difficult to change. Both forms of interactive video can be under computer control and computer graphics can be overlayed onto the video frames (Lipson, 1980-81).

So far most writers have been very positive in assessing the potential of interactive video. One of the most enthusiastic is Leveridge (1979-80), who believes that "...the videodisc represents the most significant innovation in educational technology since the invention of movable type by Gutenberg some 500 years ago." (p. 222). Molnar (1979) has stressed the need for educational technology to meet the challenge of the current expanding information boon if we were to avoid a massive "ignorance explosion." In his opinion, the videodisc affords a major means by which the human capacity for information handling may be expanded (Molnar, 1982). Kadesch (1980-81) believes that IVD would greatly improve a system's capacity to present pictorial information and rapidly access ITV segments and he speculates that these stand-alone systems employing interactive video and computer graphics will eventually emerge as the system of choice, especially for the non-traditional student. Thomas (1981) has a more skeptical view. Although he believes that IVD adds a powerful new dimension to education and training, he also maintains that a catalytic event, such as Sputnik, is needed to effect the necessary organizational changes required to stimulate the adoption of IVD. Although there seems to be widespread agreement that ITV has enormous potential, some authorities do not believe the market is ready for it (Branson & Foster, 1979-80).
Since IVT and IVD are rather recent developments, very little evidence of their effectiveness is yet available. Among the few findings now available are those from the University of Alabama, where a video-computer combination was used to teach accounting. The results suggested that this approach increased students' attention, and that retention rates, as measured by final examinations, were significantly higher than with traditional instruction. (Schmidt, 1982). At the University of Nebraska where IVD was used to teach physics, 90% of IVD students expressed a desire for more IVD classwork. WICAT used IVD to teach biology classes and found that students learned the material with 30% less study time. They also found that 30 minutes of IVD was equivalent to 10-15 hours of student learning (Molnar, 1982). IVD has also been used to teach medical students (Leveridge, 1979-80). In this application, IVD was found to be as good and probably superior to traditional instruction. In addition, students felt IVD was more enjoyable or stimulating and that it made good use of their time.

A recent Infoworld (1982) article reported on some of the industrial applications of IVD. Sears found that sales increased significantly when IVD was used in addition to its printed catalogs. IBM found that students tended to get bored after 20 minutes of CAI, but the average length of time spent with IVD was 54 minutes. The US military's use of IVD for simulating artillery and tank gunnery practice greatly improved trainees' ability to operate in the real world situation.
General Conclusions

Although CAI has not made the great impact on education its early proponents predicted, much has been learned about CAI within the last 20 years. First, the question of the effectiveness of CAI seems to have been answered. CAI does have a small positive and significant effect on student achievement, especially when used as a supplement to regular classroom instruction. The biggest and perhaps most significant finding on the effectiveness of CAI is that it can shorten learning times. CAI also seems to have a small positive impact on students' attitudes toward computers and toward instruction itself. Some evidence suggests CAI may be especially effective with low ability students and for special education.

Although a good bit has been learned about the general effects of CAI, research is still needed on the effects of specific variables that can be incorporated into instructional design. Design issues, such as the use of graphics, sound, motion, and humor, have not received very much attention, nor are we sure of how to efficiently utilize the unique characteristics of the computers themselves. Finally, we lack information on how to integrate computers into the schools and classrooms.

A recent combination of computers with video technology has received some of the same early enthusiasm that CAI did 20 years ago. Many writers agree that interactive video has great potential to affect education and training. However, many of the same problems that affected the early development of CAI now plague interactive video applications. Costs for equipment and for program development are high, and problems with incompatible hardware systems make the distribution of
software difficult. Teacher training is still an issue, and the problem of overcoming the institutional inertia that inhibits needed organizational change still looms ahead.

Rationale for Design Approach

The present project was undertaken to explore the instructional potential of new information technologies by developing and field testing theory-based prototypes of interactive computer-video instructional modules. The funding provided by the National Science Foundation was designated primarily for prototype development, so the validation efforts reported in this document represent only a first level of field testing, which should be followed by more extensive validation research with more diverse populations and age groups.

The foregoing summary of literature on computer applications in education makes it clear that computers may provide an effective means of improving performance in a variety of subjects, including mathematics. A number of reviews of research on computer based instruction (Jamison, Suppes, & Wells, 1974; Kearsley, Hunter, & Seidel, 1983; Vinsenhaler & Bass, 1972) and meta-analyses of the research literature (Burns & Bozeman, 1981; Hartley, 1977; Kulik, Bangert, & Williams, 1983) have shown quite consistent positive effects on both achievement and attitudes, and a number of these positive effects have been specific to mathematics education (Burns & Bozeman, 1981; Hartley, 1977). There is also a large body of evidence that television can serve as an effective instructional medium for teaching students from preschool through college in a variety of subject areas (Chu & Schramm, 1967).
Both CAI and instructional television have limitations. Instructional television is not interactive, while CAI is less versatile for the presentation of new content, and for providing demonstrations and examples of concepts and skill applications. The inherent limitations of both instructional television and computers may be overcome by coupling the power and interactive properties of the computer and the unique properties of television. Such an arrangement makes it possible for a microcomputer to control the presentation of video taped explanations, demonstrations, and modeling of problem-solving strategies. These capabilities are most effectively attained by interfacing the computer with the display features and high density storage capacity of the videodisc (Molnar, 1982). Although the intelligent-videodisc may ultimately be the most effective way to combine the unique instructional features of both television and the computer, this technology is not yet generally available in the schools, and its use may be approximated, although less efficiently, through the use of a video cassette recorder interfaced with a microcomputer.

This particular configuration of equipment was chosen for the present research because it seemed to offer an effective way to present theory-based instructional modules in mathematics, while requiring little equipment beyond that generally available in schools. The research was designed to test the effects of this video/computer instruction on the attainment of the specific skills and concepts that the instructional materials were designed to teach, and to determine if the treatment would affect students' attributions of success and failure in mathematics, as contrasted with other academic subjects.
The development of the instructional modules was guided by task analysis (Gagne, 1977), which produced hypothesized hierarchies of skills and concepts for each topic, and by cognitive social learning theory, which served to identify classes of variables that have been found to influence observational learning (Bandura, 1977; Rosenthal & Zimmerman, 1978). The classes of variables selected for incorporation included attentional processes, retentional processes, and motivational processes.

Attention is essential to observational learning, and a number of studies have shown that discriminative attending is enhanced when a visual display directs attention to pertinent features of a task. In addition, Rosenthal and Zimmerman (1978) have reviewed a number of studies demonstrating that affective cues influence both attention to visual displays, and willingness to perform the behaviors acquired through observation. Characteristics of the model are among those affective cues that appear to be relevant to attentional processes (Henderson & Bergen, 1976). Sex and social class (Maccoby & Wilson, 1957), age (Hicks, 1965), and ethnic status (Epstein, 1966) are among the model characteristics suggested by the literature as cues that influence attention. Accordingly, memorable and easily discriminated events and graphic images were judged to be important for incorporation into the video displays, and the models to be employed in demonstrations and dramatizations were selected to match the target audience on such dimensions as age, ethnicity, and manner of dress.

Modeled behavior must be represented symbolically by the observer if the witnessed information is to be retained (Rosenthal & Zimmerman,
The provision of summary rule statements has been used successfully to accomplish this end in research in which modeling was used to teach abstract, rule governed behavior (Henderson & Swanson, 1978; Henderson, Swanson, & Zimmerman, 1975; Brody, 1981). Variables intended to influence attention were incorporated into the modules for the present research by supplementing the modeling of rule applications with frequent presentations of verbal and written (screen display) summary rule statements. Feedback (Schimmel, Note 4) and the opportunity for active responding have also been found, at least with young children, to facilitate the acquisition and retention of mathematical rules (Swanson, Henderson & Williams, 1979). These variables were incorporated by giving students the opportunity to test their understanding of skills and concepts by actively responding to questions presented on the microcomputer. The problems posed in this fashion included both recognition and constructed response varieties. Feedback was provided by computer generated screen displays and by reviews of video material, controlled by the microcomputer.

Students who have experienced repeated failure on a given class of tasks may acquire a sense of helplessness in the face of future encounters with similar tasks. Those who manifest this pattern of "learned helplessness" are more likely than their more successful peers to attribute their failures to factors beyond their control (e.g., low ability or bad luck), than to controllable factors, such as effort (Henderson, 1982). Some research (Zimmerman & Ringle, 1981) has demonstrated that children who view models expressing confidence in their ability and attributing their failures to lack of effort or persistence
become more persistent and express increased self-efficacy. Such motivational variables were incorporated in the modules for the present study through the use of effort attribution statements by models on the video tapes, and by dramatized sequences in which models demonstrated their ability to confront and overcome failure.

In summary, there is good reason to believe that instruction presented via an interactive system that unites the microcomputer's interactive characteristics with the unique capabilities of a video cassette player to present demonstrations and dramatizations may provide a particularly effective means of teaching mathematical skills and concepts. The product validation field trials reported in the following section were designed to test the effects of theory-based instructional materials on skills identified by high school teachers as stumbling blocks for students with normal ability who had not made normal progress in mathematics learning. It was anticipated that students who used the learning modules would exhibit greater gains in the concepts and skills represented in the modules than would control group students. It was also hypothesized that students who were exposed to the materials would gain a sense of confidence in their ability to learn mathematics, and that they would attribute their performance more to effort than to ability or luck. No differences between the experimental and control groups in attributions relating to non-mathematical subjects were anticipated.

Validation Trial 1

Method

Subjects: The subjects for the field trial of the factors and prime numbers modules were selected from general mathematics and introduction
to algebra classes in a high school serving a population that included a high proportion of Hispanic students. School administrators made five classes available for a field trial of the modules and requested that pre- and posttests be administered to all classes at the same time, as a means of minimizing the disruption of classes. In order to distribute utilization of the computer/video equipment evenly across the available class periods, equal numbers of experimental students from each class were randomly assigned to the field trial participation group. Students from each class were categorized on the basis of sex and ethnicity and assignment to the experimental condition was proportional to the number of students in each of these categories. The original sample consisted of 43 control and 58 experimental students. Complete data on pre- and posttest results were available on a final sample of 36 control and 45 experimental students.

**Procedures:** All students in the classes that participated in the study were pretested on a criterion-referenced instrument to assess their knowledge and skills relating to factors and prime numbers. Students in the experimental group were told that their help was needed to determine whether or not a set of new instructional materials was effective. Students were released from their regular mathematics class to participate in the modules. All students in the experimental group went through module #1 before any student began module #2, and participation in module #2 was completed before any student participated in module #3. The computer and video equipment was housed in a separate room in the library, where students, undisturbed, could view the materials and interact with the microcomputer. During the first half of the field
trials, only one set of equipment was available, while a second setup later became available to speed up the treatment. Both sets of equipment were located in the same room and the students wore headsets for audio input. A female research assistant introduced students to the equipment and instructed them in the use of it for the first module. She sat unobtrusively at the back of the room where she could observe the actions of the students as they participated in the modules, and where she could record their spontaneous verbalizations.

At the end of each session, the students were asked in a casual manner to tell what they thought about the module they had just viewed. Each student who completed the factors module was asked the following three questions: (1) "So, how did you do?"; (2) "What did you think of all this?"; and (3) "We would like to call you out again to go through more modules in the next few weeks. Would you like that?" The questions asked following the viewing of the Primes I and II modules were: (1) "What did you think of this tape?"; (2) "Do you want to continue?"; (3) "What did you think of all of the tapes?"; and (4) "What did you think of learning from the computer?" The research assistant recorded these reactions after the student left the room.

Approximately three months were required to administer the three modules to all students in the experimental group. The posttest was administered 2 days after the completion of all modules by the last student, resulting in an intersubject range of from 2 to 16 days between completion of instruction and posttesting. The students in both experimental and control groups were also given the School Learning Questionnaire, which was administered in a posttest only design because of time
constraints imposed by the regular school program.

Instruments: A criterion-referenced test on factors and prime numbers was administered as a pre- and post-measure to students in both the experimental and control groups. Development of the instrument as guided by a systematic task analysis (see Appendix B) following Gagne (1977), resulting in a task hierarchy consisting of an hypothesized prerequisite ordering of relevant skills and concepts. Each task identified in the hierarchy was then translated into a set of task specifications (Hambleton & Eignor, Note 1) in which the demand characteristics of the tasks were identified. Task specifications were then translated into test items. The test contained a total of 32 items calling for recognition responses, and an equal number of constructed response items. (See Appendix A).

A School Learning Questionnaire (SLQ) was developed to assess success and failure attributions with reference to both mathematics and non-mathematical subjects. Items were similar in structure to those used in the Intellectual Achievement Responsibility (IAR) Questionnaire (Crandall, Katkovsky & Crandall, 1965) except that a Likert-type response format was used rather then the dichotomized forced choice format employed in the IAR. For both failure and success, subscales consisting of 5 items each were created to measure attributions to ability (internal cause), effort (internal cause), chance (external cause), and task or situational factors (external cause). These factors, based on constructs originally postulated by Heider (1958), have been hypothesized as motivational constructs (Weiner, 1972, 1979), with linkages to the development of feelings of "learned helplessness" (Mander-
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son, 1982, 1979), as variables affecting achievement motivation. The instrument yields eight subscale scores (four for failure attributions and four for success attributions) pertaining to mathematics, and a parallel set of eight subscale scores pertaining to non-mathematical school subjects. (See Appendix C).

**Equipment:** The configuration of instructional equipment included an Apple II Plus microcomputer interfaced with a videocassette recorder by means of firmware manufactured by BCD Associates. The lesson authoring software package provided by BCD was, with the consent of the company, modified extensively to eliminate computer programmer jargon and to make it more user-friendly.

This configuration made it possible to present instructional input and modeling sequences set in natural contexts (e.g. in classrooms, at home, with friends, on the telephone) by means of video-taped dramatizations. Video segments were followed by questions presented by the microcomputer. Both recognition (e.g., multiple choice) and constructed response (e.g., construct a factor tree) types of questions were chosen for use on the basis of reviews suggesting that these response forms may have differential influences on achievement, depending on the familiarity of the content to individual students (Tobias, 1976, 1982). Positive reinforcement was provided for correct responses and corrective feedback for incorrect responses was delivered by means of both computer displayed text messages and by review of materials presented on videocassette. Relevant video sequences were called up by the microcomputer. High resolution graphics were generated on a PDP 11/70 computer, using the UNIX operating system, and were then transferred to the videotapes.
Instructional Materials: The task hierarchy for factors and prime numbers served as a blueprint for the development of the instructional modules. (See Appendix D). Story-boarding based on this hierarchy guided the first draft of the script for the videocassette portion of the modules. The first draft of the script was written by the project mathematicians. The scripts were then edited for appropriateness of language level and to eliminate any content that could contribute to math anxiety or to the mystification of mathematics. Scripts were then reviewed by the project psychologists who inserted modeling scenes designed to influence attentional processes, retentional processes, and motivational processes.

Variables inserted to influence attention included devices such as computer graphics with changing colors to focus attention on relevant stimuli, and the use of models whose personal characteristics (e.g. age, dress, ethnicity) were congruent with those of the target audience. Given the special concern that the materials be relevant to women and minority students but still useful to a more general audience within the target age group, the majority of the models presented on videocassette were female (3 out of 4), and of the female models, the majority (1 chicana and 1 black) were minorities.

Variables incorporated to influence retentional processes included 1) the use of memorable images associated with concepts and processes, 2) rule statements, presented by audio and screen print, to aid in the recall of critical attributes of the concepts presented, 3) the opportunity to respond to questions driven by the microcomputer, and 4) the provision of corrective feedback. Feedback was provided in the form of
positive reinforcement for correct responses, help statements printed on
the screen for minor errors, and review of material from the video-
cassette for more serious mistakes. Where possible, video review
material provided new exemplars of previously introduced concepts or
processes, but production cost constraints necessitated the use of pre-
viously viewed material in many instances.

Variables that were incorporated to influence motivational
processes included scenes demonstrating the use of mathematics in every-
day life and scenes depicting women engaged in occupations requiring the
use of mathematics. Models were also shown making occasional errors,
then overcoming their mistakes while verbalizing attributions to effort;
E.g., "I wouldn't have made that error if I had been more careful," or
"I can get it if I just stick with it."

Results: The criterion test for factors and prime numbers yielded
three scores; a score for recognition responses, based on multiple
choice and matching items, a constructed response score, based on items
that required the production of a concept label or problem solution, and
a total score based on the sum of the scores for recognition and con-
structed responses. The reliability of the factors/prime numbers test,
as determined by the split half method and corrected for attenuation
using the Spearman-Brown formula, was .96. Descriptive statistics for
group performance on the test are presented in Tables 1 and 2.

Insert Tables 1 and 2 about here

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Recognition and constructed response scores were analyzed separately by means of 2 x 2 analyses of covariance for unequal N, following the method presented by Kerlinger & Pedhazur (1973). In each analysis, treatment groups (experimental and control) formed the first factor, with sex as the second factor. Ethnicity was not included as a separate factor because of substantial inequalities in sample size for minority and non-minority subjects. Adjustments for unequal N followed suggestions from Kerlinger (Note 2).

The analysis of recognition response scores revealed significant main effects for groups, $F(1,76) = 12.76, p < .0006$. Neither the main effect for sex nor the group x sex interaction were significant. An adjusted mean analysis (Kerlinger & Pedhazur, 1973) was applied to the post test scores. The unweighted means analysis revealed that the experimental group gained 5.98 more points than the control group. This approximates a 1 standard deviation difference, a large effect size according to Cohn (1969). An effect of this size can be interpreted as a 34% rate of misclassification regarding experimental or control group membership (Friedman, 1968), or as a 38% improvement in success rate, according to the binomial effect size display advocated by Rosenthal and Rubin (1982). A summary of the ANCOVA is presented in Table 3.

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Insert Table 3 about here
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The adjusted means for each sex were tested for treatment group differences, using the a priori t test for unequal n's (Kerlinger & Pedhazur, 1973). Male and female experimental subjects both exhibited
greater gains than their control group counterparts ($p's < .02$ and $.002$, respectively).

The ANCOVA for the constructed responses showed a significant main effect for the treatment groups, $F(1,76) = 11.26, p < .001$. No other terms were significant. The unweighted means analysis showed that the experimental group surpassed the gains made by controls by 7.36 points. The difference of slightly more than 1 standard deviation qualifies as a large effect (Cohn, 1976), indicating a 36% improvement in success rate, as determined by the binomial effect size display (Rosenthal & Rubin, 1982), or as a 35% misclassification rate (Friedman, 1982). The ANCOVA summary is presented in Table 4. The post hoc comparisons revealed that male experimental subjects made significantly larger gains than control males ($p < .0004$), and that experimental girls also made greater gains than their control group counterparts ($p < .0027$).

Data from the School Learning Questionnaire (SLQ), administered following the intervention, were available for 73 subjects, 42 experimental and 31 controls. The data were analyzed in a $2 \times 8$ ANOVA, in which treatment group constituted the between subjects factor and the within subjects factor was formed by the eight subscales of the SLQ form for mathematics. The main effects for groups was not significant, $F(1,71) = 1.289, \text{ns}$. A second $2 \times 8$ ANOVA performed on the non-mathematics form of the SLQ revealed a very similar pattern. The main effect for groups was not significant, $F(1,71) =$
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1.059, ns. Descriptive statistics for both forms of the scale are presented in Table 5.

Insert Table 5 about here

It seemed unlikely that a clear pattern of results on the SLQ would emerge because of colinearity among the 16 subscale scores. Therefore, a principal components factor analysis with varimax rotation (Nie, Hull, Jenkins, Steinbrenner & Bent, 1975) was performed for data reduction purposes. SLQ results from the second field trial (reported in the next section) showed means and standard deviations for subscale scores that were almost identical to those for the first field trial, so the SLQ data for all subjects who completed the questionnaire (N = 81) were included in the factor analysis.

Table 6 shows the four factors that emerged to account for 65% of the variance in attributions, as measured by the SLQ. The first factor had its highest loadings with OSEL, OSET, OFIA, OFEL, and OFET. This factor includes all of the non-mathematics attribution scores, with the exception of internal attributions to ability and effort in success situations and internal attributions to effort in failure situations. This factor, which explains 31% of the common variance, was interpreted as a general attribution factor, primarily external in locus, for non-mathematical school subjects. This factor was named Non-Mathematics.
The second factor, explaining 17% of the common variance, had its highest loadings with MSEL, MSET, MFIA, MFEL, and MFET. All of the variables loading on this factor were specific to mathematics, and, with the exception of MFIA, all of the variables with the highest loadings on this factor involved attributions to external causes. The pattern for variables loading on this factor was exactly parallel to the pattern for the first factor, except that this factor was specific to mathematics attributions instead of to attributions involving non-mathematical studies, as was the case for factor 1. Factor 2 was named Mathematics Attributions.

The third factor had its highest loadings with the variables that included attributions to effort; MSIE, MFIE, OFIE, OFIE. This factor, designated as Effort Attributions, accounted for 9% of the common variance.

The highest loadings for the final factor were with MSIA and OSIA. This factor, accounting for 8% of the common variance, was named Ability Attributions for Success.

Table 8 displays the intercorrelations among the attributions factor scores, the criterion scores (recognition response, constructed response, and total) for the factors/prime numbers test, and groups (experimental and control). These correlations and the MAXR analyses reported next were based on the 68 subjects for whom both SLQ data and
factor/prime number posttest results were available. The relationships of interest for this analysis were those between the attributions factor scores and the other variables. Factor III, Effort Attributions, was the only attribution factor to correlate significantly ($p < .01$) with treatment group. Given that the experimental subjects were coded 1 and controls were coded 2, the negative correlation ($-.28$) provides support for the expectation that the experimental group would surpass the controls in attributing outcomes to effort.

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Insert Table 8 about here

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Factor II, Mathematics Attributions, was significantly correlated with the total score on the factors/prime numbers test ($r = -.256, p < .035$). There was also a significant correlation between Factor III, Effort Attributions and constructed response scores ($r = .365, p < .002$), and between Factor IV, Ability Attributions and recognition responses $r = -.285, p < .018$).

The sample size was insufficient to justify a single multivariate analysis with multiple dependent variables, so the total score on the factors/prime numbers test was selected as the dependent variable of greatest interest for a regression analysis using the maximum $R^2$ improvement technique (MAXR) (SAS, 1982). Factor scores were used as the independent variables. The MAXR method first yields the best one variable model, then identifies the variable that yields the greatest increase in $R^2$ and adds it to the first variable to produce the best two variable model, and so on. The best single-variable model was Factor
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II, Mathematics Attributions, which accounted for 6.55% of the variance in factors/prime numbers total score. The best two-variable model consisted of Factor II with the addition of Factor III, Effort Attributions. The improvement in $R^2$ due to the addition of Effort Attributions to the model was .058; with 12.35% of the variance in total score on the factors/prime numbers test being explained by these two factors, and with 5.8% of the variance being uniquely explained by Effort Attributions.

The best three-variable model was comprised of Factors II and III with the addition of Factor IV, Ability Attributions for Success. This model accounted for 12.69% of the variance in the criterion measure, with only .33% of the variance being uniquely attributed to Ability Attributions for Success.

The best four-variable model added Factor I, Non-Mathematics Attributions. This model resulted in less than one one hundredth of 1% improvement over the best three variable model.

When asked how they did on the factors module, most students responded by relating the score reported by the computer. Typical responses were "I got 80%."; "I got 100%."; "Oh, okay."; "Not very good. Only 52."; "Wow! I got 100%."; "I just missed one and I knew what I did wrong right away." Typical responses to the open ended question asking what students thought about the approach were: "I did good. It was easy."; "It's hot man. This thing could really help."; "I really liked it. It was fun."; "Yes, it's weird what they can do with computers these days."; "I love it."; "Well, my dad was going to buy us one of these and he didn't. But now I'm gonna ask him to. It's good."
When asked if they would like to participate in additional modules, students were almost unanimous in giving a positive response.

Student responses to the two modules on prime numbers are represented by the following typical comments: "I got 84%. It was fun."; "I didn't do too well, 55%, but I still liked it."; "One hundred %. It's good."; "I got 76%. Is that OK?"; "It's better than coming to class. I'd like to come back."; "I'm a dummy. I didn't make it through the tape. I guess I was never cut out to be smart. I did learn what factors and primes are though."; "I really liked this one. It's fun."; "Can I do the next one?"; "Can I come in next period? Lunch?"

After the final module, students made comments such as the following: "I liked it even though I had trouble with it. I learned about factors and multiples and I never learned them in class. I liked the computer. In a way, I think you can learn just as much or more from a computer because its just you, while in the class, the teacher has to go on and pay attention to the other students." I liked learning from the computer so you didn't have to have the teacher help you."; "It's better than class."

Finally, when the students were asked, "What did you think of the module?", of the forty-two people who responded to the module on factors: 32 (76%) responded positively, 9 (22%) were neutral, and 1 (2%) was negative. For the same question for the Primes I module, of the forty-two responding: 19 (69%) were positive, 11 (26%) were neutral, and 2 (5%) were negative. When asked the same question for the Primes II module, 45 students responded in the following way: 36 of them (80%)
were positive, 8 (18%) were neutral, and one (2%) was negative.

**Validation Trial 2**

**Method**

The second field trial was conducted to provide a mini-replication of the initial field test of the factors and prime numbers modules, and a pilot test of the fractions modules. It was not possible to use a control group to guard against threats to internal validity in this test of the materials, but this trial did make it possible to examine the potential utility of such materials in a remedial education setting.

**Subjects:** The subjects for this field trial were volunteers from an alternative school special summer program for students who had failed to pass the basic skills competency test required for high school graduation. Their regular school class placement ranged from tenth through twelfth grade. The group consisted of nine boys and two girls who participated in the use of the modules for factors and prime numbers and for fractions. Complete pre- and posttest results for both instructional packages were available for eight males and one female.

**Materials and Procedures:** The instructional materials and field trial procedures for this test of the factors and prime numbers modules were identical, except for location and search assistants, to those employed during field trial 1. The fractions package consisted of 6 modules developed to teach concepts and operations involving fractions. The task analysis for fractions is presented in Appendix E. The production procedures for these modules were identical to those previously described for factors and prime numbers. The equipment configuration and field trial procedures were also the same as those employed in the
field testing of the factors and prime numbers modules.

The criterion test for factors and prime numbers was administered to the entire group prior to their viewing of the modules. The posttest was administered after all students had viewed the three modules that constituted the factors and prime numbers package. All students were then pretested with a criterion referenced instrument (See Appendix B) developed to assess attainment of the objectives that were incorporated in the six modules of the fractions series. Upon completion of all six modules by the eight students for whom complete data were available, the posttest was administered.

Results: Repeated measures analyses of variance were applied to field trial data for the remedial summer school students' performance on the recognition and constructed response items of the factors and prime numbers criterion measure. The analysis of criterion response scores revealed a significant main effect for trials, $F(1,20) = 10.99$, $p < .007$, indicating a significant change in recognition response performance from pretest (mean = 14.45) to posttest (mean = 23.64). The analysis of constructed response measures for factors and prime numbers also identified a significant main effect for trials, $F(1,20) = 22.53$, $p < .001$. The mean at the time of pretesting was 11.18, while the posttest mean was 21.09.

Repeated measures analyses of variance were also employed to assess the difference between pre- and posttest recognition and constructed response scores for the criterion test for the fractions modules. The main effects for trials were significant for both recognition, $F(1,14) = 8.75$, $p < .02$, and constructed responses, $F(1,14) = 6.10$, $p < .04$. 


Discussion

The results of the field trials show quite clearly that the computer-video instructional modules were effective in teaching or reteaching mathematical skills and concepts to secondary school students who had not made normal progress in mathematical learning. In the first field test, students who used the materials gained a statistically and practically significant advantage over controls in skills and concepts involving factors and prime numbers. The pattern of gains attributable to these modules was replicated in the second field trial, but without a control group as a safeguard to internal validity. Comparable effects were also found to be associated with the use of modules for fractions, but these results should be interpreted with caution because there was no control group. The grant providing funding for this work called for and supported only limited field testing, and more extensive testing of the fractions modules, with appropriate control conditions, is needed.

There was some support for the prediction that exposure to the materials would help students recognize that it is possible for them to learn mathematics, and that this would be reflected, in part, by positive changes in effort attributions specific to mathematics. This support was in the form of a significant correlation between the Effort Attribution factor and the categorical variable for group membership (experimental or control). It was also of interest that Mathematics Attributions were significantly related to the total score of the criterion test while Effort Attributions and Ability Attributions for Success were related to constructed responses and recognition responses, respectively. The pattern of relations between mathematics test perfor-
mance and attribution factors requires replication with samples large enough for multivariate analyses, but the present results do suggest that some students perform better on recognition type items while others do better when they are required to construct their own responses, and that these differences are related to differences in the forces to which students attribute their outcomes. These findings suggest that both kinds of test questions are necessary if the relations between attributions and performance are to be clarified.

The single most powerful predictor of mathematics performance, as reflected by the factors/prime numbers test total score, was a mathematics attribution factor. This factor consisted primarily of attributions to external causes in both success and failure situations. The one variable included in this factor that did not conform to these characteristics was that for attribution of mathematics failure to effort, an internal cause. These results tend to support the expectation that attributions of outcomes in academic subjects may be subject specific. The best two factor model added effort attributions, which uniquely explained (statistically) an additional 5.8 percent of the variance in the dependent variable, lending support to the proposition that perceptions of the importance of effort are associated with actual outcomes. This observation seems especially important in the context of the present study, because of range restrictions in mathematics achievement among the students who participated in the field tests, and, presumably, in attitudes toward mathematics as well. These findings were supported by the observational information and data from the informal exit interviews, which showed that students believed they had learned from the
experience, and that they felt very positive about it. Subsequent validity studies of the SLQ (Kachuck, 1983) have supported its content validity, and on the basis of the results of the field trials reported here, we would recommend continued work on the development and validation of the SLQ.

The materials tested in the field trials reported here were not designed as a substitute for regular classroom instruction. They were intended as a test of the feasibility of employing computer-video technology to assist students who had failed in the past to master important mathematical competencies. The goals and objectives of the materials were predetermined by the researchers, as were the instructional sequences. Some open-ended exploratory segments (e.g., a factor tree episode in which students could use trial and error) were included in the modules, but, in general, these modules were not intended to be discovery based materials. A small number of educators who have viewed the materials have expressed the view that the use of computers for instructional purposes should be limited to approaches in which the learner is given greater control and opportunities for discovery. We would be among the first to acknowledge the importance of discovery oriented materials of excellent quality, such as LOGO. At the same time, we believe there is a serious shortage of validated materials of the kind reported here. These materials go well beyond the "moving books" and "drill and practice" exercises that constitute the majority of current instructional applications of computers (Bonham, 1983). These materials combine the major unique features that some learning theorists suggest should optimize the likelihood of learning and retention. Future
applications hold the promise of making it possible to "contextualize" (Zimmerman, 1983) learning in subjects such as pre calculus. It seems important that support be provided for the continued development of instructional materials that build on the significant advances being made in research on learning and cognition, and for the validation of promising materials that take advantage of the unique potential of new technologies. The importance of theoretically guided research and product validation cannot be over emphasized, since, as George W. Bonham (1983) has noted so cogently, much of the shape of the future in technology based education is being determined by profit-oriented textbook publishers and equipment manufacturers.

When suggestions from research are pulled together to guide the development of materials, particularly those designed for delivery via educational technology, one cannot predict, nor does field testing reveal, how the several incorporated variables work in combination with each other. What is needed is a dynamic interaction among basic research in learning and cognition, the development and validation of products based on theory and basic research, and applied research designed to assess the respective contributions of product elements and their interactions. Research of an immediate nature is needed to compare the effects of microcomputer and video based instruction in isolation, and in combination with one another. Investigations are also needed to examine the efficacy of the materials used in the present work when used with students who differ in age, or in educational status. Some media research (Henderson & Swanson, 1978) has suggested crucial interactions among the nature of the concept presented, the age of the
learners, and the conditions required for learning to occur.

Finally, serious thought must be given to the role which emerging technologies will play in education in the immediate and long range future. There has been little of the comprehensive planning and vision that are necessary to guide a thoughtful and effective, rather than disorganized and fragmentary, application of new instructional technologies. Bonham (1983) has rightly commented that "there has been little critical analysis of the educational significance of what is being done" (p. 72). An assessment of the potential of developments such as those reported here, to say nothing of computer literacy, discovery oriented learning, computerized testing, and other applications that may compete for time and resources, should ultimately be judged within such a comprehensive view. It is clear, however, that effectiveness data, such as those reported here, form a necessary part of any comprehensive planning.
Reference Notes


References


Thomas, W. Interactive video. *Instructional Innovator*, 1981, 26, 19-20, 44.


Footnotes

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Table 1

DESCRIPTIVE STATISTICS:
RECOGNITION RESPONSES FOR
FACTORS AND PRIME NUMBERS MODULES

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Table 2

DESCRIPTIVE STATISTICS:
CONSTRUCTED RESPONSES FOR
FACTORS AND PRIME NUMBERS MODULES

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Table 3

SUMMARY FOR ANALYSIS OF COVARIANCE
FOR FACTORS AND PRIME NUMBERS MODULES FIELD TEST:
EFFECTS FOR GROUPS AND SEX ACROSS
RECOGNITION RESPONSE TRIALS

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Adjusted Means on Posttest

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Table 4

SUMMARY FOR ANALYSIS OF COVARIANCE
FOR FACTORS AND PRIME NUMBER MODULES FIELD TEST:
EFFECTS FOR GROUPS AND SEX ACROSS
CONSTRUCTED RESPONSE TRIALS

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Adjusted Means of Posttest

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Table 5
MEANS AND STANDARD DEVIATIONS
FOR SCHOOL LEARNING QUESTIONNAIRE:
MATHEMATICS AND NON-MATHMATICS FORMS

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<tr>
<th>Gp</th>
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<th>FIE</th>
<th>FEL</th>
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Note: Figures in parentheses designate standard deviations.

SIA = success, internal, ability
SIE = success, internal, effort
SEL = success, external, luck (chance)
SET = success, external, task (situation)
FIA = failure, internal, ability
FIE = failure, internal, effort
FEL = failure, external, luck (chance)
FET = failure, external, task (situation)
### Table 6
Four-Factor Varimax Solution for School Learning Questionnaire Subscales

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<th>II</th>
<th>III</th>
<th>IV</th>
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<td>.779</td>
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<td>MSIE</td>
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<td>.427</td>
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<td>MSET</td>
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</table>

Note: Factor I: Non-Mathematics Attributions  
Factor II: Mathematics Attributions  
Factor III: Effort Attributions  
Factor IV: Ability Attributions for Success  

- MSIA = Internal ability attribution for success in math.  
- MSIE = Internal effort attribution for success in math.  
- MSEL = External luck attribution for success in math.  
- MSET = External situation (task) attribution, success in math.  
- MFIA = Internal ability attribution for failure in math.  
- MFIE = Internal effort ability for failure in math.  
- MFEL = External luck attribution for failure in math.  
- MFET = External situation (task) attribution, failure in math.  
- OSIA = Internal ability attribution for success, non-math.  
- OSIE = Internal effort attribution for success, non-math.  
- OSEL = External luck attribution for success, non-math.  
- OSET = External situation (task) attribution, success, non-math.  
- OFIA = Internal ability attribution for failure, non-math.  
- OFIE = Internal effort attribution for failure, non-math.  
- OFEL = External luck attribution for failure, non-math.  
- OFET = External situation (task) attribution, failure, non-math.
Table 7
Means, Standard Deviations, and Intercorrelations Among Groups, Mathematics Criterion Test Scores, and Attribution Factor Scores

<table>
<thead>
<tr>
<th>Variable</th>
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<th>3</th>
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<td>(.001)</td>
<td>(.0001)</td>
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</table>

Note: Coded variable treatment group set as 1 = experimental, 2 = control.

Factor I = Non-Math Attributions; Factor II = Math Attributions; Factor III = Effort Attributions; Factor IV = Ability Attributions for Success.

Probability values for significant relationships appear in parentheses below the correlation coefficients.
Table 8
MAXR IMPROVEMENT FOR DEPENDENT VARIABLE TOTAL SCORE ON FACTORS/PRIME NUMBERS TEST

<table>
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<tr>
<th>Best Variable Model</th>
<th>Factor</th>
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<th>F</th>
<th>p</th>
<th>Removing of (F)</th>
<th>p &lt;</th>
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Note: Factor I = Non-Math Attributions; Factor II = Math Attributions; Factor III = Effort Attributions; Factor IV = Ability Attributions for Success.
## APPENDIX A

### FACTORS AND PRIMES TEST

<table>
<thead>
<tr>
<th>Name</th>
<th>School</th>
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</table>

### PART I - MULTIPLE CHOICE: Choose the one best answer to each of the following problems. Indicate your answer by circling the letter beside the best answer.

1. Given the multiplication sentence "7 x 9 = 63," which number is a FACTOR?
   - a. 7 is the only factor.
   - b. 9 is the only factor.
   - c. 126 is a factor of this sentence.
   - d. 7 and 9 are the only factors.
   - e. 7, 9, and 63 are all factors.

2. Given the multiplication sentence "2 x 40 = 80," which numbers are FACTORS?
   - a. 2 and 40 are the only factors.
   - b. 2 is the only factor.
   - c. 40 is the only factor.
   - d. 160 is a factor of this sentence.
   - e. 2, 40, and 80 are all factors.

3. Given the multiplication sentence "8 x 6 = 48," which number is the MULTIPLE?
   - a. 8 is the only multiple.
   - b. 12 is the only multiple.
   - c. 6 is the only multiple.
   - d. 48 is the only multiple.
   - e. 8 and 6 are both multiples.

4. Given the multiplication sentence "7 x 3 = 21," which number is the MULTIPLE?
   - a. 7
   - b. 21
   - c. 14
   - d. 3
   - e. 7 and 3
5. Is 12 a FACTOR of 10?
   a. yes   b. no

6. Is 4 a FACTOR of 24?
   a. yes   b. no

7. Given the list (4, 5, 9, 11), identify all of the numbers that are PRIME.
   a. 4   b. 5   c. 9   d. 11

8. Given the list (2, 7, 10, 17), identify all the numbers that are PRIME.
   a. 2   b. 7   c. 10   d. 17

9. Given the list (2, 6, 7, 10), identify all the numbers that are COMPOSITE.
   a. 2   b. 6   c. 7   d. 10

10. Given the list (5, 7, 25, 27), identify all the numbers that are COMPOSITE.
    a. 5   b. 7   c. 25   d. 27

11. Can the following factor tree be factored any further?

    \[
    \begin{array}{c}
    24 \\
    \hline
    12 \times 2
    \end{array}
    \]

   a. Yes, it can be factored further.
   b. No, it cannot be factored further.

12. Can the following factor tree be factored any further?

    \[
    \begin{array}{c}
    15 \\
    \hline
    3 \times 5
    \end{array}
    \]

   a. Yes, it can be factored further.
   b. No, it cannot be factored further.
PART II - MATCHING: For the following items, circle the appropriate letter to identify the correct answer.

13. Identify the factors and multiples in the following lists of numbers.
Circle an F if the number is a FACTOR.
Circle an M if the number is a MULTIPLE.

a. From the list \(1, 3, 5, 30, 45\), which numbers are factors of 15 and which numbers are multiples of 15?

<table>
<thead>
<tr>
<th></th>
<th>F</th>
<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
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<tr>
<td>3</td>
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<tr>
<td>5</td>
<td>F</td>
<td>M</td>
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<tr>
<td>30</td>
<td></td>
<td>M</td>
</tr>
<tr>
<td>45</td>
<td>F</td>
<td>M</td>
</tr>
</tbody>
</table>

b. From the list \(2, 3, 12, 30, 60\), which numbers are factors of 6, and which numbers are multiples of 6?

<table>
<thead>
<tr>
<th></th>
<th>F</th>
<th>M</th>
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<tbody>
<tr>
<td>2</td>
<td>F</td>
<td>M</td>
</tr>
<tr>
<td>3</td>
<td>F</td>
<td>M</td>
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<td>12</td>
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<td>M</td>
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<tr>
<td>30</td>
<td>F</td>
<td>M</td>
</tr>
<tr>
<td>60</td>
<td>F</td>
<td>M</td>
</tr>
</tbody>
</table>

14. Identify the prime and composite numbers in the following lists of numbers.
Circle a P if the number is PRIME.
Circle a C if the number is COMPOSITE.

a. From the list \(3, 6, 9, 11, 15\), which numbers are prime and which are composite?

<table>
<thead>
<tr>
<th></th>
<th>P</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>P</td>
<td>C</td>
</tr>
<tr>
<td>6</td>
<td>P</td>
<td>C</td>
</tr>
<tr>
<td>9</td>
<td>P</td>
<td>C</td>
</tr>
<tr>
<td>11</td>
<td>P</td>
<td>C</td>
</tr>
<tr>
<td>15</td>
<td>P</td>
<td>C</td>
</tr>
</tbody>
</table>

b. From the list \(2, 4, 7, 13, 20\), which numbers are prime and which are composite?

<table>
<thead>
<tr>
<th></th>
<th>P</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>P</td>
<td>C</td>
</tr>
<tr>
<td>4</td>
<td>P</td>
<td>C</td>
</tr>
<tr>
<td>7</td>
<td>P</td>
<td>C</td>
</tr>
<tr>
<td>13</td>
<td>P</td>
<td>C</td>
</tr>
<tr>
<td>20</td>
<td>P</td>
<td>C</td>
</tr>
</tbody>
</table>
PART III - CONSTRUCTED RESPONSE: Write the answer to each of the following questions in the space provided.

15. List one FACTOR of each of the following numbers (do not use the number one or the number itself to answer the questions).
   a. __________ is a FACTOR of 8.
   b. __________ is a FACTOR of 12.

16. List two FACTORS of each of the following numbers (do not use the number one or the number itself to answer the questions).
   a. __________ and __________ are FACTORS of 24.
   b. __________ and __________ are FACTORS of 10.

17. List all the FACTORS of each of the following numbers (do not use the number one or the number itself to answer the questions).
   a. __________ are all the FACTORS of 30.
   b. __________ are all the FACTORS of 28.

18. List all the PRIME numbers between 1 and 25. __________ are all the PRIME numbers between 1 and 25.

19. List two FACTORS (other than the number one and the number itself) whose product equals 20. (Just in case it slipped your mind, a product is the answer you get when you multiply two or more numbers together.) __________ and __________ are two FACTORS whose product equals 20.
20. List two FACTORS (other than the number one and the number itself) whose product equals 36.

_______    ________

and        are two FACTORS whose product

equals 36.

21. Write a factor tree for the number 20.

20

\[
\begin{array}{c}
4 \\
\times \\
5
\end{array}
\]

22. Write a factor tree for the number 36.

36

23. Finish the factor trees for the number 20.

a. Starting with:

\[
\begin{array}{c}
4 \\
\times \\
5
\end{array}
\]

b. and then starting with:

\[
\begin{array}{c}
2 \\
\times \\
10
\end{array}
\]
24. Finish the factor trees for the number 40,
   a. Starting with:
      \[ \begin{array}{c}
         40 \\
         2 \times 20 \\
      \end{array} \]
   b. and then starting with:
      \[ \begin{array}{c}
         40 \\
         5 \times 8 \\
      \end{array} \]

25. List two MULTIPLES of the following numbers:
   a. \underline{ } \underline{ } are MULTIPLES of 7.
   b. \underline{ } \underline{ } are MULTIPLES of 8.

26. List three MULTIPLES of the following numbers:
   a. \underline{ } \underline{ } \underline{ } are MULTIPLES of 5.
   b. \underline{ } \underline{ } \underline{ } are MULTIPLES of 9.

27. List five MULTIPLES of each of the following numbers:
   a. MULTIPLES of the number 3:
      \underline{ } \underline{ } \underline{ } \underline{ } \underline{ } 
   b. MULTIPLES of the number 4:
      \underline{ } \underline{ } \underline{ } \underline{ } \underline{ }
28. List one COMMON MULTIPLE of the following numbers:
   a. A COMMON MULTIPLE of the numbers 3 and 4 is
   b. A COMMON MULTIPLE of the numbers 4 and 6 is
   c. A COMMON MULTIPLE of the numbers 3 and 7 is
   d. A COMMON MULTIPLE of the numbers 5 and 9 is
   e. A COMMON MULTIPLE of the numbers 2, 3, and 9 is
   f. A COMMON MULTIPLE of the numbers 4, 6, and 9 is

29. List the LEAST COMMON MULTIPLE of the following numbers:
   a. The LEAST COMMON MULTIPLE of the numbers 23 and 20 is
   b. The LEAST COMMON MULTIPLE of the numbers 6 and 10 is
   c. The LEAST COMMON MULTIPLE of the numbers 6, 8, and 12 is
   d. The LEAST COMMON MULTIPLE of the numbers 3, 24, and 30 is
1. Your date of birth: ____________________________
   Day    Month    Year

2. Circle the number that indicates what grade you are in this year.
   6   7   8   9   10   11   12   College

3. If you are in an elementary or junior high school, please indicate what math track you are in by placing a check mark beside the appropriate designation.
   
   high math    average math    low math

4. If you are in high school or college, put a check mark beside courses you have had, including any you are taking now.
   
   General Math    Intro to Algebra    Algebra I
   Geometry    Trigonometry    Calculus
   Business Math    Other (please list)

5. Please place a check mark beside any math classes you plan to take in the future.
   
   General Math    Intro to Algebra    Algebra I
   Geometry    Trigonometry    Calculus
   Business Math    Algebra II
   Other (please list)

6. How would you rate your ability in mathematics?
   
   High    About Average    Below Average

7. How well do you like math?
   
   Like it a lot    Like it okay    Dislike it
FRACTIONS TEST

PART I - MULTIPLE CHOICE: Choose the one best answer to each of the following problems. Indicate your answer by circling the letter beside the best answer.

1. The shaded portion of the figure shown here represents
   a. $\frac{1}{8}$ of the circle.
   b. $\frac{1}{2}$ of the circle.
   c. $\frac{1}{16}$ of the circle.
   d. $\frac{1}{4}$ of the circle.

2. The shaded portion of the figure shown here represents
   a. $\frac{1}{8}$ of the circle.
   b. $\frac{1}{2}$ of the circle.
   c. $\frac{1}{16}$ of the circle.
   d. $\frac{1}{4}$ of the circle.

3. In the fraction $\frac{6}{11}$, the 6 represents the
   a. quotient.
   b. denominator.
   c. dividend.
   d. numerator.

4. In the fraction $\frac{7}{13}$, the 7 represents the
   a. dividend.
   b. quotient.
   c. numerator.
   d. denominator.

5. In the fraction $\frac{6}{4}$, the 4 represents the
   a. dividend.
   b. denominator.
   c. quotient.
   d. numerator.
6. In the fraction 9/7, the 7 represents the
   a. denominator.
   b. numerator.
   c. dividend.
   d. quotient.

7. Any fraction with 0 in the numerator is equal to
   a. 1.  b. 0.  c. undefined.  d. infinity.

8. Any fraction with 0 in the denominator is equal to
   a. 1.  b. 0.  c. undefined.  d. infinity.

9. Which of the following numbers is undefined?
   a. 0/5.  b. 3/3.  c. 6/0  d. 1/1

10. Which of the following numbers is undefined?
    a. 4/0  b. 2/2  c. 11/11  d. 0/1

11. The fraction 6/1 is equal to
    a. one sixth.  b. six.  c. one.  d. zero.

12. The fraction 5/1 is equal to
    a. zero.  b. one.  c. one-fifth.  d. five.

13. The fraction 1/4 is equal to
    a. 4/1.  b. 2/8.  c. 1/2.  d. 2/4.

14. The fraction 1/8 is equal to
    a. 2/4.  b. 8/1.  c. 1/4.  d. 2/16.

15. The fraction 5/2 is equal to
    a. more than 1.  b. less than 1.  c. exactly 1.

16. The fraction 7/16 is equal to
    a. less than 1.  b. exactly 1.  c. more than 1.
17. The reciprocal of \( \frac{7}{9} \) is
a. \( \frac{9}{7} \)  
   b. \( \frac{9}{1} \)  
   c. 7  
   d. \( \frac{1}{9} \)  

18. The reciprocal of \( \frac{5}{8} \) is
a. \( \frac{8}{1} \)  
   b. \( \frac{1}{5} \)  
   c. \( \frac{8}{5} \)  
   d. 8  

19. A fraction that has the same common denominator as \( \frac{7}{9} \) is
a. \( \frac{1}{7} \)  
   b. \( \frac{3}{9} \)  
   c. \( \frac{9}{7} \)  
   d. \( \frac{7}{10} \)  

20. A fraction that has the same common denominator as \( \frac{2}{3} \) is
a. \( \frac{8}{6} \)  
   b. \( \frac{1}{2} \)  
   c. \( \frac{7}{7} \)  
   d. 8  

PART II - CONSTRUCTED RESPONSE: Answer the questions and solve the problems in the space provided.

21. Write the number ONE as a fraction. 
   ________________  
   (27b)

22. Write a fraction that is equal to 0. 
   ________________  
   (28b)

23. Write a fraction that represents the number 3. 
   ________________  
   (29b)

24. Circle ALL of the improper fractions in the list below:
   \( \frac{3}{3} \)  
   \( \frac{2}{3} \)  
   \( \frac{4}{3} \)  
   \( \frac{17}{18} \)  
   \( \frac{7}{2} \)  
   \( \frac{5}{6} \)  
   (30b)

25. Circle ALL of the improper fractions in the list below:
   \( \frac{8}{3} \)  
   \( \frac{3}{5} \)  
   \( \frac{5}{4} \)  
   \( \frac{18}{19} \)  
   \( \frac{3}{4} \)  
   \( \frac{1}{10} \)  
   (31b)

26. Change the following fraction to a MIXED NUMBER.
   \( \frac{11}{3} = \) 
   78
27. Change the following fraction to a MIXED NUMBER.

\[
\frac{15}{14} =
\]

28. Write a fraction to represent ZERO.

\[
\frac{3}{3}
\]

29. Circle all of the mixed numbers in the following list:

\[
2 \quad \frac{5}{1} \quad 2 \frac{2}{3} \quad \frac{1}{12}
\]

30. Circle all of the mixed numbers in the following list:

\[
\frac{6}{2} \quad \frac{3}{3} \quad \frac{3}{4} \quad 2 \frac{1}{2}
\]

31. Change the following number to an improper fraction:

\[
5 \frac{2}{3} =
\]

32. Write a fraction to represent the numeral 1.

\[
\frac{1}{1}
\]

33. Change the following number to an improper fraction:

\[
6 \frac{2}{3}
\]

34. Circle all fractions that can be reduced in the following list:

\[
\frac{2}{4} \quad \frac{1}{7} \quad \frac{3}{4} \quad \frac{4}{6}
\]

35. Write a fraction that represents the number 4.

\[
\frac{4}{4}
\]

36. Circle all fractions that can be reduced in the following list:

\[
\frac{4}{8} \quad \frac{2}{5} \quad \frac{4}{5} \quad \frac{5}{7}
\]

37. Write the fraction \(\frac{10}{25}\) in its simplest form.

\[
\frac{2}{5}
\]
38. Write the fraction $\frac{12}{24}$ in its simplest form.

39. Solve the following MULTIPLICATION problems:

   a. $\frac{1}{5} \times \frac{3}{4} =$

   b. $\frac{1}{7} \times \frac{2}{5} =$

   c. $\frac{2}{3} \times \frac{4}{5} \times 1\frac{3}{4} =$

   d. $\frac{3}{4} \times \frac{5}{8} \times 1\frac{5}{16} =$

   e. $1\frac{1}{2} \times 2\frac{3}{4} =$

   f. $3 \times 2\frac{1}{11} =$

   g. $\frac{3}{4} \times 5 =$

   h. $2\frac{3}{4} \times 1\frac{1}{2} =$

40. MULTIPLY the following fractions and give your answer in simplest terms:

   a. $2 \times 3\frac{3}{8} =$

   b. $3 \times 4\frac{5}{9} =$
41. Write the reciprocal of the following:

a. \( \frac{2}{7} = \)  

b. \( \frac{4}{5} = \)  

c. \( 8 = \)  

d. \( 2 = \)  

e. \( 4 \frac{2}{3} = \)  

f. \( 3 \frac{1}{2} = \)  

42. DIVIDE the following fractions.

a. \( \frac{2}{3} - \frac{1}{4} = \)  

b. \( \frac{3}{4} - \frac{2}{5} = \)  

c. \( 3 - \frac{2}{5} = \)  

d. \( 7 - \frac{5}{9} = \)  

e. \( 9 \frac{1}{2} - \frac{3}{4} = \)
43. Find the least common denominator for the following fractions:

a. \( \frac{2}{3} \) and \( \frac{3}{7} \) LCD =

b. \( \frac{3}{4} \) and \( \frac{4}{9} \) LCD =

44. ADD the following fractions:

a. \( \frac{2}{5} + \frac{1}{5} = \)

b. \( \frac{1}{4} + \frac{2}{4} = \)

c. \( \frac{2}{20} + \frac{5}{20} + \frac{7}{20} = \)

d. \( \frac{3}{30} + \frac{6}{30} + \frac{8}{30} = \)

e. \( \frac{2}{3} + \frac{3}{5} = \)

f. \( \frac{1}{4} + \frac{3}{5} = \)

g. \( \frac{2}{3} + \frac{3}{4} = \)

h. \( \frac{2}{30} + \frac{2}{5} + \frac{1}{6} = \)

i. \( 5 + \frac{2}{9} = \)
j. \( \frac{3}{7} + \frac{6}{2/3} = \)

k. \( 6 + \frac{3}{7} = \)

l. \( 6 \frac{2/3}{5} + \frac{5/9}{9} = \)

45. SUBTRACT the following fractions:

a. \( \frac{11}{20} - \frac{10}{20} = \)

b. \( \frac{12}{25} - \frac{11}{25} = \)

c. \( \frac{9}{12} - \frac{1}{24} = \)

d. \( \frac{9}{2} - \frac{2/3}{3} = \)

e. \( 8 - \frac{3/4}{4} = \)

f. \( 5 \frac{1/5}{5} - \frac{4/7}{7} = \)

g. \( \frac{7}{10} - \frac{2/5}{5} = \)

h. \( 2 \frac{1/5}{5} - \frac{4/7}{7} = \)
Please provide the following information:

Place a check mark beside your present class in school:

- 5th grade
- 6th grade
- 7th grade
- Freshman
- Sophomore
- Other

Place a check mark beside your sex:

- Female
- Male

Place a check mark beside all of the math classes you plan to take in high school:

- General Math
- Business Math
- Intro to Algebra
- Algebra I
- Algebra II
- Geometry
- Trigonometry
- Calculus

APPENDIX C

SCHOOL LEARNING QUESTIONNAIRE

Different people have different ideas about the kinds of things that influence performance in school. The statements below represent a variety of opinions students have expressed about what things influence their work in school. We are interested in your opinions about the kinds of influences that affect your own school work.

This is not a test, and there are no right or wrong answers. We just want you to indicate whether you agree or disagree with each statement.

Your name will not be connected with answers to this questionnaire in any way. We are interested in how students in general think about these statements, and we will be looking at averages for different schools, different age groups, and so on. Each questionnaire will be assigned a number, and individual names will be deleted before the questionnaires are scored or analyzed.

The questionnaire has two parts. One part concerns mathematics, and the other section deals with learning in areas other than math. Read each statement carefully, and then circle the appropriate word below the sentence to show how much you agree or disagree with the statement.

Before turning the page to answer the questionnaire, please fill in the following information.

School you now attend: ____________________________________________

Your date of birth: Year _______ Month _______ Day _______

Ethnic group (Check one: answer is optional)

Mexican American _____ Black _____ American Indian _____

Anglo _____ Other (Please specify) ____________________________

Please write your name in this space. This section will be discarded before questionnaires are scored.

Name: ______________________, ______________________, _____________

Last _______ First _______ Middle Init.
Opinions on Mathematics Learning

(Circle the word or phrase that tells best how much you agree or disagree with each statement).

1. If I do better than usual on a math assignment, it would probably be because my ability came through especially well that day.

   Strongly agree  Agree  Disagree  Strongly disagree

2. If I do better than usual on a math assignment, it would probably be because the teacher was trying to give me encouragement.

   Strongly agree  Agree  Disagree  Strongly disagree

3. If I do better than usual on a math assignment, it would probably be because I really tried to do my best.

   Strongly agree  Agree  Disagree  Strongly disagree

4. If I do better than usual on a math assignment, it would probably be because I was overdue to have a good day.

   Strongly agree  Agree  Disagree  Strongly disagree

5. When I do poorly on a math assignment it is usually because I'm just like the rest of my family. We aren't very good at math.

   Strongly agree  Agree  Disagree  Strongly disagree

6. When I do poorly on a math assignment it is usually because it was one of those days when everything goes wrong.

   Strongly agree  Agree  Disagree  Strongly disagree

7. When I do poorly on a math assignment it is usually because the assignment was especially hard.

   Strongly agree  Agree  Disagree  Strongly disagree

8. When I do poorly on a math assignment it is usually because I didn't put in enough study time.

   Strongly agree  Agree  Disagree  Strongly disagree

9. If I get a good grade in math it is mostly because the teacher was in a good mood.

   Strongly agree  Agree  Disagree  Strongly disagree
10. If I get a good grade in math it is mostly because math is pretty easy for me.
   Strongly agree  Agree  Disagree  Strongly disagree

11. If I get a good grade in math it is mostly because I do my work carefully to avoid errors.
   Strongly agree  Agree  Disagree  Strongly disagree

12. If I get a good grade in math it is mostly because the teacher gave easy problems.
   Strongly agree  Agree  Disagree  Strongly disagree

13. If someone compliments me on my work in math it is usually because they think it will make me work harder.
   Strongly agree  Agree  Disagree  Strongly disagree

14. If someone compliments me on my work in math it is usually because they are impressed with my math ability.
   Strongly agree  Agree  Disagree  Strongly disagree

15. If someone compliments me on my work in math it is usually because luck was on my side that day.
   Strongly agree  Agree  Disagree  Strongly disagree

16. If someone compliments me on my work in math it is usually because I tried especially hard on the assignment.
   Strongly agree  Agree  Disagree  Strongly disagree

17. When I get a poor grade in math it usually happens because the teacher has it in for me.
   Strongly agree  Agree  Disagree  Strongly disagree

18. When I get a poor grade in math it usually happens because the problems were especially difficult.
   Strongly agree  Agree  Disagree  Strongly disagree

19. When I get a poor grade in math it usually happens because I wasn’t careful to check my work.
   Strongly agree  Agree  Disagree  Strongly disagree

20. When I get a poor grade in math it usually happens because math has always been hard for me.
   Strongly agree  Agree  Disagree  Strongly disagree
21. When I do well on a math assignment it is usually because I studied hard.

   Strongly agree   Agree   Disagree   Strongly disagree

22. When I do well on a math assignment it is usually because it was my lucky day.

   Strongly agree   Agree   Disagree   Strongly disagree

23. When I do well on a math assignment it is usually because the assignment wasn't very hard.

   Strongly agree   Agree   Disagree   Strongly disagree

24. When I do well on a math assignment it is usually because I have good ability in math.

   Strongly agree   Agree   Disagree   Strongly disagree

25. If the teacher is critical of my work in math it is usually because I didn't try to do my best work.

   Strongly agree   Agree   Disagree   Strongly disagree

26. If the teacher is critical of my work in math it is usually because I am not able to do better work.

   Strongly agree   Agree   Disagree   Strongly disagree

27. If the teacher is critical of my work in math it is usually because everything was going against me that day.

   Strongly agree   Agree   Disagree   Strongly disagree

28. If the teacher is critical of my work in math it is usually because I wasn't being treated fairly.

   Strongly agree   Agree   Disagree   Strongly disagree

29. If I were to fail a math test, it would probably be because math is something I just can't seem to learn.

   Strongly agree   Agree   Disagree   Strongly disagree

30. If I were to fail a math test, it would probably be because it just wasn't my day.

   Strongly agree   Agree   Disagree   Strongly disagree

31. If I were to fail a math test, it would probably be because I really didn't study very hard.

   Strongly agree   Agree   Disagree   Strongly disagree
32. If I were to fail a math test, it would probably be because luck was against me.
   Strongly agree   Agree   Disagree   Strongly disagree

33. Usually when I have trouble with math it is because the teacher didn’t explain things very well.
   Strongly agree   Agree   Disagree   Strongly disagree

34. Usually when I have trouble with math it is because I didn’t study very hard.
   Strongly agree   Agree   Disagree   Strongly disagree

35. Usually when I have trouble with math it is because math is just naturally hard for me.
   Strongly agree   Agree   Disagree   Strongly disagree

36. Usually when I have trouble with math it is because luck was against me.
   Strongly agree   Agree   Disagree   Strongly disagree

37. When I have an easy time on a math assignment, it is usually because I studied before I tried the problems.
   Strongly agree   Agree   Disagree   Strongly disagree

38. When I have an easy time on a math assignment, it is usually because math isn’t very difficult for me.
   Strongly agree   Agree   Disagree   Strongly disagree

39. When I have an easy time on a math assignment, it is usually because the problems were especially easy.
   Strongly agree   Agree   Disagree   Strongly disagree

40. When I have an easy time on a math assignment, it is usually because the problems just happened to be on things I know.
   Strongly agree   Agree   Disagree   Strongly disagree
Opinions on Learning in Subjects Other Than Mathematics

(Circle the word or phrase that tells best how much you agree or disagree with each statement. Answer by giving your general opinion on influences on learning of subjects other than math).

1. If someone compliments me on my school work, it is usually because they think it will make me work harder.
   Strongly agree  Agree  Disagree  Strongly disagree

2. If someone compliments me on my school work, it is usually because I tried especially hard on the assignment.
   Strongly agree  Agree  Disagree  Strongly disagree

3. If someone compliments me on my school work, it is usually because they are impressed with my academic ability.
   Strongly agree  Agree  Disagree  Strongly disagree

4. If someone compliments me on my school work, it is usually because luck was with me that day.
   Strongly agree  Agree  Disagree  Strongly disagree

5. When I do poorly on a school assignment it is usually because it was just one of those days when everything goes wrong.
   Strongly agree  Agree  Disagree  Strongly disagree

6. When I do poorly on a school assignment it is usually because people in my family generally don’t do well at school work.
   Strongly agree  Agree  Disagree  Strongly disagree

7. When I do poorly on a school assignment it is usually because I didn’t put in enough time studying.
   Strongly agree  Agree  Disagree  Strongly disagree

8. When I do poorly on a school assignment it is usually because the assignment was especially hard.
   Strongly agree  Agree  Disagree  Strongly disagree

9. If I get a good grade it is mostly because the teacher was in a good mood.
   Strongly agree  Agree  Disagree  Strongly disagree
10. If I get a good grade it is mostly because I do my school work carefully.

Strongly agree  Agree  Disagree  Strongly disagree

11. If I get a good grade it is mostly because the teacher gave an easy assignment.

Strongly agree  Agree  Disagree  Strongly disagree

12. If I get a good grade it is mostly because most school work is pretty easy for me.

Strongly agree  Agree  Disagree  Strongly disagree

13. If I were to fail a test, it would probably be because I really didn't study very hard.

Strongly agree  Agree  Disagree  Strongly disagree

14. If I were to fail a test, it would probably be because luck was against me.

Strongly agree  Agree  Disagree  Strongly disagree

15. If I were to fail a test, it would probably be because it just wasn't my day.

Strongly agree  Agree  Disagree  Strongly disagree

16. If I were to fail a test, it would probably be because learning school subjects is hard for me.

Strongly agree  Agree  Disagree  Strongly disagree

17. If I do better than usual on a school assignment, it would probably be because my ability came through especially well that day.

Strongly agree  Agree  Disagree  Strongly disagree

18. If I do better than usual on a school assignment, it would probably be because I really tried to do my best.

Strongly agree  Agree  Disagree  Strongly disagree

19. If I do better than usual on a school assignment, it would probably be because the teacher was trying to give me encouragement.

Strongly agree  Agree  Disagree  Strongly disagree

20. If I do better than usual on a school assignment, it would probably be because I was overdue to have a good day.
21. When I get a poor grade it usually happens because the teacher has it in for me.
   Strongly agree  Agree  Disagree  Strongly disagree
22. When I get a poor grade it usually happens because I wasn't careful to check my work.
   Strongly agree  Agree  Disagree  Strongly disagree
23. When I get a poor grade it usually happens because the problems were especially difficult.
   Strongly agree  Agree  Disagree  Strongly disagree
24. When I get a poor grade it usually happens because I don't have a good head for school work.
   Strongly agree  Agree  Disagree  Strongly disagree
25. If the teacher is critical of my school work, it is usually because I wasn't being treated fairly.
   Strongly agree  Agree  Disagree  Strongly disagree
26. If the teacher is critical of my school work it is usually because I am not able to do any better.
   Strongly agree  Agree  Disagree  Strongly disagree
27. If the teacher is critical of my school work, it is usually because I didn't try to do my best work.
   Strongly agree  Agree  Disagree  Strongly disagree
28. If the teacher is critical of my school work it is usually because everything was going against me that day.
   Strongly agree  Agree  Disagree  Strongly disagree
29. When I have an easy time on a school assignment, it is usually because the problems were especially easy.
   Strongly agree  Agree  Disagree  Strongly disagree
30. When I have an easy time on a school assignment, it is usually because I studied pretty hard.
   Strongly agree  Agree  Disagree  Strongly disagree
31. When I have an easy time on a school assignment, it is usually because I was luck enough to be assigned something I knew.
32. When I have an easy time on a school assignment, it is usually because most school work isn’t very hard for me.

Strongly agree  Agree  Disagree  Strongly disagree

33. Usually when I have trouble with school work it is because school work is just naturally hard for me.

Strongly agree  Agree  Disagree  Strongly disagree

34. Usually when I have trouble with school work it is because luck was just against me.

Strongly agree  Agree  Disagree  Strongly disagree

35. Usually when I have trouble with school work it is because I didn’t study very hard.

Strongly agree  Agree  Disagree  Strongly disagree

36. Usually when I have trouble with school work it is because the teacher didn’t explain things very well.

Strongly agree  Agree  Disagree  Strongly disagree

37. When I do well on a school assignment it is usually because I studied very hard.

Strongly agree  Agree  Disagree  Strongly disagree

38. When I do well on a school assignment it is usually because it was my lucky day.

Strongly agree  Agree  Disagree  Strongly disagree

39. When I do well on a school assignment it is usually because the assignment wasn’t very hard.

Strongly agree  Agree  Disagree  Strongly disagree

40. When I do well on a school assignment it is usually because I have some talent for school work.

Strongly agree  Agree  Disagree  Strongly disagree
### APPENDIX D

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**TABLE HIERARCHY FOR MULTIPLICATION AND DIVISION OF FRACTIONS**

<table>
<thead>
<tr>
<th>Multiplication</th>
<th>Division</th>
</tr>
</thead>
<tbody>
<tr>
<td>(27) Multiply integer and fraction &amp; reduce to simplest form</td>
<td>(33) Divide mixed number by fraction</td>
</tr>
<tr>
<td>(26) Multiply integer and fraction</td>
<td>(32) Divide integer by fraction</td>
</tr>
<tr>
<td>(25) Multiply two mixed numbers</td>
<td>(31) Divide two fractions</td>
</tr>
<tr>
<td>(24) Multiply three fractions</td>
<td>(30) Produce reciprocal of mixed number</td>
</tr>
<tr>
<td>(23) Multiply two fractions</td>
<td>(29) Produce reciprocal of integer</td>
</tr>
<tr>
<td>(22) Convert a mixed number to an improper fraction</td>
<td>(28) Produce reciprocal of fraction</td>
</tr>
<tr>
<td>(21) From a list, identify mixed numbers</td>
<td></td>
</tr>
<tr>
<td>(20) Write '0' as a fraction</td>
<td></td>
</tr>
</tbody>
</table>
(19) Write $N$ as a fraction

(18) Write "$1" as a fraction

(17) Given a fraction, identify the denominator

(16) Given a fraction, identify the numerator

(15) Given a pie diagram, identify fractional part represented

(14) Given a set of 2 or 3 numbers, produce the least common multiple

(13) Given a set of 2 or 3 numbers, produce a common multiple

(12) Given a number, produce a multiple(s) of that number

(11) Given a number, produce a factor tree

(10) Given a number, produce primes

(9) Given a number, produce factors(s)

(8) Given a list of numerals, discriminate between prime and composite numbers

(7) Given a list of numerals, discriminates between factors and multiples of a given number

(6) Determine if a factor tree can be factored further
Given a list of numerals, identify composite numbers

(4) Given a list of numerals, identify prime numbers

(3) Discriminate instances and non-instances of factors

(2) Identify multiple in multiplication sentence

(1) Identify factors in multiplication sentence

Note: * = facilitating but not prerequisite
** = more difficult but not prerequisite
APPENDIX E

TASK HIERARCHY FOR ADDITION AND SUBTRACTION OF FRACTIONS

<table>
<thead>
<tr>
<th>Addition</th>
<th>Subtraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>(34) Subtract a fraction from a mixed number</td>
<td>(34) Subtract a fraction from a mixed number</td>
</tr>
<tr>
<td>(33) Subtract a fraction from an integer</td>
<td>(33) Subtract a fraction from an integer</td>
</tr>
<tr>
<td>(32) Subtract fractions without a common denominator</td>
<td>(32) Subtract fractions without a common denominator</td>
</tr>
<tr>
<td>(31) Subtract fractions with a common denominator</td>
<td>(31) Subtract fractions with a common denominator</td>
</tr>
<tr>
<td>(30) Add a mixed number and a fraction</td>
<td>(30) Add a mixed number and a fraction</td>
</tr>
<tr>
<td>(29) Add an integer and a fraction</td>
<td>(29) Add an integer and a fraction</td>
</tr>
<tr>
<td>(28) Add three fractions without a common denominator</td>
<td>(28) Add three fractions without a common denominator</td>
</tr>
<tr>
<td>(27) Add two fractions without a common denominator</td>
<td>(27) Add two fractions without a common denominator</td>
</tr>
<tr>
<td>(26) Add three fractions with a common denominator</td>
<td>(26) Add three fractions with a common denominator</td>
</tr>
<tr>
<td>(25) Add two fractions with a common denominator</td>
<td>(25) Add two fractions with a common denominator</td>
</tr>
<tr>
<td>(24) Given a set of two fractions, produce the least common denominator</td>
<td>(24) Given a set of two fractions, produce the least common denominator</td>
</tr>
<tr>
<td>(23) Convert a mixed number to an improper fraction</td>
<td>(23) Convert a mixed number to an improper fraction</td>
</tr>
</tbody>
</table>

(22) Subtract fractions with a common denominator
(22) From a list, identify mixed numbers

(21) Write '0' as a fraction

(20) Write N as a fraction

(19) Write '1' as a fraction

(18) Given a fraction, identify a fraction with a common denominator

(17) Given a fraction, identify the denominator

(16) Given a fraction, identify the numerator

(15) Given a pie diagram, identify the fractional part represented

(14) Given a set of 2 or 3 numbers, produce the least common multiple.

(13) Given a set of 2 or 3 numbers, produce a common multiple

(12) Given a number, produce multiple(s) of that number

(11) Given a number, produce a factor tree

(10) Given a number, produce primes

(9) Given a number, produce a factor(s)
(8) Given a list of numerals, discriminate between prime and composite numbers.

(7) Given a list of numerals, discriminate between factors and multiples of a given number.

(6) Determine if a factor tree can be factored further.

(5) Given a list of numerals, identify composite factors.

(4) Given a list of numerals, identify prime numbers.

(3) Discriminate instances and non-instances of factors.

(2) Identify multiple in multiplication sentence.

(1) Identify factors in multiplication sentence.

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Note: This structure represents an instructional sequence rather than a prerequisite ordering of tasks. Some steps are included to facilitate learning, but they were not hypothesized to be prerequisite to subsequent steps. At some points additional complexity is introduced, thereby increasing the difficulty level of the task, but the added complexity does not contribute to performance of the next step in the sequence.

* = facilitating but not prerequisite.
** = more difficult but not prerequisite.